Precision Quantum Chromodynamics at the LHC

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Dawn of a New Era

- The July 2012 announcement of the discovery of a Higgs-like boson at CERN by ATLAS and CMS completed our discovery of the Standard $SU(3) \times SU(2) \times U(1)$ Model

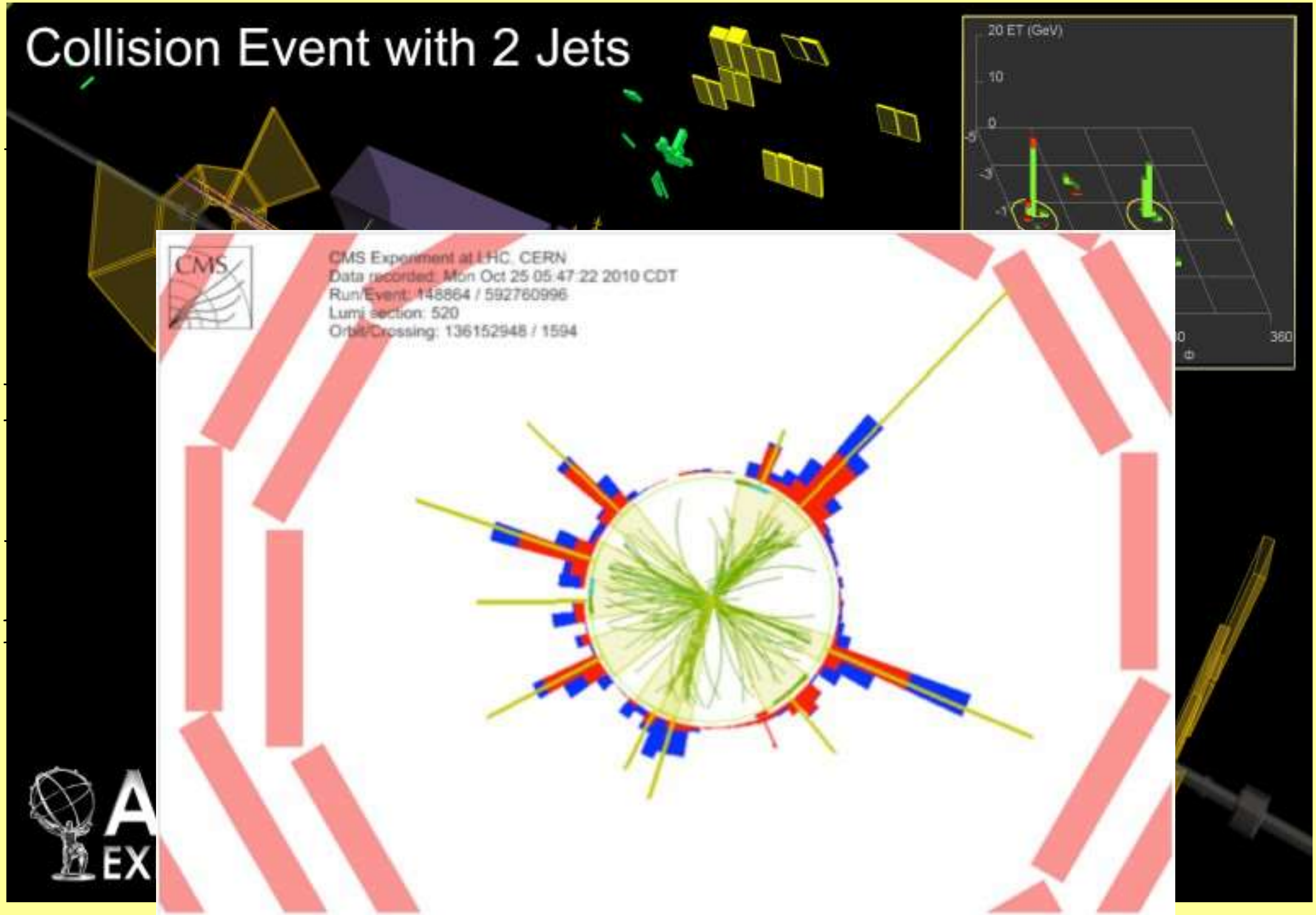
- Captures three of the four known forces

- Misses dark matter — most of the matter in the universe!

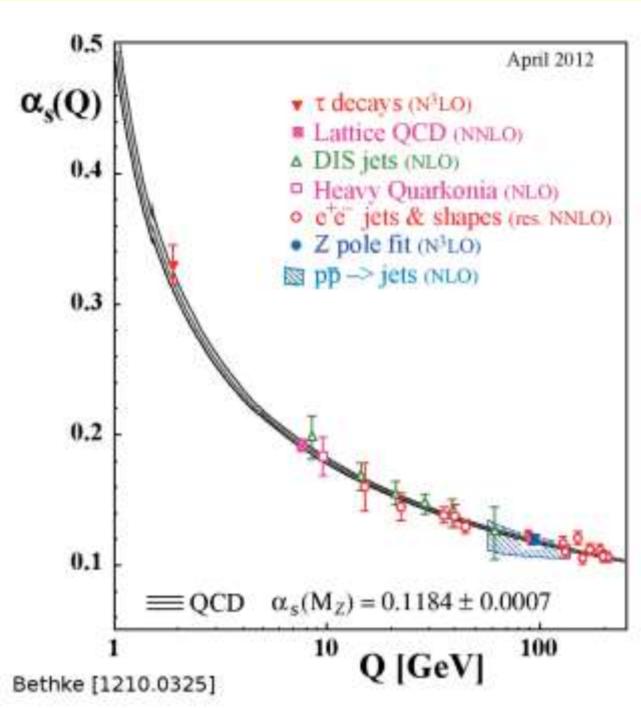
- Tantalizingly incomplete in other ways: just an effective low-energy theory?

Experiments

- Need input from experiments
- Direct searches for physics Beyond the Standard Model
- Indirect searches
  - Precision measurements of the Higgs; of top quarks; of electroweak vector bosons
  - Rare decays: K, D, B
  - Muon magnetic moment
  - Neutrino mixing
- Theory complement: precision calculations of signals and backgrounds


Collision Event with 2 Jets
20 ET (GeV)
CMS Experiment at LHC, CERN
Data recorded: Mon Oct 25 05:47:22 2010 CDT
Run/Event: 148864 / 592760996
Lumi section: 520
Orbit/Crossing: 136152948 / 1594

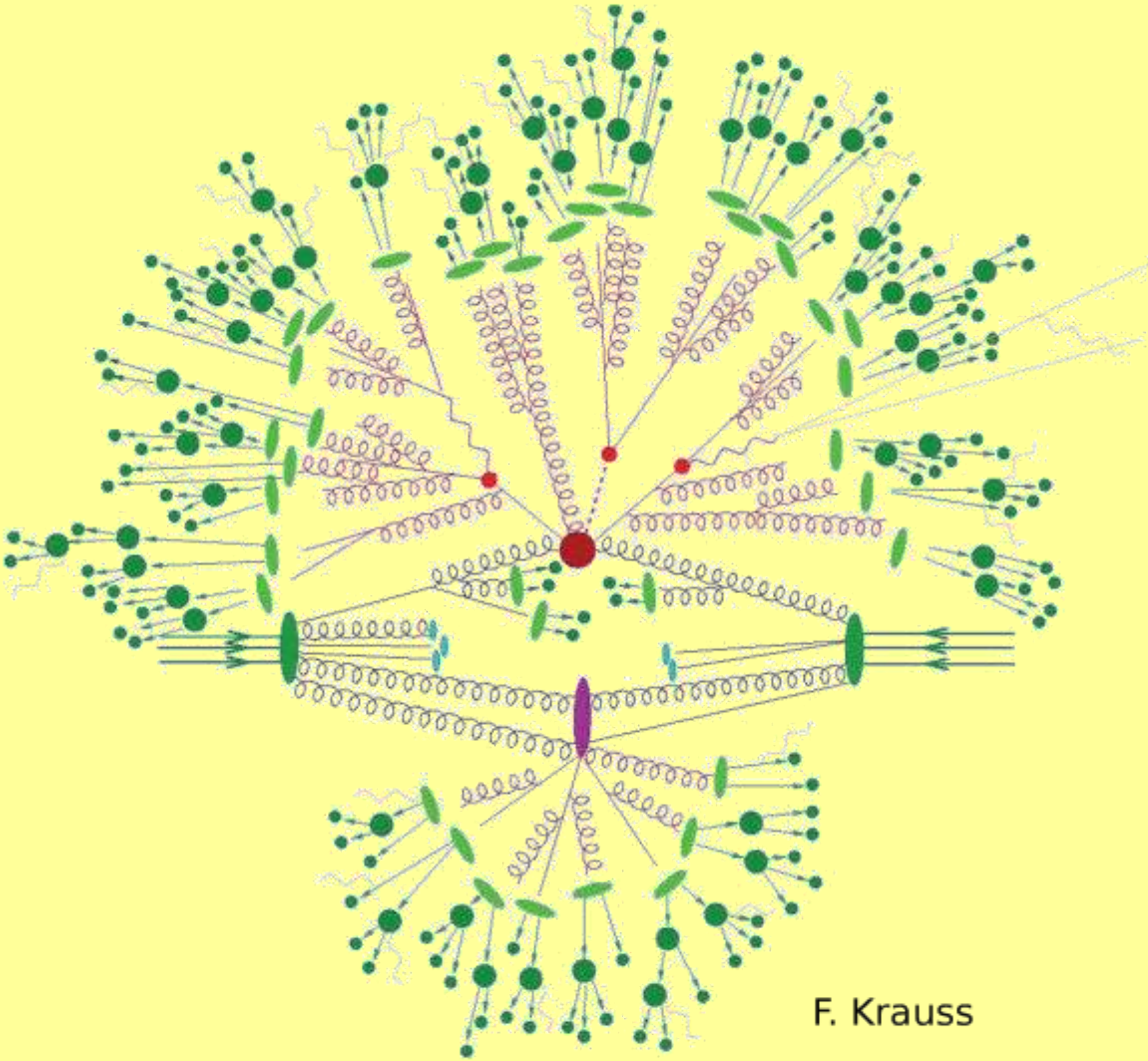
QCD



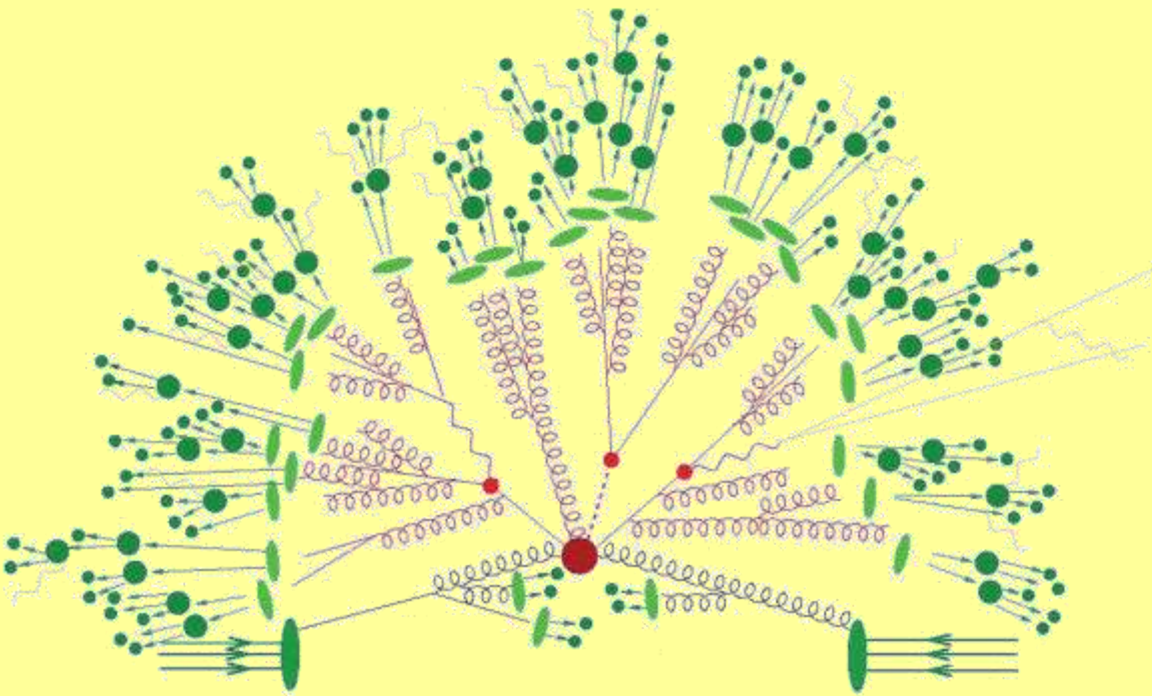
- Describes proton structure

- Source of dominant backgrounds to measurements and searches

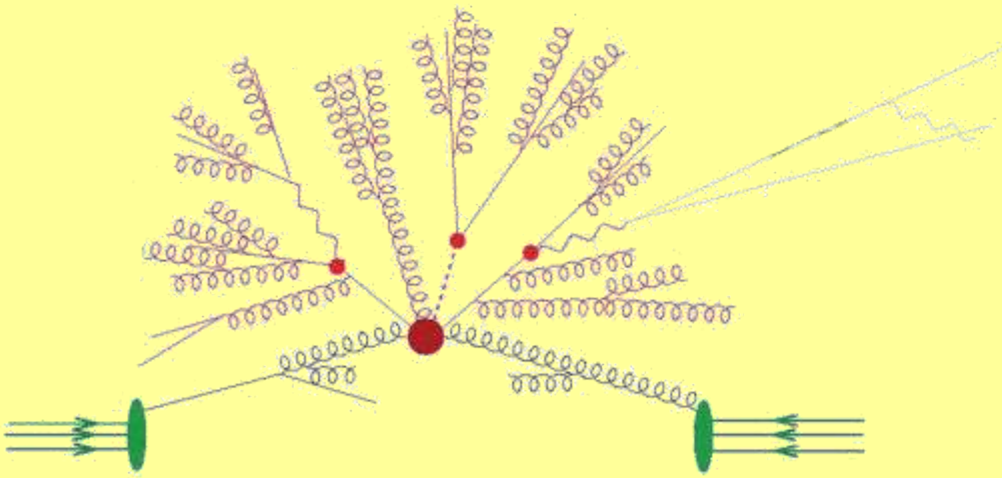
- Strong coupling is not small: $\alpha_s(M_Z) \approx 0.12$ and running is important
  - $\Rightarrow$ events have high multiplicity of hard clusters (jets)
  - $\Rightarrow$ each jet has a high multiplicity of hadrons
  - $\Rightarrow$ higher-order perturbative corrections are important



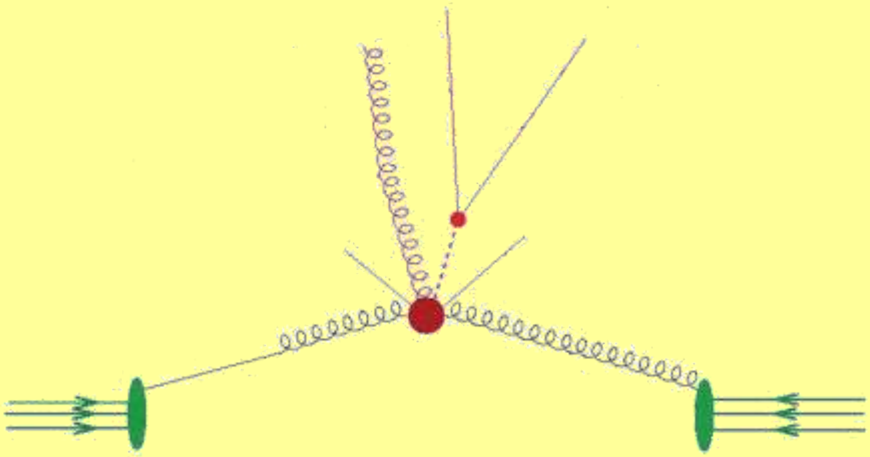
F. Krauss



F. Krauss



F. Krauss



F. Krauss

Fixed-Order Calculations

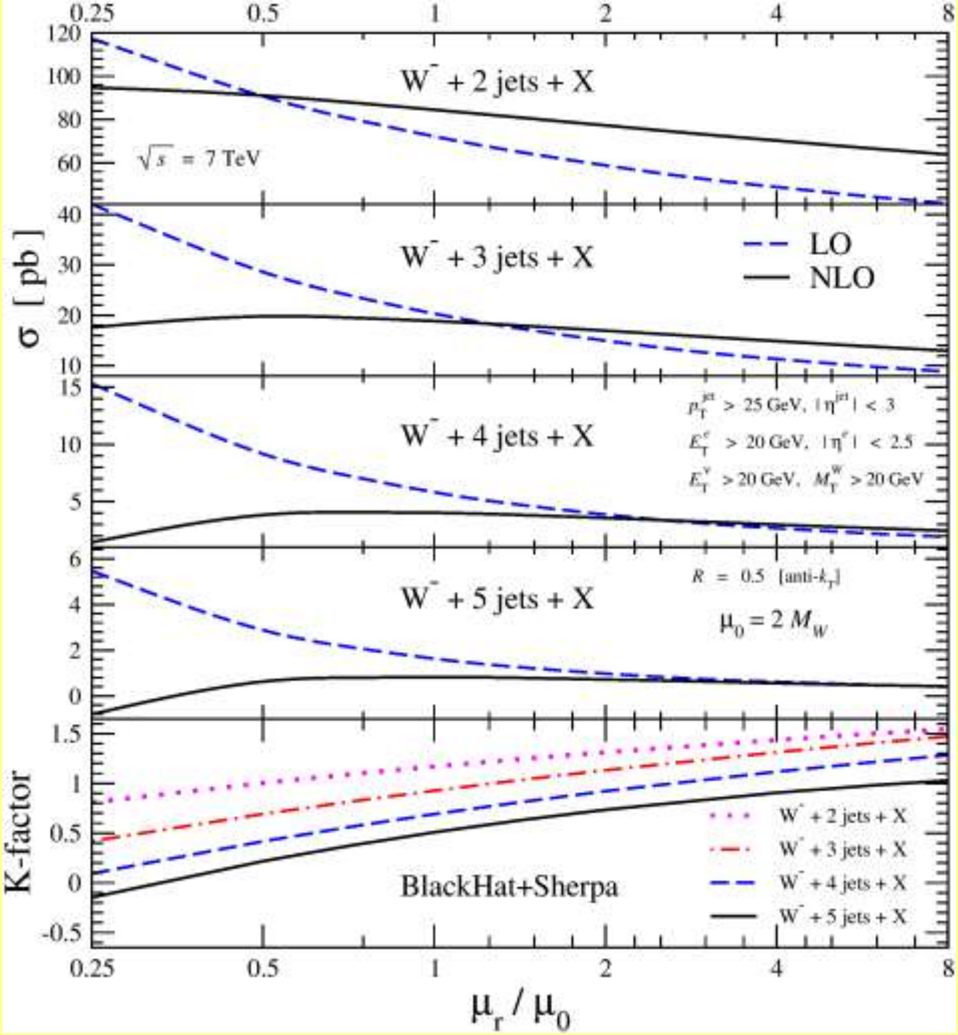
- Simplify to essentials:
  - Focus on jets
  - Numerical jet programs: general observables
  - Systematic to higher order/high multiplicity in perturbation theory
  - Parton-level, approximate jet algorithm; match detector events only statistically
- Every sensible observable has an expansion in α_s

$$\frac{d\sigma^{W+3\,\mathrm{jet}}}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}} = \alpha_s^3(\mu)\frac{d\hat{\sigma}^{\mathrm{LO}}}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}} + \alpha_s^4(\mu)\frac{d\hat{\sigma}^{\mathrm{NLO}}(\mu)}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}} + \alpha_s^5(\mu)\frac{d\hat{\sigma}^{\mathrm{NNLO}}(\mu)}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}}$$

Theory for Many Jets

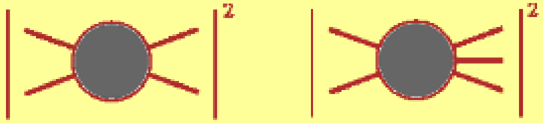
- Want quantitative predictions
- Renormalization scale needed to define α_s; factorization scale to separate long-distance physics
- Physical observables should be independent of scales; truncated perturbation theory isn't
- LO has large dependence
- NLO reduces this dependence
- NLO importance grows with increasing number of jets

- Expect predictions reliable to 10–15%
- <5% predictions will require NNLO



Ingredients for NLO Calculations

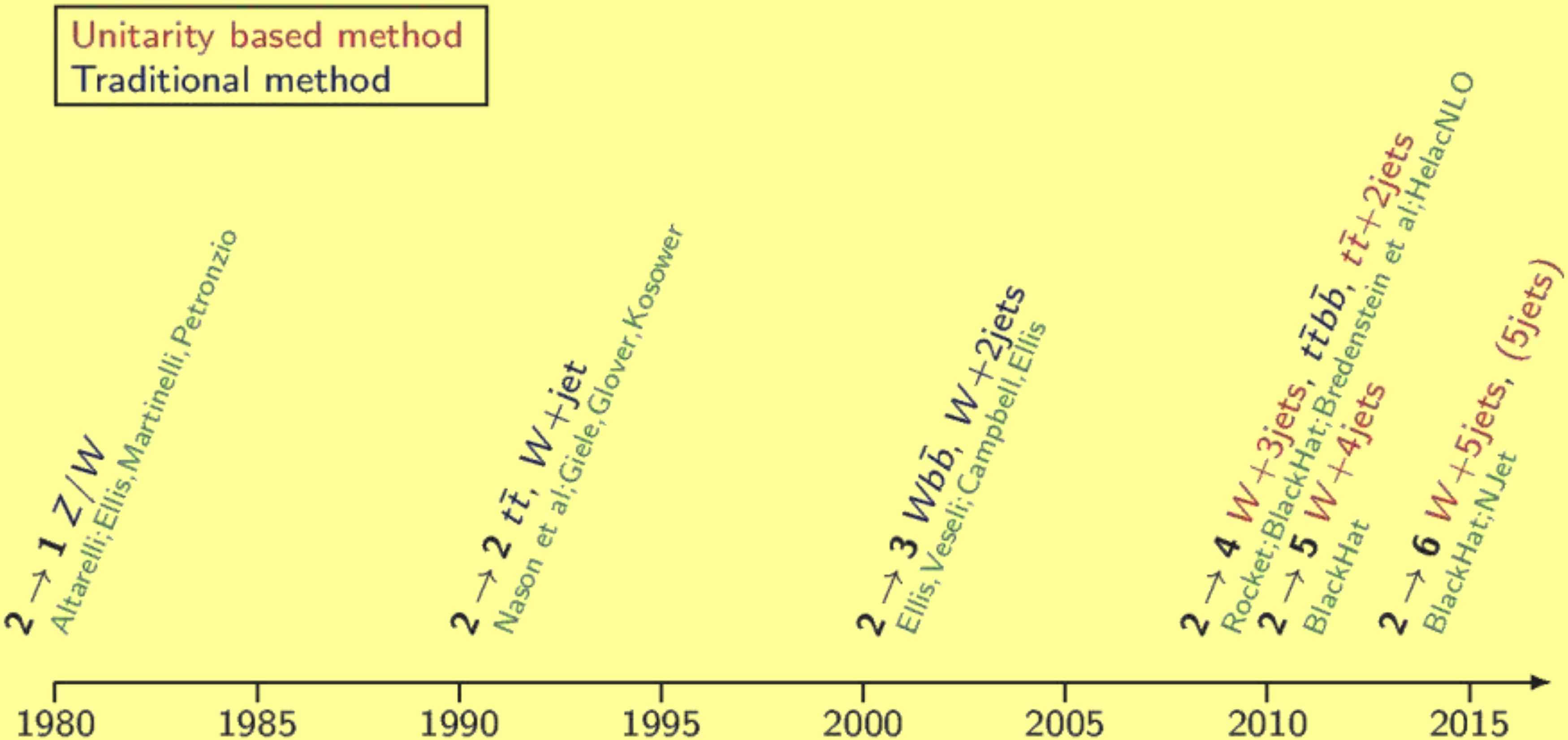
- Tree-level matrix elements for LO and real-emission terms



- Singular behavior of tree-level amplitudes, integrals, initial-state collinear behavior
- NLO parton distributions (MSTW,CTEQ,NNPDF,…)
- General framework for numerical real–virtual cancellations (Catani–Seymour subtraction is most popular) & its automation
- One-loop amplitudes

- On-shell methods have enabled the NLO Revolution

The NLO revolution

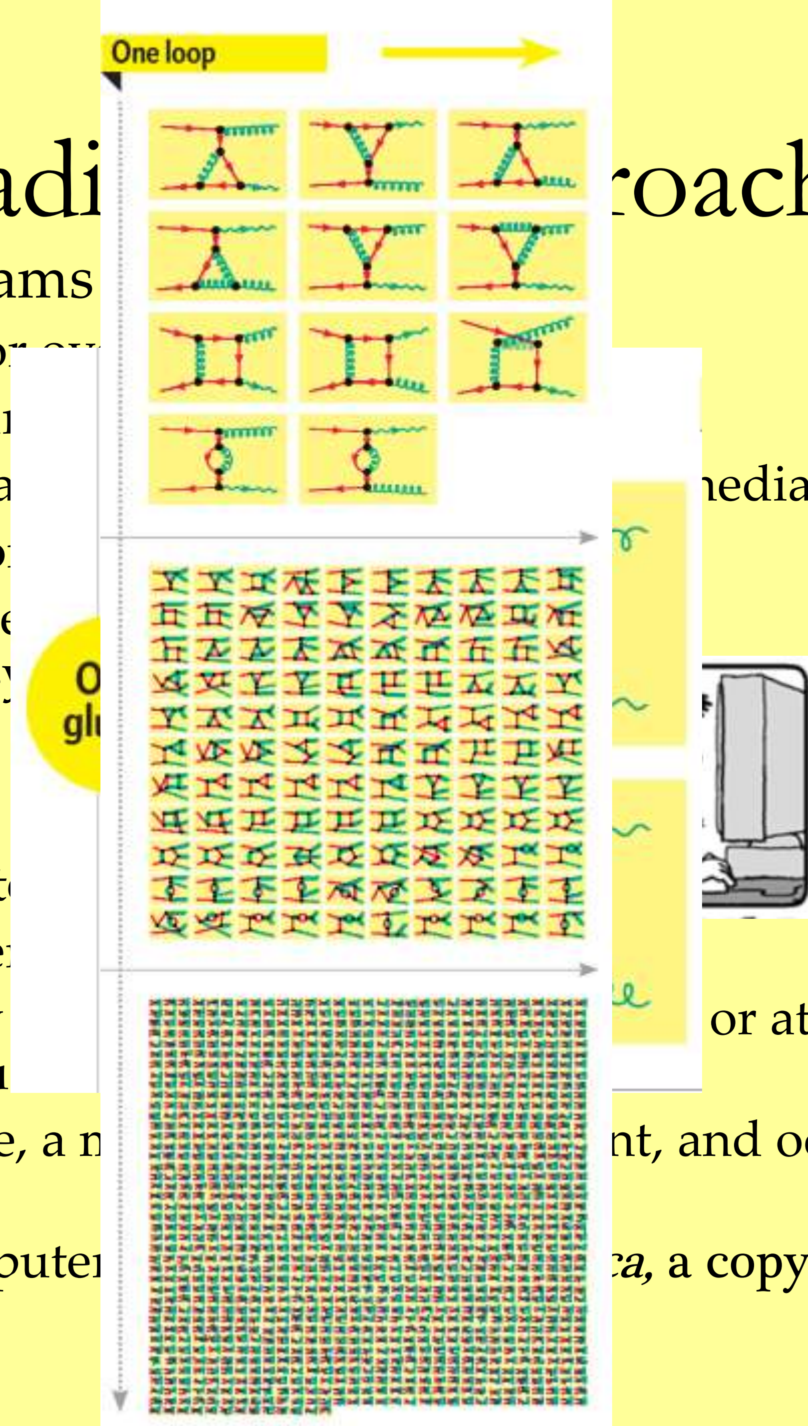


S. Höche

NLO Revolution

- Lots of revolutionaries roaming the world
  - BLACKHAT: Bern, Dixon, Febres Cordero, Hoeche, Ita, Lo Presti, DAK, Maitre
  - HELAC-NLO: Ossola, Papadopoulos, Pittau, Actis, Bevilacqua, Czakon, Draggiotis, Garzelli, van Hameren, Mastrolia, Worek & their clients
  - Rocket: Ellis, Giele, Kunszt, Lazopoulos, Melnikov, Zanderighi
  - GoSam/Samurai: Mastrolia, Ossola, Reiter,Tramontano,Cullen, Greiner, Heinrich, Luisoni
  - NJet/NGluon: Badger, Biedermann, & Uwer + Sattler & Yundin
  - MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, & Pittau
  - Analytics: Anastasiou, Britto, Duhr, Feng, Henn
  - Loop-Subtraction: Weinzierl, Becker, Goetz, Reuschle

Tradi… roach

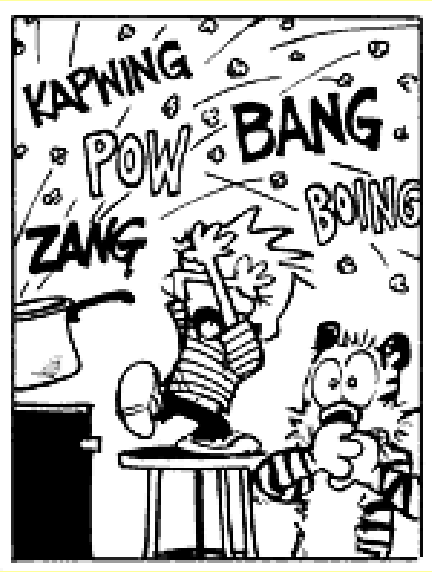


- Feynman Diagrams
  - Widely used for ov
  - Heuristic pictur
  - Introduces idea … nediate states
  - Precise rules fo
  - Classic successe discovery of asy
- How it works
  - Pick a process
  - Grab a graduat
  - Lock him or he
  - Provide a copy … or at least of Peskin & Schroeder's Qu
  - Supply caffeine, a n … nt, and occasional instructions
  - Provide a compute … *ca,* a copy of FORM & a C++ compiler

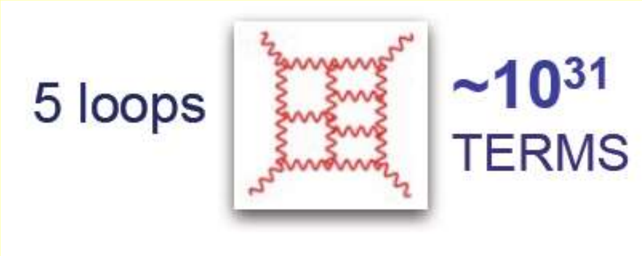
A Difficulty

- Huge number of diagrams in calculations of interest — factorial growth with number of legs or loops
- $2 \to 6$ jets: 34300 tree diagrams, ~ $2.5 \cdot 10^7$ terms

  ~$2.9 \cdot 10^6$ 1-loop diagrams, ~ $1.9 \cdot 10^{10}$ terms



- In gravity, it's even worse



Results Are Simple!

- Parke–Taylor formula for A^{MHV}

$$i\frac{\langle m_1\, m_2\rangle^4\,\delta^4(\sum_i k_i)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle (n-1)\,n\rangle\,\langle n\,1\rangle}$$

Parke & Taylor; Mangano, Parke, & Xu

$$\sim \sqrt{2k_1\cdot k_2}$$

Even Simpler in $\mathcal{N}$=4 Supersymmetric Theory

- Nair–Parke–Taylor form for MHV-class amplitudes

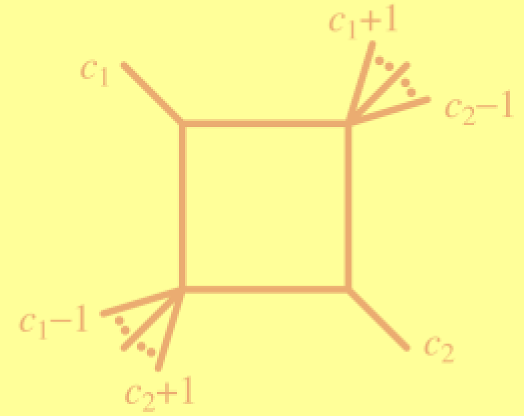
$$i\frac{\delta^{4|8}\left(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \middle| \sum_i \lambda_i^\alpha \eta_i^A\right)}{\langle 1\,2\rangle\,\langle 2\,3\rangle \cdots \langle (n-1)\,n\rangle\,\langle n\,1\rangle}$$

$$\sim \sqrt{p_i^\mu}$$

Answers Are Simple At Loop Level Too

One-loop in $\mathcal{N} = 4$:

$$-A^{\text{tree}}(1^+, \ldots, i^-, \ldots, j^-, \ldots, n^+)$$
$$\times \sum_{\text{easy 2 mass}} \text{Box} \cdot \tfrac{1}{2}(\text{its denominator})$$



- All-n QCD amplitudes for MHV configuration on a few Phys Rev D pages

Calculation is a Mess

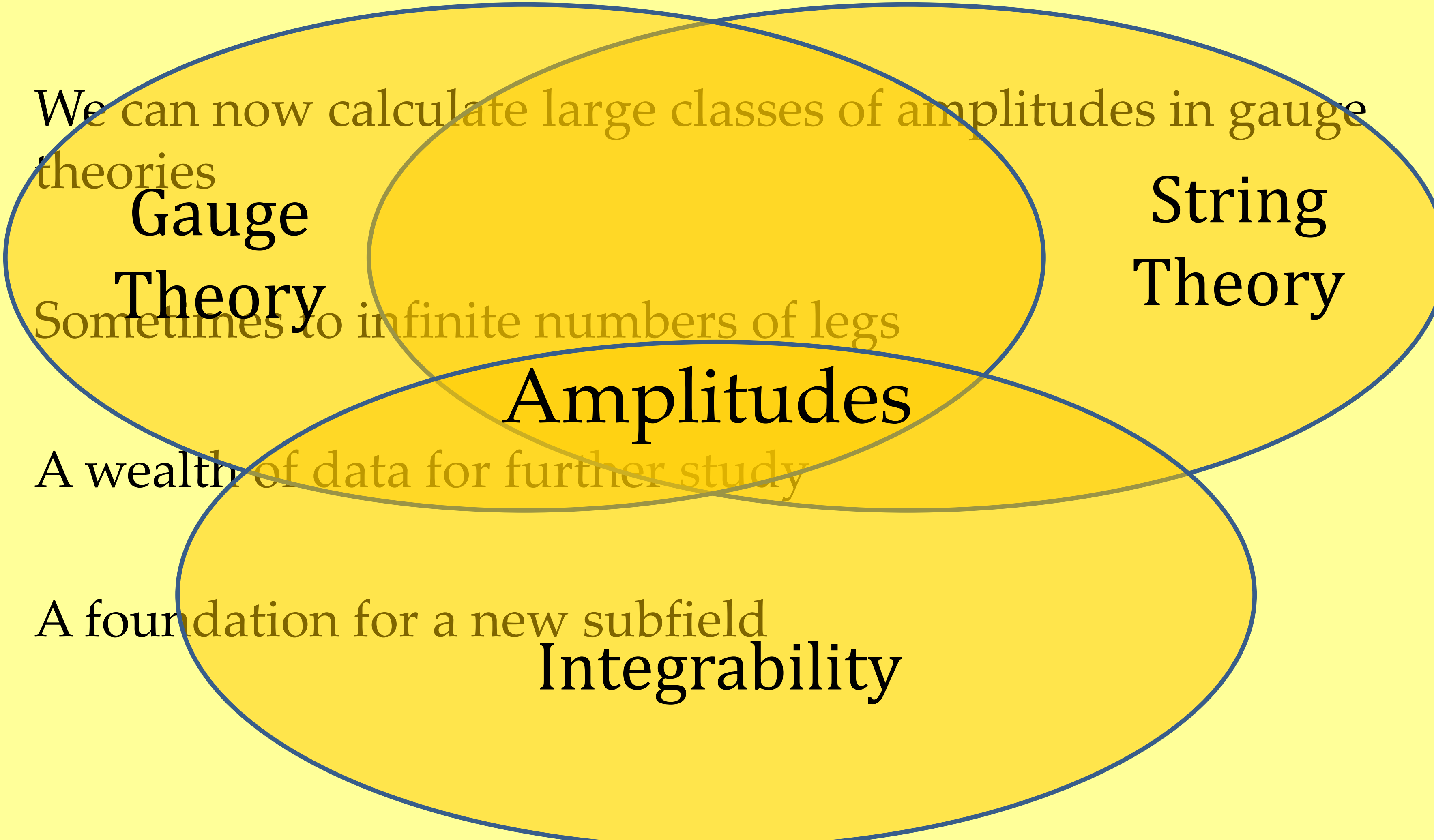
- Diagram insides involve unphysical states
- Each diagram does not respect symmetry of the theory ("not gauge-invariant") — huge cancellations of gauge-noninvariant, redundant, parts are to blame (exacerbated by algebra)
- There is almost no information in any given diagram

We can now calculate large classes of amplitudes in gauge theories

Sometimes to infinite numbers of legs

A wealth of data for further study

A foundation for a new subfield

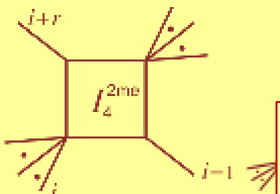


On-Shell Methods

- All physical quantities computed
  - From basic interaction amplitude: A_3^{tree}
  - Using only information from physical on-shell states
  - Avoid size explosion of intermediate terms due to unphysical states
  - Without need for a Lagrangian
- Properties of amplitudes become tools for calculating
  - Kinematics
    - Spinor variables
  - Underlying field theory
    - Integral basis
  - Factorization
    - On-shell recursion relations (BCFW) for tree-level amplitudes
    - Control infrared divergences in real-emission contributions to higher-order calculations
  - Unitarity
    - Unitarity and generalized unitarity for loop calculations

Integral Basis

- At one loop
  - Tensor reductions Brown–Feynman, Passarino–Veltman
  - Gram determinant identities
  - Boxes, triangles, bubbles, tadpoles



Integrals expressible in terms of logarithms, dilogarithms, rational functions of invariants

- At higher loops
  - Tensor reductions & Gram determinant identities
  - Irreducible numerators: Integration by parts Chetyrkin–Tkachov
  - Laporta algorithm
  - AIR (Anastasiou,Lazopoulos), FIRE (Smirnov,Smirnov), Reduze (Manteuffel, Studerus), LiteRed (Lee)
  - `Four-dimensional basis': integrals with up to 4 L propagators

BCFW On-Shell Recursion Relations

Britto, Cachazo, Feng, Witten (2005)

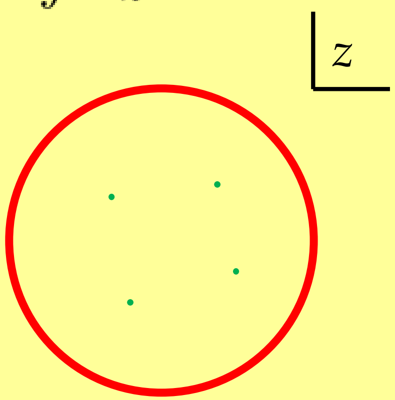
- Define a shift $[j, l\rangle$ of spinors by a complex parameter z

$$
\begin{aligned}
|j^-\rangle &\rightarrow |j^-\rangle - z|l^-\rangle, \\
|l^+\rangle &\rightarrow |l^+\rangle + z|j^+\rangle
\end{aligned}
$$

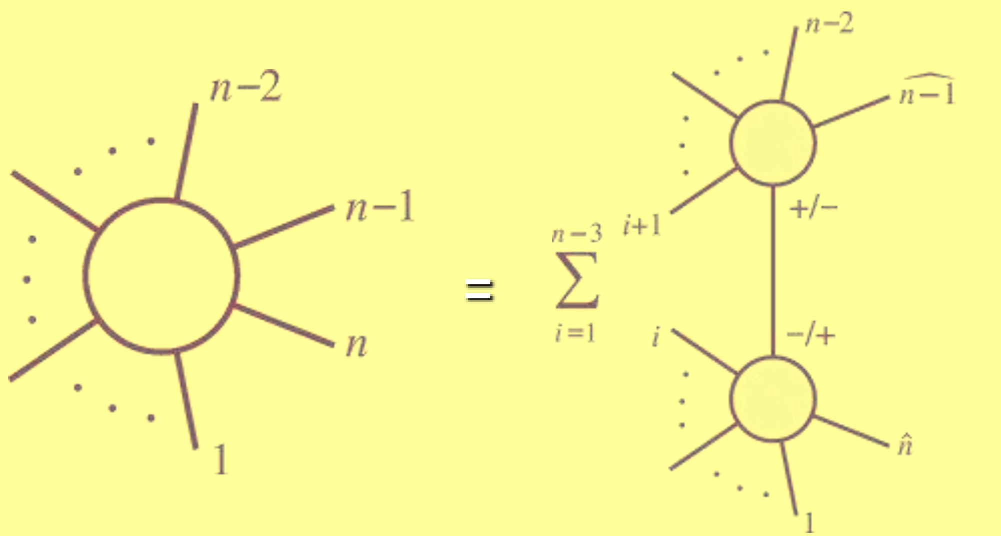
which defines a z-dependent continuation of the amplitude $A(z)$

- Assume that $A(z) \rightarrow 0$ as $z \rightarrow \infty$;
  look at contour integral

$$\oint \frac{dz}{z} A(z)$$



Factorization in *complex* momenta



NLO Revolution: On-Shell Methods

Master equation

$$\mathrm{Ampl} = \sum_{j \in \mathrm{Basis}} c_j \, \mathrm{Int}_j + \mathrm{Rational}$$

Known integral basis

Unitarity in $D = 4$

On-shell Recursion;
D-dimensional unitarity
via $\int$ mass

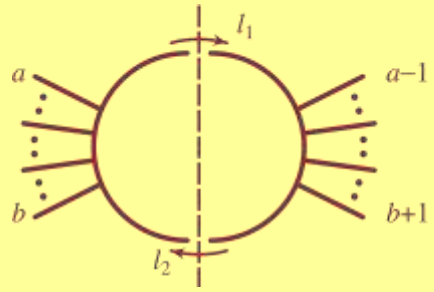
On general grounds, from knowledge that there's an underlying field theory

Unitarity

- Conservation of probability
- At the diagram or amplitude level, corresponds to Cutskosky rule: “cut” a pair of propagators

$$\frac{1}{\ell^2 - m^2 + i\delta} \longrightarrow -2\pi i \delta^{(+)}(\ell^2 - m^2)$$
$$= -2\pi i \delta(\ell^2 - m^2)\Theta(\ell^0)$$

- Reconstruct coefficients from the cuts — which are tree amplitudes
- No loop diagrams involved



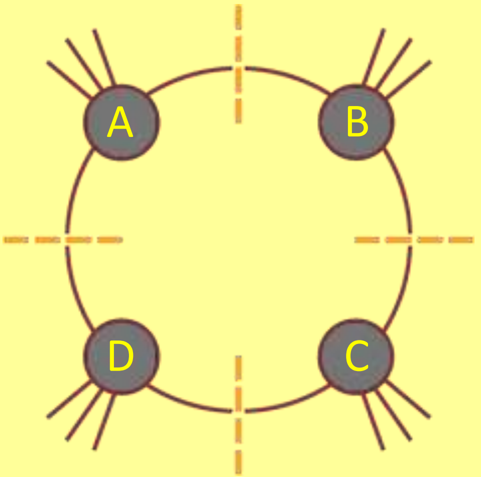
Generalized Unitarity

- "Cut" more propagators — with appropriate contour integration
- Each contour integration imposes an on-shell condition
- For the box integral, four on-shell conditions freeze the loop momentum completely

$$\ell^2 = 0\,, \quad -2\ell\cdot k_1 + k_1^2 = 0\,, \quad -2\ell\cdot k_2 + K_{12}^2 - k_1^2 = 0\,, \quad 2\ell\cdot k_4 + k_4^2 = 0\,.$$

- Solutions are complex momenta!
- Coefficient expressed in terms of tree amplitudes evaluated at these momenta

$$\text{Box coefficient} = \frac{1}{2}\sum_{\text{solutions}}\ \sum_{\substack{\text{species}\\ \text{helicities}}}\ \prod_J A_J^{\text{tree}}$$



- No algebraic reductions needed: suitable for pure numerics

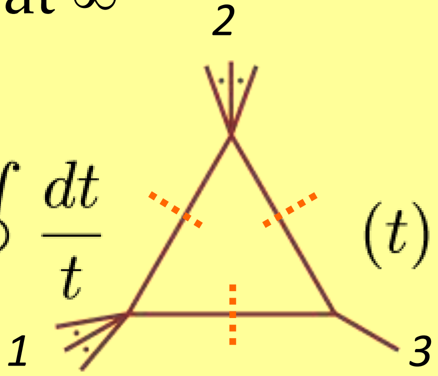
Triangle Cuts

Unitarity leaves one degree of freedom in triangle integrals.

Coefficients are the residues at ∞

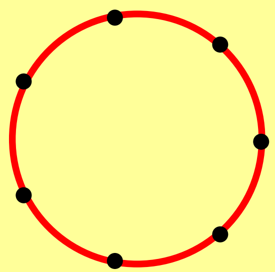
Forde (2007)

$$\text{coeff} = \frac{1}{2\pi i}\oint \frac{dt}{t}$$



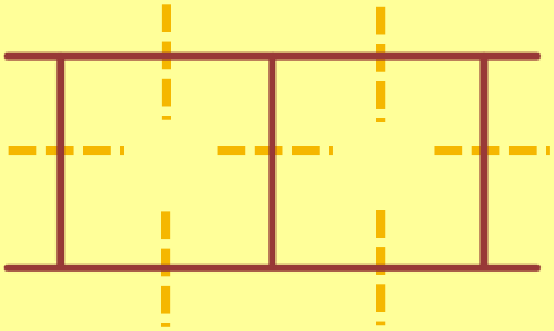
(t)

Evaluate numerically using a discrete Fourier projection (exact!)



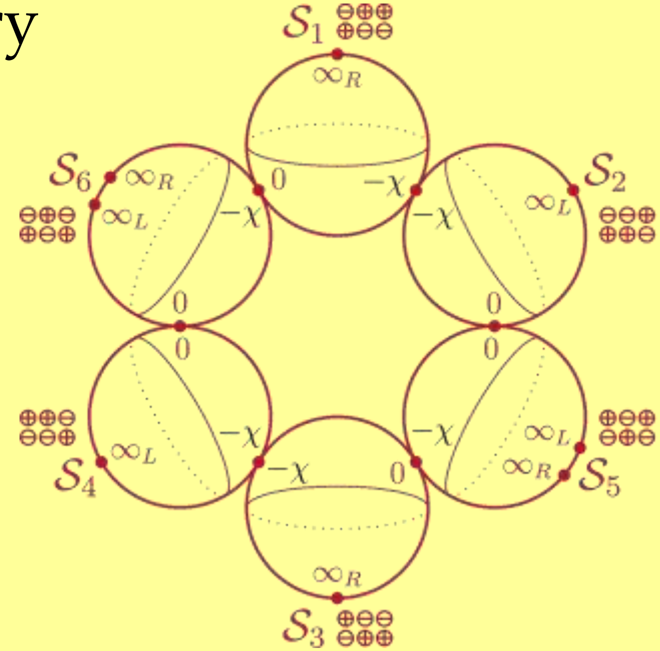
Higher Loops

- Same master equation
- Formulas for coefficients still under development



- Connections to algebraic geometry

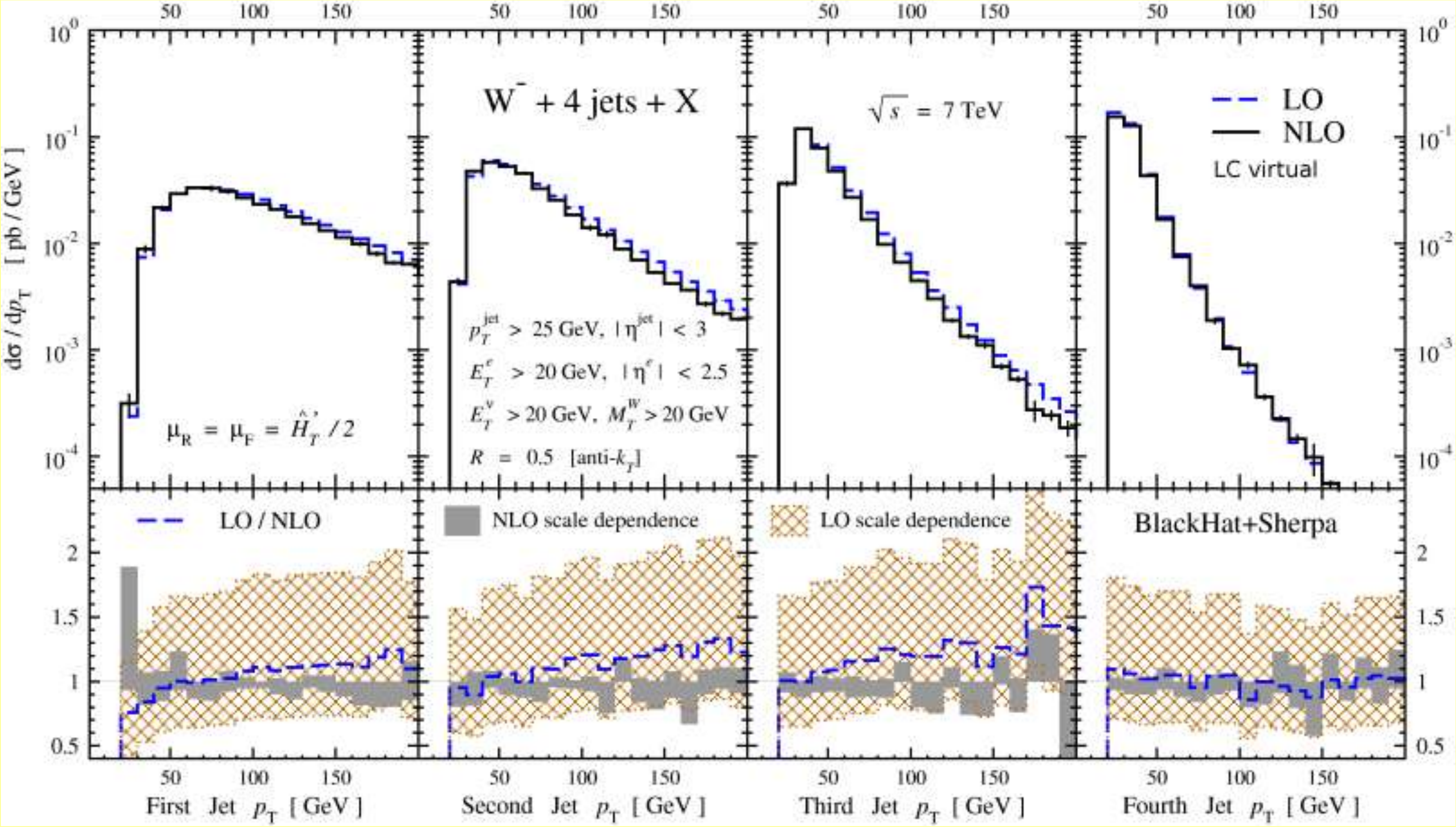
with Larsen & Johansson (2011–4)



BLACKHAT

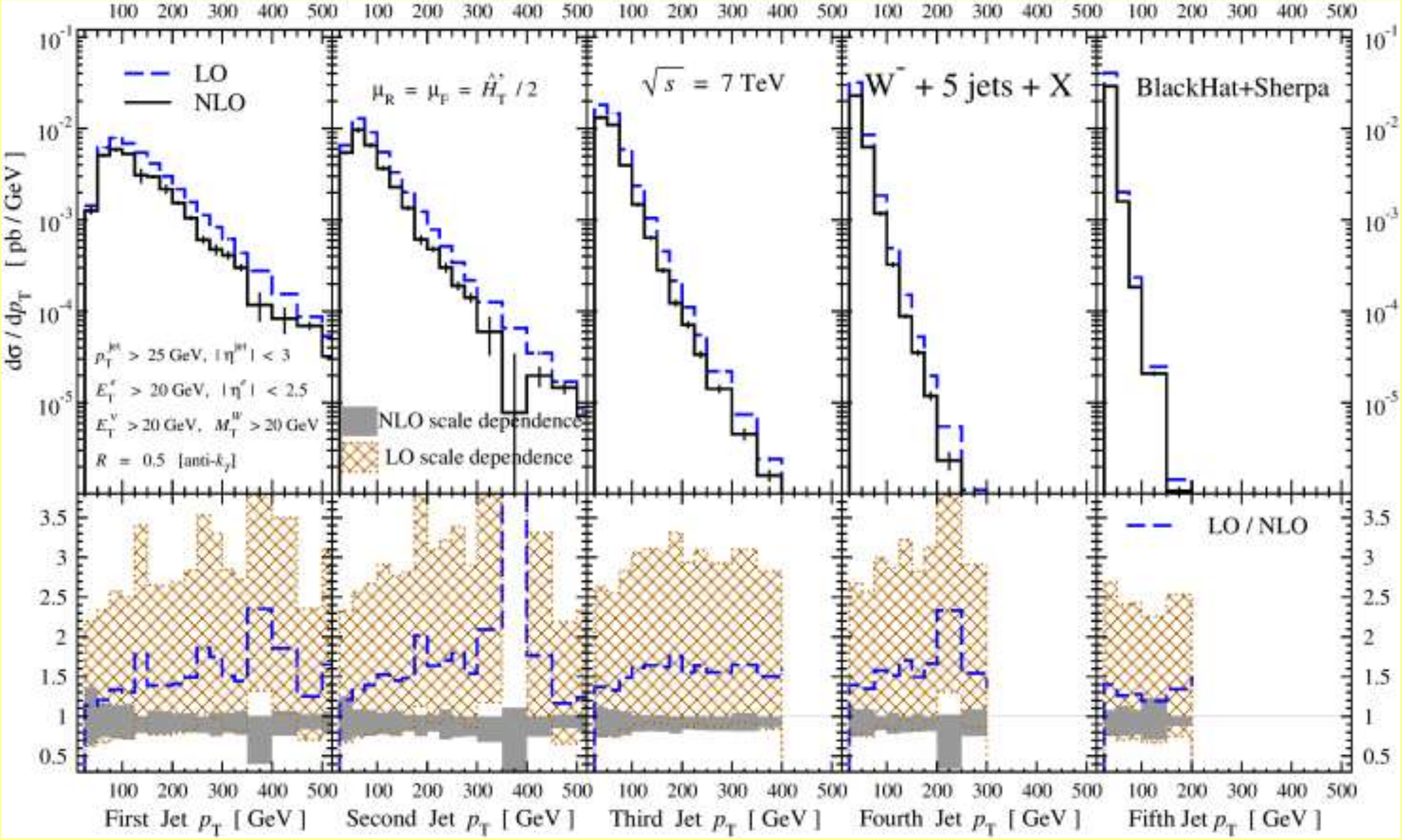
- BLACKHAT (Bern, Dixon, Febres Cordero, Hoeche, Ita, Lo Presti, DAK, Maitre)
  - One-loop matrix elements
  - Software library and its eponymous collaboration
  - Automated, numerical implementation of unitarity method
- COMIX
  - Born & real-emission matrix elements
  - Subtraction terms for real–virtual cancellation (Catani–Seymour)
- SHERPA
  - Process Management
  - Phase-space integration
  - *No* showering

W+4 Jets



- Scale variation reduced substantially at NLO; central scale $\widehat{H}'_{\mathrm{T}}/2$
- Successive jet distributions fall more steeply
- Shapes of 4th jet distribution unchanged at NLO — but first three are slightly steeper

W+5 Jets

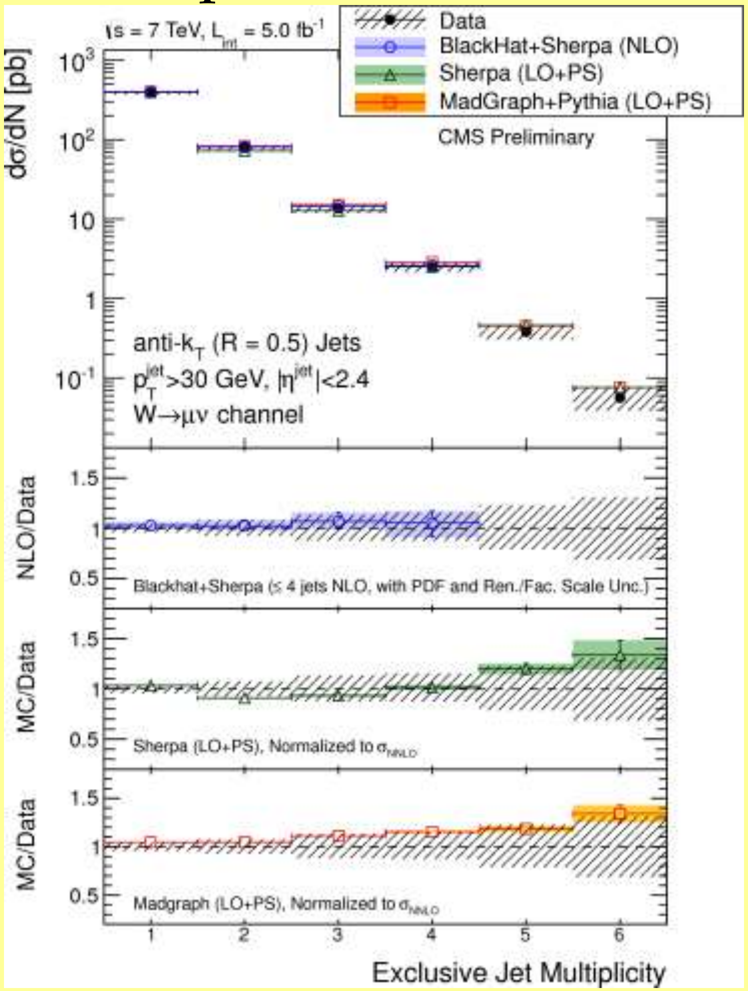


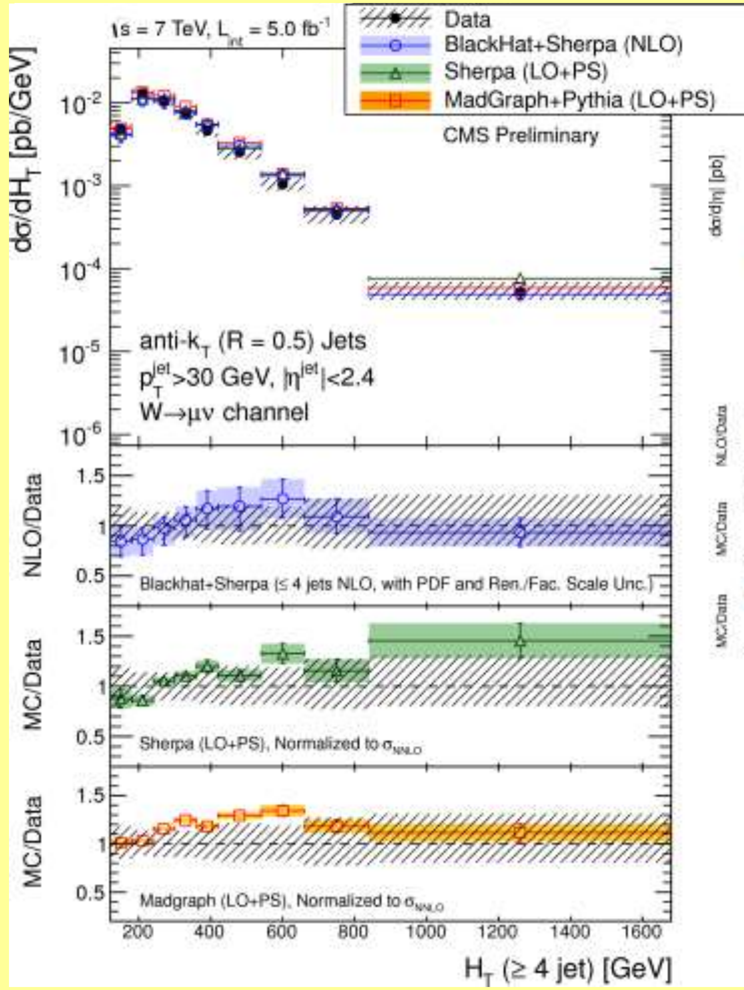
Scale-uncertainty bands shrink dramatically

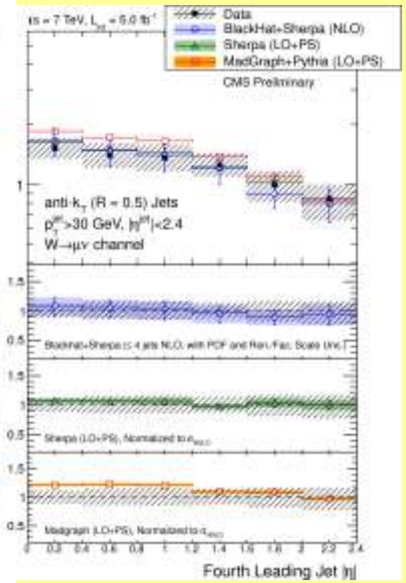
Last jet shape is stable, harder jets have steeper spectrum at NLO

Last three jet shapes look similar, just getting steeper

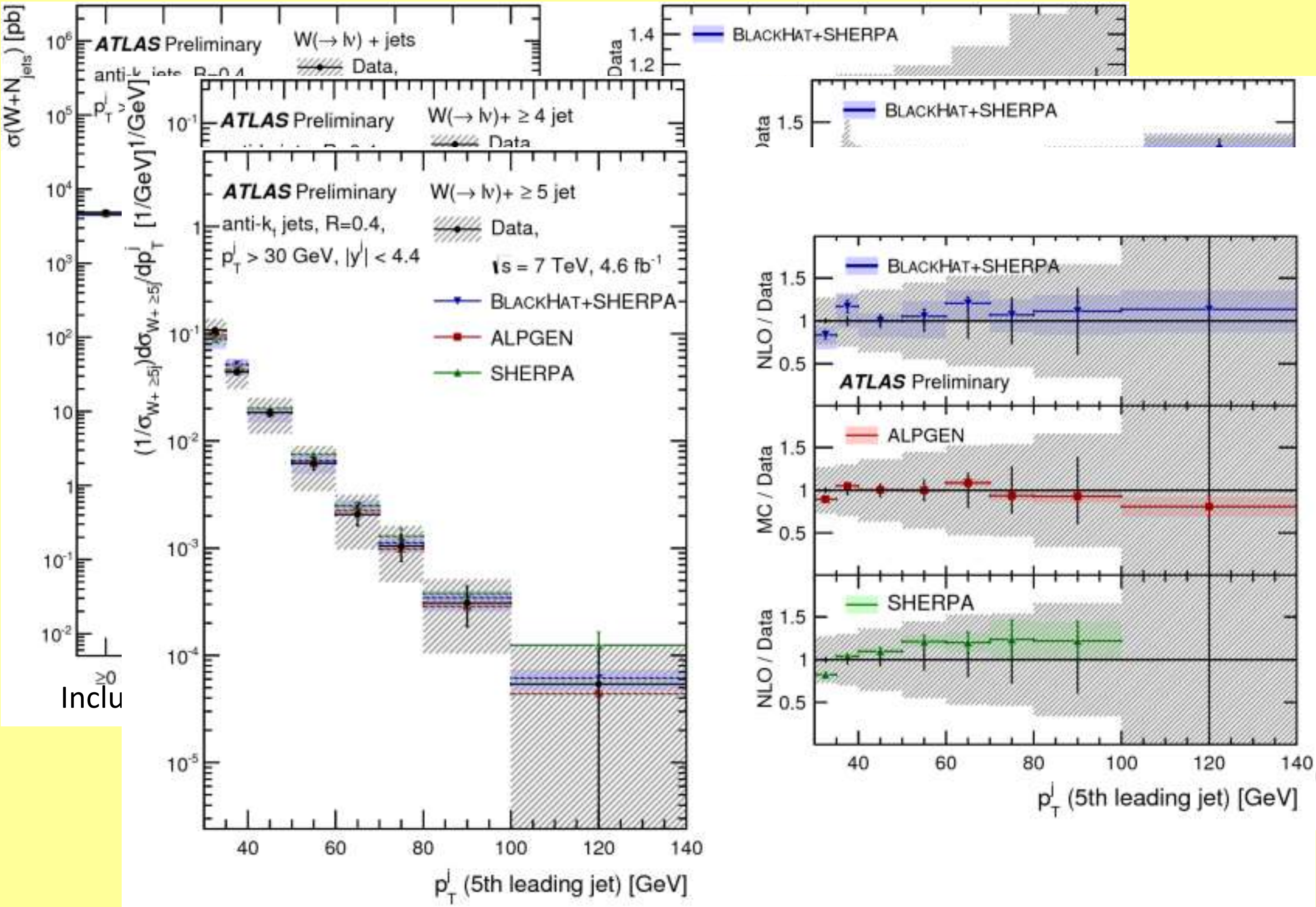
Comparison to recent 7 TeV results from CMS (March 2014)







- Comparison to recent ATLAS results at 7 TeV



Jet-Production Ratios

- Ratios reduce uncertainties both in experiment and theory (hadronization, jet energy scale,…)

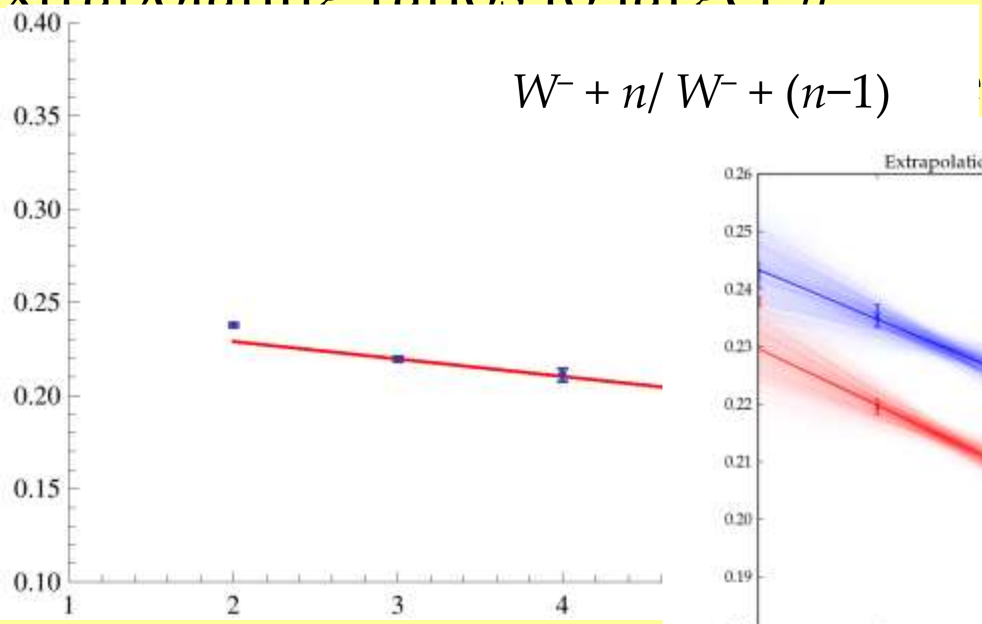
- Ratio has interesting behavior as a function of jet & W p_{T}, and total energy in jets $H_{\mathrm{T}}^{\mathrm{jets}}$

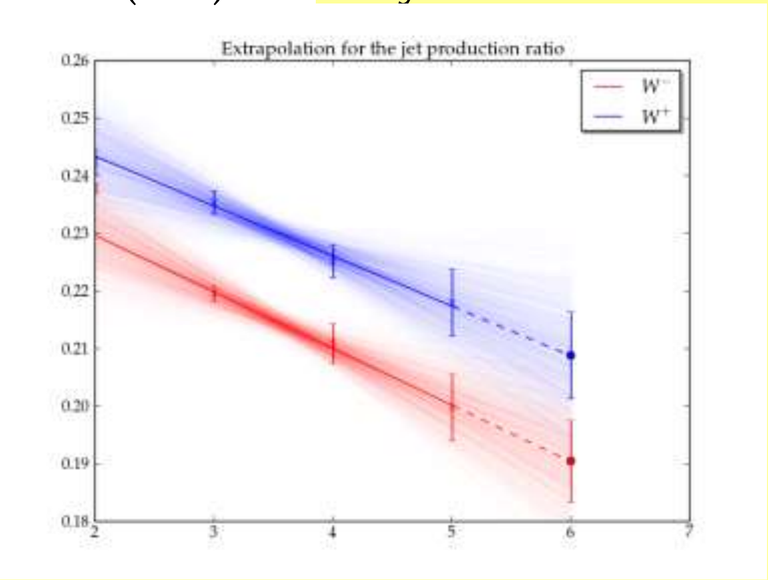
- W+1: missing subprocesses
- W+2: kinematic restrictions (W cannot be close to leading jet)

- Similarities of shapes of ratios for W+3 or more jets $\Rightarrow$ try extrapolating

Extrapolations

- Let's try extrapolating ratios to larger *n*
- We know ... ntly from $W+n/W+($ ...
  missing p ...
- We could ...
  with two ...
  that?





- With the W+5/W+4 ratio, a linear fit (with excellent χ^2/dof) makes the extrapolation meaningful:

W^- + 6 jets: 0.15 ± 0.01 pb

W^+ + 6 jets: 0.30 ± 0.03 pb

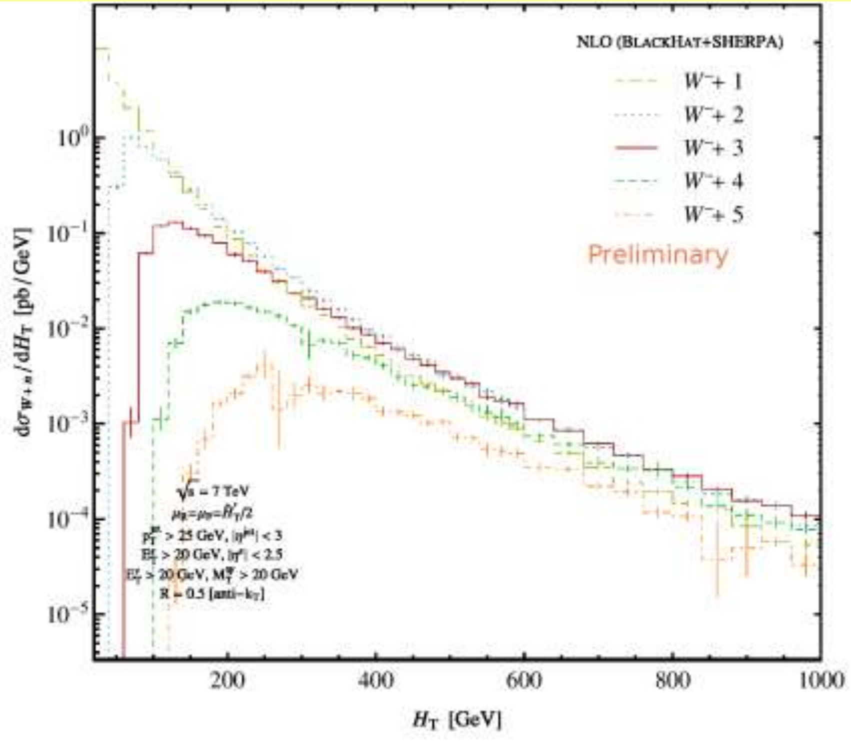
Uncertainty estimates from Monte-Carlo simulation of synthetic data

$H_\mathrm{T}^\mathrm{jet}$ Distribution

- Look at distribution of total transverse energy in jets: good probe into high-p_T physics

Different peaks

Different thresholds



- Let's try to extrapolate the distribution to W+6 jets

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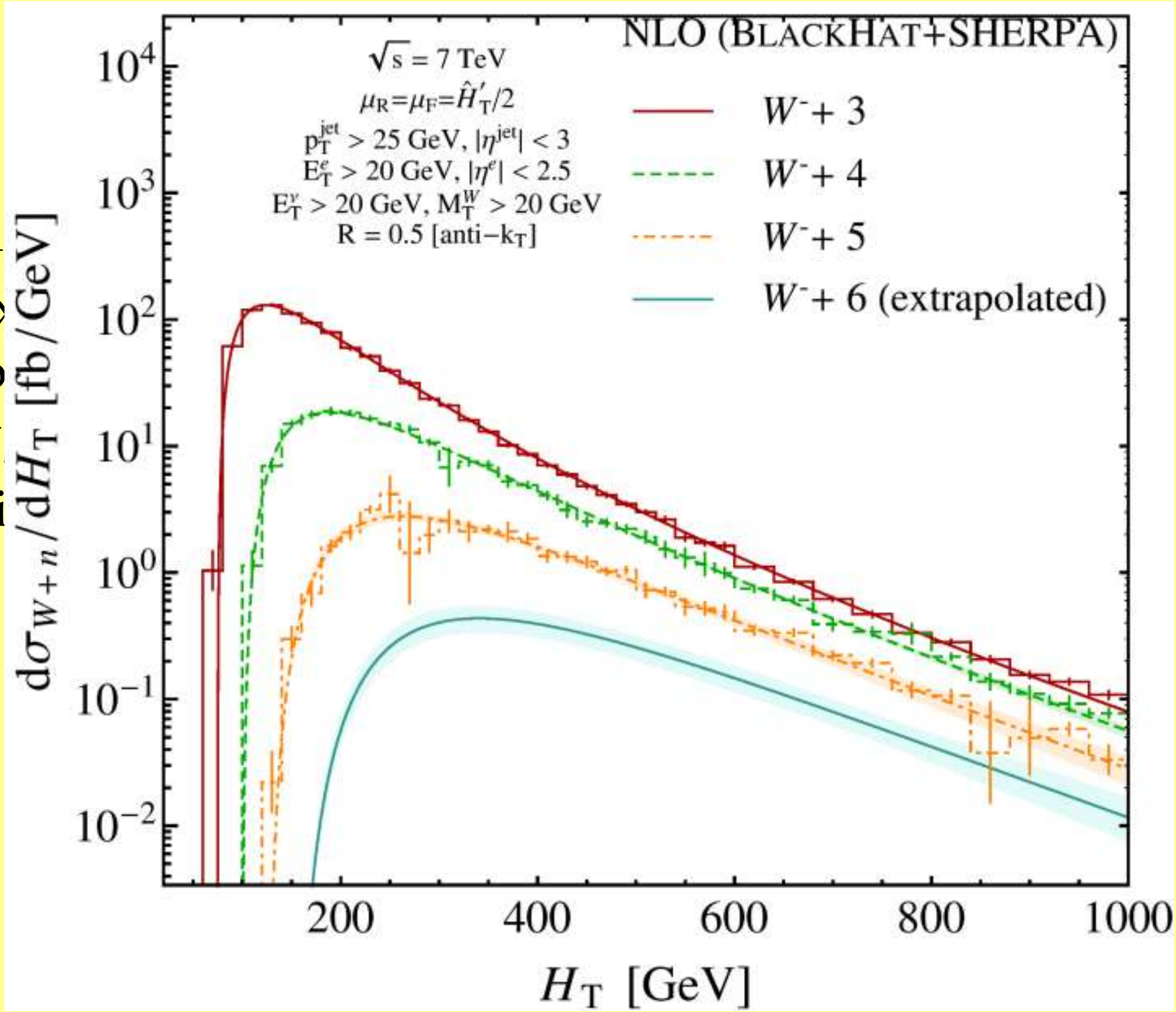

Summary

- NLO calculations are the first step to precision theory at the LHC
- On-shell methods have allowed us to push these calculations to multiplicities that seemed hopelessly out of reach 15 years ago
- Strong foundation for increasing precision and reach to match upcoming experimental improvements