On-Shell Methods, Amplitudes and Collider Physics

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Tools for Computing Amplitudes

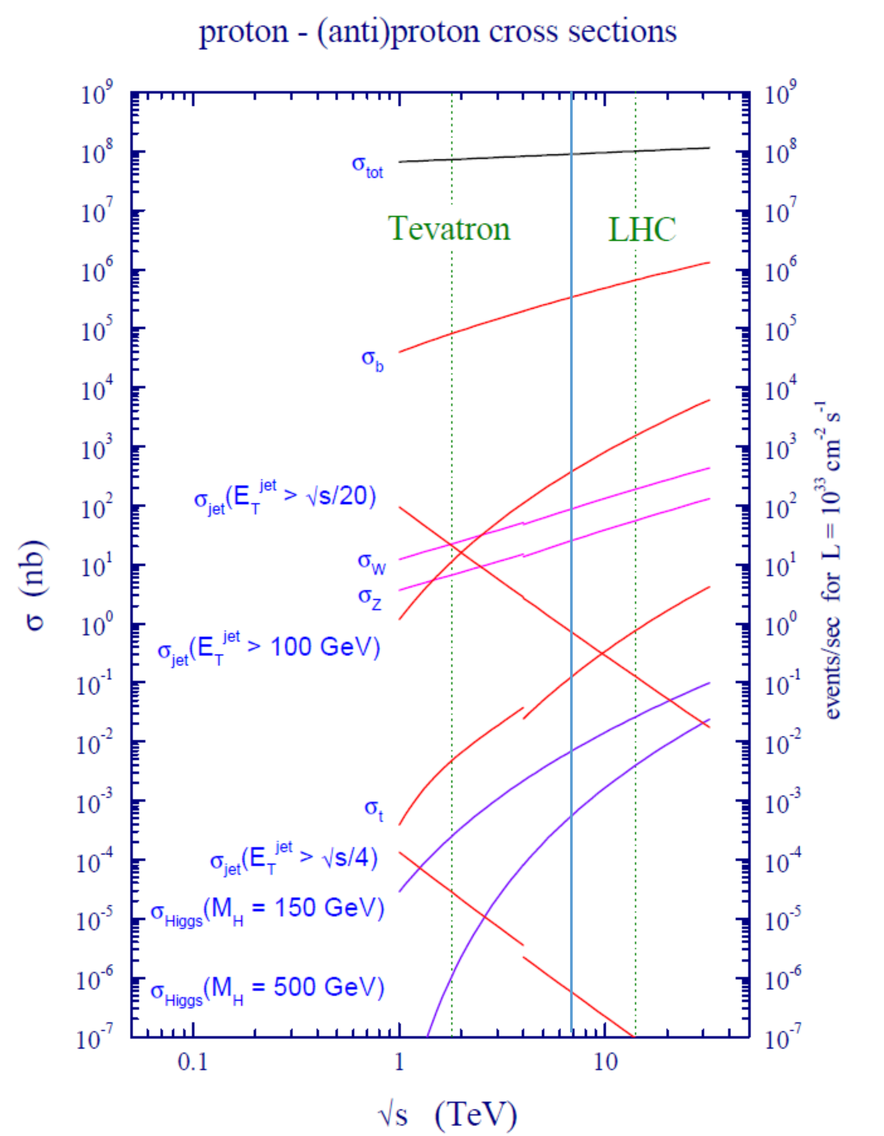
- New tools for computing in gauge theories — the core of the Standard Model

- Motivations and connections
  - Particle physics: $SU(3) \times SU(2) \times U(1)$
  - $\mathcal{N} = 4$ supersymmetric gauge theories and AdS/CFT
  - Witten's twistor string
  - Grassmanians
  - $\mathcal{N} = 8$ supergravity

- The particle content of the Standard Model is now complete, with the announcement in 2012 a Higgs-like boson by the ATLAS and CMS collaborations, looking more and more SM-like

- Every discovery opens new doors, and raises new questions

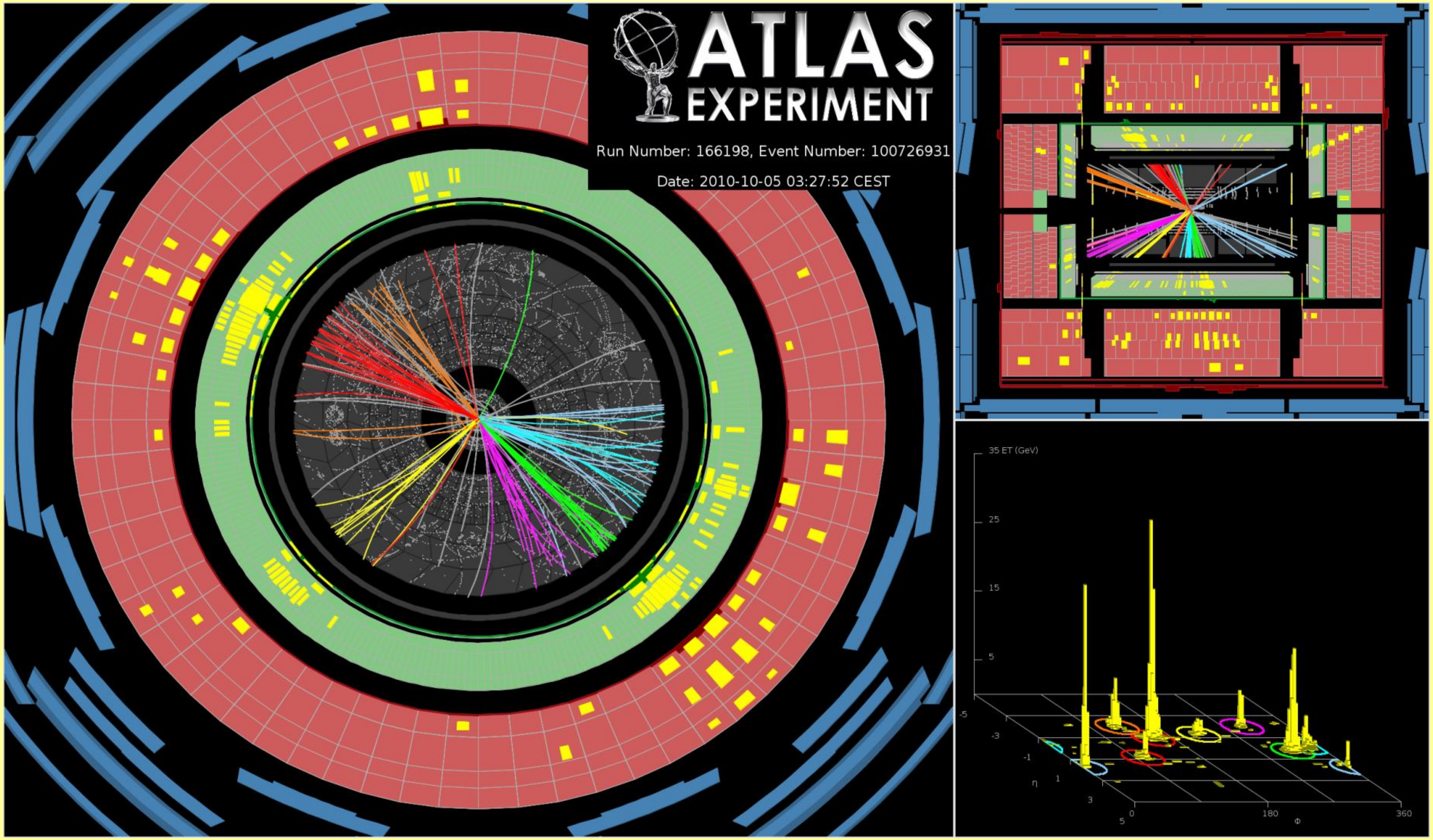
- How Standard-Model-like is the new boson?
  - We'll need precision calculations to see
- Is there anything else hiding in the LHC data?
  - We'll need background calculations to know



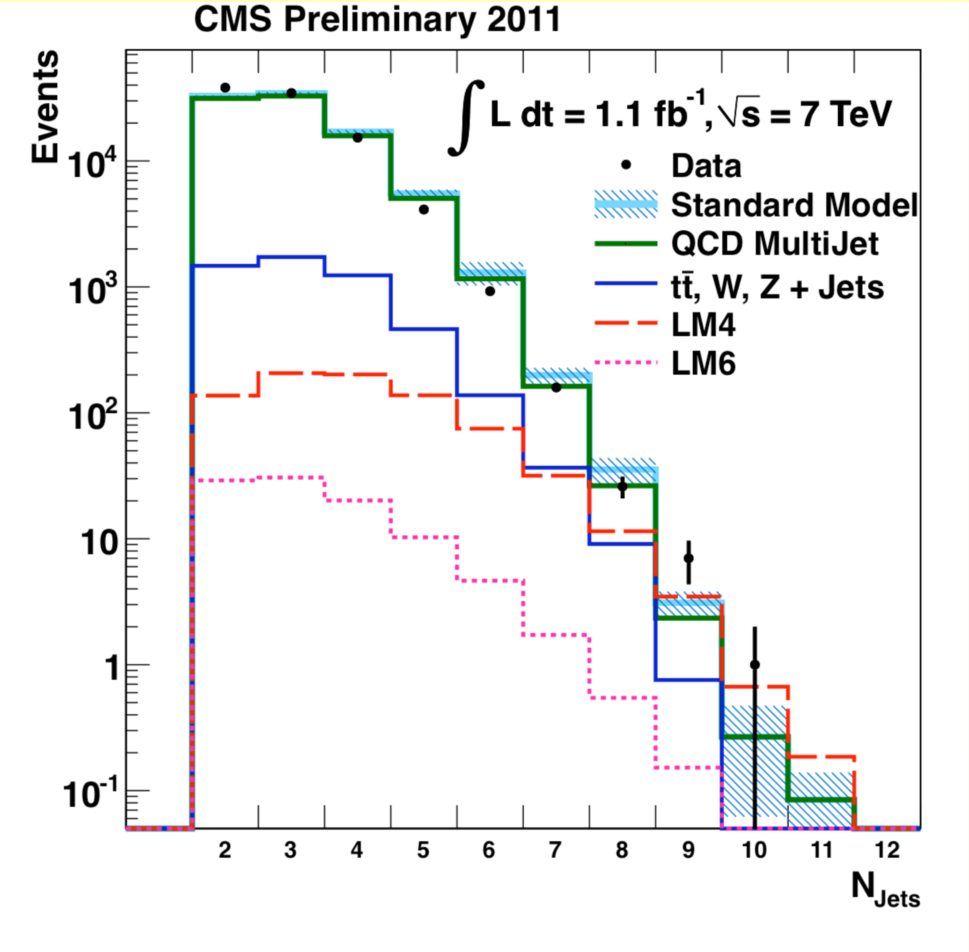
Campbell, Huston & Stirling '06

Jets are Ubiquitous

An Eight-Jet Event



Jets are Ubiquitous



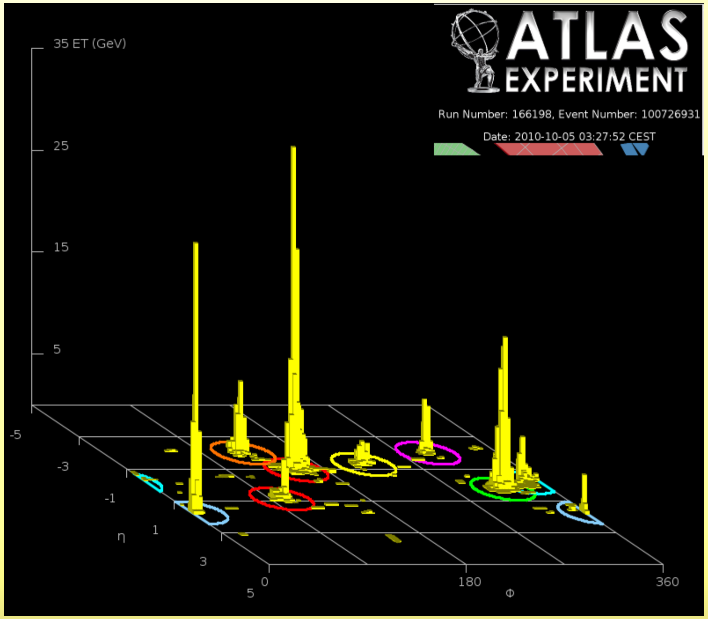
- Complexity is due to QCD
- Perturbative QCD:

  Gluons & quarks → gluons & quarks
- Real world:

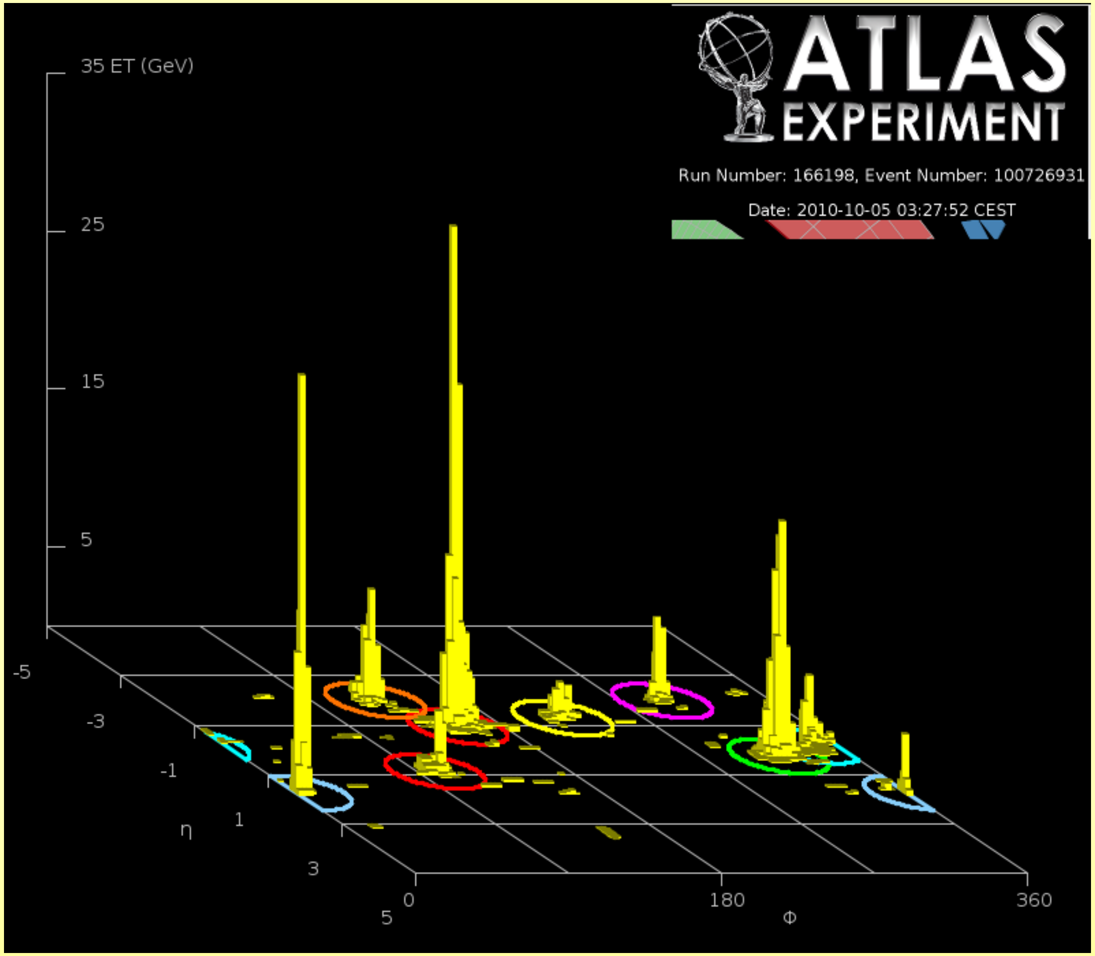
  Hadrons → hadrons with hard physics described by pQCD
- Hadrons → jets *narrow nearly collimated streams of hadrons*

Jets

- Defined by an experimental resolution parameter
  - originally by invariant mass in e^+e^- (JADE), later by relative transverse momentum (Durham, Cambridge, …)
  - cone algorithm in hadron colliders: cone size $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$ and minimum E_T: modern version is seedless (SISCone, Salam & Soyez)
  - (anti-)k_T algorithm: essentially by a relative transverse momentum



Atlas eight-jet event

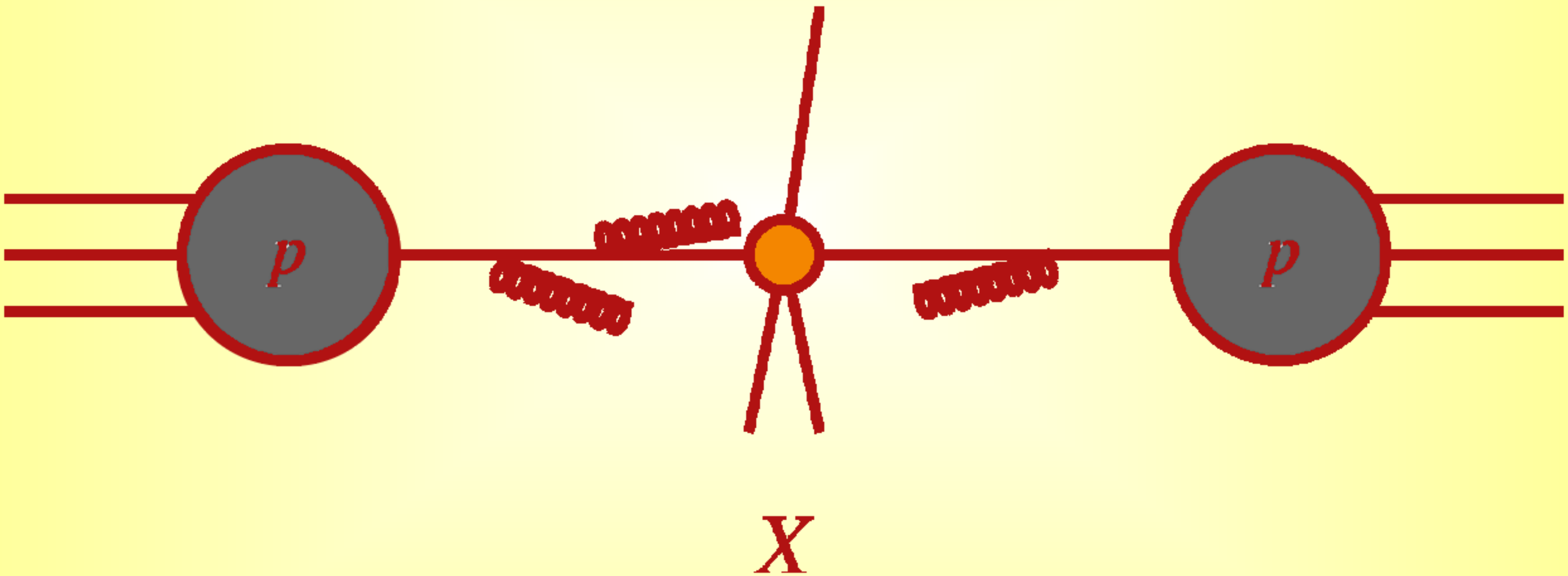

ATLAS EXPERIMENT
Run Number: 166198, Event Number: 100726931
Date: 2010-10-05 03:27:52 CEST
35 ET (GeV)
25
15
5
-5
-3
-1
1
3
5
η
0
180
360
Φ

In theory, theory and practice are the same.

In practice, they are different

— Yogi Berra

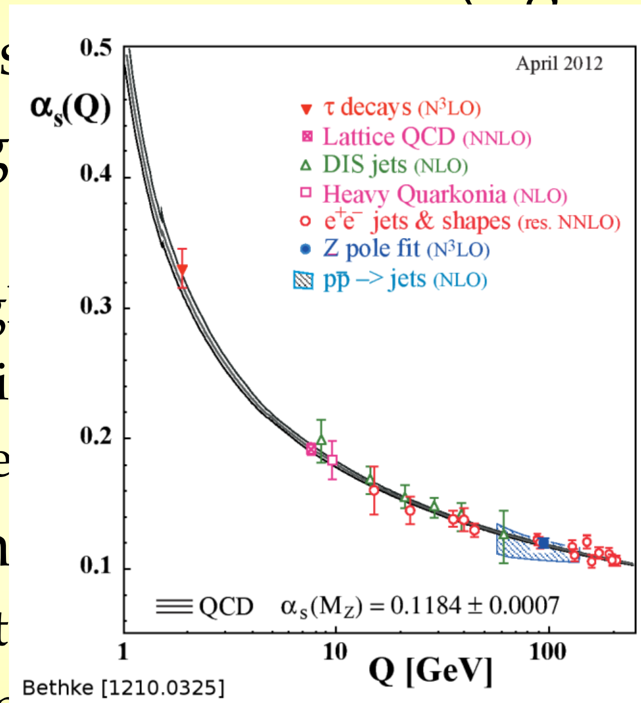
QCD-Improved Parton Model



$$\sum_{a,b} \int dx_a dx_b \, d\text{Phase} \; f_a f_b \, \sigma_{ab} \delta(v - \text{Observable})$$

The Challenge

- Everything at a hadron collider (signals, backgrounds, luminosity meas… …D
- Strong coupling … …0.12 and running is important
  - ⇒ events have hig… …ers (jets)
  - ⇒ each jet has a hi…
  - ⇒ higher-order pe… …nportant
- Processes can in… …$p_T(W)$ & M_W
  - ⇒ need resummat…



- Confinement introduces further issues of mapping partons to hadrons, but for suitably-averaged quantities (infrared-safe) avoiding small E scales, this is not a problem (power corrections)

Approaches

- General parton-level fixed-order calculations
  - Numerical jet programs: general observables
  - Systematic to higher order/high multiplicity in perturbation theory
  - Parton-level, approximate jet algorithm; match detector events only statistically
- Parton showers
  - General observables
  - Leading- or next-to-leading logs only, approximate for higher order/high multiplicity
  - Can hadronize & look at detector response event-by-event
  - Understood how to match to matrix elements at leading order
- Semi-analytic calculations/resummations
  - Specific observable, for high-value targets
  - Checks on general fixed-order calculations

Renormalization Scale

- Needed to define the coupling
- Physical quantities should be independent of it
- Truncated perturbation theory isn't
- Dependence is ~ the first missing order * logs
- Similarly for factorization scale — define parton distributions

Every sensible observable has an expansion in α_s

$$\frac{d\sigma}{d\mathcal{O}} = \alpha_s^{n_0}(\mu)\frac{d\hat{\sigma}^{\mathrm{LO}}}{d\mathcal{O}} + \alpha_s^{n_0+1}(\mu)\frac{d\hat{\sigma}^{\mathrm{NLO}}(\mu)}{d\mathcal{O}} + \alpha_s^{n_0+2}(\mu)\frac{d\hat{\sigma}^{\mathrm{NNLO}}(\mu)}{d\mathcal{O}}$$

Examples

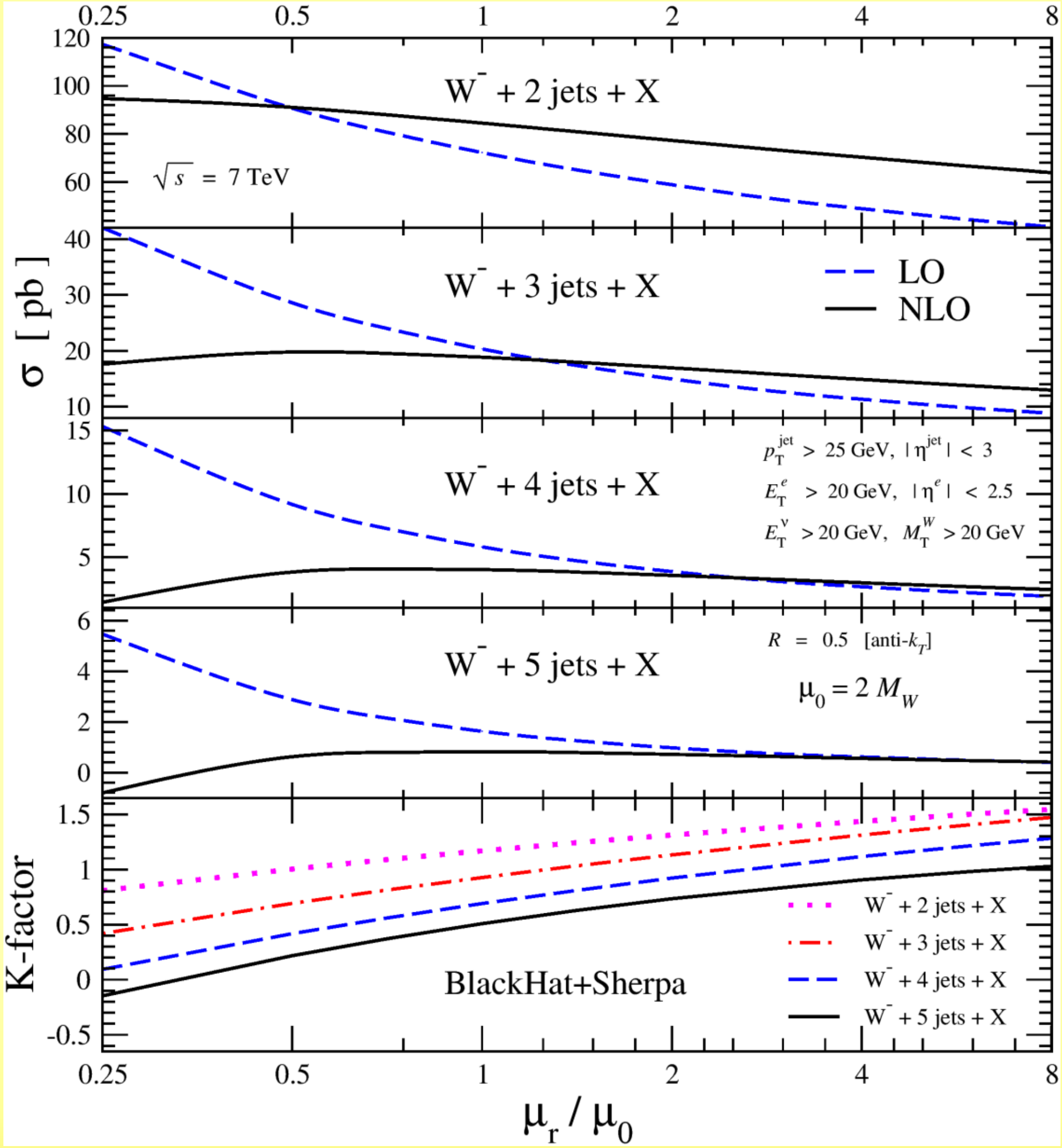
$$\frac{d\sigma^{W+1\,\mathrm{jet}}}{dp_{\mathrm{T}}^{\mathrm{jet}}} = \alpha_s(\mu)\frac{d\hat{\sigma}^{\mathrm{LO}}}{dp_{\mathrm{T}}^{\mathrm{jet}}} + \alpha_s^2(\mu)\frac{d\hat{\sigma}^{\mathrm{NLO}}(\mu)}{dp_{\mathrm{T}}^{\mathrm{jet}}} + \alpha_s^3(\mu)\frac{d\hat{\sigma}^{\mathrm{NNLO}}(\mu)}{dp_{\mathrm{T}}^{\mathrm{jet}}}$$

$$\frac{d\sigma^{W+2\,\mathrm{jet}}}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}} = \alpha_s^2(\mu)\frac{d\hat{\sigma}^{\mathrm{LO}}}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}} + \alpha_s^3(\mu)\frac{d\hat{\sigma}^{\mathrm{NLO}}(\mu)}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}} + \alpha_s^4(\mu)\frac{d\hat{\sigma}^{\mathrm{NNLO}}(\mu)}{dp_{\mathrm{T}}^{2^{\mathrm{nd}}\,\mathrm{jet}}}$$

Leading-Order, Next-to-Leading Order

- QCD at LO is not quantitative

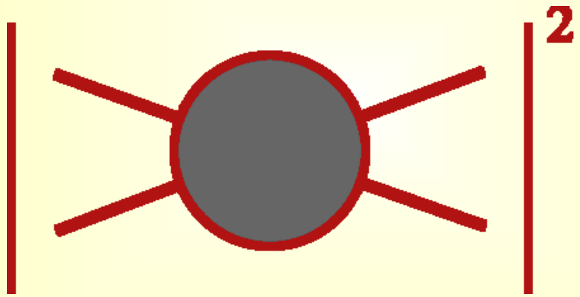
- LO: Basic shapes of distributions
  but: no quantitative prediction — large dependence on unphysical
  renormalization and factorization scales
  missing sensitivity to jet structure & energy flow
- NLO: First quantitative prediction, expect it to be reliable to 10–15%
  improved scale dependence — inclusion of virtual corrections
  basic approximation to jet structure — jet = 2 partons
  importance grows with increasing number of jets
- NNLO: Precision predictions
  small scale dependence
  better correspondence to experimental jet algorithms
  understanding of theoretical uncertainties
  will be required for <5% predictions for future precision measurements



W⁻ + 2 jets + X
$\sqrt{s}$ = 7 TeV
W⁻ + 3 jets + X
LO
NLO
σ [pb]
W⁻ + 4 jets + X
p_T^{jet} > 25 GeV, $|\eta^{jet}|$ < 3
E_T^e > 20 GeV, $|\eta^e|$ < 2.5
E_T^ν > 20 GeV, M_T^W > 20 GeV
W⁻ + 5 jets + X
R = 0.5 [anti-k_T]
$\mu_0 = 2\, M_W$
K-factor
BlackHat+Sherpa
W⁻ + 2 jets + X
W⁻ + 3 jets + X
W⁻ + 4 jets + X
W⁻ + 5 jets + X
μ_r / μ_0

What Contributions Do We Need?

- Short-distance matrix elements to 2-jet production at leading order: tree level amplitudes



- Short-distance matrix elements to 2-jet production at next-to-leading order: tree level + one-loop amplitudes + real emission