CMB When is last scattering?

Approximate answer:

When equilibrium abundance of electrons drops to 10% of baryons. $e + p \leftrightarrow H + \gamma$

Saha approximation: equilibrium balance of chemical potentials yields ratio $\frac{n_e n_p}{n_H} = \frac{n_e^{(0)} n_p^{(0)}}{n_H^{(0)}}$

Define $X_e = \frac{n_e}{n_p + n_H} = \frac{n_e}{n_b}$

$$\frac{X_e^2}{1-X_e} = \left[\frac{(m_e c^2)(K_B T)}{2\pi}\right]^{3/2} \frac{e^{-B/K_B T}}{n_b (\hbar c)^3}$$

Find $T = T_0(1+z)$ at $X_e = 0.1$

$$n_b = \frac{3}{8\pi G} H_0^2 \Omega_b \frac{1}{m_b c^2} \times \left(\frac{T}{T_0}\right)^3$$

$B = 13.6\,\text{eV}$, $\Omega_b = 0.045$, $T_0 = 2.7\,\text{K}$.

Get $z_{rec} \simeq 1250$

Not bad for recombination

Correct answer? Solve Boltzmann eq'n.

CMB OPTICAL DEPTH

$$\tau_\gamma = \int_0^{t(z)} c\,dt\ n_e \sigma$$

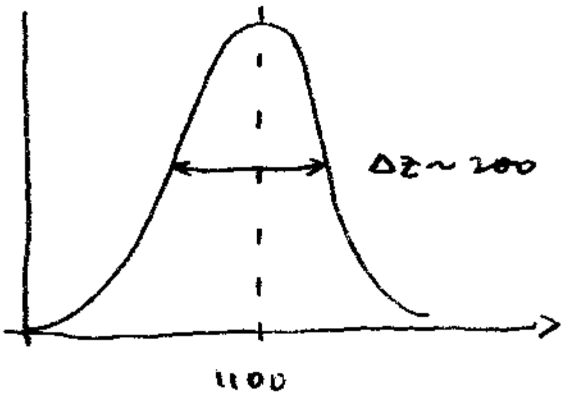
VISIBILITY

$$g = -\tau_\gamma' e^{-\tau_\gamma}$$

probability for a photon to be scattered in interval $\tau, \tau + d\tau$

where $\int_0^{\tau_0} g(\tau)\,d\tau = 1$

g



WMAP: $z \sim 1089 \pm 1$

$\Delta z \sim 195 \pm 2$

["SUDDEN" APPROXIMATION : $g(z) = \delta(z - z_{LS})$]

LARGE SCALE CMB in "SUDDEN" LIMIT

SACHS-WOLFE EFFECT

Temperature of CMB $\propto$ Photon Energy

$$\frac{T_{OBS}}{T_{EMIT}} = \frac{\sqrt{-g_{00}(\text{emit})}}{\sqrt{-g_{00}(\text{obs})}} = \frac{a(t_{EMIT})}{a(t_{OBS})}\sqrt{\frac{(1+2\phi_e)}{(1+2\phi_0)}}$$

$$\simeq \frac{a_e}{a_0}\left(1+\phi_e(t_e,\vec{x}_e)-\phi_0(t_0,\vec{x}_0)\right)$$

$$T_{OBS} = \bar{T}_0 + \Delta T$$

But T_{emit} depends on direction

"EMIT" WHEN ρ_γ = particular value

$$\bar{\rho}_0\left(\frac{a_0}{a_e}\right)^4 = \bar{\rho}_0\left(\frac{T_e}{\bar{T}_0}\right)^4$$

$$\left(\frac{t_0}{t_e+\delta t}\right)^{8/3} = \left(\frac{T_e}{\bar{T}_0}\right)^4$$

$$\Big\downarrow \quad t_e+\delta t = t_e(1+\phi_e)$$

$$\left(\frac{t_0}{t_e}\right)^{2/3}\left(1-\frac{2}{3}\phi_e\right) = \frac{T_e}{\bar{T}_0}$$

$$T_{OBS} = \bar{T}_0 + \Delta T = T_e\frac{a_e}{a_0}(1+\phi_e) = \bar{T}_0\left(1-\frac{2}{3}\phi_e\right)(1+\phi_e)$$

$$\simeq \bar{T}_0\left(1+\frac{1}{3}\phi_e\right)$$

CMB

$$\frac{\Delta T}{T_0} = \frac{1}{3}\phi_e = \frac{1}{3}\phi(\tau=\tau_{LS}, \vec{x}=(\tau_0-\tau_{LS})\hat{n})$$

INFLATION INFORMS US OF THE STATISTICS OF CMB $\frac{\Delta T}{T}$.

$$\frac{\Delta T}{T}(\hat{n}) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})$$

$$\left\langle \frac{\Delta T}{T}(\hat{n}) \frac{\Delta T}{T}(\hat{n}')\right\rangle = C(\theta)$$

$$= \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta)$$

$$C_\ell = \frac{1}{2\ell+1}\sum_m |a_{\ell m}|^2$$

or $\langle a_{\ell m} a_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell$

R is Gaussian distributed w/ zero mean, variance $\langle R^2\rangle$

so is ϕ, so is $\frac{\Delta T}{T}$, $a_{\ell m}$

$$\left\langle \frac{\Delta T}{T}(\hat{n}) \frac{\Delta T}{T}(\hat{n}') \right\rangle = \frac{1}{9} \langle \Phi(\tau_{ls}, \lambda \hat{n}) \, \Phi(\tau_{ls}, \lambda \hat{n}') \rangle$$

$$\lambda = \tau_0 - \tau_{ls}$$

recall $\Phi = \frac{3}{5} R$ in matter era, for long wavelengths

$$= \frac{1}{25} \langle R(\tau_{ls}, \lambda \hat{n}) \, R(\tau_{ls}, \lambda \hat{n}') \rangle$$

$$= \frac{1}{25} \int \frac{d^3k}{(2\pi)^3} \, e^{i\vec{k}\cdot\lambda(\hat{n}-\hat{n}')} \, P_R(k)$$

identity $e^{i\vec{k}\cdot\vec{r}} = 4\pi \sum_\ell i^\ell j_\ell(kr) \sum_m Y^*_{\ell m}(\hat{n}) \, Y_{\ell m}(\hat{k})$

= after a few steps

$$= \frac{1}{25} \int \frac{k^2 dk}{(2\pi)^3} (4\pi)^2 P_R \sum_\ell \frac{2\ell+1}{4\pi} |j_\ell(k\lambda)|^2 P_\ell(\cos\theta)$$

$$= \sum_\ell \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta)$$

so that $C_\ell = \frac{1}{25} \int_0^\infty \frac{k^2 dk}{(2\pi)^3} P_R \, (4\pi)^2 \, j_\ell(k\lambda)^2$

$$P_R = \frac{2\pi^2}{k^3} A \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$= 4\pi \frac{A}{25} \int_0^\infty \frac{dk}{k} \left(\frac{k}{k_0}\right)^{n_s-1} j_\ell(k\lambda)^2$$

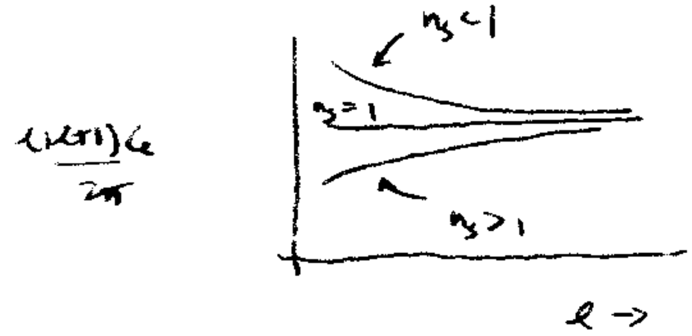
$$C_\ell = \frac{A}{25} \quad \frac{2\pi}{\ell(\ell+1)} \qquad \text{for } n_s = 1$$

$$\ell(\ell+1) C_\ell / 2\pi = \frac{A}{25}$$

or more generally

$$\ell(\ell+1) C_\ell / 2\pi = \frac{A}{25} \times \frac{\sqrt{\pi}}{2} \left(k_0 \lambda\right)^{1-n_s} \frac{\Gamma\left(\frac{3-n_s}{2}\right)\Gamma\left(\frac{2\ell-1+n_s}{2}\right)}{\Gamma\left(\frac{4-n_s}{2}\right)\Gamma\left(\frac{2\ell+5-n_s}{2}\right)}$$

$$\times\ \ell(\ell+1)$$



"Sachs-Wolfe plateau"

Q: Can one distinguish model universes based on such behavior?

A: Beware of COSMIC VARIANCE

$\langle C_\ell \rangle$ is predicted, with variance

$$\langle (C_\ell - \langle C_\ell \rangle)^2 \rangle = \frac{2}{2\ell+1} \langle C_\ell \rangle^2$$

← WIDTH DIMINISHES AT INCREASING ℓ.

CMB

PHOTONS $\qquad f_0 = \left[\exp\left(\frac{E}{T}\right) - 1\right]^{-1}$

INHOMOGENEOUS UNIVERSE

$$f(\bar{x}, \tau, \hat{n}, E) = f_0 + F(\bar{x}, \tau, \hat{n}, E)$$

evolve/solve with Boltzmann Eq'n

$$\frac{Df}{dt} = C[f, f_b]$$

$$\frac{\partial f}{\partial t} + \frac{dx^i}{dt}\frac{\partial f}{\partial x^i} + \frac{dE}{dt}\frac{\partial f}{\partial E} + \frac{dn^i}{dt}\frac{\partial f}{\partial n^i} = C[f, f_b]$$

lhs: perturbed RW metric

rhs: photons scatter off e, p

$$T(\tau, \bar{x}, \hat{n}) = T_0(\tau)\left(1 + \Theta(\tau, \bar{x}, \hat{n})\right)$$

$$F = -E\frac{\partial f_0}{\partial E}\Theta$$

Heroic calculation! see texts (eg. Dodelson)

SNAPSHOTS

$$\Theta' + \hat{n}\cdot\vec{\nabla}(\Theta+\Psi) - \Phi' = n_e\sigma_T a(\Theta_0 - \Theta + \hat{n}\cdot\vec{v}_b)$$

– ignore angular dependence of scattering
→ NO POLARIZATION

In Fourier space, where $\mu = \hat{k}\cdot\hat{n}$

$$\Theta' + ik\mu(\Theta+\Psi) - \Phi' = -\tau_\gamma'(\Theta_0 - \Theta + \mu v_b)$$

optical depth: $\tau_\gamma' = -n_e\sigma_T a$
monopole: $\Theta_0 = \frac{1}{2}\int_{-1}^{1} d\mu\, \Theta$

Integral solution along line of sight

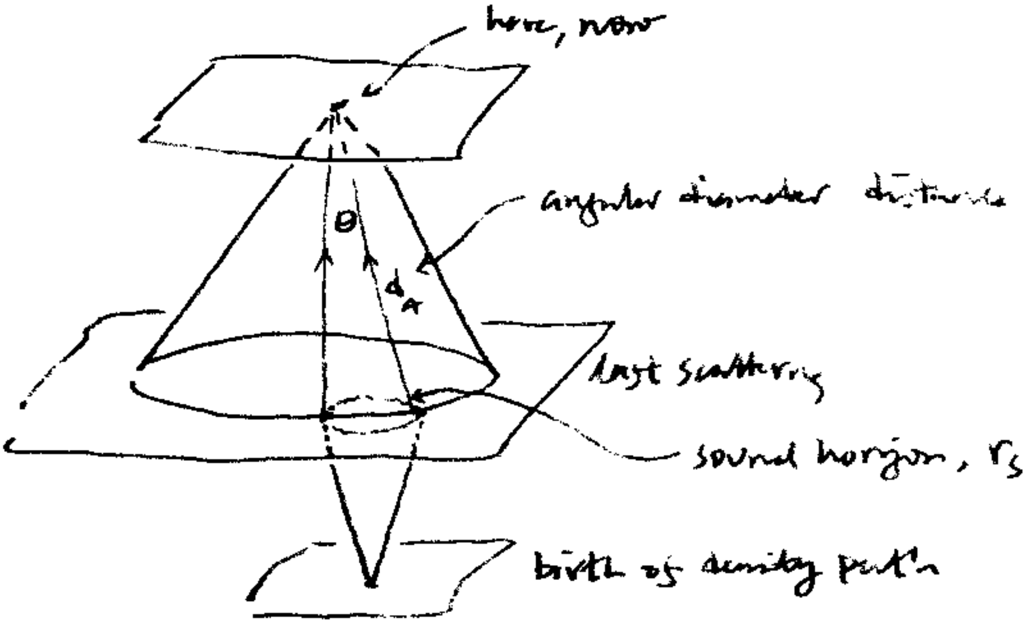
$$\Theta(\tau_0,\mu,k) = \int_0^{\tau_0} d\tau\, e^{ik\mu(\tau-\tau_0)}\, S(\tau,\mu,k)$$

$$S = e^{-\tau_\gamma}(\Phi'+\Psi') + g(\Theta_0 + \Psi + \mu v_b)$$

ISW SW Doppler

Invert FT to get $\frac{\Delta T}{T}(\hat{n}) = \Theta(\hat{n})$

CMB GEOMETRY



Size of CMB feature, on the sky $\theta = \dfrac{r_s}{d_A}$

COSMOLOGIES WITH SAME:

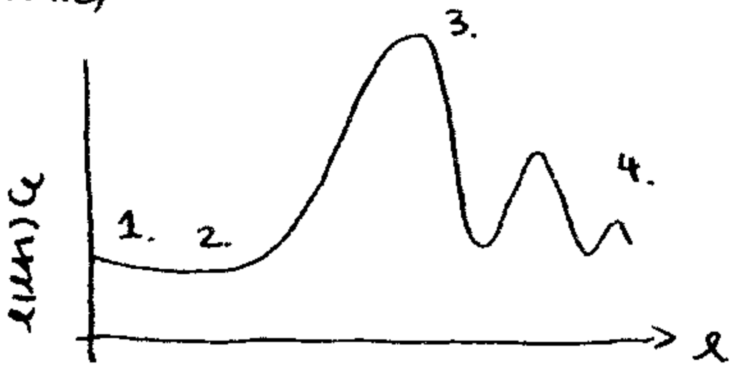
θ, $\Omega_b h^2$, $\Omega_m h^2$, r_s ...

HAVE (NEARLY) THE SAME CMB POWER SPECTRA OUT TO SMALL ANGULAR SCALES ($\ell \lesssim 1000$)

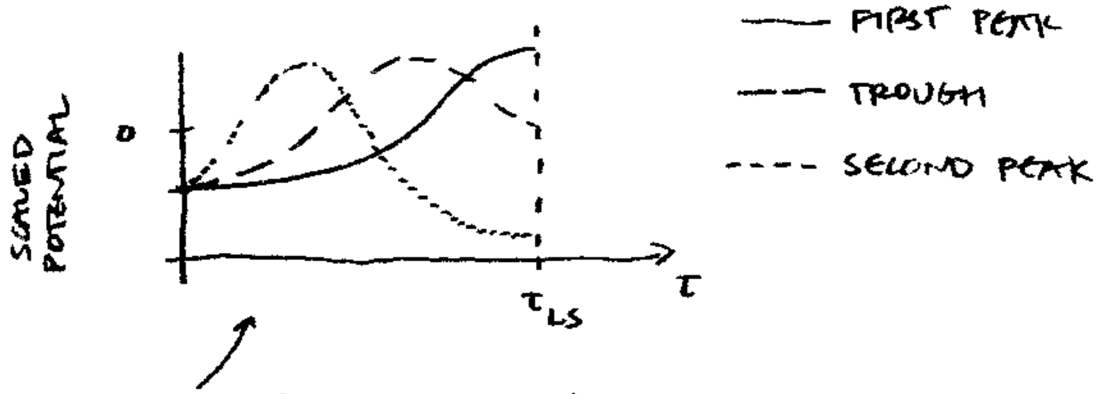
MORE CMB EFFECTS

SACHS-WOLFE, INTEGRATED SACHS-WOLFE,
PHOTON DENSITY PERTURBATIONS,
DOPPLER SHIFT,
POLARIZATION,
LENSING, ...

CMB Features



1. INTEGRATED SACHS-WOLFE
2. SACHS-WOLFE
3. ACOUSTIC PEAKS:



POTENTIALS "ENTER HORIZON" AND GROW UNTIL RAD. PRESSURE BECOMES IMPORTANT

4. DAMPING OF SMALL SCALE ANISOTROPIES:

γ NOT PERFECTLY COUPLED TO e, p

MEAN FREE PATH WASHES OUT SMALLER SCALE FEATURES $K > \frac{2\pi}{\lambda_{MFP}}$