Lecture 5

A Gravitational Wave Bestiary

Jolien Creighton

University of Wisconsin–Milwaukee

July 19, 2012

bes·ti·ar·y /ˈbɛstɪərɪ/

*noun (pl. **-ar·ies**)*

a descriptive or anecdotal treatise on various real or mythical kinds of animals, esp. a medieval work with a moralizing tone.

Sources of continuous gravitational waves

Sources of gravitational wave bursts

- Binary coalescence sources
- Gravitational collapse

Sources of a stochastic background of gravitational radiation

Sources of continuous gravitational waves

Sources of gravitational wave bursts

- Binary coalescence sources
- Gravitational collapse

Sources of a stochastic background of gravitational radiation

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:
  - Distorted neturon stars with ellipticity $\varepsilon > 0$

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:
  - Distorted neturon stars with ellipticity $\varepsilon > 0$
  - Wobbling neutron stars (free precession)

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:
  - Distorted neturon stars with ellipticity $\varepsilon > 0$
  - Wobbling neutron stars (free precession)
  - Unstable modes under gravitational radiation (Chandrasekhar-Friedman-Schutz instability), e.g. r-modes

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:
  - Distorted neturon stars with ellipticity $\varepsilon > 0$
  - Wobbling neutron stars (free precession)
  - Unstable modes under gravitational radiation (Chandrasekhar-Friedman-Schutz instability), e.g. r-modes
- Search categories:

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:
  - Distorted neturon stars with ellipticity $\varepsilon > 0$
  - Wobbling neutron stars (free precession)
  - Unstable modes under gravitational radiation (Chandrasekhar-Friedman-Schutz instability), e.g. r-modes
- Search categories:
  - Known pulsars (targeted searches): position and phase evolution known, e.g. Crab pulsar

Sources of continuous gravitational waves

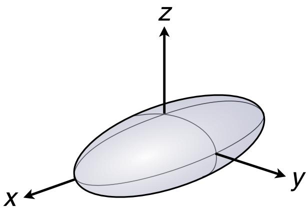
- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:
  - Distorted neturon stars with ellipticity $\varepsilon > 0$
  - Wobbling neutron stars (free precession)
  - Unstable modes under gravitational radiation (Chandrasekhar-Friedman-Schutz instability), e.g. r-modes
- Search categories:
  - Known pulsars (targeted searches): position and phase evolution known, e.g. Crab pulsar
  - Known/suspected neutron stars (directed searches): position known, e.g. Sco X1, Cas A

Sources of continuous gravitational waves

- At high frequencies, 1 Hz to 1000 Hz, expect main sources of continuous waves to be rapidly rotating neutron stars:
  - Isolated rapidly rotating neutron stars
  - Neutron stars in low-mass X-ray binaries (Sco X1?)
- Signal may be produced by:
  - Distorted neturon stars with ellipticity $\varepsilon > 0$
  - Wobbling neutron stars (free precession)
  - Unstable modes under gravitational radiation (Chandrasekhar-Friedman-Schutz instability), e.g. r-modes
- Search categories:
  - Known pulsars (targeted searches): position and phase evolution known, e.g. Crab pulsar
  - Known/suspected neutron stars (directed searches): position known, e.g. Sco X1, Cas A
  - Blind search for unknown isolated neutron stars

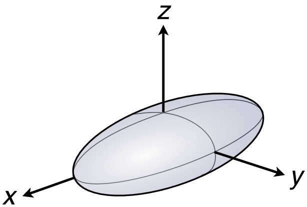
Sources of continuous gravitational waves

Targeted pulsar search example: Crab



Sources of continuous gravitational waves

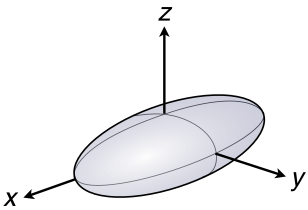
Targeted pulsar search example: Crab



Observed spin-down limits gravitational wave strain

Sources of continuous gravitational waves

Targeted pulsar search example: Crab

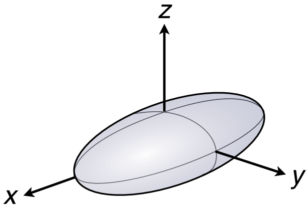


Observed spin-down limits gravitational wave strain

- Recall, for Crab pulsar: $h \lesssim 10^{-25}$

Sources of continuous gravitational waves

Targeted pulsar search example: Crab

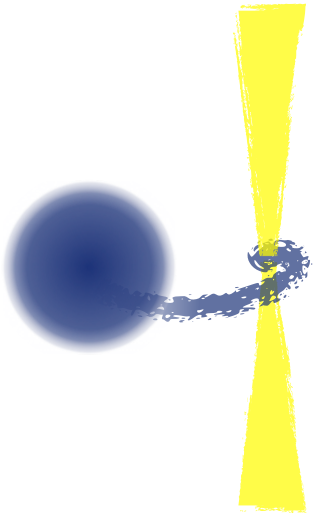


Observed spin-down limits gravitational wave strain

- Recall, for Crab pulsar: $h \lesssim 10^{-25}$
- Could be arbitrarily small!

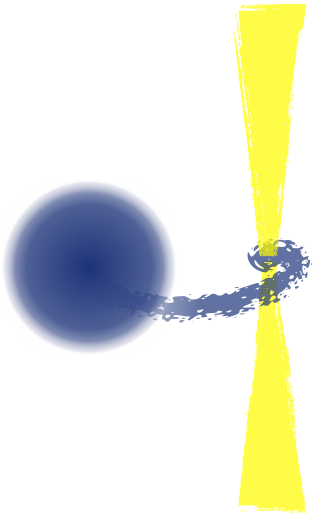
Sources of continuous gravitational waves

Directed search example: Sco X1



Sources of continuous gravitational waves

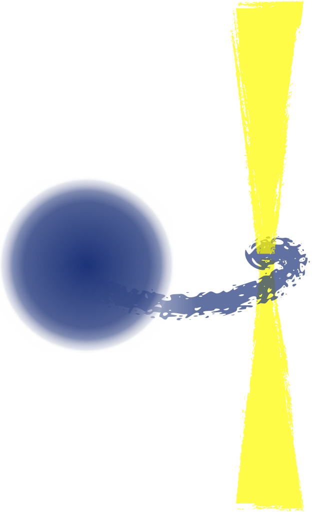
Directed search example: Sco X1



Assume that gravitational radiation balances torque from accretion

Sources of continuous gravitational waves

Directed search example: Sco X1

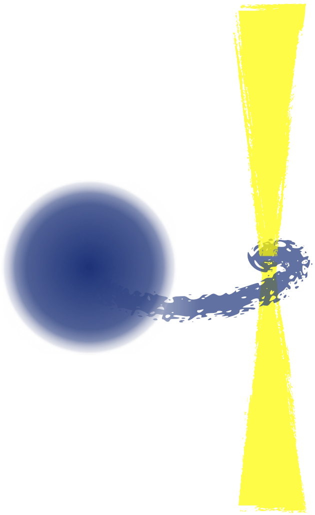


Assume that gravitational radiation balances torque from accretion

- Accretion rate: $\dot{M}$

Sources of continuous gravitational waves

Directed search example: Sco X1

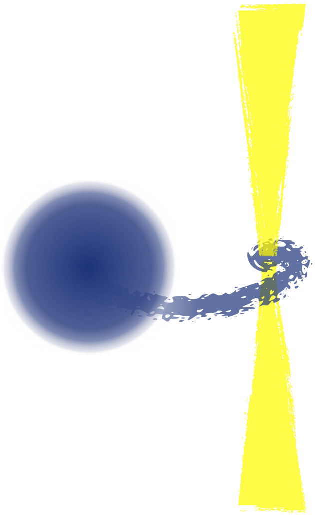


Assume that gravitational radiation balances torque from accretion

- Accretion rate: $\dot{M}$
- Torque: $\dot{M}\sqrt{GMR}$

Sources of continuous gravitational waves

Directed search example: Sco X1

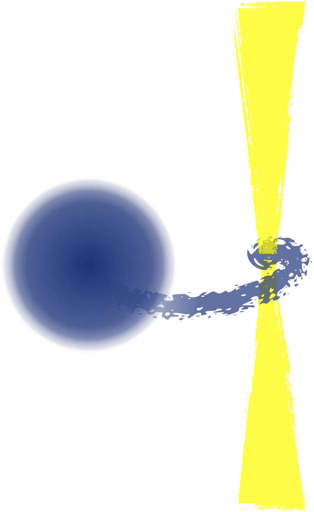


Assume that gravitational radiation balances torque from accretion

- Accretion rate: $\dot{M}$
- Torque: $\dot{M}\sqrt{GMR}$
- X-ray luminosity: $L_X \approx GM\dot{M}/R$

Sources of continuous gravitational waves

Directed search example: Sco X1



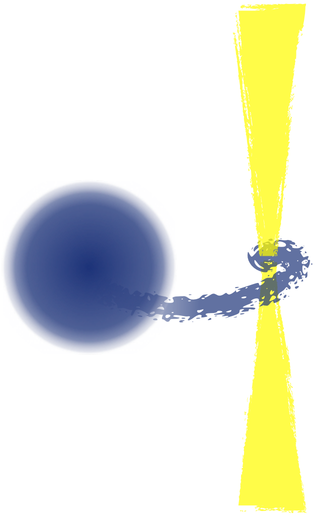
Assume that gravitational radiation balances torque from accretion

- Accretion rate: $\dot{M}$
- Torque: $\dot{M}\sqrt{GMR}$
- X-ray luminosity: $L_X \approx GM\dot{M}/R$
- Observed X-ray flux: $F_X = L_X/(4\pi r^2) = 2 \times 10^{-10}\,\mathrm{W\,m^{-2}}$

$$h \sim \left(\frac{GP}{c^3}\right)^{1/2} \left(\frac{R^3}{GM}\right)^{1/4} F_X^{1/2}$$

Sources of continuous gravitational waves

Directed search example: Sco X1



Assume that gravitational radiation balances torque from accretion

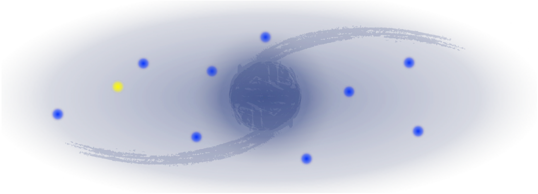
- Accretion rate: $\dot{M}$
- Torque: $\dot{M}\sqrt{GMR}$
- X-ray luminosity: $L_X \approx GM\dot{M}/R$
- Observed X-ray flux: $F_X = L_X/(4\pi r^2) = 2 \times 10^{-10}\,\mathrm{W\,m^{-2}}$

$$h \sim \left(\frac{GP}{c^3}\right)^{1/2} \left(\frac{R^3}{GM}\right)^{1/4} F_X^{1/2}$$
$$\sim 3 \times 10^{-26}$$

for $P = 4\,\mathrm{ms}$, $M = 1.4\,M_\odot$, $R = 10\,\mathrm{km}$.

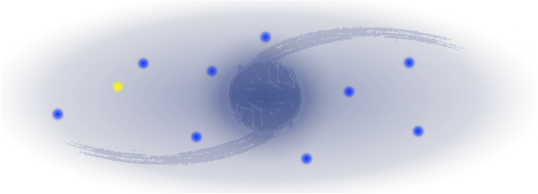
Sources of continuous gravitational waves

Blandford's argument for unknown "gravitars"



Sources of continuous gravitational waves

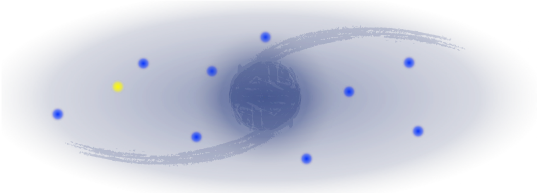
Blandford's argument for unknown "gravitars"



- Assume a disk population of gravitars

Sources of continuous gravitational waves

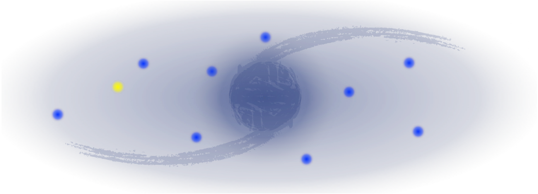
Blandford's argument for unknown "gravitars"



- Assume a disk population of gravitars
- Galactic disk birth rate per unit area: $\mathcal{R}$

Sources of continuous gravitational waves

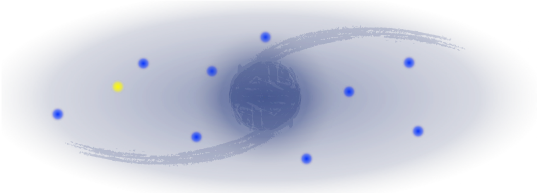
Blandford's argument for unknown "gravitars"



- Assume a disk population of gravitars
- Galactic disk birth rate per unit area: $\mathcal{R}$
- Spindown time-scale: $\tau = P/\dot{P}$ where $\dot{P} \sim G I_3 P^{-3} \varepsilon^2 / c^5$

Sources of continuous gravitational waves

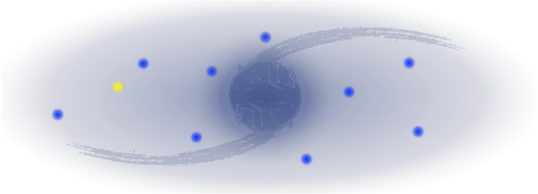
Blandford's argument for unknown "gravitars"



- Assume a disk population of gravitars
- Galactic disk birth rate per unit area: $\mathcal{R}$
- Spindown time-scale: $\tau = P/\dot{P}$ where $\dot{P} \sim G I_3 P^{-3} \varepsilon^2 / c^5$
- Nearest gravitar: $r_{\text{nearest}} \sim (\mathcal{R}\tau)^{-1/2}$

Sources of continuous gravitational waves

Blandford's argument for unknown "gravitars"



- Assume a disk population of gravitars
- Galactic disk birth rate per unit area: $\mathcal{R}$
- Spindown time-scale: $\tau = P/\dot{P}$ where $\dot{P} \sim G I_3 P^{-3} \varepsilon^2 / c^5$
- Nearest gravitar: $r_{\text{nearest}} \sim (\mathcal{R}\tau)^{-1/2}$
- Strongest source: $h_{\text{largest}} \sim G I_3 P^{-2} \varepsilon / (c^4 r_{\text{nearest}}) \sim \sqrt{G I_3 \mathcal{R} / c^3}$
  Depends only on birth rate!

$h \sim 3 \times 10^{-24}$ for $I_3 = 10^{38}\,\text{m}^2\,\text{kg}$, $\mathcal{R} = (30\,\text{yr})^{-1} / [4\pi (10\,\text{kpc})^2]$

Sources of continuous gravitational waves

Sources of gravitational wave bursts

- Binary coalescence sources
- Gravitational collapse

Sources of a stochastic background of gravitational radiation

Binary coalescence sources

- At high frequencies, 1 Hz to 1000 Hz, expect main binary coalescence sources to be:

Binary coalescence sources

- At high frequencies, 1 Hz to 1000 Hz, expect main binary coalescence sources to be:
  - Coalescences of binary neturon star systems

Binary coalescence sources

- At high frequencies, 1 Hz to 1000 Hz, expect main binary coalescence sources to be:
  - Coalescences of binary neturon star systems
  - Coalescences of neutron-star + black-hole binaries

Binary coalescence sources

- At high frequencies, 1 Hz to 1000 Hz, expect main binary coalescence sources to be:
  - Coalescences of binary neturon star systems
  - Coalescences of neutron-star + black-hole binaries
  - Coalescences of binary black hole systems with $M \lesssim 100\,M_\odot$
- Anticipated rates:

System	Rate density ($\mathrm{Myr}^{-1}\,\mathrm{Mpc}^{-3}$)
NSNS	0.1 to 10
NSBH	6×10^{-4} to 1
BHBH	1×10^{-4} to 0.3

Abadie et al. (2010) *Class. Quantum Grav.* **27** 173001

Binary coalescence sources

- Advanced LIGO will have a typical range of $\sim 100\,\mathrm{Mpc}$ for NSNS systems

Binary coalescence sources

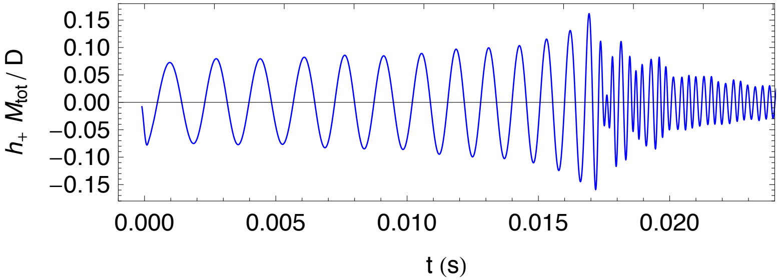
- Advanced LIGO will have a typical range of $\sim 100\,\mathrm{Mpc}$ for NSNS systems
- Expect ~ 40 events per year
  (But more than an order of magnitude uncertainty!)

Binary coalescence sources

- Advanced LIGO will have a typical range of $\sim 100\,\mathrm{Mpc}$ for NSNS systems
- Expect ~ 40 events per year
  (But more than an order of magnitude uncertainty!)
- NSBH and BHBH detection rates are even more uncertain, but could be as high

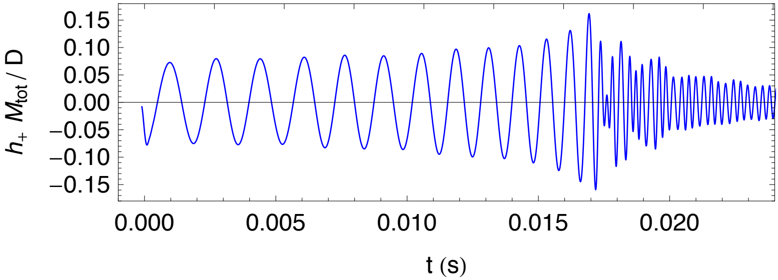
Binary coalescence sources

Binary neutron star inspral + merger + post-merger



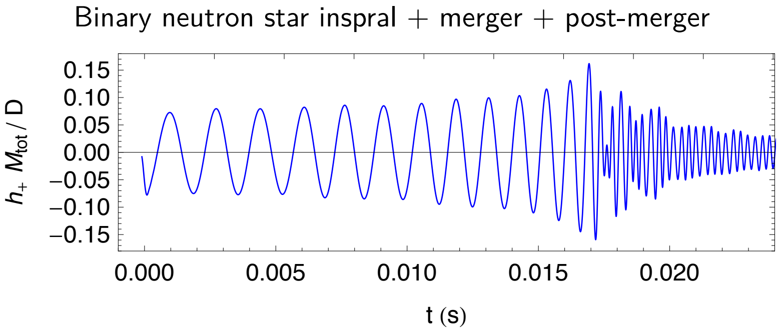
Binary coalescence sources

Binary neutron star inspral + merger + post-merger



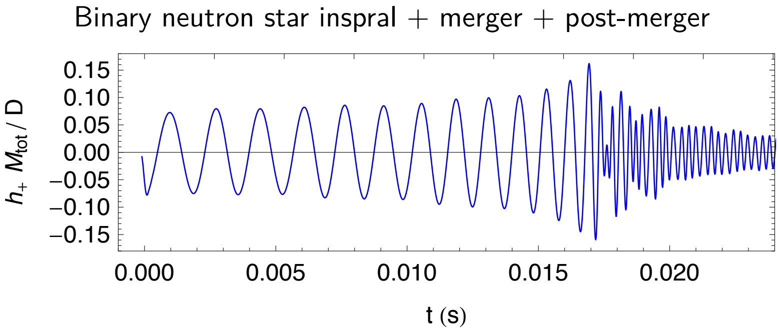
- Inspiral: described by post-Newtonian theory

Binary coalescence sources



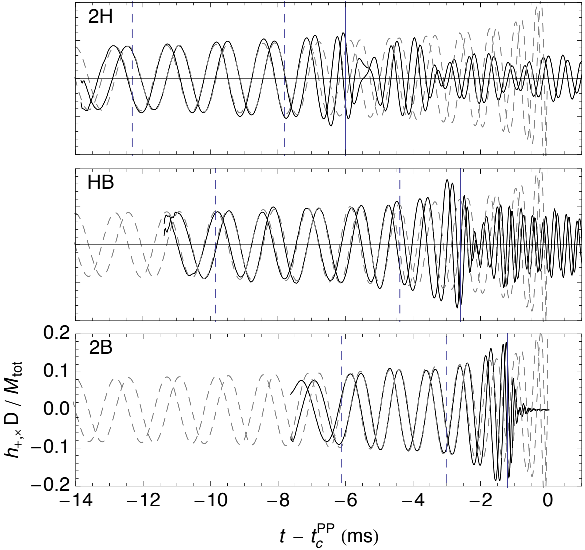
- Inspiral: described by post-Newtonian theory
- Late inspiral / merger / post-merger oscillations: computed using Numerical Relativity

Binary coalescence sources



- Inspiral: described by post-Newtonian theory
- Late inspiral / merger / post-merger oscillations: computed using Numerical Relativity
- Tidal effects: size of the star encoded in waveform

Binary coalescence sources



Stiff

Medium

Soft

Binary coalescence sources

Science goals:

Binary coalescence sources

Science goals:

- Determine population of NSNS, NSBH, BHBH systems

Binary coalescence sources

Science goals:

- Determine population of NSNS, NSBH, BHBH systems
  - Informs models of stellar evolution

Binary coalescence sources

Science goals:

- Determine population of NSNS, NSBH, BHBH systems
  - Informs models of stellar evolution
  - How do supermassive black holes form?

Binary coalescence sources

Science goals:

- Determine population of NSNS, NSBH, BHBH systems
  - Informs models of stellar evolution
  - How do supermassive black holes form?
- Determine progenitors of short gamma-ray bursts

Binary coalescence sources

Science goals:

- Determine population of NSNS, NSBH, BHBH systems
  - Informs models of stellar evolution
  - How do supermassive black holes form?
- Determine progenitors of short gamma-ray bursts
- Measure neutron star equation of state

Binary coalescence sources

Science goals:

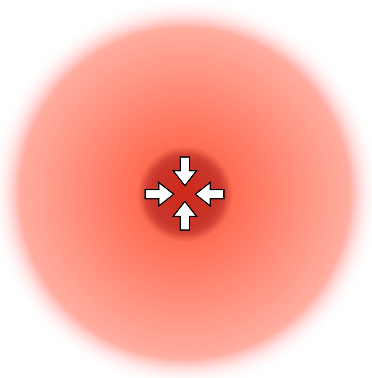
- Determine population of NSNS, NSBH, BHBH systems
  - Informs models of stellar evolution
  - How do supermassive black holes form?
- Determine progenitors of short gamma-ray bursts
- Measure neutron star equation of state
  - Independent Distance-Redshift relationship

Binary coalescence sources

Science goals:

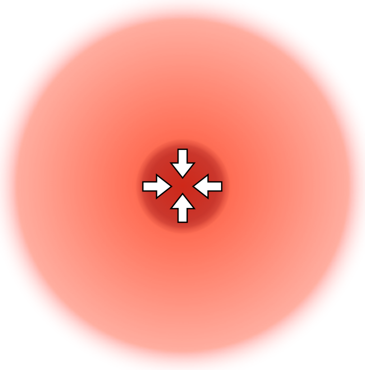
- Determine population of NSNS, NSBH, BHBH systems
  - Informs models of stellar evolution
  - How do supermassive black holes form?
- Determine progenitors of short gamma-ray bursts
- Measure neutron star equation of state
  - Independent Distance-Redshift relationship
- Tests of General Relativity

Core collapse supernovae



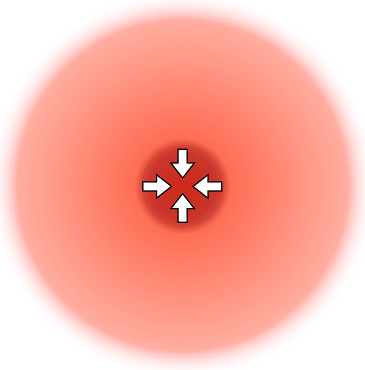
- At end of life, massive stars have cores of Fe “ash” supported by electron degeneracy

Core collapse supernovae



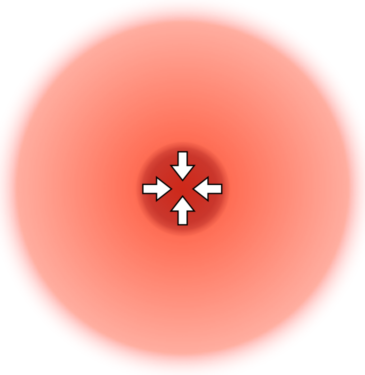
- At end of life, massive stars have cores of Fe “ash” supported by electron degeneracy
- Collpse when $M_{\text{core}} > 1.4\,M_\odot$

Core collapse supernovae



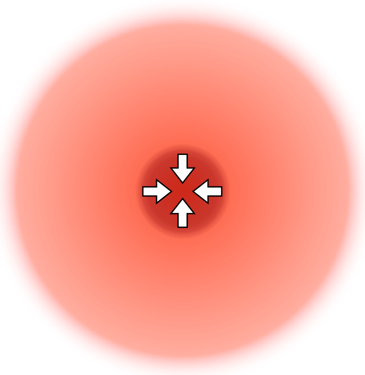
- At end of life, massive stars have cores of Fe "ash" supported by electron degeneracy
- Collpse when $M_{\text{core}} > 1.4\, M_{\odot}$
- Core becomes NS

Core collapse supernovae



- At end of life, massive stars have cores of Fe "ash" supported by electron degeneracy
- Collpse when $M_{\text{core}} > 1.4\, M_{\odot}$
- Core becomes NS
- Rebound shock powered by neutrinos from photo-dissociation of nuclei

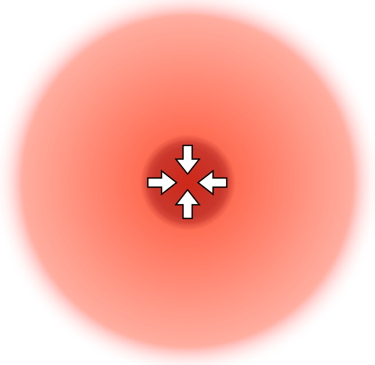
Core collapse supernovae



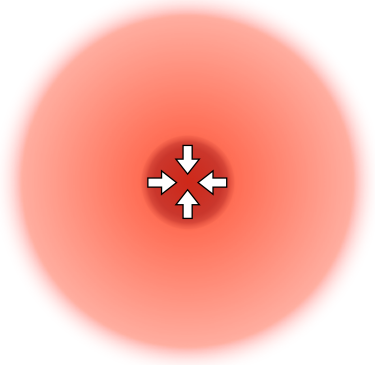
- At end of life, massive stars have cores of Fe “ash” supported by electron degeneracy
- Collpse when $M_{\text{core}} > 1.4\,M_\odot$
- Core becomes NS
- Rebound shock powered by neutrinos from photo-dissociation of nuclei
- Remnant is a NS or BH

Core collapse supernovae

At end of collapse



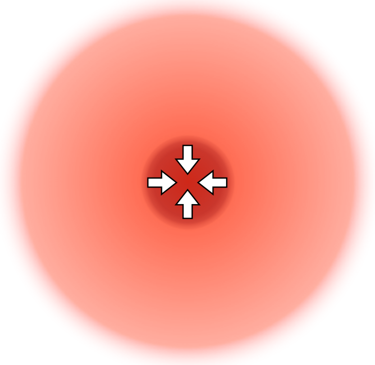
Core collapse supernovae



At end of collapse

- Dynamical timescale is
  $$t_{ff} = \sqrt{\frac{3\pi}{32}\frac{1}{G\rho}}$$

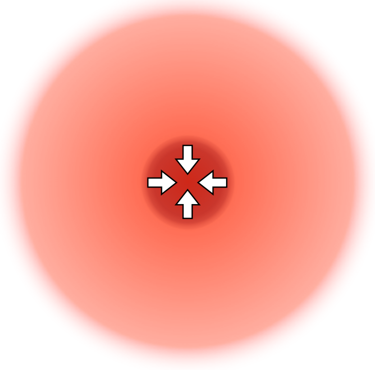
Core collapse supernovae



At end of collapse

- Dynamical timescale is
  $$t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32}\frac{1}{G\rho}} \sim 0.1\,\mathrm{ms}$$
  for $\rho \sim 10^{18}\,\mathrm{kg\,m^{3}}$

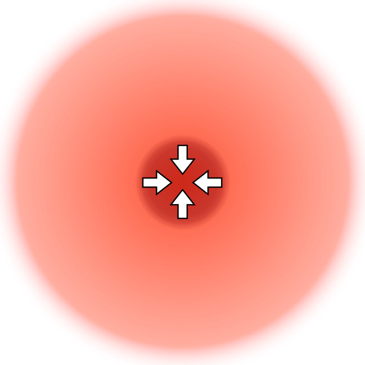
Core collapse supernovae



At end of collapse

- Dynamical timescale is
$$t_{\rm ff} = \sqrt{\frac{3\pi}{32}\frac{1}{G\rho}} \sim 0.1\,\mathrm{ms}$$
for $\rho \sim 10^{18}\,\mathrm{kg\,m^3}$
- Axisymmetric collapse has $h \sim G(\ddot{I}_{22} - \ddot{I}_{33})/c^4 r$ where $I_{11} = I_{22} = (1 - \frac{1}{2}e^2)I_{33}$

Core collapse supernovae

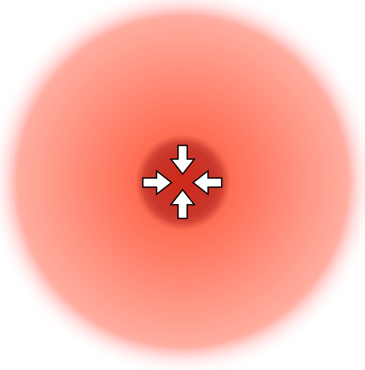


At end of collapse

- Dynamical timescale is
  $$t_{\rm ff} = \sqrt{\frac{3\pi}{32}\frac{1}{G\rho}} \sim 0.1\,{\rm ms}$$
  for $\rho \sim 10^{18}\,{\rm kg\,m^3}$
- Axisymmetric collapse has $h \sim G(\ddot{I}_{22} - \ddot{I}_{33})/c^4 r$ where $I_{11} = I_{22} = (1 - \frac{1}{2}e^2)I_{33}$
- During collapse, $e \propto R^{-1/2}$

$$h \sim \frac{GM}{c^2 r}\left(\frac{eR}{ct_{\rm ff}}\right)^2$$

Core collapse supernovae



At end of collapse

- Dynamical timescale is
  $$t_{\rm ff} = \sqrt{\frac{3\pi}{32}\frac{1}{G\rho}} \sim 0.1\,{\rm ms}$$
  for $\rho \sim 10^{18}\,{\rm kg\,m^3}$
- Axisymmetric collapse has $h \sim G(\ddot{I}_{22} - \ddot{I}_{33})/c^4 r$ where $I_{11} = I_{22} = (1 - \frac{1}{2}e^2)I_{33}$
- During collapse, $e \propto R^{-1/2}$

$$h \sim \frac{GM}{c^2 r}\left(\frac{eR}{ct_{\rm ff}}\right)^2 \sim 10^{-20}$$

for $M \sim 1\,M_\odot$, $R \sim 15\,{\rm km}$, $e \sim 0.1$, $r \sim 10\,{\rm kpc}$

Sources of continuous gravitational waves

Sources of gravitational wave bursts

- Binary coalescence sources
- Gravitational collapse

Sources of a stochastic background of gravitational radiation

Sources of a stochastic background of gravitational radiation

See Robert Caldwell's Lectures