Problem Set 3

Problem 1: Newtonian tides

In Newtonian gravity, show that the tidal acceleration between two freely-falling bodies separated by the displacement vector $\boldsymbol{\zeta}$ is given by

$$\frac{d^2\zeta_i}{dt^2} = -\mathcal{E}_{ij}\zeta^j$$

where

$$\mathcal{E}_{ij} = \frac{\partial\Phi}{\partial x^i \partial x^j}$$

is the tidal tensor field.

Compute the tidal tensor field in Cartesian coordinates at some position $\mathbf{r} = (0, 0, z)$ above the North pole of the Earth.

Problem 2: The TT Gauge

Let $\bar{h}_{\alpha\beta} = \bar{h}_{\alpha\beta}(t - z/c)$ be a plane wave solution in the Lorenz gauge in which

$$\frac{\partial}{\partial x^\mu}\bar{h}^{\mu\alpha} = 0.$$

Let $\xi^\alpha = \xi^\alpha(t - z/c)$ be a vector field that genrates a gauge transformation that preserves the Lorenz gauge by virtue of the fact that

$$\Box\xi^\alpha = 0.$$

Find the form of ξ^α that brings us into the TT gauge via

$$\bar{h}'_{\alpha\beta} = h_{\alpha\beta} - \frac{\partial\xi_\alpha}{\partial x^\beta} - \frac{\partial\xi_\beta}{\partial x^\alpha} + \eta_{\alpha\beta}\eta^{\mu\nu}\frac{\partial\xi_\mu}{\partial x^\nu}$$

in which $\bar{h}'_{\alpha 0} = 0$ (spatial), $\bar{h}'_{\alpha 3} = 0$ (transverse), and $\eta^{\mu\nu}\bar{h}'_{\mu\nu} = 0$ (trace-free).