

Towards brane-antibrane inflation in type IIA string theory: the holographic MQCD model

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Summary:

- 1. Introduction
- 2. Problems of embedding inflation in string theory
- 3. KKLT/KKLMMT scenario
- 4. Holographic MQCD
- 5. Two inflation set-ups from holographic MQCD
- 6. The potential and consequences for cosmology
- 7. Conclusions

1. Introduction

- Inflation \rightarrow need a scalar field with a flat potential \rightarrow slow roll ($\epsilon = M_P^2(V'/V)^2 \ll 1$ and $\eta = M_P^2 V''/V \ll 1$)
- But quantum corrections can spoil it. Also, need a fundamental theory near M_{Pl} to **derive** $V(\phi)$.
- Natural to look for string theory potentials. But, \nexists fully consistent embedding of inflation in string theory yet.
- String theory has *many* light scalars (moduli) \rightarrow need to stabilize most. Stabilization and/or quantum corrections generically spoil slow roll.
- One standard model \rightarrow KKLT/KKLMMT in IIB string theory.

- But it has same problems generically.
- Also, IIB easier to build model. How about the T-dual type IIA? No good model yet.
- Here: model based on a T-dual analog of KKLMNT \rightarrow a first step towards IIA inflation.
- Based on a holographic MQCD gauge theory.
- We find the shape of the scalar potential and argue that it can fit experimental constraints.

2. Problems of embedding inflation in string theory

- String theory: quantum theory of gravity, living in 10d (IIB) or 11d (IIA \rightarrow M theory at strong coupling)
- Need to do something with extra dimensions \rightarrow Kaluza-Klein reduction: small compact space, or or transverse \rightarrow a brane (matter restricted to a 4d model)
- \exists many moduli (light scalars) \rightarrow parametrize the "size" of the space (including the overall volume) \rightarrow Kähler structure moduli and the "shape" of the space (like the ratio of radii of a torus) \rightarrow complex structure moduli.
- As well: related fields (e.g. B_{ij} components)
- In brane scenario, brane positions in transverse space are also light scalars.

- Need to stabilize (most of) the moduli, at least now → we don't see light scalars. But also, for simplest inflation → we want just one scalar to roll → inflaton.
- Stabilization mechanisms → quantum (often nonperturbative) potentials → hard to calculate.
- Easiest: fluxes of antisymmetric tensor fields in extra dimensions give stabilizing superpotentials.
- Example of flux: 11d supergravity "spontaneous compactification" to 4d on $AdS_4 \times S^7$ (Freund-Rubin): $F_{\mu\nu\rho\sigma} \propto \epsilon_{\mu\nu\rho\sigma}$ ($\mu, \nu, \rho, \sigma = 1, \dots, 4$ and F_{MNPQ} =4-form fields strength in 11d)
- GVW superpotential for G-flux: $G = F^{RR} - \tau H^{NS}$

$$W = \int_{K_6} \Omega \wedge G$$

where Ω =holomorphic 3-form on G .

- Generic potential in string theory has $V''/V \sim 1/M_{Pl}^2 \Rightarrow \eta \sim 1 \rightarrow$ "the eta problem in string theory" \rightarrow naturally, quantum corrections for scalar mass \Rightarrow run to cut-off scale (M_P): $\Delta\eta \sim \Delta m^2/3H^2 \geq 1$
- One can choose non-generic points in moduli space \rightarrow but then how to justify initial conditions.
- We still need *full quantum potential* to be sure.

- Stabilizing potentials by flux \rightarrow generically the last field to be stabilized: volume \rightarrow introduce steep regions in potential \Rightarrow spoils slow-roll.
- Recent progress \rightarrow towards computing the full quantum potential.
- Popular example: brane, or brane-antibrane potential \rightarrow brane motion in extra dimensions \rightarrow motion in 4d scalar field space \rightarrow inflation.
- Another big problem: de Sitter (dS) background is very difficult in string theory \rightarrow susy requires AdS \Rightarrow need some susy breaking, but still to have stability.

3. KKLT/KKLMMT scenario

- → based on brane-antibrane inflation.

- In flat space $D - \bar{D}$ potential is

$$V(y) = A - \frac{B}{y^{d_{\perp}-2}}$$

where $A = 2T_p V_{\parallel}$ → free branes, and interaction potential → attractive.

- Canonically normalize ⇒

$$\eta \sim -\beta(d_{\perp} - 1)(d_{\perp} - 2) \left(\frac{r_{\perp}}{V}\right)^{d_{\perp}}$$

- On a torus however, \exists images of branes ⇒ \exists metastable positions ($D - \bar{D} - D - \bar{D} - \dots$) ⇒

$$V(z) = A - Cz^4$$

where z = displacement from metastable positions ⇒ inflation only if we start near $z = 0$.

- KKLT: mechanism for de Sitter vacua in string theory.
- Warped throat geometry of Klebanov-Strassler (KS) type → cigar type

$$\begin{aligned}
 ds^2 &= e^{2A} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n \\
 g_{mn} dy^m dy^n &= dr^2 + r^2 ds_{X_5}^2
 \end{aligned}$$

($X_5 = T^{1,1}$) glued to a CY_3 to cap off the throat → compactification.

- \exists G-gflux \Rightarrow tree level superpotential $W = W_0$, and nonperturbative superpotential $W = Ae^{ia\rho}$, ρ =volume modulus (Euclidean D3-brane instanton or gluino condensation on D7-branes).
- Tree-level Kähler potential

$$K = -3 \ln[-i(\rho - \bar{\rho})]$$

\Rightarrow supersymmetric potential \Rightarrow AdS vacuum.

- But we can introduce an anti-D3-brane at tip of cigar (\exists exponentially small warping there) $\Rightarrow \delta V = D/(\text{Im}\rho)^3$ nonsusy potential \Rightarrow dS vacuum.

- KKLMNT: proposal for brane-antibrane inflation, where $\bar{D}3$ is the above in KKLT (fixed) and \exists moving D3 on warped throat geometry.

- KS \sim log-corrected AdS. For AdS: consider D3-brane as another D3-source, modifying the harmonic function $h(r) = e^{-4A}$:

$$\frac{R^4}{r^4} \rightarrow \frac{R^4}{r^4} + \frac{R^4/N}{(r-r_1)^4} \simeq R^4 \left(\frac{1}{r^4} + \frac{1}{N r_1^4} \right)$$

- Calculate DBI action of $\bar{D}3$ -brane in this background \Rightarrow potential

$$V(\phi) = \frac{4\pi^2 \phi_0^4}{N} \left(1 - \frac{1}{N} \frac{\phi_0^4}{\phi^4} \right)$$

- flat potential \rightarrow though volume stabilization generically spoils it. And could be quantum corrected.

4. Holographic MQCD

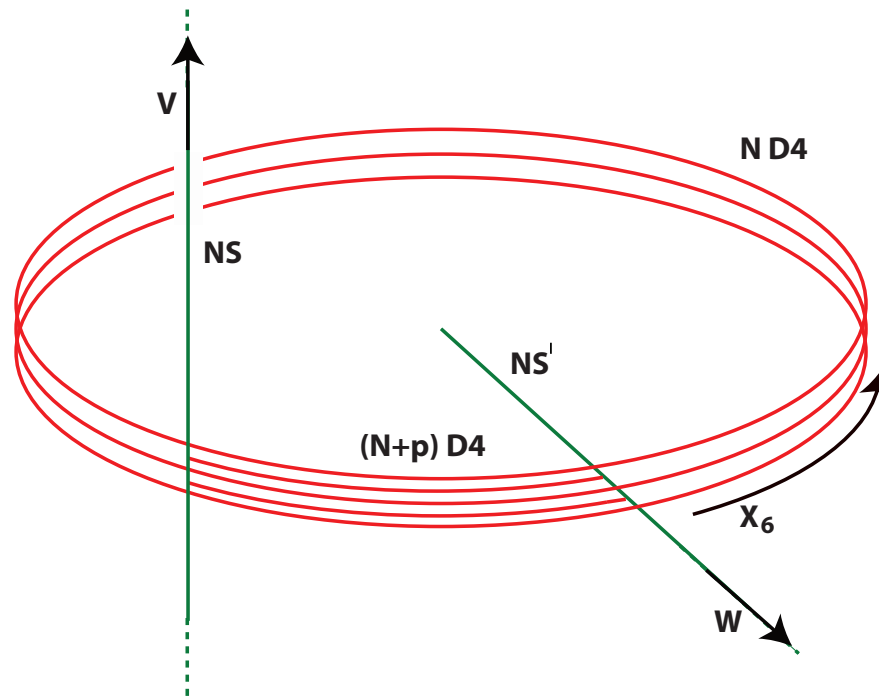
- MQCD \rightarrow generalization of 4d $\mathcal{N} = 1$ SYM theory in M theory with one extra parameter (g_s , or radius of 11th dimension)
- Constructed by Witten (1997) as: 2 NS5-branes, \parallel in 3+1 dimensions (x^0, \dots, x^3) and extended in 2 new different directions: x^4, x^5 with $v = x^4 + ix^5$, vs. x^7, x^8 , with $w = x^7 + ix^8$, and separated in x^6 by a length L .
- Linked by p D4-branes of length L .
- Has a solution in M theory: \exists single M5-brane $\mathbb{R}^4 \times \Sigma$, with $w = \zeta v^{-1}$, $v^n = t$, $w^n = \zeta n t^{-1}$. ($t = e^{-z}$, $z = R^{-1}x^6 + ix^{10}$)

- Holographic MQCD \rightarrow gravitational description of MQCD, where we add N D4-branes \parallel the p D4-brane plane, with $N \geq p$ - probe approximation \rightarrow replace the N D4-branes by their gravity background

$$ds^2 = H^{-1/3}(dx_\mu^2 + dx_6^2 + dx_{11}^2) + H^{2/3}(|dv|^2 + |dw|^2 + dx_7^2)$$

$$C_6 = H^{-1}d^4x \wedge dx_6 \wedge dx_{11}, \quad H = 1 + \frac{\pi \lambda_N l_s^2}{|\vec{r} - \vec{r}_0|^3}$$

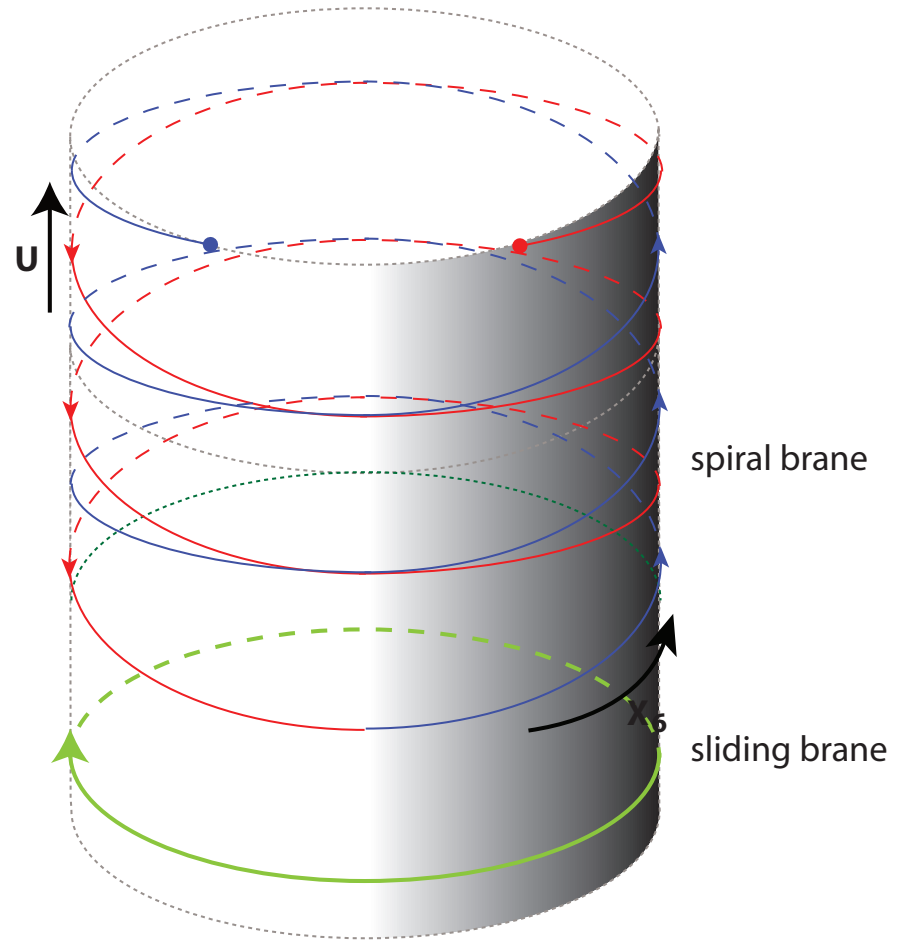
- NS5-branes and p D4-branes form a single curved M5-brane in the background.
- Compact $x_6 \Rightarrow$ M5-brane on cylinder (x_6, u) spirals down from $u = \infty$ to u_0 and then back up \rightarrow carries p units of D4-brane charge.

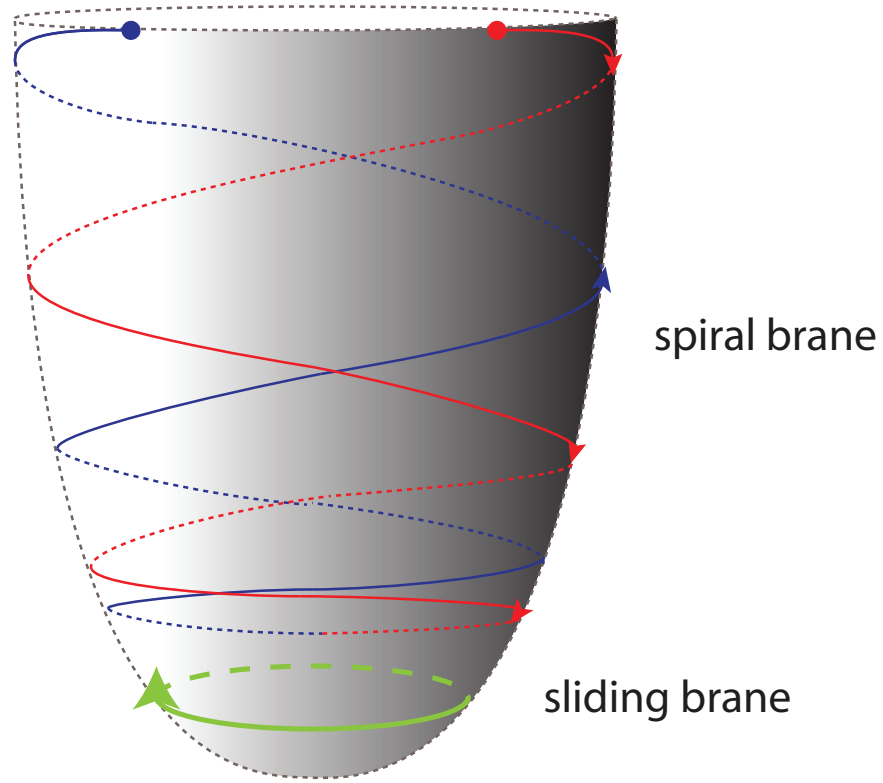


- Is T-dual to KKLT susy set-up.

- Cigar topology: non-supersymmetric model \rightarrow double Wick rotated near-extremal D4-brane \Rightarrow cigar instead of cylinder

$$ds^2 = H^{-1/3}[-dt^2 + dx_i^2 + f(r)dx_6^2 + dx_{11}^2] + H^{2/3}[(f^{-1}(r)-1)dr^2 + |dv|^2 + |dw|^2 + dx_7^2]$$





5. Two set-ups for holographic MQCD

Model 1

- Consider the nonsusy deformation of holo MQCD set-up: in T-dual KKLT, \exists anti-D3 at tip of KS \rightarrow take Wick-rotated near-extremal D4-brane \rightarrow make $\bar{D}3$ perturbation of background.
- Then: semi-infinite cigar in r over S^4 , for $r_{min} = r_H$, and cigar in r over x_6 . Cut-off cigar at r_{max} , gluing another space \rightarrow compactify.
- (x_6, r) is T-dual to $(r, T^{1,1})$ in KKLT, glued onto $CY_3 \Rightarrow (x_6, r, S^4)$ glued onto CY_3 space W , T-dual to M . (susy breaking $\Rightarrow \exists$ small additional energy at gluing region)
- Add also sliding D4-brane on cigar \rightarrow like KKLMMT.

- Action for D4-brane in double-Wick rotated near-extremal D4-brane \Rightarrow

$$S = -\frac{T_p}{g_s} \int H_p^{-1}(r) \left[\sqrt{f_p + \frac{H_p(r)}{f_p} g^{\mu\nu} d_\mu r d_\nu r} - 1 \right]$$

- Then, the potential is

$$V_4(r) = +\frac{T_4 R}{g_s} H^{-1}(r) [\sqrt{f(r)} - 1] = +\frac{T_4 R}{g_s} \frac{1}{1 + \alpha_4 \left(\frac{r_4}{r}\right)^3} \left[\sqrt{1 - \frac{r_H^3}{r^3}} - 1 \right] < 0$$

- Going from $V(\infty) = 0$ to $V(r_H) \simeq -T_4 R (r_H/r_4)^3 / g_s$.
- \exists a plateau for $r_H \ll r \ll r_4$,

$$V(r) \simeq -\frac{T_4 R}{2g_s \alpha_4} \left(\frac{r_H}{r_4}\right)^3 \left[1 + \frac{1}{4} \frac{r_H^3}{r^3} - \frac{r^3}{\alpha_4 r_4^3} \right]$$

- The spiralling brane has a negligible effect on the potential (calculated)

Model 2

- consider a sliding anti-brane in a susy (cylinder) background \Rightarrow cylinder in (x_6, r) and semi-infinite in (r, S^4) .
- Space terminates at $r = 0$ however \Rightarrow by adding a CY_3 , the space is again truly compactified.
- Calculation of potential is similar: consider the action of an anti-brane in a susy background.
- Find

$$V_4(r) = \frac{2T_4 R}{g_s} \frac{1}{1 + \frac{r_4^3}{r^3}}$$

- Potential now varies between $V(0) = 0$ and

$$V(\infty) = \frac{2\pi R}{g_s}$$

with an infinite derivative at 0:

$$V'_4(r) = \frac{6T_4 R}{g_s r} \frac{r_4^3/r^3}{1 + r_4^3/r^3} \rightarrow \infty (r \rightarrow 0)$$

- Neglect again contribution of spiral, though now form of correction is less rigorous \rightarrow again only interaction between brane and antibrane via modified background, and no backreaction.

6. The potential and consequences for cosmology

- **Model 1:** The potential on the plateau is

$$V(\phi) \simeq \frac{T_4 R}{g_s} \left(\frac{\phi_H}{\phi_4} \right)^3 \left[N - \frac{1}{2\alpha_4} \left(1 + \frac{\phi_H^3}{4\phi^3} - \frac{\phi^3}{\alpha_4 \phi_4^3} \right) \right]$$

and gives for the slow-roll parameters and the number of e-folds

$$\begin{aligned} \epsilon &\equiv \frac{1}{2} \left(M_P \frac{V'}{V} \right)^2 \simeq \frac{1}{2} \left[\frac{3M_P}{2N\phi} \left(\frac{1}{4} \frac{\phi_H^3}{\phi^3} + \frac{\phi^3}{\phi_4^3} \right) \right]^2 \\ \eta &\equiv M_P^2 \frac{V''}{V} = \frac{3M_P^2}{2N\phi^2} \left(-\frac{\phi_H^3}{\phi^3} + 2\frac{\phi^3}{\phi_4^3} \right) \\ \mathcal{N} &= \frac{8N}{15} \frac{\phi_{in}^5 - \phi_{end}^5}{M_P^2 \phi_H^3} \end{aligned}$$

- Note that ϕ_4/M_P is generically large, so we are generically on the plateau $\phi_H \ll \phi \ll \phi_4$ (for generic ϕ , of order M_P), for a red spectrum we need $\eta < 0$, or $\phi < \sqrt{2^{-1/3} \phi_H \phi_4}$ (since $\epsilon \ll |\eta|$), and we can easily get 60 e-folds.

- Also, COBE normalization puts a constraint on V at horizon exit, which gives

$$V_p = N \left(\frac{T_4 R}{g_s} \right) \left(\frac{\phi_H}{\phi_4} \right)^3 \sim 12\pi\epsilon \times 10^{-10} M_P^4$$

- Reheating: $V'(\phi)$ seems finite at $\phi = 0$, but divergent kinetic term $\sim \sqrt{(d\phi)^2/(r - r_H)}$, so effective description breaks down \Rightarrow usual brane-antibrane reheating mechanism.

- **Model 2:** the potential on the plateau is

$$V_4(\phi) = \frac{2T_4 R}{g_s} \frac{1}{1 + \frac{\phi_4^3}{\phi^3}}$$

leading to

$$\begin{aligned} \epsilon &= \frac{1}{2} \left[\frac{3M_P}{\phi} \frac{\phi_4^3/\phi^3}{1 + \phi_4^3/\phi^3} \right]^2 \\ \eta &= -\frac{M_P^2 \phi_4^3/\phi^3 (32 + 30\phi_4^3/\phi^3)}{\phi^2 (1 + \phi_4^3/\phi^3)^2} \\ \mathcal{N} &\simeq \frac{\phi_{in}^5 - \phi_{end}^5}{3M_P^2 \phi_4^3} \end{aligned}$$

so we always have a red spectrum ($\epsilon > 0$ and $\eta < 0$, and $n_s - 1 = -6\epsilon + 2\eta$), but we can have slow-roll for non-generic $\phi \gg M_P$, or if $\phi \sim M_P$, we need $\phi_4 \ll M_P$, needing the non-generic $V_5 m^4 / (\sqrt{\alpha'} N^{2/3}) \gg 1$. However, reheating is simpler since now $V'(\phi) = \infty$ at $\phi = 0$.

- COBE normalization gives

$$V_f \sim \frac{2T_4 R}{g_s} \sim 12\pi\epsilon \times 10^{-10} M_P^4$$

7. Conclusions

- Inflation is difficult to obtain in string theory. A standard model (try) is KKLT/KKLMMT in IIB string theory.
- It is based on the KS throat geometry, compactified by gluing a CY_3 .
- We can write a gravity dual to MQCD, T-dual to the KKLT construction.
- Based on it, we presented two models in IIA string theory, one with a compactified non-susy cigar geometry, and one with a compactified cylinder geometry, but with a sliding antibrane.
- The cosmology coming from it is consistent with current experiments, but it has the usual problems of KKLMMT.