Revisiting the S-matrix approach to the open superstring low energy effective lagrangian

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(... and work in progress ...)

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A. (Extremely) Brief review of the tree level open string scattering amplitudes

(Tree level) N-point amplitude of gauge bosons

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$$\mathcal{A}_N = i(2\pi)^D \delta^D(k_1 + \ldots + k_N) \times$$
$$\times \left[\, \mathrm{tr}(\lambda^{a_1}\lambda^{a_2}\ldots\lambda^{a_N}) \; A(1,2,\ldots,N) + \begin{pmatrix} \text{non-cyclic} \\ \text{permutations} \end{pmatrix} \, \right]$$

(Tree level) N-point amplitude of gauge bosons

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$$A(1,2,\ldots,N) =$$
$$g^{N-2} \int d\mu(z) \ < \hat{V}_1(z_1,k_1)\hat{V}_2(z_2,k_2) \ \ldots \hat{V}_N(z_N,k_N) > \ ,$$

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$\hat{V}_j(z_j, k_j) \rightarrow$ Open string vertex operator.

For the **BOSONIC** open string:

$$A(1,2,\ldots,N) =$$
$$g^{N-2}\int_0^1 dx_{N-2}\int_0^{x_{N-2}} dx_{N-3}\ \ldots \int_0^{x_3} dx_2 \prod_{i<j}^{N}(x_j - x_i)^{2\alpha' k_i\cdot k_j}$$
$$\times\ \exp\left(\sum_{i<j}^{N}\frac{2\alpha'\zeta_i\cdot\zeta_j}{(x_j-x_i)^2} - \sum_{i\neq j}^{N}\frac{2\alpha' k_j\cdot\zeta_i}{(x_j-x_i)}\right)\Big|_{linear}$$

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$$\times\ \exp\left(\sum_{i<j}^{N}\frac{2\alpha'\zeta_i\cdot\zeta_j}{(x_j-x_i)^2} - \sum_{i\neq j}^{N}\frac{2\alpha' k_j\cdot\zeta_i}{(x_j-x_i)}\right)\Bigg|_{linear}$$

For the **SUPERSYMMETRIC** open string:

$$A(1,2,\ldots,N) = 2\frac{g^{N-2}}{(2\alpha')^2}(x_{N-1}-x_1)(x_N-x_1)\times$$

$$\int_0^{x_{N-1}} dx_{N-2}\int_0^{x_{N-2}} dx_{N-3}\ \ldots \int_0^{x_3} dx_2\ \times$$

$$\times \int d\theta_1\ldots d\theta_{N-2}\prod_{i<j}^{N}(x_j - x_i - \theta_j\theta_i)^{2\alpha' k_i\cdot k_j}\ \times$$

$$\int d\phi_1\ldots d\phi_N\ \times$$

$$\exp\left(\sum_{i\neq j}^{N}\frac{(2\alpha')^1(\theta_j-\theta_i)\phi_j(\zeta_j\cdot k_i) - 1/2\ (2\alpha')^1\phi_j\phi_i(\zeta_j\cdot\zeta_i)}{x_j - x_i - \theta_j\theta_i}\right)$$

Example 1 : 3-point amplitude

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i) Open BOSONIC string

$$A_b(1,2,3) =$$
$$2g \Big[(\zeta_1 \cdot k_2)(\zeta_2 \cdot \zeta_3) + (\zeta_2 \cdot k_3)(\zeta_3 \cdot \zeta_1) + (\zeta_3 \cdot k_1)(\zeta_1 \cdot \zeta_2) \Big] +$$

$$+ 2g\ (2\alpha')\ (\zeta_1 \cdot k_2)(\zeta_2 \cdot k_3)(\zeta_3 \cdot k_1)$$

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i) Open BOSONIC string

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$$+\,2g\;(2\alpha')\;(\zeta_1\cdot k_2)(\zeta_2\cdot k_3)(\zeta_3\cdot k_1)$$

ii) Open SUPERSYMMETRIC string

$$A_s(1,2,3) = 2g\left[\,(\zeta_1\cdot k_2)(\zeta_2\cdot\zeta_3)+(\zeta_2\cdot k_3)(\zeta_3\cdot\zeta_1)+(\zeta_3\cdot k_1)(\zeta_1\cdot\zeta_2)\,\right]$$

Example 2 : **4-point amplitude**

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i) Open BOSONIC string

$$A_b(1,2,3,4) = 8\ g^2\ \alpha'^2\ \frac{\Gamma(-1-\alpha' s)\Gamma(-1-\alpha' t)}{\Gamma(2-\alpha' s-\alpha' t)}\ \times$$
$$\times\ K_b(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4; \alpha')$$

Example 2 : 4-point amplitude

i) Open BOSONIC string

$$A_b(1,2,3,4) = 8\ g^2\ \alpha'^2\ \frac{\Gamma(-1-\alpha' s)\Gamma(-1-\alpha' t)}{\Gamma(2-\alpha' s-\alpha' t)}\ \times$$
$$\times\ K_b(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4;\alpha')$$

ii) Open SUPERSYMMETRIC string

$$A_s(1,2,3,4) = 8\ g^2\ \alpha'^2 \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}\ \times$$
$$\times\ K_s(\zeta_1,k_1;\zeta_2,k_2;\zeta_3,k_3;\zeta_4,k_4)\ ,$$

where

$$s = -(k_1 + k_2)^2 \ , \quad t = -(k_1 + k_4)^2 \ , \quad u = -(k_1 + k_3)^2$$

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are the Mandelstam variables and

$$\begin{aligned} K_s = -\frac{1}{4} \Big[& ts(\zeta_1 \cdot \zeta_3)(\zeta_2 \cdot \zeta_4) + \\ & +su(\zeta_2 \cdot \zeta_3)(\zeta_1 \cdot \zeta_4) + ut(\zeta_1 \cdot \zeta_2)(\zeta_3 \cdot \zeta_4) \Big] + \\ + \frac{1}{2} \, s \, \Big[& (\zeta_1 \cdot k_4)(\zeta_3 \cdot k_2)(\zeta_2 \cdot \zeta_4) + (\zeta_2 \cdot k_3)(\zeta_4 \cdot k_1)(\zeta_1 \cdot \zeta_3) + \\ & +(\zeta_1 \cdot k_3)(\zeta_4 \cdot k_2)(\zeta_2 \cdot \zeta_3) + (\zeta_2 \cdot k_4)(\zeta_3 \cdot k_1)(\zeta_1 \cdot \zeta_4) \Big] + \\ + \frac{1}{2} \, t \, \Big[& (\zeta_2 \cdot k_1)(\zeta_4 \cdot k_3)(\zeta_3 \cdot \zeta_1) + (\zeta_3 \cdot k_4)(\zeta_1 \cdot k_2)(\zeta_2 \cdot \zeta_4) + \\ & +(\zeta_2 \cdot k_4)(\zeta_1 \cdot k_3)(\zeta_3 \cdot \zeta_4) + (\zeta_3 \cdot k_1)(\zeta_4 \cdot k_2)(\zeta_2 \cdot \zeta_1) \Big] + \\ + \frac{1}{2} \, u \, \Big[& (\zeta_1 \cdot k_2)(\zeta_4 \cdot k_3)(\zeta_3 \cdot \zeta_2) + (\zeta_3 \cdot k_4)(\zeta_2 \cdot k_1)(\zeta_1 \cdot \zeta_4) + \\ & +(\zeta_1 \cdot k_4)(\zeta_2 \cdot k_3)(\zeta_3 \cdot \zeta_4) + (\zeta_3 \cdot k_2)(\zeta_4 \cdot k_1)(\zeta_1 \cdot \zeta_2) \Big] \ . \end{aligned}$$

α' expansion of the **4-point momentum factor**:

$$\alpha'^2 \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} =$$

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$$\alpha'^2 \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} =$$

$$\frac{1}{st} - \frac{\pi^2}{6}\alpha'^2 - \zeta(3)(s+t)\ \alpha'^3 - \frac{\pi^4}{360}(4s^2+st+4t^2)\ \alpha'^4 +$$

$$+ \left[\frac{\pi^2}{6}\zeta(3)\ st(s+t) - \zeta(5)(s^3+2s^2t+2st^2+t^3)\right]\alpha'^5 +$$

$$+ \left[\frac{1}{2}\zeta(3)^2 st(s+t)^2 - \right.$$

$$\left. - \frac{\pi^6}{15120}(16s^4+12s^3t+23s^2t^2+12st^3+16t^4)\right]\alpha'^6 +$$

$$+ \mathcal{O}(\alpha'^7)\ .$$

B. (Extremely) Brief review of the low energy effective lagrangian

General Structure

$$\mathcal{L}_{\text{eff}} = \frac{1}{g^2}\,\mathrm{tr}\Big[\ F^2 + (2\alpha')F^3 + (2\alpha')^2F^4 + (2\alpha')^3\big(F^5 + D^2F^4\big) + \\ +(2\alpha')^4\big(F^6 + D^2F^5 + D^4F^4\big) + \mathcal{O}\big((2\alpha')^4\big)\ \Big]$$

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$$+\ (\text{fermions})$$

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$$+(2\alpha')^4\big(F^6 + D^2F^5 + D^4F^4\big) + \mathcal{O}\big((2\alpha')^4\big)\;\Big] \;+$$
$$+\;({\rm fermions})$$

i) Low energy effective lagrangian up to ${\alpha'}^2$ terms

$$\mathcal{L}_{\rm eff} = \frac{1}{g^2}{\rm tr}\Bigg[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (2\alpha')^1 a_1 F_\mu{}^\lambda F_\lambda{}^\nu F_\nu{}^\mu +$$
$$+ (2\alpha')^2\Big(a_3F^{\mu\lambda}F^\nu{}_\lambda F_\mu{}^\rho F_{\nu\rho} + a_4F^\mu{}_\lambda F_\nu{}^\lambda F^{\nu\rho}F_{\mu\rho} +$$
$$+ a_5F^{\mu\nu}F_{\mu\nu}F^{\lambda\rho}F_{\lambda\rho} + a_6F^{\mu\nu}F^{\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\Big) +$$
$$+ O((2\alpha')^3)\;\Bigg]\;.$$

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→ 1 coefficient at α'^1 order : a_1

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→ **1 coefficient at ${\alpha'}^1$ order :** a_1

→ **4 coefficients at ${\alpha'}^2$ order:** a_3, a_4, a_5, a_6.

ii) Low energy effective lagrangian at α'^3 order

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}{}^{(3)} = \frac{(2\alpha')^3}{g^2} \\
\operatorname{tr}\Big[\ & a_{10}\ F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\rho F_\rho{}^\sigma F_\sigma{}^\mu + a_{11}\ F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\rho F_\sigma{}^\mu F_\rho{}^\sigma + \\
& + a_{12}\ F_\mu{}^\nu F_\nu{}^\lambda F_\sigma{}^\mu F_\lambda{}^\rho F_\rho{}^\sigma + a_{13}\ F_\mu{}^\nu F_\rho{}^\sigma F_\nu{}^\lambda F_\sigma{}^\mu F_\lambda{}^\rho \\
& + a_{14}\ F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\mu F_\rho{}^\sigma F_\sigma{}^\rho + a_{15}\ F_\mu{}^\nu F_\nu{}^\lambda F_\rho{}^\sigma F_\lambda{}^\mu F_\sigma{}^\rho + \\
& + a_{16} \left(D_\mu F_\nu{}^\lambda\right) \left(D^\mu F_\lambda{}^\rho\right) F_\sigma{}^\nu F_\rho{}^\sigma + \\
& + a_{17} \left(D_\mu F_\nu{}^\lambda\right) F_\sigma{}^\nu \left(D^\mu F_\lambda{}^\rho\right) F_\rho{}^\sigma + \\
& + a_{18} \left(D_\mu F_\nu{}^\lambda\right) \left(D^\mu F_\lambda{}^\nu\right) F_\rho{}^\sigma F_\sigma{}^\rho + \\
& + a_{19} \left(D_\mu F_\nu{}^\lambda\right) F_\rho{}^\sigma \left(D^\mu F_\lambda{}^\nu\right) F_\sigma{}^\rho + \\
& + a_{20} \left(D_\sigma F_\mu{}^\nu\right) F_\lambda{}^\rho \left(D^\mu F_\nu{}^\lambda\right) F_\rho{}^\sigma + \\
& + a_{21} F_\mu{}^\nu \left(D^\mu F_\nu{}^\lambda\right) F_\rho{}^\sigma \left(D_\sigma F_\lambda{}^\rho\right) + \\
& + a_{22} F_\mu{}^\nu \left(D^\mu F_\lambda{}^\rho\right) \left(D_\sigma F_\nu{}^\lambda\right) F_\rho{}^\sigma\ \Big]
\end{aligned}
$$

ii) Low energy effective lagrangian at α'^3 order

$$\mathcal{L}_{\rm eff}{}^{(3)} = \frac{(2\alpha')^3}{g^2}$$

$$\begin{aligned}
\mathrm{tr}\Big[\, & a_{10}\, F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\rho F_\rho{}^\sigma F_\sigma{}^\mu + a_{11}\, F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\rho F_\sigma{}^\mu F_\rho{}^\sigma + \\
& + a_{12}\, F_\mu{}^\nu F_\nu{}^\lambda F_\sigma{}^\mu F_\lambda{}^\rho F_\rho{}^\sigma + a_{13}\, F_\mu{}^\nu F_\rho{}^\sigma F_\nu{}^\lambda F_\sigma{}^\mu F_\lambda{}^\rho \\
& + a_{14}\, F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\mu F_\rho{}^\sigma F_\sigma{}^\rho + a_{15}\, F_\mu{}^\nu F_\nu{}^\lambda F_\rho{}^\sigma F_\lambda{}^\mu F_\sigma{}^\rho + \\
& + a_{16} \left(D_\mu F_\nu{}^\lambda\right) \left(D^\mu F_\lambda{}^\rho\right) F_\sigma{}^\nu F_\rho{}^\sigma + \\
& + a_{17} \left(D_\mu F_\nu{}^\lambda\right) F_\sigma{}^\nu \left(D^\mu F_\lambda{}^\rho\right) F_\rho{}^\sigma + \\
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\end{aligned}$$

→ 13 coefficients at α'^3 order

C. Basics of the S-matrix approach to the (tree level) open string low energy effective lagrangian

Matching the open string amplitudes with the ones from the LEL:

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Bosonic string: $a_1 = -i/3.$

Supersymmetric string: $a_1 = 0.$

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Coefficient	Bosonic open string theory	Supersymmetric open string theory
a_3	$\pi^2/12$	$\pi^2/12$
a_4	$\pi^2/24$	$\pi^2/24$
a_5	$-\pi^2/48 - 1/8$	$-\pi^2/48$
a_6	$-\pi^2/96 + 1/8$	$-\pi^2/96$

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It is *exactly* the <u>SAME</u> method applied to two <u>different</u> expressions for

$$A(1, 2, \ldots, N) \ .$$

D. Our **revisited** S-matrix approach to the (tree level) open string low energy effective lagrangian

Main observation

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This is probably due to
D=10 Supersymmetry

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→ At α'^2 order there is only 1 coefficient: $\mathbf{a_6}$.

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$$a_3 = -8\ \mathbf{a_6} \quad , a_4 = -4\ \mathbf{a_6} \quad , a_5 = 2\ \mathbf{a_6} \quad .$$

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$\rightarrow$ At α'^2 order there is only 1 coefficient: $\mathbf{a_6}$.

$$\mathbf{a_6} = -\frac{\pi^2}{96}$$

ii) ${\alpha'}^3$ calculation

ii) α'^3 calculation

$$-2a_{16} = -2a_{17} = 8a_{19} = -a_{20} = \mathbf{a_{22}} \ ,$$
$$a_{18} = a_{21} = 0 \ .$$

$$a_{11} = a_{13} = -2a_{15} = -i \ \mathbf{a_{22}} \ ,$$
$$a_{10} = a_{12} = a_{14} = 0 \ .$$

ii) α'^3 calculation

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\mathcal{L}_{\rm eff}{}^{(3)} &= -\frac{(2\alpha')^3 \ \mathbf{a_{22}}}{g^2} \times \\
&\times \ {\rm tr}\Big[\ i \ F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\rho F_\sigma{}^\mu F_\rho{}^\sigma + i \ F_\mu{}^\nu F_\rho{}^\sigma F_\nu{}^\lambda F_\sigma{}^\mu F_\lambda{}^\rho - \\
&- \frac{i}{2} \ F_\mu{}^\nu F_\nu{}^\lambda F_\rho{}^\sigma F_\lambda{}^\mu F_\sigma{}^\rho + \frac{1}{2}\left(D_\mu F_\nu{}^\lambda\right)\left(D^\mu F_\lambda{}^\rho\right) F_\sigma{}^\nu F_\rho{}^\sigma \\
&+ \frac{1}{2}\left(D_\mu F_\nu{}^\lambda\right) F_\sigma{}^\nu \left(D^\mu F_\lambda{}^\rho\right) F_\rho{}^\sigma - \frac{1}{8}\left(D_\mu F_\nu{}^\lambda\right) F_\rho{}^\sigma \left(D^\mu F_\lambda{}^\nu\right) F_\sigma{}^\rho - \\
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$$\mathbf{a_{22}} = 2\zeta(3)$$

iii) α'^4 calculation:

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$$\mathcal{L}_{\text{eff}}{}^{(4)} = \frac{(2\alpha')^4 \pi^4}{g^2} \left(\mathcal{L}_{F^6} + \mathcal{L}_{D^2F^5} + \mathcal{L}_{D^4F^4} \right) ,$$

where

$$\mathcal{L}_{F^6} = \frac{1}{46080} \times$$
$$\times\ t_{(12)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4\mu_5\nu_5\mu_6\nu_6}\ \mathrm{tr}\Big(F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5}F_{\mu_6\nu_6}\Big)\ ,$$

$$\begin{aligned}
\mathcal{L}_{D^2F^5} = \frac{56\ i}{46080} t_{(10)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4\mu_5\nu_5}\ \times \\
\times\ \mathrm{tr}\Big(F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}D^\alpha F_{\mu_4\nu_4}D_\alpha F_{\mu_5\nu_5}\Big) + \\
+\ \frac{i}{46080}\ (\eta\cdot t_{(8)})^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4\mu_5\nu_5} \times \\
\times\ \mathrm{tr}\Big(-169\ D^\alpha F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}F_{\mu_4\nu_4}D_\alpha F_{\mu_5\nu_5} + \\
+68\ D^\alpha F_{\mu_1\nu_1}D_\alpha F_{\mu_2\nu_2}F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5} + \\
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+16\ F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}D^\alpha F_{\mu_4\nu_4}D_\alpha F_{\mu_5\nu_5}\ \Big) - \\
-\ \frac{i}{5760}\ t_{(8)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\ \times \\
\times\ \Big\{17\ \mathrm{tr}\Big(D^{\mu_5}F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}D^{\nu_5}F_{\mu_4\nu_4}F_{\mu_5\nu_5}\Big) + \\
+2\ \mathrm{tr}\Big(F_{\mu_1\nu_1}D^{\mu_5}F_{\mu_2\nu_2}D^{\nu_5}F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5}\Big)\Big\}\ ,
\end{aligned}$$

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\mathcal{L}_{D^4F^4} = -\frac{1}{11520}\ t_{(8)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}\ \times \\
\times\ \mathrm{tr}\Big(D^\alpha F_{\mu_1\nu_1}D_{(\alpha}D_{\beta)}F_{\mu_2\nu_2}D^\beta F_{\mu_3\nu_3}F_{\mu_4\nu_4} + \\
+8\ D^\alpha F_{\mu_1\nu_1}D_\alpha F_{\mu_2\nu_2}D^\beta F_{\mu_3\nu_3}D_\beta F_{\mu_4\nu_4}\ \Big)\ .
\end{aligned}$$

E. Summary of *our* **revisited** S-matrix method

1. There now is an *improved* (revisited) version of the S-matrix approach to the Open Superstring low energy effective lagrangian, $\mathcal{L}_{\text{eff}}$.

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2. It incorporates a kinematic requiremenent in the gauge boson scattering amplitudes, (presumably) due to Supersymmetry.

3. It has been successfully used to obtain the bosonic terms of $\mathcal{L}_{\text{eff}}$ up to α'^4 order.

4. Our *revisited* S-matrix method suggests that, demanding:

I. The kinematical constraints (absence of $(\zeta \cdot k)^N$ terms)
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JHEP10 (2012) 108

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→ It is due to Koerber and Sevrin (2001-2004).

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The method of BPS configurations

→ It is due to Koerber and Sevrin (2001-2004).

→ Our results agree with the ones of Koerber and Sevrin up to ${\alpha'}^3$ order and probably also agree at ${\alpha'}^4$ order.

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$$A(1,2,\ldots,N) = 2\frac{g^{N-2}}{(2\alpha')^2}(x_{N-1}-x_1)(x_N-x_1)\times$$

$$\int_0^{x_{N-1}} dx_{N-2}\int_0^{x_{N-2}} dx_{N-3}\ \ldots\int_0^{x_3} dx_2\ \times$$

$$\times\int d\theta_1\ldots d\theta_{N-2}\prod_{i<j}^{N}(x_j-x_i-\theta_j\theta_i)^{2\alpha' k_i\cdot k_j}\times$$

$$\int d\phi_1\ldots d\phi_N\times$$

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Mafra, Schlotterer, Stieberger, arXiv:0909.0256

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(Both expressions are equivalent)

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$$F_1^{(5)} = \alpha_{12}\ \alpha_{34} \int_0^1 dx_3 \int_0^{x_3} dx_2\ {x_2}^{2\alpha'\alpha_{12}-1} {x_3}^{2\alpha'\alpha_{13}} (1-x_2)^{2\alpha'\alpha_{24}} \\ (1-x_3)^{2\alpha'\alpha_{34}-1} (x_3-x_2)^{2\alpha'\alpha_{23}}$$

$$F_1^{(5)} = 1 - (2\alpha')^2\ \zeta(2)\ (\alpha_{12}\ \alpha_{34} - \alpha_{34}\ \alpha_{45} - \alpha_{12}\ \alpha_{51}) - \\ - (2\alpha')^3\ \zeta(3)\ \Big(\alpha_{12}^2\ \alpha_{34} + 2\ \alpha_{12}\ \alpha_{23}\ \alpha_{34} + \alpha_{12}\ \alpha_{34}^2 - \\ - \alpha_{34}^2\ \alpha_{45} - \alpha_{34}\ \alpha_{45}^2 - \alpha_{12}^2\ \alpha_{51} - \alpha_{12}\ \alpha_{51}^2\Big) \\ + \mathcal{O}((2\alpha')^4)$$

Mafra, Schlotterer and Stieberger's result:

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Examples:

iii) N=6 :

$$A(1,2,3,4,5,6) =$$
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What can the *revisited* S-matrix method say about Mafra-Schlotterer-Stieberger's result,

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Answer:

It can be re-derived from it !

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} Properties of gauge boson amplitudes

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G. Global summary

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$$\mathcal{L}_{\rm eff} = \frac{1}{g^2}\,{\rm tr}\Big[\ F^2 + (2\alpha')F^3 + (2\alpha')^2 F^4 + (2\alpha')^3\big(F^5 + D^2F^4\big) + +(2\alpha')^4\big(F^6 + D^2F^5 + D^4F^4\big) + \mathcal{O}\big((2\alpha')^4\big)\ \Big]$$

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like, for example, for N=6 :

$$\int_0^1 dx_4 \int_0^{x_4} dx_3 \int_0^{x_3} dx_2 \; {x_2}^{2\alpha'\alpha_{12}} \; {x_3}^{2\alpha'\alpha_{13}} \; {x_4}^{2\alpha'\alpha_{14}-1} \; (x_3 - x_2)^{2\alpha'\alpha_{23}-1} \; (x_4 - x_2)^{2\alpha'\alpha_{24}} \; (x_4 - x_3)^{2\alpha'\alpha_{34}-1} (1 - x_2)^{2\alpha'\alpha_{25}-1} \; (1 - x_3)^{2\alpha'\alpha_{35}} \; (1 - x_4)^{2\alpha'\alpha_{45}}$$

$$\int_0^1 dx_4 \int_0^{x_4} dx_3 \int_0^{x_3} dx_2 \; {x_2}^{2\alpha'\alpha_{12}} \; {x_3}^{2\alpha'\alpha_{13}} \; {x_4}^{2\alpha'\alpha_{14}-1} \; (x_3-x_2)^{2\alpha'\alpha_{23}-1}$$
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$$=$$

$$\frac{1}{(2\alpha')^3}\left[\left(\frac{1}{\alpha_{23}\alpha_{16}t_{234}}+\frac{1}{\alpha_{34}\alpha_{56}t_{234}}\right)+\left(\frac{1}{\alpha_{34}\alpha_{16}t_{234}}+\frac{1}{\alpha_{23}\alpha_{56}t_{234}}\right)\right]+$$
$$+\frac{\zeta(2)}{(2\alpha')^1}\left[-\left(\frac{\alpha_{16}}{\alpha_{23}t_{234}}+\frac{\alpha_{56}}{t_{234}\alpha_{34}}\right)+\left(\frac{1}{\alpha_{56}}+\frac{1}{\alpha_{16}}\right)-\left(\frac{\alpha_{23}}{t_{234}\alpha_{56}}+\frac{\alpha_{34}}{t_{234}\alpha_{16}}\right)+\right.$$
$$+\left(\frac{1}{\alpha_{23}}+\frac{1}{\alpha_{34}}\right)-\left(\frac{\alpha_{23}}{t_{234}\alpha_{16}}+\frac{\alpha_{34}}{t_{234}\alpha_{56}}\right)-\left(\frac{\alpha_{12}}{\alpha_{56}\alpha_{34}}+\frac{\alpha_{45}}{\alpha_{23}\alpha_{16}}\right)-$$
$$\left.-\left(\frac{\alpha_{56}}{\alpha_{23}t_{234}}+\frac{\alpha_{16}}{t_{234}\alpha_{34}}\right)-\left(\frac{t_{345}}{\alpha_{34}\alpha_{16}}+\frac{t_{123}}{\alpha_{23}\alpha_{56}}\right)\right]+$$

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Obrigado !