Geometric Deformation in the Braneworld and (microscopic)Black Holes

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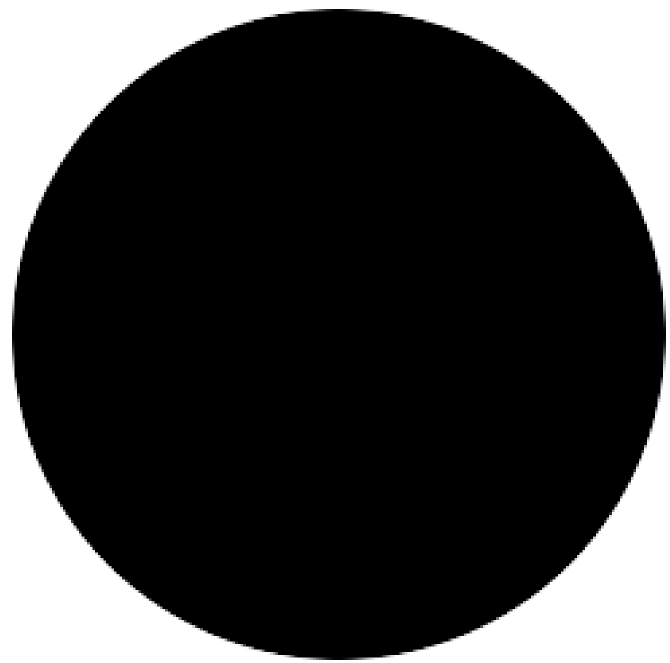
Extra Dimensional Gravity

- Braneworld
  - Introduccion.
  - Astrophysics in the brane world.
  - Minimal geometric deformation approach.
  - Some results about extra dimensional consequences on compact self-gravitational systems.
- Black Holes in the Braneworld
  - Introduccion.
  - Black Holes in the Brane world
  - Micro Black Holes in the Brane world
  - Black holes limit and miimum mass

Beyond Einstein...

- Motivation
  - Due to its inconsistency with quantum mechanics, it is not possible to ensure that General Relativity keeps its original structure at high energies.
  - In extra dimensional gravity the fundamental scale of gravity can be as low as TeV range. Hence the production of Black Holes at the Large Hadron Collider (LHC) could be allowed.
  - One of the goals of the current study is to see what features of theories beyond Einstein could be relevant in the description/production of (micro) Black Holes
  - In this talk: Astrophysics in the brane world
  - Micro Black Holes in the Braneworld

Black holes, neutron stars, quark stars



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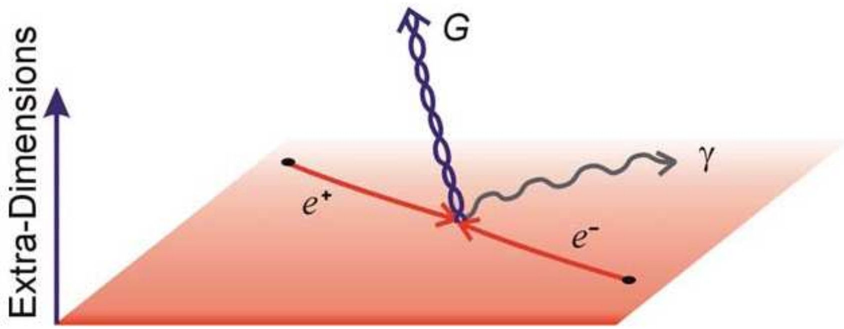
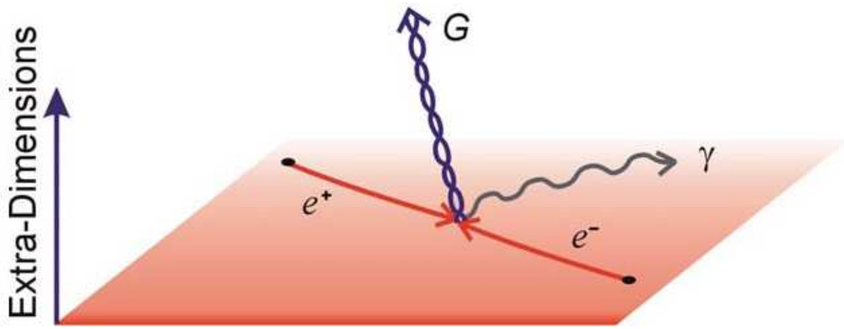
Extra dimension



- **Large Extra Dimension** (ADD theory) Arkani-Hamed, Dimopoulos, Dvali (1998)
- **Braneworld** (RS theory) L. Randall and R. Sundrum (1999)
  - Both models explain the hierarchy problem
  - ADD: **Many flat** extra dimensions
  - Braneworld: **Only one** extra dimension with a **warped geometry**.

Extra dimension



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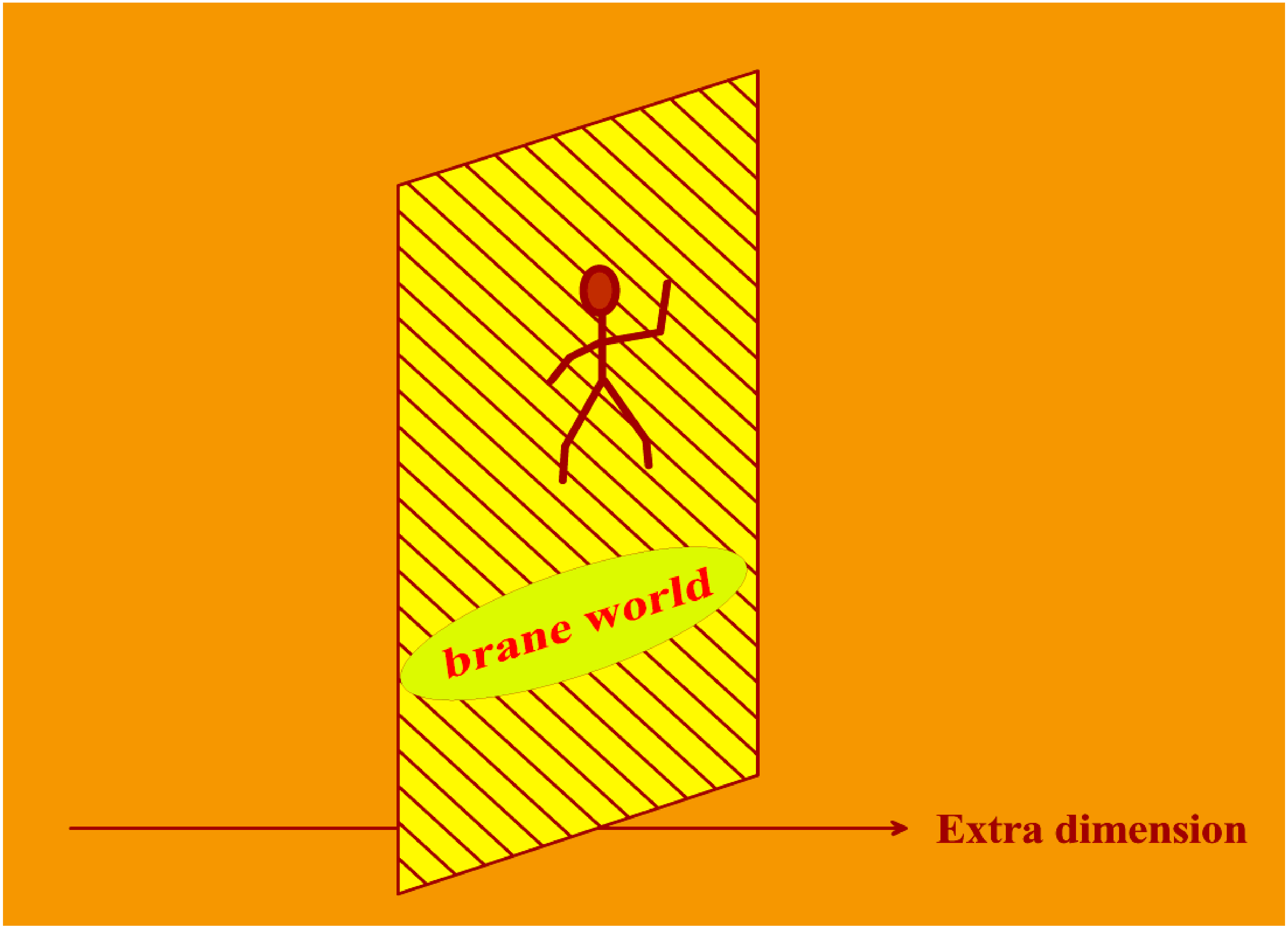
  - Both models explain the hierarchy problem
  - ADD: **Many flat** extra dimensions
  - Braneworld: **Only one** extra dimension with a **warped geometry**.
- **No experimental evidence for extra dimensions so far:**
  - **LEP:** LEP Exotica Working Group, LEP Exotica WG 2004-03;
  - **Tevatron:** CDF Collaboration, Phys. Rev. Lett. 101 (2008) 181602; D0 Collaboration, Phys. Rev. Lett. 101 (2008) 011601.
  - **LHC:** ATLAS Collaboration, Phys. Lett. B 705 (2011) 294; Phys. Lett. B 709 (2012) 322.
  - **LHC:** CMS Collaboration, Phys. Rev. Lett. 107 (2011) 201804.
  - Recently: **LHC:** ATLAS collaboration, arXiv:1204.4646v2[hep-ex] Sep.2012.

The Braneworld



Einstein field equations on the brane

The Einstein field equations on the brane may be written as a modification of the standard field equations [Shiromizu et al 2002]

5D Einstein equations:

$$G_{ab} + \Lambda_5 g_{ab} = \kappa_5^2 T_{ab}; \quad \kappa_5 = 8\pi G_5 \quad a = 0, ...4 \quad (Bulk)$$

$$G_{\mu\nu} = -8\pi T^T_{\mu\nu} - \Lambda g_{\mu\nu}, \quad \mu = 0, ...3 \quad (Brane)$$

where the energy-momentum tensor has new terms carrying bulk effects onto the brane:

$$T_{\mu\nu} \to T_{\mu\nu}{}^T = T_{\mu\nu} + \frac{6}{\sigma} S_{\mu\nu} + \frac{1}{8\pi} \mathcal{E}_{\mu\nu}$$

Here σ is the brane tension

The new terms and are the high-energy corrections $S_{\mu\nu}$ and the projection of the bulk Weyl tensor on the brane $\mathcal{E}_{\mu\nu}$

$$S_{\mu\nu} = \frac{1}{12} T_\alpha^{\ \alpha} T_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T^\alpha_{\ \nu} + \frac{1}{24} g_{\mu\nu} \left[3 T_{\alpha\beta} T^{\alpha\beta} - (T_\alpha^{\ \alpha})^2 \right]$$

$$-8\pi \mathcal{E}_{\mu\nu} = -\frac{6}{\sigma} \left[\mathcal{U} (u_\mu u_\nu + \frac{1}{3} h_{\mu\nu}) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_{(\mu} u_{\nu)} \right]$$

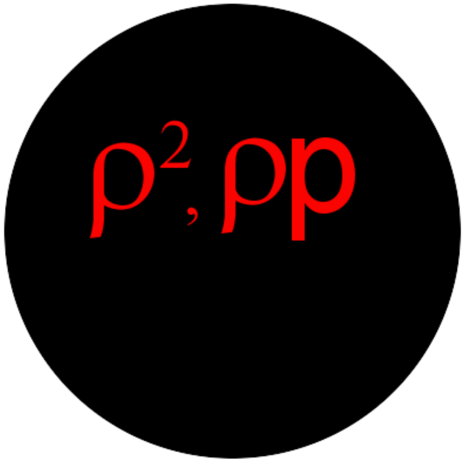
$$\begin{array}{rcl} \mathcal{U} & \rightarrow & Dark\ \ radiation \\ \mathcal{P}_{\mu\nu} & \rightarrow & Anisotropic\ \ stress \\ \mathcal{Q}_\mu & \rightarrow & Energy\ \ flux \end{array}$$

The interior: the simplest distribution

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

Perfect fluid

The interior: the simplest distribution



$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

Perfect fluid+high energy terms

Too complicated!

The interior: the simplest distribution

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

Perfect fluid+high energy terms+dark radiation/pressure

Too complicated!
THERE IS NOT SOLUTION! (Indefinity system)

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Perfect fluid+high energy terms+dark radiation/pressure

Too complicated!
THERE IS NOT SOLUTION! (Indefinity system)
However .... we found a general effective 4D solution!
===>The Minimal Geometric Deformation approach (MGD)

JO
Mod.Phys.Lett.A**23**38(2008)3247;Int.Jour.Mod.Phys.D,**18**,5(2009)837;Mod.Phys.Lett.A,**25**39(2010)

Spherically symmetric static distribution

Schwarzschild-like coordinates

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

A perfect fluid (General Relativity)+high energy corrections

$$-8\pi\left(\rho + \frac{1}{\sigma}\left(\frac{\rho^2}{2}\qquad\right)\right) = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right),$$

$$-8\pi\left(-p - \frac{1}{\sigma}\left(\frac{\rho^2}{2} + \rho p\qquad\right)\qquad\right) = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right),$$

$$-8\pi\left(-p - \frac{1}{\sigma}\left(\frac{\rho^2}{2} + \rho p\qquad\right)\qquad\right) = \frac{1}{4}e^{-\lambda}\left[2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{(\nu' - \lambda')}{r}\right],$$

$$p' = -\frac{\nu'}{2}(\rho + p).$$

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$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

A perfect fluid (General Relativity)+high energy corrections+Weyl functions

$$-8\pi \left(\rho + \frac{1}{\sigma} \left(\frac{\rho^2}{2} + 6\mathcal{U} \right) \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) + \frac{\mathcal{P}}{\sigma} \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) - \frac{\mathcal{P}}{2\sigma} \right) = \frac{1}{4} e^{-\lambda} \left[2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{(\nu' - \lambda')}{r} \right],$$

$$p' = -\frac{\nu'}{2}(\rho + p).$$

Minimal geometric deformation

Let us see the "solution" for the geometric function

$$e^{-\lambda} = 1 - \frac{8\pi}{r}\int_0^r r^2 \left[\rho + \frac{1}{\sigma}\left(\frac{\rho^2}{2} + \frac{6}{k^4}\mathcal{U}\right)\right] dr,$$

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It can be written as

$$e^{-\lambda} = \underbrace{1 - \frac{8\pi}{r}\int_0^r r^2 \rho dr}_{General\ Relativity} \quad \textbf{+ "DEFORMATIONS"}$$

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The deformation undergone by the geometric function λ produces anisotropic consequences, as can be seen through

$$\frac{8\pi}{k^4}\frac{\mathcal{P}}{\sigma} = \frac{1}{6}\left(G_1^1 - G_2^2\right),$$

An exact solution

Let us pick a general relativistic solution:

$$\rho(r) = \frac{C\ \left(9 + 2\,C\,r^2 + C^2\,r^4\right)}{7\,\pi\,(1 + C\,r^2)^3}; \quad p(r) = \frac{2C(2 - 7Cr^2 - C^2r^4)}{7\pi(1 + Cr^2)^3}; \quad e^{\nu} = A(1 + Cr^2)^4$$

The braneworld solution is found through

$$e^{-\lambda(r)} = 1 - \frac{2\tilde{m}(r)}{r}$$

where the interior mass function is given by

$$\begin{aligned} \tilde{m}(r) \quad &= \quad m(r) - \frac{1}{\sigma}\left(\frac{2}{7}\right)^2 \frac{Cr}{2\pi}\left[\frac{240 + 589Cr^2 - 25C^2r^4 - 41C^3r^6 - 3C^4r^8}{3(1 + Cr^2)^4(1 + 3Cr^2)}\right. \\ &\qquad \left. - \frac{80}{(1 + Cr^2)^2}\,\frac{arctg(\sqrt{C}r)}{(1 + 3Cr^2)\sqrt{C}r}\right], \end{aligned}$$

$$m(r) = \int_0^r 4\pi r^2 \rho dr = \frac{4}{7}Cr^3\frac{(3 + Cr^2)}{(1 + Cr^2)^2}, \quad GR\ \ mass\ \ \ function.\ \ Durgapal{-}Fuloria\,(1983).$$

An exact solution

the interior Weyl functions are

$$
\begin{aligned}
\mathcal{P}(r) \quad = \quad & \frac{32}{441r^3(1+Cr^2)^6(1+3Cr^2)^2}\Big[Cr\left(180+2040Cr^2+8696C^2r^4\right.\\
& +16533C^3r^6+12660C^4r^8+146C^5r^{10}-120C^6r^{12}+9C^7r^{14}\Big)\\
& -60\sqrt{C}(1+Cr^2)^3(3+26Cr^2+63C^2r^4)arctg(\sqrt{C}r)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{U}(r) \quad = \quad & \frac{32}{441r(1+Cr^2)^6(1+3Cr^2)^2}\Big[C^2r\left(795+4865Cr^2+10044C^2r^4\right.\\
& +6186C^3r^6-373C^4r^8-219C^5r^{10}-18C^6r^{12}\Big)\\
& -240C^{3/2}(1+Cr^2)^3(5+9Cr^2)arctg(\sqrt{C}r)\Big].
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& \left. + 16533 C^3 r^6 + 12660 C^4 r^8 + 146 C^5 r^{10} - 120 C^6 r^{12} + 9 C^7 r^{14} \right) \\
& - 60 \sqrt{C} (1+Cr^2)^3 (3 + 26 Cr^2 + 63 C^2 r^4) arctg(\sqrt{C} r) \Big] ,
\end{aligned}
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& \left. + 6186 C^3 r^6 - 373 C^4 r^8 - 219 C^5 r^{10} - 18 C^6 r^{12} \right) \\
& - 240 C^{3/2} (1+Cr^2)^3 (5 + 9 Cr^2) arctg(\sqrt{C} r) \Big] .
\end{aligned}
$$

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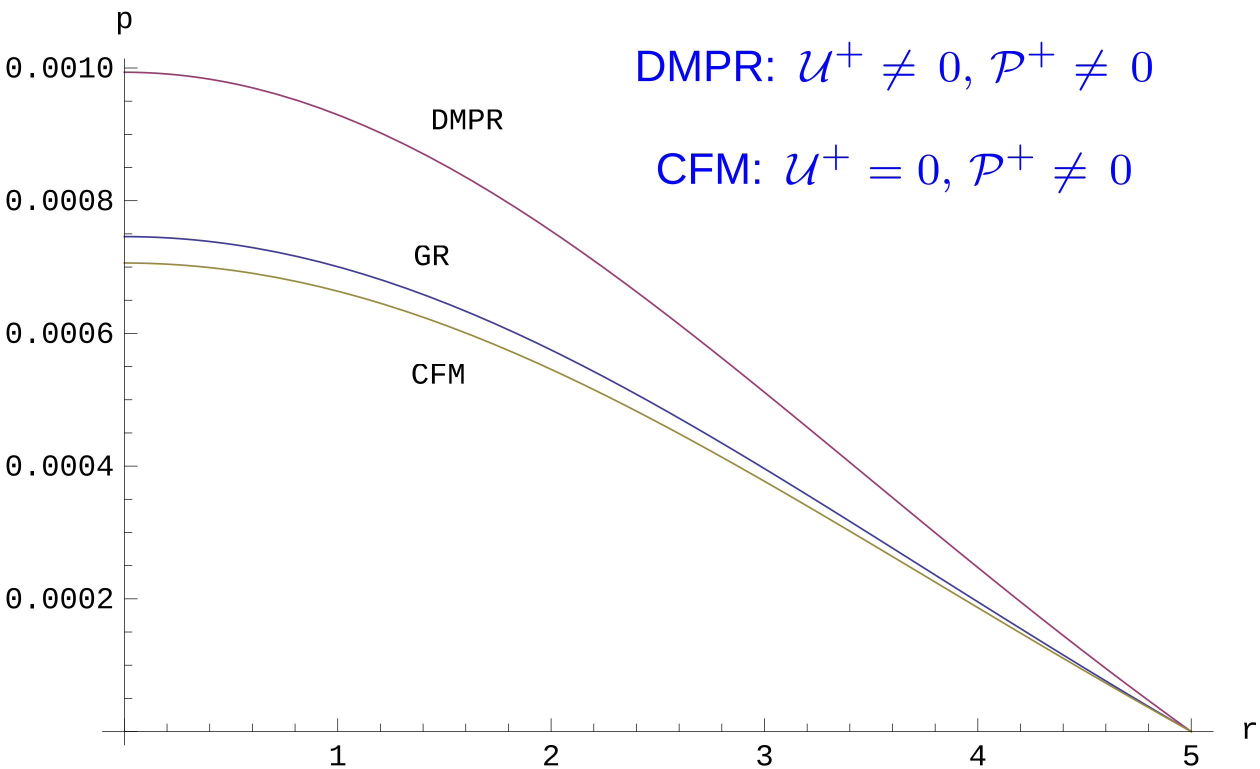
Also: JO + F. Linares (Guanajuato University) “The Tolman IV Braneworld Star: an Exact Solution” (in progress)

Role of dark radiation and dark pressure



- The exterior dark radiation $\mathcal{U}^+$ always increases both the pressure and the compactness of the stellar structures.
- The exterior dark pressure $\mathcal{P}^+$ always reduces them.

JO, A. Sotomayor (Antofagasta), A. Pascua (Trieste) (2012)

Minimal geometric deformation

- THE MGD WORKS!
- When a solution of the four-dimensional Einstein equations is considered as a possible solution of the BW system, the geometric deformation produced by extra-dimensional effects is minimized, and the open system of effective BW equations is automatically satisfied.
- This approach was successfully used to generate physically acceptable interior solutions for stellar systems JO Mod. Phys. Lett. **A23**, 3247 (2008); Mod. Phys. Lett. **A25**, 3323 (2010). and even exact solutions were found:
  - JO Int. J. Mod. Phys. D **18**, 837 (2009);
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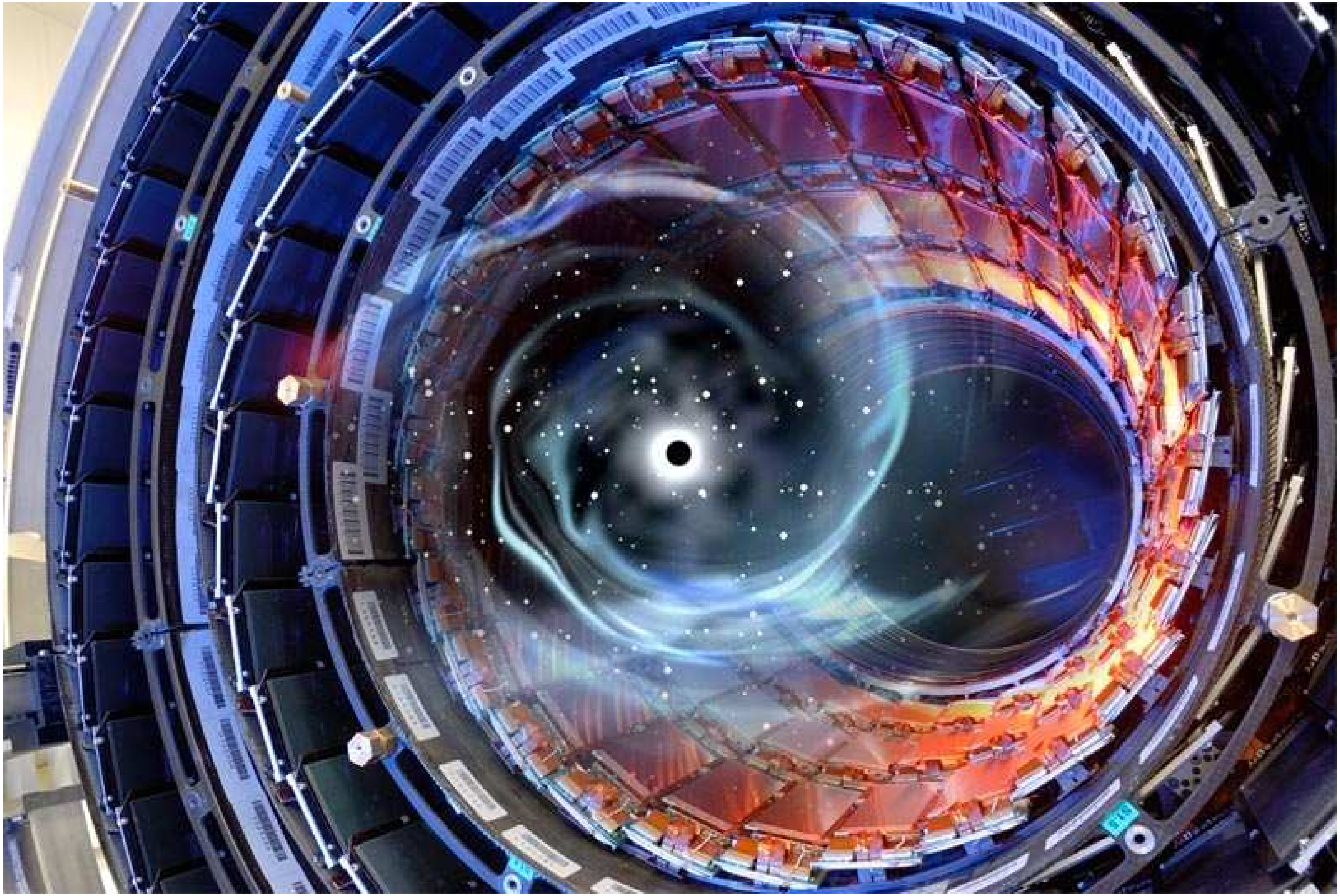
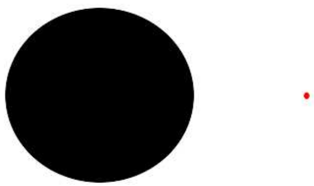
Black Holes in the Braneworld



Black Holes in 4D

Schwarzschild-like coordinates $ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right)$



$$e^{\nu} = e^{-\lambda} = 1 - \frac{2\,G\,M}{r} \Rightarrow h = 2\,G\,M$$

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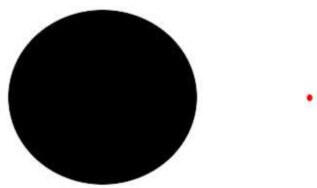

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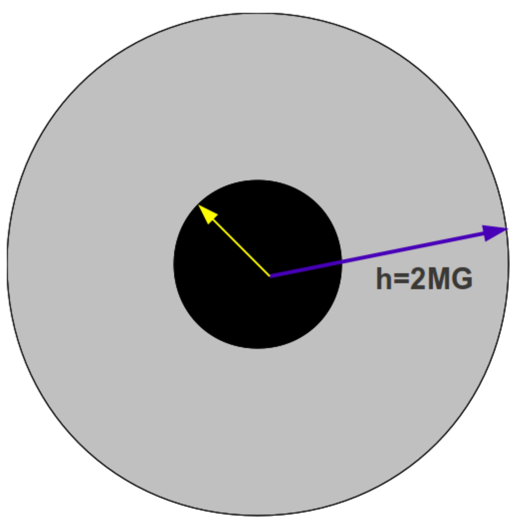
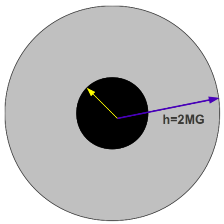


$$R < 2\,M\,G = 2\,M\,\frac{l_p}{M_p}$$

Black Holes in 4D



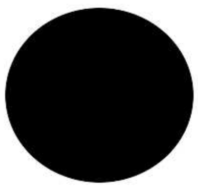
$$R < 2\,M\,G = 2\,M\,\frac{l_p}{M_p}$$

- **Micro Black Holes in 4D**

$$R \Rightarrow \lambda_C \qquad \lambda_C \simeq \frac{\hbar}{M} = \frac{l_p\,M_p}{M}$$

$$\frac{l_p\,M_p}{M} \lesssim 2\,M\,\frac{l_p}{M_p} \Rightarrow M_c \approx M_p \qquad \text{(Minimum possible mass)}$$

Black Holes in the Braneworld

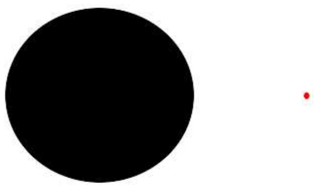


$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

- Dadhich,Maartens,Papadopoulos and Rezania (DMPR Solution):

$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\mathcal{M}}{r} + \frac{q}{r^2}, \quad \mathcal{U}^+ = -\frac{\mathcal{P}^+}{2} = \frac{4}{3}\pi q \sigma \frac{1}{r^4},$$

Black Holes in the Braneworld



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- Casadio, Fabbri and Mazzacurati (CFM Solution)

$$e^{\nu^+} = \left[\frac{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}}{1+\eta}\right]^2, \quad e^{\lambda^+} = \left[1 - \frac{2\mathcal{M}}{r}(1+\eta)\right]^{-1},$$

$$\frac{16\pi\mathcal{P}^+}{k^4\sigma} = -\frac{\mathcal{M}(1+\eta)\eta}{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}}\frac{1}{r^3}, \quad \mathcal{U}^+ = 0,$$

Micro Black Holes in the BW

- The tidally charged metric

$$ds^2 = e^\nu \, dt^2 - e^\lambda \, dr^2 - r^2(d\theta^2 + sin^2\theta \, d\phi^2); \qquad e^\nu = e^{-\lambda} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,r} - \frac{q}{r^2}$$

$$\text{Its horizon} \quad h = \ell_{\rm P}\left[\frac{\mathcal{M}}{M_{\rm P}} + \sqrt{\frac{\mathcal{M}^2}{M_{\rm P}^2} + q\,\frac{M_{\rm P}^2}{M_G^2}}\,\right]$$

$$\text{Micro Black Holes :} \quad \lambda_C \lesssim h \;\Rightarrow\; \frac{\ell_{\rm P}\,M_{\rm P}}{M} \lesssim \ell_{\rm P}\left[\frac{\mathcal{M}}{M_{\rm P}} + \sqrt{\frac{\mathcal{M}^2}{M_{\rm P}^2} + q\,\frac{M_{\rm P}^2}{M_G^2}}\,\right]$$

We consider black holes near their minimum possible mass $\mathrm{M} \sim \mathcal{M} \approx \mathrm{M_G} \ll \mathrm{M_P}$

$$\rightarrow \; M_c \approx \frac{M_G}{\sqrt{q}} \quad G.L.Alberghi, R.Casadio, O.Micu, and A.Orlandi, JHEP\,\mathbf{1109}, 023(2011)$$

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We consider black holes near their minimum possible mass $\mathrm{M} \sim \mathcal{M} \approx \mathrm{M_G} \ll \mathrm{M_P}$

$\rightarrow\; M_c \approx \dfrac{M_G}{\sqrt{q}}$ $\quad G.L.Alberghi, R.Casadio, O.Micu, and A.Orlandi, JHEP\,\mathbf{1109}, 023(2011)$

But q is unknown!!! We need the complete $5D$ solution, which is unknown so far. However...

Finding the tidal charge q

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2\,\ell_{\mathrm{P}}\,\mathcal{M}}{M_{\mathrm{P}}\,r} - \frac{q}{r^2}$$

- What is the relationship between $\mathcal{M}$ and q?
- We need the complete 5D solution (unknown).

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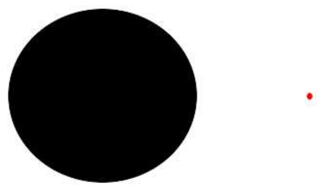
- What is the relationship between $\mathcal{M}$ and q?
- We need the complete 5D solution (unknown).
- We have to consider an alternative way: the $Minimal\ Geometric\ Deformation$
  - In the GR limit $\sigma^{-1} \to 0$, the tidal charge q must vanish.
  - We expect $\mathcal{M} = 0 \implies q = 0$

$$\text{Hence } \mathrm{q} = \mathrm{q}(\mathcal{M}, \sigma)$$

Finding the tidal charge q

- Exterior



$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,r} - \frac{q}{r^2}$$

Finding the tidal charge q

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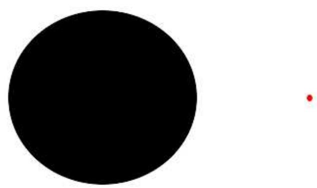

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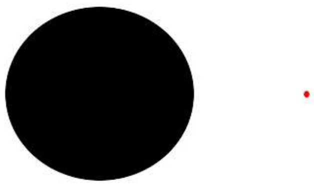
- Interior

$$e^{-\lambda^-} = 1 - \frac{2\,\tilde{m}(r)}{r}$$

where the interior mass function $\tilde{m}$ is given by $\tilde{m}(r) = m(r) - \frac{r}{2}\,f^*(r)$, with $f^*(r)$ *the minimal geometric deformation.*

Finding the tidal charge q

- Exterior



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- Interior

$$e^{-\lambda^-} = 1 - \frac{2\,\tilde{m}(r)}{r}$$

where the interior mass function $\tilde{m}$ is given by $\tilde{m}(r) = m(r) - \frac{r}{2}\,f^*(r)$, with $f^*(r)$ *the minimal geometric deformation.*

- Matching conditions at $r = R$

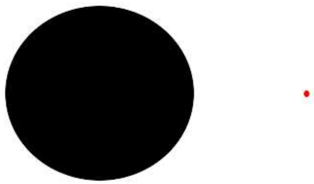
$$e^{\nu_R} = 1 - \frac{2\,\ell_{\rm P}\,\mathcal{M}}{M_{\rm P}\,R} - \frac{q}{R^2}$$

$$\frac{2\,\mathcal{M}}{R} = \frac{2\,M}{R} - \frac{M_{\rm P}}{\ell_{\rm P}}\left(f^* + \frac{q}{R^2}\right)$$

$$\frac{q}{R^4} = \left(\frac{\nu'_R}{R} + \frac{1}{R^2}\right) f^* + 8\,\pi\,\frac{\ell_{\rm P}}{M_{\rm P}}\,p_R$$

Finding the tidal charge q

- Exterior



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- Matching conditions at $r = R$
  - We then ontain the tidal charge as

$$\frac{M_{\rm P}}{\ell_{\rm P}}\,q = \left(\frac{R\,\nu'_R + 1}{R\,\nu'_R + 2}\right)\left(\frac{2\,M}{R} - \frac{2\,\mathcal{M}}{R}\right)R^2 + \frac{8\,\pi\,p_R\,R^4}{2 + R\,\nu'_R}$$

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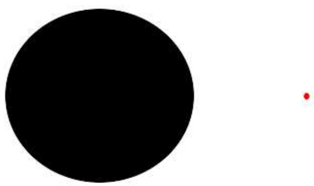

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  - We need an interior solution to evaluate ν'_R and then to find $q = q(\mathcal{M}, \sigma)$

Finding the tidal charge q

Taking $p_R = 0$ and imposing the boundary constraint

$$R\,\nu'_R = -\frac{(M-\mathcal{M}) - \frac{2\,\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}{(M-\mathcal{M}) - \frac{\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}$$

where K is a (dimensionful) constant we can fix later, we obtain a simple relation between q and $\mathcal{M}$ given by (R. Casadio, JO, Phys. Lett. B, 715, 251-255 (2012)).

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- it vanishes for $\mathcal{M} \to 0$ and for $\sigma^{-1} \to 0$, and
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As the pressure does not need to vanish at the surface in the BW, we can get the same simple $q = q(\mathcal{M},\sigma)$ solution by

$$4\,\pi\,R^3\,p_R = \frac{M_{\rm P}\,\mathcal{M}\,K}{\ell_{\rm P}\,\sigma\,R^2}\left(2 + R\,\nu'_R\right) - (M-\mathcal{M})\left(1 + R\,\nu'_R\right)$$

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In our solution $(*)$ R is still a free parameter. We need an interior solution to fix it!

Finding the tidal charge q

Let us consider the exact interior solution (JO Int. J. Mod. Phys. D **18**, 837 (2009))

$$e^{\nu} = A\left(1 + C\,r^2\right)^4$$

$$\rho = C_\rho\left(\frac{M_{\rm P}}{\ell_{\rm P}}\right)\frac{C\left(9 + 2\,C\,r^2 + C^2\,r^4\right)}{7\,\pi\,(1 + C\,r^2)^3}$$

where $C_\rho = C_\rho(K)$ is a constant to be determined for consistency, and

$$p_R = \left(\frac{M_{\rm P}}{\ell_{\rm P}}\right)\frac{2\,C\left(2 - 7\,C\,R^2 - C^2\,R^4\right)}{7\,\pi\,(1 + C\,R^2)^3} = 0.$$

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$$R = 2\,n\left(\frac{\ell_{\rm P}}{M_{\rm P}}\right)\frac{M}{C_\rho}; \qquad K = \left(\frac{M_{\rm P}}{M_{\rm G}}\right)^2 \frac{\ell_{\rm G}}{M_{\rm G}}; \qquad C_\rho = (M_{\rm G}/M_{\rm P})^4$$

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$$\mathcal{M} = \frac{M^3}{M^2 + n_1\,M_{\rm G}^2} \qquad q = \frac{\ell_{\rm G}^2\,M^2}{n\,\left(M^2 + n_1\,M_{\rm G}^2\right)}$$

where we used $\sigma \simeq \ell_{\rm G}^{-2}$.

Black hole limit and minimum mass

The ADM mass $\mathcal{M}$ and tidal charge q do not explicitly depend on the star radius R, and we can therefore assume they are valid in the limit $R \to 0$ (or, more cautiously, $R \ll \ell_{\rm G}$). Hence

$$e^{\nu} = 1 - \frac{2\,\ell_{\rm P}\,M^3}{M_{\rm P}\left(M^2 + n_1\,M_{\rm G}^2\right)r}\left(1 + \frac{\ell_{\rm G}^2\,M_{\rm P}}{2\,n\,\ell_{\rm P}\,M\,r}\right)$$

which can be used to describe a BH of “bare” (or proper) mass M.

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which can be used to describe a BH of “bare” (or proper) mass M. Using the dimensionless proper mass $\bar{M} = M/M_{\rm G}$ we have

$$\bar{\mathcal{M}} = \frac{\mathcal{M}}{M_{\rm G}} = \frac{\bar{M}^3}{\bar{M}^2 + n_1} \simeq \frac{\bar{M}^3}{0.1 + \bar{M}^2}$$

and

$$\bar{q} = \frac{q}{\ell_{\rm G}^2} = \frac{\bar{M}^2}{n\,(n_1 + \bar{M}^2)} \simeq \frac{\bar{M}^2}{0.2 + 1.6\,\bar{M}^2}$$

Black hole limit and minimum mass

We obtain the horizon radius

$$h = \frac{\ell_{\mathrm{P}}}{M_{\mathrm{P}}}\left(\mathcal{M} + \sqrt{\mathcal{M}^2 + q\,\frac{M_{\mathrm{P}}^2}{\ell_{\mathrm{P}}^2}}\right)$$

and the classicality condition $h \gtrsim \lambda_M$ reads

$$\frac{M}{M_{\mathrm{P}}^2}\left(\mathcal{M} + \sqrt{\mathcal{M}^2 + q\,\frac{M_{\mathrm{P}}^2}{\ell_{\mathrm{P}}^2}}\right) \gtrsim 1$$

We expand for $M \sim \mathcal{M} \simeq M_{\mathrm{G}} \ll M_{\mathrm{P}}$, thus obtaining

$$\frac{h^2}{\lambda_C^2} \simeq \frac{M^2}{M_{\mathrm{P}}^2}\,\frac{q}{\ell_{\mathrm{P}}^2} \simeq \frac{M_{\mathrm{G}}^2}{M_{\mathrm{P}}^2}\,\bar{M}^2\,\bar{q}\,\frac{\ell_{\mathrm{G}}^2}{\ell_{\mathrm{P}}^2} \simeq \bar{M}^2\,\bar{q} \simeq 1$$

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$$\frac{h^2}{\lambda_C^2} \simeq \frac{M^2}{M_{\mathrm{P}}^2} \frac{q}{\ell_{\mathrm{P}}^2} \simeq \frac{M_{\mathrm{G}}^2}{M_{\mathrm{P}}^2} \bar{M}^2 \, \bar{q} \, \frac{\ell_{\mathrm{G}}^2}{\ell_{\mathrm{P}}^2} \simeq \bar{M}^2 \, \bar{q} \simeq 1$$

or $\bar{M}^4 \simeq n \left(n_1 + \bar{M}^2 \right)$, which yields

$$M_c \simeq 1.3 \, M_{\mathrm{G}}$$

This can be viewed as the minimum allowed mass for a semiclassical BH in the BW.

Two more BW astrophysical solutions

- A non-uniform BW solution (with $M/R \simeq 0.38\frac{M_{\rm P}}{\ell_{\rm P}}$)

$$e^{-\lambda(r)} = 1 - \frac{3\,C\,r^2}{2\,(1+C\,r^2)} + f^*(r); \quad e^{\nu(r)} = A(1+Cr^2)^3$$

$$\rho(r) = \frac{3\,C\,\left(3+Cr^2\right)}{2\,k^2\,(1+Cr^2)^2}; \quad p(r) = \frac{9\,C\,\left(1-Cr^2\right)}{2\,k^2\,(1+Cr^2)^2}$$

$$M_c \simeq 1.22\,M_{\rm G}$$

- The Schwarzschild solution (with $M/R \simeq 0.28\frac{M_{\rm P}}{\ell_{\rm P}}$)

$$e^{-\lambda} = 1 - \frac{r^2}{C^2} + f^*; \quad e^{\nu} = \left(A - B\sqrt{1-\frac{r^2}{C^2}}\right)^2$$

$$\rho = \frac{3}{k^2\,C^2}; \quad p(r) = \frac{\rho}{3}\left[\frac{3\,B\,\sqrt{1-\frac{r^2}{C^2}} - A}{A - B\,\sqrt{1-\frac{r^2}{C^2}}}\right]$$

$$M_c \simeq 1.9\,M_{\rm G}$$

Astrophysical consequences for M_c

From $\simeq \lambda_C$, we find

$$\bar{M}^2 \bar{q} \simeq 1$$

which yields

$$M_c^2 \simeq \frac{n}{2}\left(1 + \sqrt{1 + \frac{4\,n_1}{n}}\right)\, M_G^2$$

Now, from GR we know that the compactness of any stable stellar distribution of mass M and radius R must satisfy the constraint $M/R < 4/9$. This bound leads to $n > 9/8$, and, correspondingly,

$$M_c{}^2 \simeq \frac{n}{2}\underbrace{\left(1 + \sqrt{1 + \frac{4\,n_1}{n}}\right)}_{>2}\, M_G^2$$

Always a critical mass M_c above M_G JO, R. Casadio, arXiv:1212.0409 [gr-qc] (2012).

Conclusions

- We have analyzed analytical descriptions of stars in the BW, with the tidal charge as an explicit function of the ADM mass and brane tension, which was still an open problem.
- Different astrophysical solutions lead to different critical mass.
- By using the general relativistic constraint $M/R < 4/9$ we found that the minimum mass of a semiclassical microscopic black hole M_c is always above M_G
- A more general solution regarding charged black holes will be considered (in progress).

THANK YOU!