Some consequences of a proposed local and gauge invariant version of QCD

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The motivations of a local and gauge invariant version of QCD for massive fermions proposed in a previous work, will be reviewed. It will be underlined that its action includes new vertices which eventually could had been overlooked before since at first sight, they look as breaking power counting renormalizability. However, the fact that these terms also modify the quark propagators to become more convergent at large momenta, strongly suggests that theory is renormalizable. Accepting this view, it also can be argued that all the four fermions terms constituting the Nambu-Jona-Lasinio models, could be included as counterterms in a slightly generalized renormalization procedure for massless QCD. Then, a way for exploring the possibility of generating the quark mass hierarchy Is identified.

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Overview

1. Review the mass generation effects followed from a previously proposed non local modified version of massless QCD including gluon and quark condensates.

2. How to “move” the gluon and quark condensate effects to be represented by vertex terms in a modified action.

3. The proposed form for a local and gauge invariant version for the QCD Lagrangian.

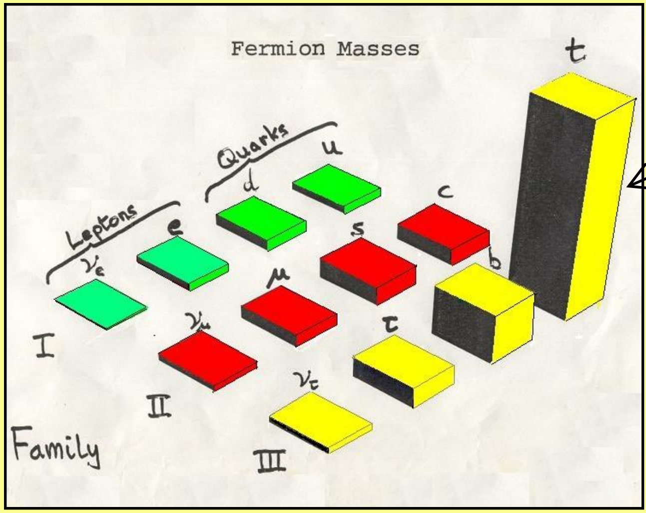
4. The pure “quark” condensate case.

5. The possibility that the scheme can dynamically describe the quark mass spectrum.

6. Remarks arguing that the proposed model could be an effective model being equivalent to massless QCD.

7. Summary

1. **Review the mass generation effects followed from a previously proposed non local modified version of massless QCD including gluon and quark condensates.**



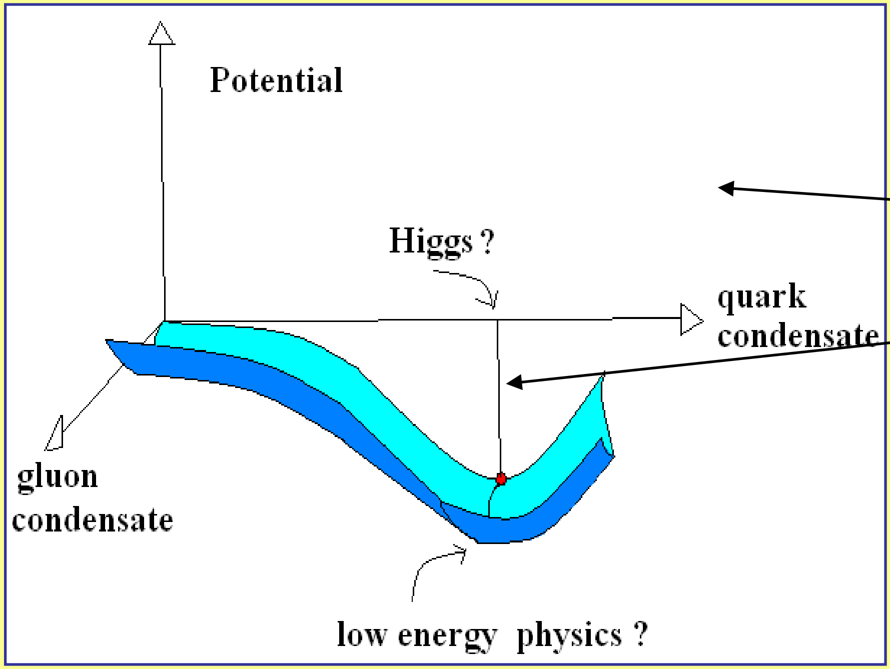
Determining the origin of the wide range of values spanned by the **quark masses**, and more generally, the structure of the **lepton and quark mass spectrum** (illustrated in the figure on the left, but not at a correct scale) is one of the central problems of **High Energy Physics**.

In former works (**Mod. Phys. Lett. A10, 2413 (1995), Phys. Rev. D 62 074018 (2000), Eur. Phys. J. C23, 289 (2002), JHEP (04), 044 (2003), Eur. Phys. J. C47, 95 (2006), Eur. Phys. J. C47, 355 (2006), Eur. Phys. J. C 55, 85 (2008), Eur. Phys. J. C 64: 133 (2009), Eur. Phys. J. C 71, 1620, (2011)**), the formulation and implications of a modified version of the **PQCD** have been explored.

Close related analysis discussions had been considered by various authors starting even before our activity. We refer only few of these authors:

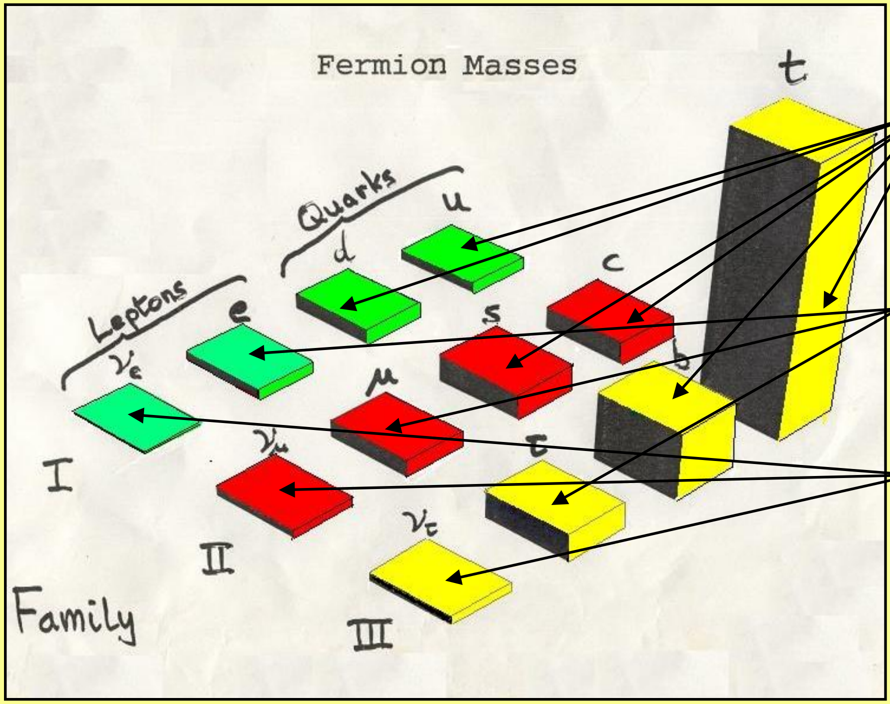
L. S. Celenza and C. M. Sakin, Phys. Rev. D34, 1591 (1986),
H. J. Munczek and A. M. Nemirovski, Phys. Rev. D28, 171(1983) ,
P. C. Tandy, Prog. Part. Nucl. Phys. 39, 117 (1997),
C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. 33, 477 (1994) ,
R.T. Cahill and S.M. Gunner, Fizika B,171 (1998),
A. Natale, Mod.Phys.Lett. A14, 2049 (1999),
H.-P. Pavel, D. Blaschke, V.N. Perbushin and G. Ropke, Int. J. Mod. Phys. A14, 205(1999),
P. Hoyer, Act. Phys. Pol. B34, 3121 (2003).

The general motivation for this previous work was created by the suspicion about that the strong degeneration of the non-interacting QCD vacuum (the state which is employed for the construction of the standard Feynman rules of **PQCD**) in combination with the strong forces carried by the QCD fields, could produce a large **dimensional transmutation effect,** implying the generation of large quark and gluon condensates, and correspondingly **large quak masses and confinement**. Historically, the first motivation was the aim in to develop a sort of improved “Savvidi Chromomagnetic field model” not showing the known symmetry difficulties, which affected that scheme, which was one the first models indicating the existence of confinement in QCD.



If the search turns to be well oriented, the following picture could is speculated to arise:

- A sort of the Top condensate model might be the effective action for massless QCD.
- A Top quark condensate arising at the same SM model level could play the role of the Higgs.
- The SM could be closed by generating all the masses within its proper framework as follows:



The six quarks could get their masses thanks to the mentioned flavor symmetry breaking.

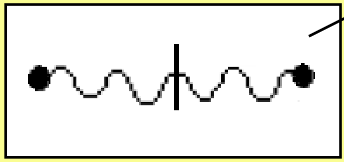
The electron, muon and tau leptons, would receive their intermediate mass values due to radiative corrections mediated by their electromagnetic interactions with quarks.

Finally, only the weak interacting character of the three neutrinos with all the particles could determine their even smaller mass values.

$$|\phi\rangle = \exp \sum_a \left[C_1(p) A^{a+}_{p,1} A^{a+}_{p,1} + C_2(p) A^{a+}_{p,2} A^{a+}_{p,2} + C_3(p) \right.$$
$$\left. \times (B^{a+}_p A^{L,a+}_p + i\bar{c}^{a+}_p c^{a+}_p)\right]|0\rangle,$$

The **BCS like initial state** for the derivation of the modified Feynman rules for the case of gluon condensation in the absence of quark pair condensation. **(Phys. Rev. D 62 074018 (2000))**

$$G^{ab}_{\mu\nu}(p) = \left(\frac{1}{p^2 + i\epsilon} - i\delta(p)C\right)\delta^{ab} g^{\mu\nu}$$

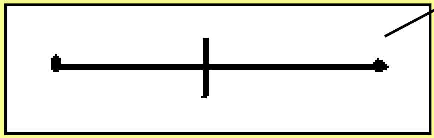


The originally proposed modified gluon propagator reflecting the condensation of zero momentum gluons, was reproduced after choosing appropriate values for the parameters in the initial **BCS** like state. (**Mod. Phys. Lett. A10, 2413 (1995)**)

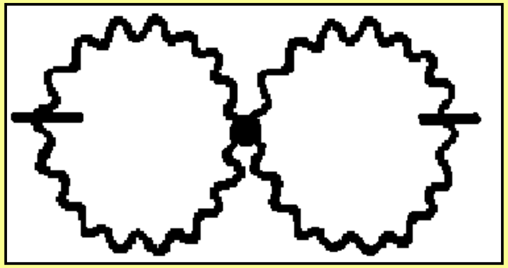
$$|\phi^*\rangle = \lim_{p\to 0} \exp\left(\sum_{f_1 f_2} \tilde{C}_q^{f_1 f_2}(p)\, \bar{q}^+_{f_1}(p)\, q^+_{f_2}(p)\, |\phi\rangle\right)$$

The results for gluons motivated the idea of also considering the quarks as massless and search for the possibility of generate their masses dynamically, thanks to the condensation of quark pairs. For this purpose the **BCS like initial state** was generalized to include the quark pair condensates in massless QCD. (**JHEP (04), 044 (2003), Eur. Phys. J. C 47, 95–112 (2006)**)

$$G_q^{i_1 i_2; f_1 f_2}(p) = \left(-\frac{\gamma^\mu p_\mu \delta^{f_1 f_2}}{p^2 + i\epsilon} + i\delta(p) C^{f_1 f_2}\right)\delta^{i_1 i_2}$$



In this case a covariant **free-quark propagator** was following .



The scheme determined a gluonic Lagrangian mean value in the simplest approximation in terms of *C* and interaction coupling *g.*

$$\langle G^2 \rangle = \frac{288 g^2 C^2}{(2\pi)^8},$$

Then, making use of an estimate for the mean value of the gluon Lagrangian, the parameter ***g²C*** was evaluated.

$$\langle g^2 G^2 \rangle \cong 0.5 \left(\mathrm{GeV}/c^2\right)^4$$

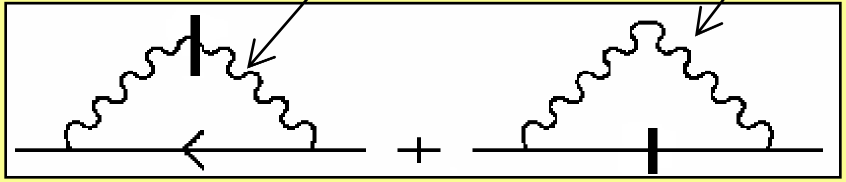
$$g^2 C = 64.9394 \left(\mathrm{GeV}/c^2\right)^2$$

$$\frac{1}{p^2}\left(-p^2 p_\mu \gamma^\mu \left(1 - \frac{M^2}{p^2}\right)\delta^{f_1 f_2} - \frac{4 g^2 C_F}{(2\pi)^4} C^{f_1 f_2}\right)\Psi_i^{f_2}(p) = 0\,,$$

Dyson equation for quarks in the simplest condensate dependent approximation.

$$M^2 = \frac{2 g^2 C_F C}{(2\pi)^4} = 0.1111 \left(\mathrm{GeV}/c^2\right)^2$$

The quark self-energy in the lowest order in the power expansion in the condensate parameters.



$Quark\ q$	$m_{qLow}^{Exp}(MeV)$	$m_{qUp}^{Exp}(MeV)$	$m_q^{Theo}(MeV)$
u	1.5	5	333–0
d	3	9	333–0
s	60	170	339–326–13
c	1100	1400	1255
b	4100	4400	4233
t	168600	17900	173500

1) Disregarding the gluon condensate, the quark condensate matrix was fixed in a diagonal form by reproducing the measured Lagrangian quark masses as the solutions of the Dyson equation.
2) After that, the solution of the Dyson equation including the value of ***g²C*** through the constant *M* **,** furnished the "constituent" values of 1/3 of the nucleon mass for the light quarks obtained in Ref. **Eur. Phys. J C23, 289 (2002).**

2. How to "move" the gluon and quark condensate effects to be represented by vertex terms in a modified action.

The general Feynman diagram (or perturbative) expansion of the modified theory was expressed by the generating functional of Green functions of the form

$$Z[j,\eta,\overline{\eta},\xi,\overline{\xi}] = \frac{I[j,\eta,\overline{\eta},\xi,\overline{\xi}]}{I[0,0,0,0]},$$

$$I[j,\eta,\overline{\eta},\xi,\overline{\xi}] = \exp(V^{int}[\frac{\delta}{\delta j},\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\frac{\delta}{\delta\overline{\xi}},\frac{\delta}{-\delta\xi}])\times$$

$$\exp(\int\frac{dk}{(2\pi)^D}j(-k)\frac{1}{2}D(k)j(k))\times$$

$$\exp(\sum_f\int\frac{dk}{(2\pi)^D}\overline{\eta}_f(-k)G_f(k)\eta_f(k))\times$$

$$\exp(\int\frac{dk}{(2\pi)^D}\overline{\xi}(-k)G_{gh}(k)\xi(k)),$$

$$f = 1,2,...6.$$

where *D, G_f* and G_{gh} are the previously mentioned modified gluon, quark and ghost propagators.

The Lagrangian defining the vertex part V^{int} of the Wick expansion was the usual QCD one.

$$S_g = \int dx(-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{2\alpha}\partial_\mu A^a_\mu\partial_\nu A^a_\nu + \overline{c}^a\overleftarrow{\partial}_\mu D^{ab}_\mu c^b - \sum_f\overline{\Psi}^i_f\ i\gamma_\mu D^{ij}_\mu\Psi^j_f),$$

$$V^{int} = S_g - S_0,$$

with the conventions for the gluon fields

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu,$$

$$D^{ij}_\mu = \partial_\mu\delta^{ij} + ig\ A^a_\mu T^{ij}_a, \qquad D^{ab}_\mu = \partial_\mu\delta^{ab} + gf^{abc}\ A^c_\mu,$$

$$\{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu}, \qquad [T_aT_b] = if^{abc}T_c.$$

The already mentioned gluon, quark and ghost propagators had the forms (***Eur. Phys. J. C* 47**, 95–112 (2006)):

$$D^{ab}_{\mu\nu}(k) = \delta^{ab}\left(\frac{1}{k^2}\left(\delta_{\mu\nu} - (1-\alpha)\frac{k_\mu k_\nu}{k^2}\right)\theta_N(k) + C_g \delta^D(k)\delta_{\mu\nu}\right),$$

$$G^{ij}_f(k) = \delta^{ij}\left(\frac{\theta_N(k)}{m + \gamma_\mu k_\mu} + C_f \delta^D(k) I\right),$$

$$G^{ab}_{gh}(k) = \delta^{ab}\frac{\theta_N(k)}{k^2}.$$

These are basically the same Feynman propagators of PQCD, after adding the Dirac delta functions at zero momentum (Munczek and Nemirovsky) that represent the gluon and quark condensates. They included a Heaviside function (introduced by Nakanishi to solve difficulties in the quantization of the free gauge theory). They make the Feynman contribution to the propagator vanish in a small neighborhood of the zero value of the momentum. The introduction of this function allowed to get rid of various singular contributions to the Feynman expansion that could have been appeared, due to the distributional character of the Dirac delta function modifications of the propagators (***Eur. Phys. J. C* 47**, 95–112 (2006)).

"Moving" the gluon and quark condensate effects to be represented by vertex terms in a modified action.

Noting that the gluon and quark terms modifying the propagators are quadratic forms in the zero momentum component of the spatial Fourier transforms of the sources:

○ The exponential of those terms in ***Z*** were expressed as Gaussian integrals over auxiliary vector and fermion parameters $\alpha, \chi, \overline{\chi}$ which makes the dependence of the Feynman integrands linear in the gluon and fermion sources.

○ Then, after evaluating the simple commutator of an exponential having an argument being linear in the sources with the exponential of the vertex part ***V****int* as expressed in terms of the functional derivatives over the sources, ***Z*** was expressed as

$$
\begin{aligned}
Z[j,\eta,\overline{\eta},\xi,\overline{\xi}] &= \frac{1}{\mathcal{N}}\int\int d\alpha d\overline{\chi} d\chi \exp[-\sum_f \overline{\chi}^i_{f,\ r}\chi^i_{f,\ r} - \frac{\alpha^a_\mu \alpha^a_\mu}{2}] \\
&\exp\,[\overline{S}^*_g[\frac{\delta}{\delta j},\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\alpha,\chi,\overline{\chi}]]\times \\
&\int \mathcal{D}[A,\overline{\Psi},\Psi,\overline{c},c]\exp\,[\int dx\ (j(x)A(x)+\sum_f(\overline{\eta}_f(x)\Psi_f(x)+\overline{\Psi}_f(x)\eta_f(x)))] \\
&= \frac{1}{\mathcal{N}}\int\int d\alpha d\overline{\chi} d\chi \exp\,[-\sum_f \overline{\chi}^i_{f,\ r}\chi^i_{f,\ r} - \frac{\alpha^a_\mu \alpha^a_\mu}{2}]\times \\
&\int \mathcal{D}[A,\overline{\Psi},\Psi,\overline{c},c]\exp\,[\int dx\ (\overline{S}^*_g[A,\Psi,\overline{\Psi},c,\overline{c},\alpha,\chi,\overline{\chi}]+ \\
&j(x)A(x)+\sum_f(\overline{\eta}_f(x)\Psi_f(x)+\overline{\Psi}_f(x)\eta_f(x)))].
\end{aligned}
$$

The new action appearing can be written in the form

$$
\begin{aligned}
\overline{S}^*_g &= \overline{S}^*_g[A,\Psi,\overline{\Psi},c,\overline{c},\alpha,\chi,\overline{\chi}] \\
&= \int dx[-\frac{1}{4}F^a_{\mu\nu}(A+(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\alpha)F^a_{\mu\nu}(A+(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\alpha) \\
&- \frac{1}{2\alpha}\partial_\mu A^a_\mu \partial_\nu A^a_\nu - \overline{c}^a\partial_\mu D^{ab}_\mu(A+(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\alpha)c^b \\
&- \sum_f (\overline{\Psi}^i_f\ i\gamma_\mu D^{ij}_\mu(A+(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\alpha)\Psi^j_f + \overline{\Psi}^i_f\ i\gamma_\mu D^{ij}_\mu(A)\chi^j_f(\frac{C_f}{(2\pi)^D})^{\frac{1}{2}} + (\frac{C_f}{(2\pi)^D})^{\frac{1}{2}}\overline{\chi}^i_f\ i\gamma_\mu D^{ij}_\mu(A)\Psi^j_f \\
&+ (\frac{C_f}{(2\pi)^D})(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\overline{\chi}^i_f\ i\gamma_\mu\ ig\alpha^a_\mu T^{ij}_a\chi^j_f)].
\end{aligned}
$$

It is needed to underline that the parameters resulted as constants independent of the space time coordinates. This was a consequence of the simple perturbative modifications of the free QCD vacuum employed to connect the interaction for the construction of the Wick expansion. Therefore, all the space time derivatives of these parameters in the above formula, in fact vanish, which makes that the expression can not be explicitly seen as a quantized gauge invariant theory. This represents a limitation of the previous version of the modified massless QCD considered up to now. However, the above written form of the action suggests a direct solution of this problem to be examined in what follows.

3. The proposed form for a local and gauge invariant version for the QCD Lagrangian.

$$Z[j,\eta,\overline{\eta},\xi,\overline{\xi}]=\frac{1}{\mathcal{N}}\int\int\mathcal{D}[\alpha,\overline{\chi},\chi]\exp[-\sum_f\overline{\chi}^i_{f,\ r}(x)\chi^i_{f,\ r}(x)-\frac{\alpha^a_\mu(x)\alpha^a_\mu(x)}{2}]\times$$
$$\int\mathcal{D}[A,\overline{\Psi},\Psi,\overline{c},c]\exp[\int dx\ (S^*_g[A,\Psi,\overline{\Psi},c,\overline{c},\alpha,\chi,\overline{\chi}]+$$
$$j(x)A(x)+\sum_f(\overline{\eta}_f(x)\Psi_f(x)+\overline{\Psi}_f(x)\eta_f(x)))],$$

The solution suggested starts by "promoting" the $\alpha,\chi,\overline{\chi}$ parameters to be full space-time dependent functions.

$$\begin{aligned}
S^*_g &= S^*_g[A,\Psi,\overline{\Psi},c,\overline{c},\alpha,\chi,\overline{\chi}]\\
&=\int dx[-\frac{1}{4}F^a_{\mu\nu}(A+(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\alpha)F^a_{\mu\nu}(A+(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\alpha)\\
&-\frac{1}{2\alpha}\partial_\mu A^a_\mu\partial_\nu A^a_\nu-\overline{c}^a\partial_\mu D^{ab}_\mu(A)c^b)\\
&-\sum_f\overline{\Psi}^i_f\ i\gamma_\mu D^{ij}_\mu(A+(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\alpha)\ \Psi^j_f\\
&-\sum_f\overline{\Psi}^i_f\ i\gamma_\mu D^{ij}_\mu(A)\chi^j_f\ (\frac{C_f}{(2\pi)^D})^{\frac{1}{2}}-\sum_f(\frac{C_f}{(2\pi)^D})^{\frac{1}{2}}\overline{\chi}^i_f\ i\gamma_\mu D^{ij}_\mu(A)\Psi^j_f\\
&-\sum_f(\frac{C_f}{(2\pi)^D})(\frac{2C_g}{(2\pi)^D})^{\frac{1}{2}}\overline{\chi}^i_f\ i\gamma_\mu\ ig\alpha^a_\mu T^{ij}_a\chi^j_f].
\end{aligned}$$

The new action (without considering the FP gauge fixing terms) is taken as the same one as in the previous modified QCD, in which now the new fields $\alpha,\chi,\overline{\chi}$ transform in a homogeneous way under gauge changes.

4. The pure “quark” condensate case

In the limit of vanishing gluon condensate parameter, the Z functional reduces to

$$Z[j,\eta,\overline{\eta},\xi,\overline{\xi}] = \frac{1}{\mathcal{N}}\int\int \mathcal{D}[\alpha,\overline{\chi},\chi]\exp[-\sum_f \overline{\chi}^i_{f,\ r}(x)\chi^i_{f,\ r}(x) - \frac{\alpha^a_\mu(x)\alpha^a_\mu(x)}{2}]\times$$
$$\int \mathcal{D}[A,\overline{\Psi},\Psi,\overline{c},c]\exp[\int dx\ (S^*_g[A,\Psi,\overline{\Psi},c,\overline{c},\chi,\overline{\chi}]+$$
$$j(x)A(x)+\sum_f(\overline{\eta}_f(x)\Psi_f(x)+\overline{\Psi}_f(x)\eta_f(x)))],$$

where the action now adopts the simpler form

$$S^*_g = S^*_g[A,\Psi,\overline{\Psi},c,\overline{c},\chi,\overline{\chi}]$$
$$=\int dx\ [-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{2\alpha}\partial_\mu A^a_\mu\partial_\nu A^a_\nu - \overline{c}^a\partial_\mu D^{ab}_\mu c^b)$$
$$-\sum_f(\overline{\Psi}^i_f\ i\gamma_\mu D^{ij}_\mu\Psi^j_f + \overline{\Psi}^i_f\ i\gamma_\mu D^{ij}_\mu\chi^j_f\ (\frac{C_f}{(2\pi)^D})^{\frac{1}{2}} + (\frac{C_f}{(2\pi)^D})^{\frac{1}{2}}\overline{\chi}^i_f\ i\gamma_\mu D^{ij}_\mu\Psi^j_f)],$$

which is basically the massless QCD action plus two linear terms in the new fermion fields $\chi,\overline{\chi}$

The Gaussian integral over the auxiliary functions $\chi, \overline{\chi}$ can be evaluated by solving the equations

$$\frac{\delta S_g^*[A,\overline{\Psi},\Psi,\overline{c},c,\overline{\chi},\chi]}{\delta\overline{\chi}_f^i} = -\chi_f^i - (\frac{C_q}{(2\pi)^D})^{\frac{1}{2}} i\gamma_\mu D_\mu^{ij}\Psi_f^j = 0,$$

$$\frac{\delta S_g^*[A,\overline{\Psi},\Psi,\overline{c},c,\overline{\chi},\chi]}{\delta\chi_f^i} = \overline{\chi}_f^i + \overline{\Psi^j}_f \; i\gamma_\mu \overleftarrow{D}_\mu^{ji}(\frac{C_q}{(2\pi)^D})^{\frac{1}{2}} = 0,$$

$$D_\mu^{ji} = \delta^{ji}\partial + igA_\mu^a T_a^{ji}, \quad \overleftarrow{D}_\mu^{ji} = -\delta^{ji}\overleftarrow{\partial} + igA_\mu^a T_a^{ji},$$

Then Z reduces to the form

$$Z = \frac{1}{\mathcal{N}}\int \mathcal{D}[A,\overline{\Psi},\Psi,\overline{c},c,] \exp[S[A,\overline{\Psi},\Psi,\overline{c},c]],$$

$$S[A,\overline{\Psi},\Psi,\overline{c},c] = S_{mqcd}[A,\overline{\Psi},\Psi,\overline{c},c] + \; S^q[A,\overline{\Psi},\Psi]$$

$$S_{mqcd}[A,\overline{\Psi},\Psi,\overline{c},c] = \int dx(-\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a - \frac{1}{2\alpha}\partial_\mu A_\mu^a \partial_\nu A_\nu^a - \sum_f \overline{\Psi}_f^i \; i\gamma_\mu D_\mu^{ij}\Psi_f^j - \overline{c}^a\partial_\mu D_\mu^{ab}c^b),$$

In which the action is the massless QCD one with added local terms of the form

$$S^q[A,\overline{\Psi},\Psi] = -\sum_{f\ f'} \frac{C_{f\ f'}}{(2\pi)^D}\int dx \overline{\Psi}_f^j \; i\gamma_\mu \overleftarrow{D}_\mu^{ji} \; i\gamma_\nu D_\nu^{ik}\Psi_{f'}^k.$$

which are gauge invariant and relevantly also do not disturb power counting renormalizability because they also make the quark propagator to decrease with the square of the momentum at large values.

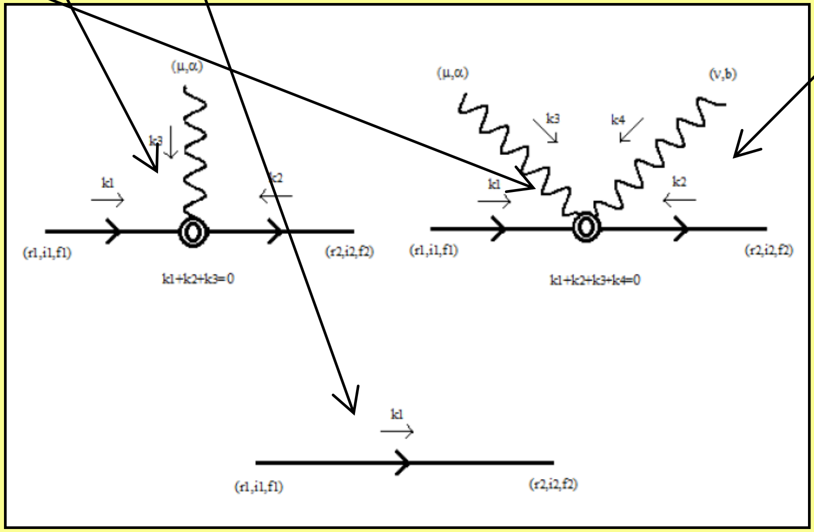
The quark propagator of the new expansion takes the form

$$S_f(p) = \frac{1}{-\gamma_\nu p^\nu - \frac{C_f}{(2\pi)^D} p^2} \equiv \frac{(-\gamma_\nu p^\nu - \frac{C_f}{(2\pi)^D} p^2)^{rr'} \delta^{ii'}}{p^2(1 - (\frac{C_f}{(2\pi)^D})^2 p^2)}$$
$$= \frac{m_f}{(m_f^2 - p^2)} - \frac{m_f^2}{(m_f^2 - p^2)} \frac{\gamma_\nu p^\nu}{p^2} = S_f^{(s)}(p) + S_f^{(f)}(p).$$

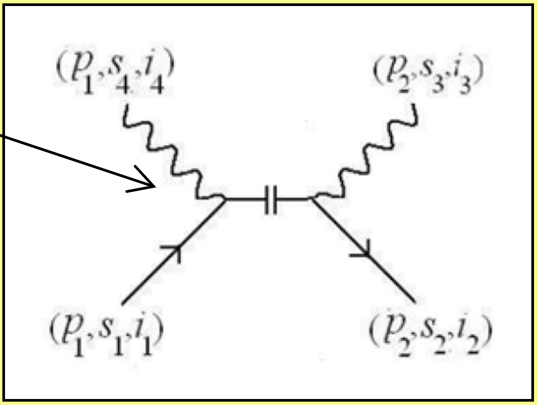
which clearly shows its decreasing with the square of the momentum, and decomposes in the sum of a scalar like and a Dirac like components and determines masses for the quarks.

The new action terms also create two new vertices in the modified Feynman expansion

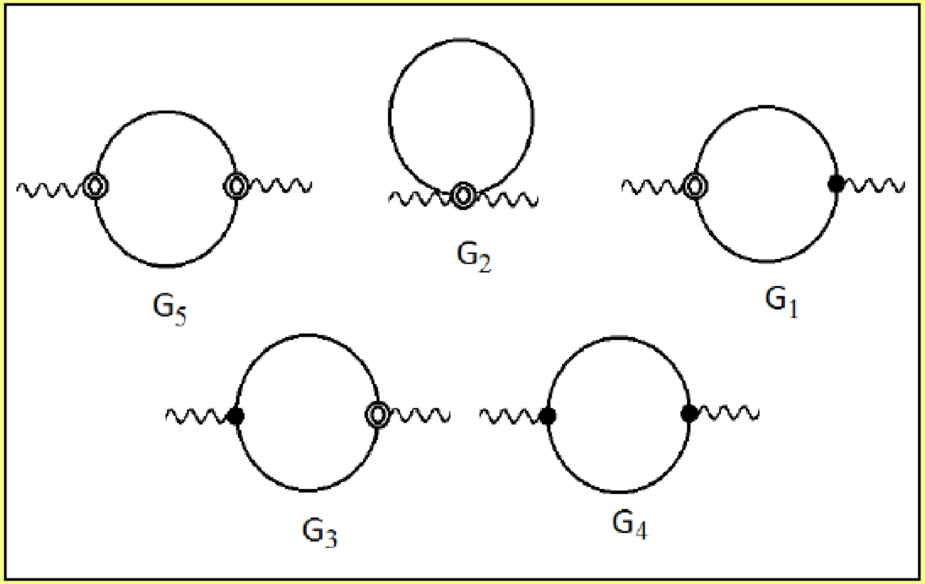
$$V^{(3)(\mu,a)}_{(r_1,i_1,f_1)((r_2,i_2,f_2)}(k_1,k_2,k_3) = g\frac{C_{f_1 f_2}}{(2\pi)^D} T_a^{i_1 i_2}(-(k_{1\alpha}\gamma^\alpha)^{r_1 s}(\gamma^\mu)^{s r_2} + (\gamma^\mu)^{r_1 s}(k_{2\alpha}\gamma^\alpha)^{s r_2}),$$
$$V^{(4)\ (\mu,a)(\nu,b)}_{(r_1,i_1,f_1)((r_2,i_2,f_2)}(k_1,k_2,k_3,k_4) = g^2\frac{C_{f_1 f_2}}{(2\pi)^D} T_a^{i_1 i} T_b^{i i_2}(\gamma^\mu)^{r_1 s}(\gamma^\nu)^{s r_2}.$$



The four legs vertex is the local counterpart of the non local one appearing in the previous modified expansion



The contributions to the polarization operator determined by the new vertices



The gluon and ghost one loop contributions are the same as in massless QCD

The result became transversal, thus satisfied the Ward identity associated to the gauge invariance

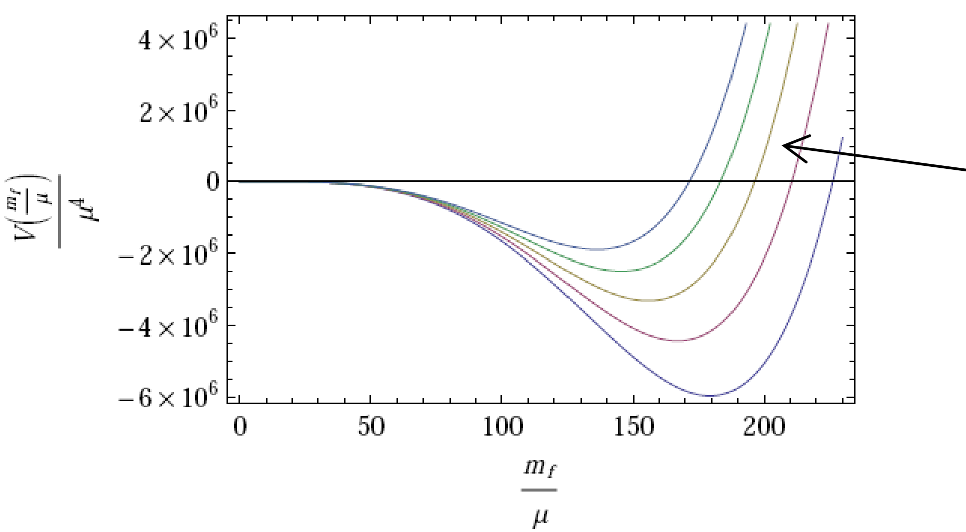
$$\Pi^{ab}_{\mu\nu}(q) = \Pi^{ab}_{T}(q)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}),$$

$$\begin{aligned}\Pi^{ab}_{T}(q) &= \Pi^{ab}_{5T}(q) + \Pi^{ab}_{2T}(q) + \Pi^{ab}_{1T}(q) + \Pi^{ab}_{3T}(q) + \Pi^{ab}_{4T}(q)\\ &= -\frac{g^2\delta^{ab}}{D(D-1)}(D(D-26)+8)L_m(m_f) + \frac{g^2\delta^{ab}(2-D)}{(D-1)}q^2 L_{12}(q) +\\ &+ \frac{g^2\delta^{ab}}{2(D-1)}(-2D(D+17)m_f^2 - 9D(2+D)q^2)L_{34}(q,m_f).\end{aligned}$$

The effective potential up to the two loops approximation, is determined by the integral over the momentum of the evaluated polarization operator after contracted with the free gluon propagator

$$V(m_f) = 0.0656145\ m_f^4\ + 273.18\ m_f^4\ \frac{g_0^2}{4\pi} - (0.0379954 + 322.47\ \frac{g_0^2}{4\pi})\ m_f^4\ Log(\frac{m_f}{\mu}) + 132.527\ m_f^4\ \frac{g_0^2}{4\pi}\ Log^2(\frac{m_f}{\mu}).$$

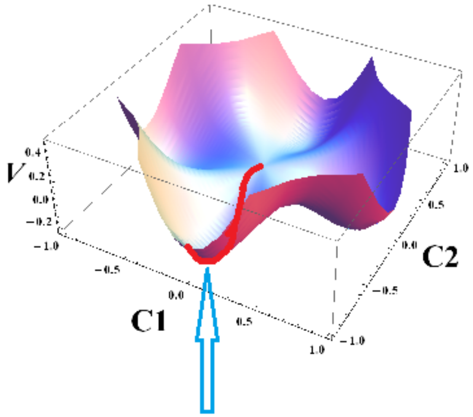
The calculated expression for the finite part of the effective potential as a function of the fermion mass (condensate parameter), the dimensional regularization parameter and the coupling.



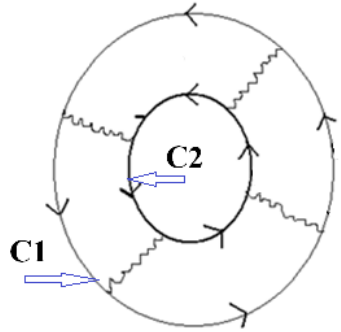
The figure illustrates the effective potential as a function of the ratio between the fermion mass m_f and the dimensional regularization parameter μ. At μ = 1 GeV, various small values of the coupling constant around g_0 = 0.0271828 were chosen only to illustrate that a minimum of the potential can be fixed at a m_f being close to the top quark mass of 173 GeV. For higher values of the coupling the minimum tends to disappear in this one loop approximation.

The results indicate that, assumed that the coupling is small, the treatment produces a mass generation effect. This conclusion could be of interest for studying the possibility of a dynamical generation of a weak interaction sector as in the SM model.

5) The possibility that the scheme can dynamically describe the quark mass spectrum.



The hierarchy: an approximate minimum for a single non vanishing condensate



It can only follow in a three or higher loop approximation where more than two types of quark flavours can "interfere" in a diagram.

○ It can be noted that the two loop result for the effective potential in the case of strong coupling is insufficient to decide about the possibility or not of a dynamical breaking of the flavor symmetry in this theory.

○ By example, in the two loop case, in a given diagram only one kind of quark propagator can appear. Thus, the results for the potential as a function of their condensate parameters are identical for all the quark types (flavors). Therefore, the minimal potential is simply the sum of six identical contributions, which does not show any flavor symmetry breaking.

○ However, in three and higher loop contributions, as the one illustrated in the above figure, two or more types of quark propagators can participate. Those diagrams are increasingly important at higher coupling values.

○Therefore, these terms of the potential could be able to generate minima of the potential as functions of the six quark condensates, appearing around a finite value of a particular quark condensate, but for nearly null or smaller values of the other condensate types. Henceforth, the appearance of this effect could furnish an explanation for the top quark mass. Afterwards, if further analogous steps occur, the suspected hierarchical behavior of the quark mass spectrum could be predicted.

6) Remarks arguing that the proposed model could be an effective theory being equivalent to massless QCD.

○ The fact that the new two gluon two quarks vertices can be included in the renormalized action, also leads to conceive that additional counterterms could also become allowed countertems by the new highly decreasing behavior of the quark propagator at large momenta.

○ Then, it follows the surprising conclusion, that almost all the Lagrangian terms which define the Nambu-Jona–Lasinio (NJL) models are allowed to be considered as counterterms! .

○ Moreover, since the theory seems to be power counting renormalizable, the mass generation properties embodied in the usual non-renormalizable phenomenological NJL models, could dynamically appear now in the context of a specially renormalized massless QCD!.

$$S_g = \int dx \ \Big\{ -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2\alpha} \partial_\mu A^{a\mu} \partial_\nu A^{a\nu} + \overline{\Psi^i}_q \, i\gamma^\mu D^{ij}_\mu \Psi^j_q + \overline{c}^a \overleftarrow{\partial}_\mu D^{ab\mu} c^b +$$

$$+ \sum_{f=1,\ldots,6} \frac{C_f}{(2\pi)^D} \overline{\Psi^j}_f \, \gamma_\mu \overleftarrow{D}^{ji\mu} \, \gamma_\nu D^{ik\nu} \Psi^k_f +$$

$$+ \sum_{\xi,\xi',\varsigma,\varsigma'} \overline{\Psi}_\xi \, \overline{\Psi}_{\xi'} \, \Gamma^{\xi,\xi'}_{\varsigma,\varsigma'} \Psi^{\varsigma} \Psi^{\varsigma'} \Big\} \qquad \xi,\xi',\varsigma,\varsigma' = (f,s,c)$$

The sum of fourth order terms in the quark fields should be the most general expression of this kind being invariant under the same symmetries of QCD, to allow the cancelation of the divergences.

7. Summary

1) We reviewed a recently proposed improved version of the modified massless QCD discussed in previous works. The model shows local gauge invariance and include the same kind of gluon and fermion condensate parameters.
2) The analysis done for constructing the proposal suggests its equivalence with massless QCD. In the pure quark condensate case this equivalence is indicated after considering that the new vertices might be classified as allowed counterterms within a generalized renormalization procedure of massless QCD.
3) In the gluodynamic limit, the appearance of Gaussian means over color fields suggests the possibility of a first principles derivation of the linear confining effects predicted by the stochastic vacuum models of QCD .
4) In the case of only retaining the quark condensate parameters, the appearing fermion auxiliary functions can be integrated leading to a QCD described by an alternative Lagrangian. It is given by the massless QCD one, to which six new gauge invariant terms, one for each quark flavor, are added.
5) The new terms determine masses for all the six quarks which are given by the reciprocal of the new flavor condensate couplings linked with each quark type. The strength of the condensate couplings decreases with the masses of the associated quarks.
6) The gluon self-energy was evaluated up to the second order in the gauge coupling including all orders of the flavor condensate ones. The result, satisfies the transversality condition as required by the gauge invariance.
7) The transversal part of the self energy is used to evaluate the two loop contribution to the vacuum energy at zero mean fields, as a function of the flavor condensate couplings. The transversality and gauge parameter independence of the gluon self-energy, also determines the gauge parameter independence of the result for the potential.

8) The effective potential is evaluated in the two loop approximation. The result in this case is able to predict the dynamic generation of quark masses, but only for sufficiently small values of the QCD coupling with no flavor symmetry breaking

9) However, it can be noted that in this two loop order, the considered effective potential diagrams can not yet incorporate contributions associated to two different kinds of flavors. Thus, in this approximation, the minimal energy simply corresponds to all the quark condensates for the different flavors minimizing their independent contributions to the effective potential.

10) Therefore, for the different flavor condensates to interfere one with other, at least three loops corrections to the effective potential are required. Thus, in order to explain the quark mass hierarchy as a dynamical flavor symmetry breaking, such "interference" like effects in the vacuum energy corrections should exist. In them, the contributions of diagrams showing two or more kinds of fermion lines might tend to rise the energy of the configurations with equal values of the quark condensates, making them more energetic that one in which a single quark condensate parameter gets a large value, and the others take hierarchical lower ones. We have the impression that the considered framework seems appropriate to realize the *Democratic Symmetry Breaking* properties of the mass hierarchy remarked by H. Fritzsch.

10) Its is underlined that the results indicate that a special renormalization scheme of massless QCD can lead to an effective action incorporating the NJL fermion estructure in the renormalized action. This conclusion further supports the starting idea that the massless QCD can generate a large dimensional transmutation effect.