QCD effective charge and the structure function F_2 at small-x

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- $F_2(x, Q^2)$ at low Q^2 has been measured in the previously unexplored small-x regime at the HERA collider

  The present data of F_2 imply a steep gluon at small-x

  This steep behavior can be generated from a flat-x gluon distribution at some initial low Q_0^2 scale

- At sufficiently small x we must resum the power series in $\alpha_s \ln(1/x)$ via BFKL equation

  The result of this procedure is sensitive to the infrared k_T region and, for running α_s , it is found that $\tilde{g}(x, k_T^2) \sim C(k_T^2)\, x^{-\lambda}$ where $\lambda \sim 0.5$

  Here $\tilde{g}(x, k_T^2)$ is the *unintegrated* gluon distribution and hence the resummation program requires knowledge of the gluon for all k_T^2 including the infrared region.

- There are some successful descriptions of $F_2(x, Q^2)$ by means of DGLAP evolution in the NLO approximation

  However, there is no reason to expect that they are reliable in the very small-x region

  Moreover, in the infrared region the BFKL equation is not expected to be valid

  Ultimately, with decreasing x, the singular behavior must be suppressed by non-perturbative effects

  The low Q^2 and small-x region bring us into a kinematical region where non-perturbative QCD effects becomes essential in order to understand the proton constitution

How to address this question?

The generalized DAS approximation - *first step*

- Our task of calculating infrared contributions to the QCD description of the HERA data on F_2 can succeed in a consistent way by analyzing exclusively the small-x region

  In this limit some of the existing analytical solutions of the DGLAP equation can be directly used [Ball, Forte, Kotikov, ...]

  In this approach the HERA data at small-x is interpreted in terms of the double-asymptotic-scaling (DAS) phenomenon

  The analytical solutions can be extended in order to include the subasymptotic part of the Q^2 evolution

  $\Rightarrow$ *generalized* DAS approximation [Mankiewicz, Saalfeld, Weigl, Kotikov, Illarionov, ...]

  $\Rightarrow$ parton distributions evolved from flat x distributions at some starting point Q_0^2 for the DGLAP evolution

The generalized DAS approximation - *first step*

- In the *generalized* approach, at NLO, the twist-two (leading) term of $F_2(x, Q^2)$ is given by

$$F_2(x, Q^2) = e \left[f_q(x, Q^2) + \frac{4T_R n_f}{3} \frac{\alpha_s(Q^2)}{4\pi} f_g(x, Q^2) \right],$$

where $e = \sum_i^f e_i^2 / n_f$ and $f_a(x, Q^2) = f_a^+(x, Q^2) + f_a^-(x, Q^2)$

The "$+$" and "$-$" representation above follows from the solution, at leading twist approximation, of the DGLAP equation in the Mellin moment space:

$$f_a^-(x, Q^2) = A_a^-(Q^2, Q_0^2) \exp\left[-d_-(1)s - D_-(1)p\right] + \mathcal{O}(x),$$

$$f_g^+(x, Q^2) = A_g^+(Q^2, Q_0^2) \, \tilde{I}_0(\sigma) \, \exp\left[-\bar{d}_+(1)s - \bar{D}_+(1)p\right] + \mathcal{O}(\rho),$$

$$f_q^+(x,Q^2) = A_q^+(Q^2,Q_0^2)\left[\left(1-\bar{d}^q_{+-}(1)\,\frac{\alpha_s(Q^2)}{4\pi}\right)\rho\,\tilde{I}_1(\sigma)\right.$$

$$\left. + \frac{20C_A}{3}\,\frac{\alpha_s(Q^2)}{4\pi}\,\tilde{I}_0(\sigma)\right],$$

$$\times \exp\left[-\bar{d}_+(1)s - \bar{D}_+(1)p\right] + \mathcal{O}(\rho),$$

where

$$s = \ln\left[\alpha_s(Q_0^2)/\alpha_s(Q^2)\right], \qquad p = \left[\alpha_s(Q_0^2) - \alpha_s(Q^2)\right]/4\pi,$$

$$D_\pm(n) = d_{\pm\pm}(n) - (\beta_1/\beta_0)d_\pm(n), \qquad \sigma = 2\sqrt{\left(\hat{d}_+ s + \hat{D}_+ p\right)\ln x}$$

and $$\rho = \sqrt{\left(\hat{d}_+ s + \hat{D}_+ p\right)/\ln x} = \sigma/2\ln(1/x)$$

For details about the components of the anomalous dimensions see [Luna, dos Santos, Natale, Phys. Lett. B **698** (2011) 52]

- This problem can be properly addressed by bringing up information about the infrared properties of QCD

  More specifically, by considering the possibility that the non-perturbative dynamics of QCD generate an effective gluon mass at very slow Q^2 region

  The dynamical gluon mass is intrinsically related to an infrared finite strong coupling constant

  Its existence is strongly supported by recent QCD lattice simulations as well as by phenomenological results

  Phenomenological infrared modifications of the strong-coupling constant are quite usual in the literature

  Nevertheless the fact that an infrared finite coupling constant appears as a consequence of a dynamically generated gluon mass is much less known

- Recent studies of a non-linear Schwinger-Dyson equation for the gluon self-energy show that $m^2(Q^2)$ may in fact have two distinct asymptotic behaviors [Aguilar, Papavassiliou]

  First, the dynamical gluon mass runs as an inverse power of a logarithm:

$$m^2(Q^2) = m_g^2 \left[\frac{\ln\left(\frac{Q^2+\rho m_g^2}{\Lambda^2}\right)}{\ln\left(\frac{\rho m_g^2}{\Lambda^2}\right)} \right]^{-1-\gamma_1},$$

  where $\gamma_1 = -6(1 + c_2 - c_1)/5$ with $c_1 \in [0.15, 0.4]$ and $c_2 \in [-1.07, -0.92]$.

Second, $m^2(Q^2)$ drops as an inverse power of momentum:

$$m^2(Q^2) = \frac{m_g^4}{Q^2 + m_g^2}\left[\frac{\ln\left(\frac{Q^2+\rho m_g^2}{\Lambda^2}\right)}{\ln\left(\frac{\rho m_g^2}{\Lambda^2}\right)}\right]^{\gamma_2 - 1},$$

where $\gamma_2 = (4 + 6c_1)/5$, with $c_1 \in [0.7, 1.3]$; the ρ and m_g parameters are constrained to lie in the same interval as the logarithm case, namely $\rho \in [1.0, 8.0]$ and $m_g \in [300, 800]$ MeV

Deep-inelastic scattering structure function F_2 in the IR region

- The small-x region is a very interesting kinematical domain for testing new QCD theoretical ideas

- NLO pQCD can describe in a reasonable way the evolution of F_2 data and its derivatives down to Q^2 values ~ 2 GeV^2

  The infrared region is not covered

- For analysing exclusively the small-x region we can adopt some analytical solutions of the DGLAP equations in the small-x limit [Ball, Forte, Parente, Kotikov, ...]

  However, non-perturbative effects are expected to give a substantial contribution to F_2

Task: we have to develop a consistent dynamical NLO approach

Deep-inelastic scattering structure function F_2 in the IR region

- Solution (first step): to adopt one analytical solution of the DGLAP equations in the small-x limit

  Small-x data from HERA can be interpreted in terms of the so-called double-asymptotic-scaling (DAS) phenomenon[Ball,Forte]

  Subasymptotic part of the Q^2 can be included $\Rightarrow$ *generalized DAS approximation* [Saalfeld,Weigl,Kotikov,...]

- Solution (second step): to propose one ansatz for the behaviour of the effective (dynamical) coupling beyond leadind order

- Solution (third step): adopt at NLO the leading term of F_2, given by

$$F_2(x,Q^2) = e\left[f_q(x,Q^2) + \frac{4T_R n_f}{3}\frac{\alpha_s(Q^2)}{4\pi}f_g(x,Q^2)\right]$$

- Using the leading-twist approximation of the Wilson OPE with dynamical $\bar{\alpha}_s^{NLO}$ coupling leads to good agreement with experimental data of DIS scattering at DESY-HERA [Luna, dos Santos, Natale, Phys. Lett. B **698** (2011) 52]

  The data analysis using the canonical (dynamical) NLO coupling gives $\chi^2/DoF = 2.88$ ($\chi^2/DoF = 1.87$)

  We obtain a dynamical gluon mass $m_g = 364 \pm 26$ MeV

  The dynamical coupling has a frozen value $\alpha_s(0) \sim 0.58$: as pointed by Cvetic *et al.*, this value reproduces succesfully the experimental value of r_τ, the well-measured semihadronic τ decay ratio

Deep-inelastic scattering structure function F_2 in the IR region

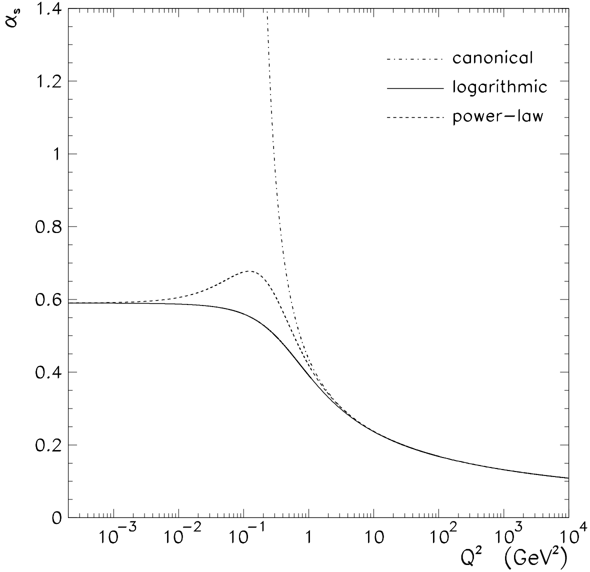


Figure I: *The canonical (perturbative) and dynamical couplings*

Deep-inelastic scattering structure function F_2 in the IR region

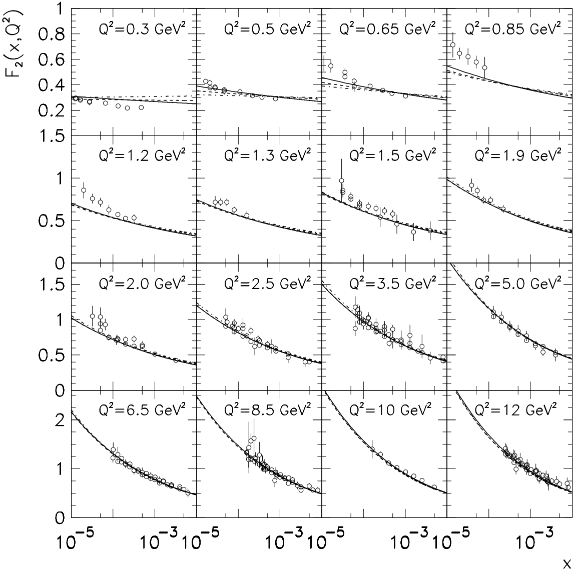
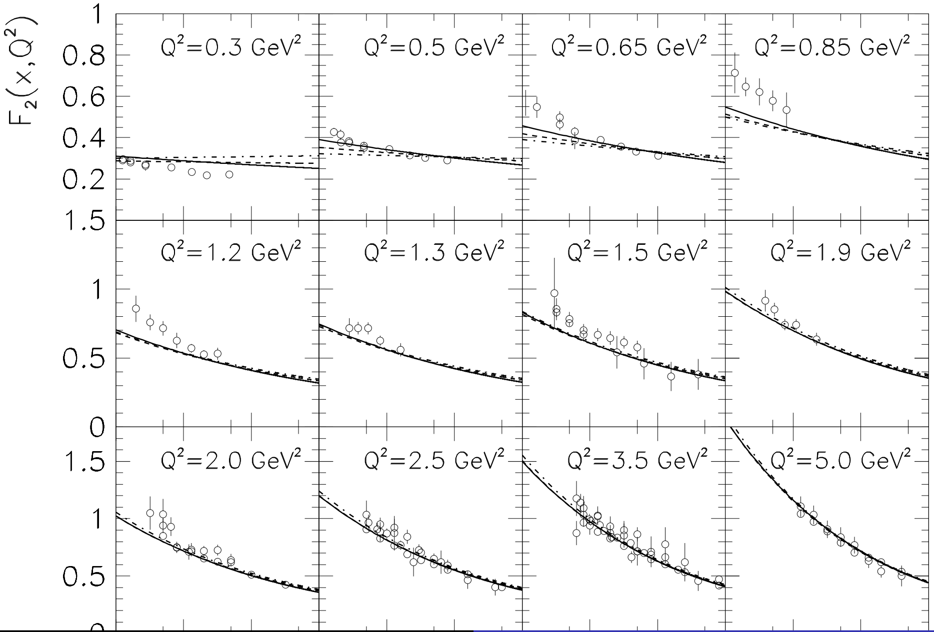


Figure II: *x-dependence of* $F_2(x, Q^2)$

Deep-inelastic scattering structure function F_2 in the IR region



Conclusions

- The small-x region is a very interesting kinematical domain for testing new QCD theoretical ideas
- We have computed the structure function $F_2(x, Q^2)$ of the proton by means of the *generalized* DAS approximation with a QCD effective charge
  [Luna, dos Santos, Natale, PLB **698** (2011) 52]

  We propose one ansatz for its behavior at NLO

  The coupling has a frozen value $\alpha_s(0) \sim 0.58$: this value reproduces succesfully the experimental value of r_τ, the well-measured semihadronic τ decay ratio [Cvetic *et al.*]
- The flat x initial condition in DGLAP equations is naturally related to what is expected from QCD with dynamically generated masses

Conclusions

- Through global fits to F_2 data we obtain the best values of the infrared mass scale for the logarithmic (power-law) running mass: $m_g = 364 \pm 26$ MeV ($m_g = 355 \pm 27$ MeV)

  These infrared values are of the same order of magnitude as the values obtained in other calculations of strongly interacting processes

- Our results show that the leading-twist approximation of the Wilson operator product expansion is quite accurate on the description of the structure function F_2 data

  However, in order to differentiate the logarithmic from the power-law running mass, a higher-twist study is necessary

  An analysis using higher twist corrections to the expansion of $F_2(x, Q^2)$ is in progress...