S_4 Flavored CP Symmetry for Neutrinos

Mohapatra, Nishi, PRD**86**, 073007(2012)

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Topics

Tribimaximal mixing

- Harrison, Perkins, Scott ('02) $\rightarrow$ TBM

$$V_{\mathrm{MNS}} \approx U_{\mathrm{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Very good approximation until recently!
- Daya-Bay, RENO, 2012: $\theta_{13} \approx 9^\circ$
- Assume still good approximation in some limit
- TBM from flavor symmetry?
- 3 families $\rightarrow$ groups with 3-dim irreps

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Tribimaximal mixing & flavor symmetries

- $V_{\rm MNS} = U_l^\dagger U_\nu$

  U_l diagonalizes $\bar{M}_l = M_l M_l^\dagger$
  U_ν diagonalizes M_ν (Majorana)

- Fix a basis → Flavor basis → diagonal $\bar{M}_l = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$
- Symmetries of TBM

$G_l \simeq U(1) \times U(1)$ of $\bar{M}_l$: $T^\dagger \bar{M}_l T = \bar{M}_l$

$$T = \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{-i(\alpha_1+\alpha_2)} \end{pmatrix}$$

$G_\nu \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$ of M_ν: $G_i^T M_\nu G_i = M_\nu$

$$G_2 = -\frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix} \qquad G_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad G_1 = G_2 G_3$$

"magic" symmetry $\qquad$ μ-τ symmetry

- automatic (Grimus,Lavoura,Ludl,'09)
- specific to TBM

Tribimaximal mixing & finite flavor symmetries

- Restrict to finite groups $\rightarrow$ finite T
- Model: $G_F \rightarrow G_l$ or G_ν in each sector
- Flavor group G_F containing $G_l, G_\nu\big|_{\mathrm{TB}}$ $\rightarrow$ should contain S_4

Lam,'08

- $G_F = S_4$

$$G_l = \langle T \rangle \simeq \mathbb{Z}_3 \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \qquad \omega \equiv e^{i2\pi/3}$$

$$G_\nu = \langle G_2, G_3 \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \qquad G_2 = -\frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}$$

$$G_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Tribimaximal mixing & S_4

- In another basis of $G_F = S_4$

$$G_l = \langle T \rangle \simeq \mathbb{Z}_3 \qquad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$G_\nu = \langle G_2, G_3 \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \qquad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

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usual **3** representation of S_4

- can be generated by T and $S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

Tribimaximal mixing & finite flavor symmetries

- Flavor symmetries v.s. residual symmetries of $\bar{M}_l$ and M_ν
- Some residual symmetries may be accidental
- Different groups $A_4, S_4, \Delta(27), T_7, \ldots$
- $\theta_{13} \neq 0 \implies$ TBM is not exact!
  - Include corrections
  - Different symmetries
- μ-τ symmetry $\implies \theta_{13} = 0$
- μ-τ reflection $\implies \theta_{23} = 45^\circ, \delta_D = \pm 90^\circ$ if $\theta_{13} \neq 0$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} M_\nu{}^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = M_\nu$$

μ-τ interchange w/ c.c.
Harrison & Scott, '02
Grimus & Lavoura, '03

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We need CP!

Incorporating CP in S_4

S_4 generated by $S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ $T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$$T^3 = S^4 = 1, \quad ST^2S = T$$

$\tilde{S}_4$ generated by $\tilde{S} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \mathrm{CP}$ $T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

- $\tilde{S}_4 \simeq S_4$ once we factor $\mathrm{CP}^2 = -1$ for fermions
- recipe: extract $\tilde{S}_4$ subgroup from $S_4 \otimes \langle \mathrm{CP} \rangle$
- irreps of $\tilde{S}_4$: $\mathbf{3}, \mathbf{1}, \mathbf{1}_\omega, \mathbf{1}_{\omega^2}$ from
- irreps of S_4 : $\mathbf{3}, \mathbf{3}', \mathbf{1}, \mathbf{1}', \mathbf{2}$

The model

3 families of leptons L_i, l_i

3 Higgs doublets ϕ_i

4 Higgs triplets Δ_0, Δ_i $\sim (3,2)$

Transforming under $\tilde{S}_4$ as

$L_i \sim \mathbf{3}$ $\quad L_i(x) \xrightarrow{\tilde{S}} S_{ij} C L_j^*(\hat{x}), \quad L_i(x) \xrightarrow{T} T_{ij} L_j(x)\,;$ $\quad \hat{x} = (x_0, -\mathbf{x})$

$\phi_i \sim \mathbf{3}$ $\quad \phi_i(x) \xrightarrow{\tilde{S}} S_{ij} \phi_j^*(\hat{x}), \quad \phi_i(x) \xrightarrow{T} T_{ij} \phi_j(x)\,;$ $\quad \sim \quad \Delta_i \sim \mathbf{3}$

$l_1 \sim \mathbf{1}$ $\quad l_1(x) \xrightarrow{\tilde{S}} C l_1^*(\hat{x}), \quad l_1(x) \xrightarrow{T} l_1(x)\,;$ $\quad \sim \quad \Delta_0 \sim \mathbf{1}$

$l_2 \sim \mathbf{1}_\omega$ $\quad l_2(x) \xrightarrow{\tilde{S}} C l_2^*(\hat{x}), \quad l_2(x) \xrightarrow{T} \omega\, l_2(x)\,;$ $\quad \omega = e^{i2\pi/3}$

$l_3 \sim \mathbf{1}_{\omega^2}$ $\quad l_3(x) \xrightarrow{\tilde{S}} C l_3^*(\hat{x}), \quad l_3(x) \xrightarrow{T} \omega^2\, l_3(x)\,;$

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The model: charged leptons

- Yukawa Lagrangian for charged leptons

$$-\mathcal{L}_Y^l = y_1(\bar{L}_1\phi_1 + \bar{L}_2\phi_2 + \bar{L}_3\phi_3)l_1 + y_2(\bar{L}_1\phi_1 + \omega^2\bar{L}_2\phi_2 + \omega\bar{L}_3\phi_3)l_2 + y_3(\bar{L}_1\phi_1 + \omega\bar{L}_2\phi_2 + \omega^2\bar{L}_3\phi_3)l_3 + h.c.,$$

y_i *real* due to $\tilde{S}$

- doublet vevs $\langle\phi_i\rangle = \frac{v}{\sqrt{3}}(1,1,1)$ $\qquad \tilde{S}_4 \to \tilde{S}_3$

- mass matrix $M_l = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \mathrm{diag}(m_e, m_\mu, m_\tau)$

$U_\omega^* \implies U_l^\dagger = U_\omega$

The model: neutrinos

- Type II seesaw

$$-\mathcal{L}^{\nu} = \frac{1}{2} f_0 \overline{L_i^c} \epsilon \Delta_0 L_i + f_1 \left(\overline{L_2^c} \epsilon \Delta_1 L_3 + \overline{L_3^c} \epsilon \Delta_2 L_1 + \overline{L_1^c} \epsilon \Delta_3 L_2 \right) + h.c.,$$

f_0, f_1 real due to $\tilde{S}$

- Triplet vevs $\langle \Delta_0^{(0)} \rangle = u_0 \, , \ \ \langle \Delta_i^{(0)} \rangle = u_i$

- Mass matrix $M_\nu = \begin{pmatrix} a & f & e \\ f & a & d \\ e & d & a \end{pmatrix}$ a, d, e, f real

- In flavor basis $U_\omega^\dagger M_\nu U_\omega^* = \begin{pmatrix} x & z & z^* \\ z & -2z^* & y \\ z^* & y & -2z \end{pmatrix}$

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The model: predictions

- Decompose $V_{\rm MNS} = U_l^\dagger U_\nu = U_{\rm TB}\,{\rm diag}(1,1,i) U_\epsilon$ $\quad U_\nu = U U_\epsilon$
- U_ϵ diagonalizes $M'_\nu = U^{\rm T} M_\nu U = \begin{pmatrix} a+d & b & 0 \\ b & a & c \\ 0 & c & a-d \end{pmatrix}$

  $U = U^*_\omega U_{\rm TB}\,{\rm diag}(1,1,i)$ $\quad b = \frac{e+f}{\sqrt{2}},\ c = \frac{e-f}{\sqrt{2}}$
- TBM : $b = c = 0$

  $m_1 = |d| - a\,,\quad m_2 = a\,,\quad m_3 = a + |d|$
- $c \neq 0$ controls $\theta_{13} \neq 0$
- maximal Dirac CP phase $\leftrightarrow$ one maximal Majorana phase
- approximate sum-rule $m_3 - 2m_2 - m_1 \approx 0$ no phases
- $a, b, c, d \rightarrow \theta_{12}, \theta_{13}, m_1, m_2, m_3$

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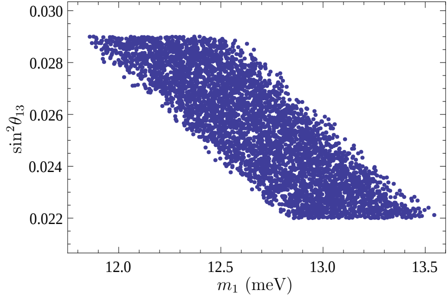
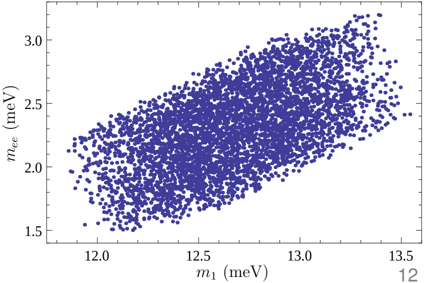
- $a, b, c, d \rightarrow \theta_{12}, \theta_{13}, m_1, m_2, m_3$ correlations!

The model: predictions





$(m_1, m_2, m_3) \approx (13, 16, 52)\,\text{meV}$

- pts compatible within 1-σ
- "large masses", small m_{ee}
- similar to Ishimori & Ma ('12) but more restrictive

Relation with more general approaches

- Holthausen, Lindner, Schmidt, arXiv:1211.6953
  - General theory: **CP** $\longleftrightarrow$ outer automorphism of G_F
  - Clarification of geometric/calculable **CP** phases
  - Analysis of several relevant cases $A_4, \Delta(27), T_7, \ldots$
  - Consequence to our work:
    $\tilde{S}_4$ is the only consistent **CP** extension of A_4 (if $\mathbf{3}, \mathbf{1}'$ are present)
- Feruglio, Hagedorn, Ziegler, arXiv:1211.5560
  - General consequences of including **CP**
  - Focus on residual **CP** symmetry in neutrino sector G_ν
  - Dirac and Majorana phases depending on one angle
  - Specific analysis for S_4 (not a complete model)

Conclusions

- A consistent way of incorporating **CP** into flavor groups
- The only way of defining **CP** to A_4 (Holthausen, *et.al.*,'12)
- $\tilde{S}_4$ example, similar to A_4 but more constraining
- Can be further explored for flavor model-building
- Accidental μ-τ reflection leading to maximal δ_D, θ_{23}
- NH and approximate sum-rule (10-15%)
- One maximal Majorana phase