Top quark effects in composite vector pair production at the LHC

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Introduction.

A 125 GeV Higgs boson has been found at the LHC. It remains to determine whether Electroweak Symmetry Breaking (EWSB) is weakly or strongly coupled.

- Weakly coupled, as the Standard Model (SM), 2HDM, Multi Higgs Models, Little Higgs, Gauge Higgs Unification models, $SU(3)_C \otimes SU(3)_L \otimes U(1)_Y$ models (3-3-1 models), GUT and their supersymmetric extensions.
- Strongly coupled, as Technicolor, Composite Higgs, Strongly Interacting Light Higgs (SILH), Composite Higgs with composite Vectors, Randall-Sundrum (RS) models, 10^{32} SM copies, Full Hierarchy Quiver Theories (FHQT).

The hierarchy problem provides a plausible motivation for considering strongly coupled theories of EWSB.

Chiral Lagrangian with massive spin one fields, scalar singlet and SM fermions.

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{v^2}{4}\left\langle D_\mu U \left(D^\mu U\right)^\dagger \right\rangle - \frac{1}{2g^2}\left\langle W_{\mu\nu}W^{\mu\nu}\right\rangle - \frac{1}{2g'^2}\left\langle B_{\mu\nu}B^{\mu\nu}\right\rangle \\
& -\frac{1}{4}\left\langle \hat{V}^{\mu\nu}\hat{V}_{\mu\nu}\right\rangle + \frac{M_V^2}{2}\left\langle V^\mu V_\mu \right\rangle - \frac{ig_V}{2\sqrt{2}}\left\langle \hat{V}_{\mu\nu}[u^\mu, u^\nu]\right\rangle \\
& -\frac{g_V}{\sqrt{2}}\left\langle \hat{V}_{\mu\nu}\left(uW^{\mu\nu}u^\dagger + u^\dagger B^{\mu\nu}u\right)\right\rangle - \frac{1}{8}\left\langle [V_\mu, V_\nu][u^\mu, u^\nu]\right\rangle \\
& +\frac{i}{2}\left\langle V_\mu V_\nu \left(uW^{\mu\nu}u^\dagger + u^\dagger B^{\mu\nu}u\right)\right\rangle + \frac{ig_K}{4\sqrt{2}}\left\langle \hat{V}_{\mu\nu}[V^\mu, V^\nu]\right\rangle \\
& +\frac{g_V^2}{8}\left\langle [u_\mu, u_\nu][u^\mu, u^\nu]\right\rangle + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{m_h^2}{2}h^2 \\
& +\frac{v^2}{4}\left\langle D_\mu U \left(D^\mu U\right)^\dagger\right\rangle \left(2a\frac{h}{v} + b\frac{h^2}{v^2}\right) + \frac{dv}{8g_V^2}h\left\langle V_\mu V^\mu\right\rangle \\
& -\frac{v}{\sqrt{2}}\sum_{i,j}\left(\bar{u}_L^{(i)} d_L^{(i)}\right) U \left(1 + c\,\frac{h}{v}\right)\begin{pmatrix}\lambda_{ij}^u u_R^{(j)} \\ \lambda_{ij}^d d_R^{(j)}\end{pmatrix} + h.c.
\end{aligned}
$$

$$U(x) = e^{i\hat{\pi}(x)/v}\,, \qquad \hat{\pi}(x) = \tau^a \pi^a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad u \equiv \sqrt{U}$$

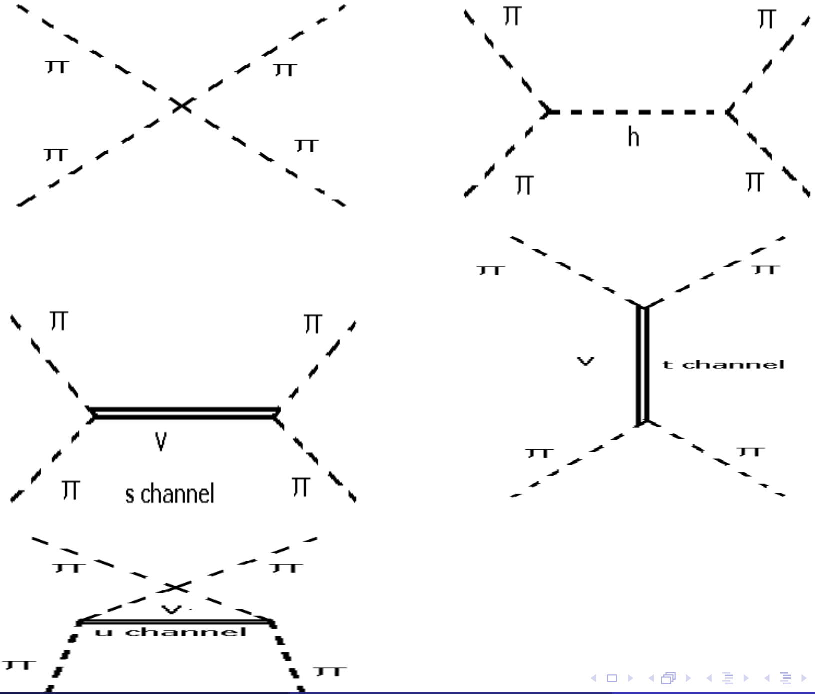
$$D_\mu U = \partial_\mu U - iB_\mu U + iUW_\mu\,, \qquad W_\mu = \frac{g}{2}\tau^a W_\mu^a\,, \qquad B_\mu = \frac{g'}{2}\tau^3 B_\mu^0\,,$$

$$V_\mu = \frac{1}{\sqrt{2}}\tau^a V_\mu^a\,, \qquad \hat{V}_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu, \qquad u_\mu = u_\mu^\dagger = iu^\dagger D_\mu U u^\dagger,$$

$$\nabla_\mu V = \partial_\mu V + [\Gamma_\mu, V]\,, \qquad \Gamma_\mu = \frac{1}{2}\left[u^\dagger\left(\partial_\mu - iB_\mu\right)u + u\left(\partial_\mu - iW_\mu\right)u^\dagger\right]$$

Here the following assumptions have been made:

1. Before weak gauging, the Lagrangian responsible for EWSB has a $SU(2)_L \times SU(2)_R$ global symmetry
2. The strong dynamics produces a composite triplet of heavy vectors and a composite scalar singlet under $SU(2)_{L+R}$.
3. Only one vector triplet V_μ^a of the $SU(2)_{L+R}$ group has a mass below the cut-off $\Lambda \approx 3\,\text{TeV}$. The new V states couple to fermions only via SM gauge interactions.
4. The light scalar singlet of mass $m_h = 125$ GeV interacts with the Standard Model gauge bosons and fermions only via weak gauging and (proto)-Yukawa couplings, respectively.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The various amplitudes have the following asymptotic behaviour (Barbieri-Carcamo-Corcella-Trincherini-Torre 2010, Carcamo-Torre 2010):

$$A\left(W_L W_L \to W_L W_L\right) \sim \frac{s}{v^2}, \qquad A\left(W_L W_L \to V_L V_L\right) \sim \frac{s^2}{v^2 M_V^2}$$

$$A\left(W_L W_L \to hh\right) \sim \frac{s}{v^2}, \qquad A\left(W_L W_L \to V_L h\right) \sim \frac{s}{v^2}$$

$$A\left(W_L W_L \to f\bar{f}\right) \simeq \frac{m_f\left(1-ac\right)\sqrt{s}}{v^2}.$$

The choice:

$$a=\sqrt{1-\frac{3G_V^2}{v^2}}\,, \qquad G_V \equiv g_V M_V\,, \qquad G_V \leq v/\sqrt{3}.$$

guarantees a good asymptotic behavior of elastic $W_L W_L$ scattering, while $g_V g_K = 1$ ensures that $\mathcal{A}\left(\pi^a \pi^b \to V_L^c V_L^d\right)$ grows at most like s/v^2. The gauge model scenario is defined by (Carcamo-Torre 2010):

$$a=\frac{1}{2}\,, \quad b=\frac{1}{4}\,, \quad d=1\,, \quad g_K=\frac{1}{g_V}\,, \quad g_V=\frac{v}{2M_V} \quad c=\frac{1}{a}.$$

Gluon fusion production amplitudes

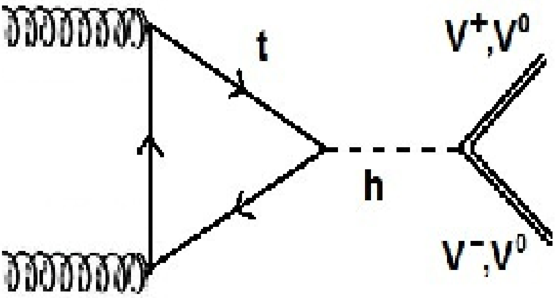

Figure: Leading order diagram, containing a hVV coupling vertex, of the vector pair production through the gluon fusion process. Crossing the gluon legs yields a second diagram.

$$
\begin{aligned}
A\left(gg\to V^{+}V^{-}\right) &= -\frac{\alpha_S}{\pi}\left(\frac{cd}{8g_V^2\left(s-M_h^2\right)}\right)\varepsilon_\mu\left(p,\chi\right)\varepsilon_\nu\left(k,\chi'\right) \\
&\quad\times\delta_{ab}\left[g^{\mu\nu}\left(p\cdot k\right)-p^{\nu}k^{\mu}\right]I\left(\frac{s}{m_t^2}\right) \\
&\quad\times g^{\rho\sigma}\varepsilon_\rho\left(l,\xi\right)\varepsilon_\sigma\left(q,\xi'\right),
\end{aligned}
$$

$$
I\left(\frac{s}{m_t^2}\right)=\int_0^1 dx\int_0^{1-x}dy\,\frac{1-4xy}{1-\frac{s}{m_t^2}xy},
$$

$$
\begin{aligned}
\sum_{a,b,\chi,\chi',\xi,\xi'}\left|A\left(gg\to V^{+}V^{-}\right)\right|^2 &= \frac{1}{4}\sum_{a,b,\chi,\chi',\xi,\xi'}\left|A\left(gg\to V^{0}V^{0}\right)\right|^2 \\
&= \frac{c^2d^2\alpha_S^2s^2}{16\pi^2g_V^4\left(s-M_h^2\right)^2} \\
&\quad\times\left|I\left(\frac{s}{m_t^2}\right)\right|^2\left(\frac{s^2}{4M_V^4}-\frac{s}{M_V^2}+3\right).
\end{aligned}
$$

Vector pair production total cross sections via gluon fusion

$$
\begin{aligned}
\sigma_{pp\to gg\to V^+V^-(V^0V^0)}(S) &= \int_{\sqrt{\frac{2M_V^2}{S}}}^{1} dx \int_{\sqrt{\frac{2M_V^2}{S}}}^{1} dy f_{p/g}\left(x,\mu^2\right) f_{p/g}\left(y,\mu^2\right) \\
&\quad \times \sigma_{gg\to V^+V^-(V^0V^0)}(s),
\end{aligned}
$$

where $s = xyS$ and the choice $\mu = 2M_V$ is made.

$$
\begin{aligned}
\sigma_{gg\to V^+V^-}(s) &= \frac{1}{4}\sigma_{gg\to V^0V^0}(s) \\
&= \frac{1}{4}\times\frac{1}{64}\times\frac{1}{16\pi s^2} \\
&\quad \times \int_{t_{\min}}^{t_{\max}} \sum_{a,b,\chi,\chi',\xi,\xi'} \left|A\left(gg\to V^+V^-\right)\right|^2 d\widehat{t},
\end{aligned}
$$

$$
\begin{aligned}
t_{\min} &= -\left(\sqrt{\frac{s}{4}} + \sqrt{\frac{s}{4} - M_V^2}\right)^2, \\
t_{\max} &= -\left(\sqrt{\frac{s}{4}} - \sqrt{\frac{s}{4} - M_V^2}\right)^2.
\end{aligned} \tag{1}
$$

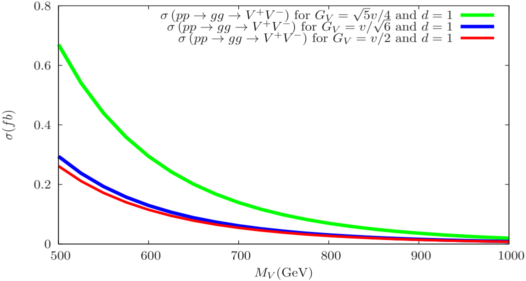

Figure 1: Total cross sections for the V^+V^- production via the gluon fusion mechanism at the LHC for $\sqrt{S} = 14$ TeV and $d = 1$ as functions of the heavy vector mass M_V for different values of the G_V parameter.

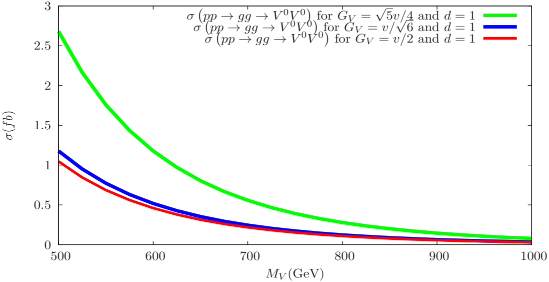

Figure 2: Total cross sections for the V^0V^0 production via the gluon fusion mechanism at the LHC for $\sqrt{S} = 14$ TeV and $d = 1$ as functions of the heavy vector mass M_V for different values of the G_V parameter.

Same-sign di-lepton and tri-lepton events.

Since the heavy vector have dominant decay mode into pair of SM Gauge bosons (with branching ratio very close to one), the vector pair production by gluon fusion will lead to 4 SM gauge bosons in the final state.

Decay Mode	di-leptons (%)	tri-leptons (%)
$V^0 V^0 \to W^+ W^- W^+ W^-$	8.9	3.2

Table: Dominant decay mode and cumulative branching ratios for the $V^0 V^0$ charge configuration.

G_V	a	di-leptons	tri-leptons
$\sqrt{5}v/4$	$1/4$	24	9
$v/2$	$1/2$	9	3
$v/\sqrt{6}$	$1/\sqrt{2}$	11	4

Table: Total number of same-sign di-lepton and tri-lepton events (e or μ from W decays) for the vector pair production via gluon fusion at the LHC for $\sqrt{S} = 14$ TeV and $\int \mathcal{L} dt = 100$ fb^{-1} at $M_V = 500$ GeV, $M_h = 125$ GeV and $m_t = 171.3$ GeV for different values of the parameter G_V and for $d = 1$.

Signal	**Number of Events**
$pp \to V^0 V^0 \to W^+ W^- W^+ W^- \to 2l4j\not{E}_T$	24
Backgrounds	
$t\bar{t}W \to WWW2j \to 2l4j\not{E}_T$	$\sim 6.5 \times 10^3$
$HH \to WWWW \to 2l4j\not{E}_T$	$\sim 1.6 \times 10^3$
$WWW2j \to 2l4j\not{E}_T$	~ 253
$HW2j \to WWW2j \to 2l4j\not{E}_T$	~ 100
$HWZ \to WWW2j \to 2l4j\not{E}_T$	~ 1.5
$HWW \to WWW2j \to 2l4j\not{E}_T$	~ 6
$WWWW \to 2l4j\not{E}_T$	~ 3

Table: Number of same-sign di-lepton events and estimation of the corresponding backgrounds at the LHC for $\sqrt{S} = 14$ TeV and $\int \mathcal{L}dt = 100$ fb^{-1}. The signal corresponds to the case $M_V = 500$ GeV, $M_h = 125$ GeV, $m_t = 171.3$ GeV, $a = 1/4$, $d = 1$ and $G_V = \sqrt{5}v/4$.

Signal	Number of Events
$pp \to V^0 V^0 \to W^+ W^- W^+ W^- \to 3l2j\not{E}_T$	9
Backgrounds	
$t\bar{t}W \to WWW2j \to 3l2j\not{E}_T$	$\sim 2 \times 10^3$
$HH \to WWWW \to 3l2j\not{E}_T$	$\sim 10^3$
$WZZ \to 3l2j\not{E}_T$	$\sim 10^2$
$WWW2j \to 3l2j\not{E}_T$	~ 80
$HW2j \to WWW2j \to 3l2j\not{E}_T$	~ 30
$HWW \to WWW2j \to 2l4j\not{E}_T$	~ 4
$WWZZ \to 3l2j\not{E}_T$	~ 2
$WWWW \to 3l2j\not{E}_T$	~ 2
$HWZ \to WWW2j \to 3l2j\not{E}_T$	~ 0.5

Table: Number of tri-lepton events and estimation of the corresponding backgrounds at the LHC for $\sqrt{S} = 14$ TeV and $\int \mathcal{L}dt = 100$ fb^{-1}. The signal corresponds to the case $M_V = 500$ GeV, $M_h = 125$ GeV, $m_t = 171.3$ GeV, $a = 1/4$, $d = 1$ and $G_V = \sqrt{5}v/4$.

Conclusions

- New particles need are needed to stabilize the weak scale.
- Strongly coupled EWSB can be described by an Effective Lagrangian with massive spin-1 fields + 1 singlet scalar.
- The total cross sections for the production of the V^+V^- and V^0V^0 final states at the LHC by the gluon fusion mechanism are of the order of few *fb*. For $\int \mathcal{L}dt = 100$ fb^{-1}, the $N_{multileptonevents} \sim 10$.
- Signals are very suppressed by a factor 10^{-2} with respect to the $t\bar{t}W$ and HH backgrounds.
- The LHC will shed light in the dynamics responsible for EWSB and the stabilization of EW scale.

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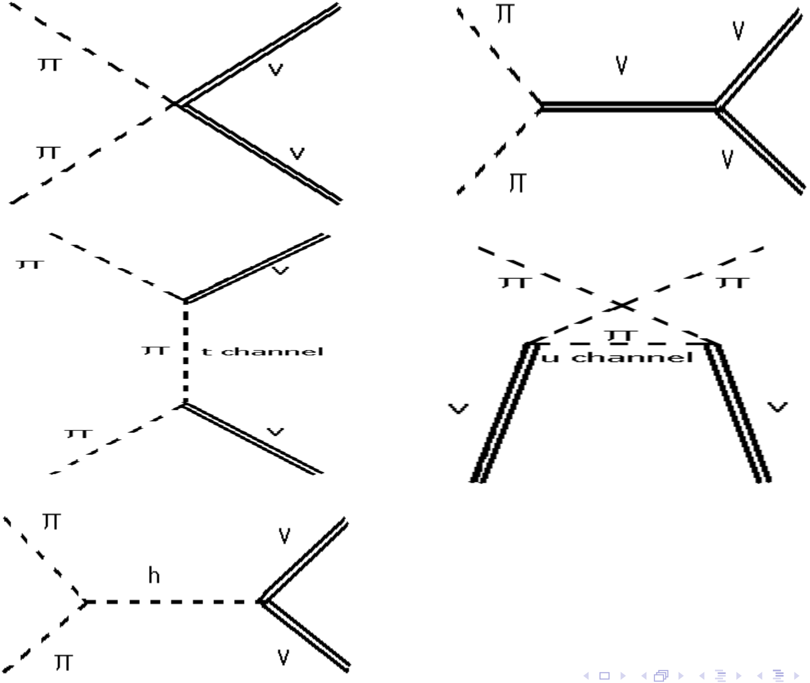
Extra Slides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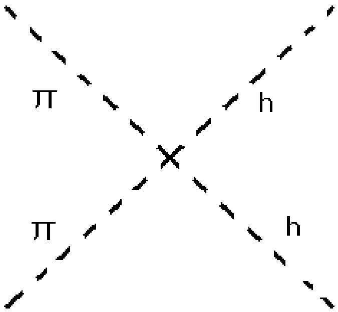
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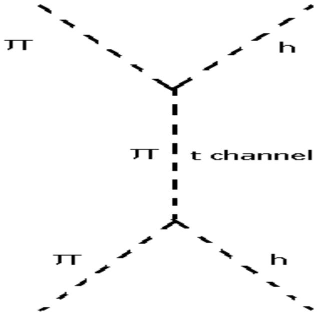
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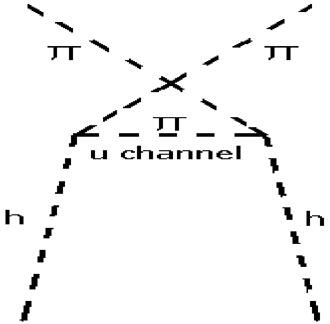
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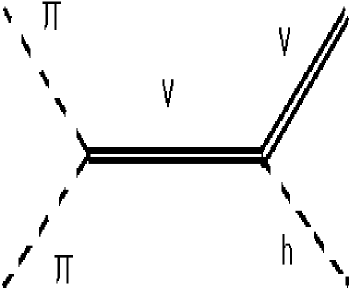
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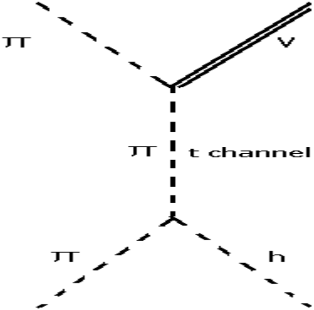
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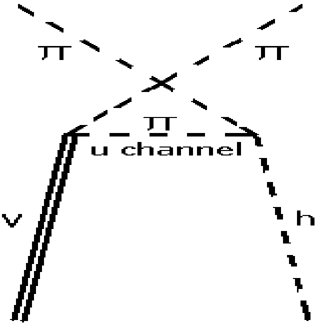
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G_V	a	d	V^+V^- (fb)	V^0V^0 (fb)
$\sqrt{5}v/4$	$1/4$	0	0	0
$\sqrt{5}v/4$	$1/4$	1	0.67	2.68
$\sqrt{5}v/4$	$1/4$	2	2.68	11.72
$v/2$	$1/2$	0	0	0
$v/2$	$1/2$	1	0.26	1.04
$v/2$	$1/2$	2	1.04	4.16
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.29	1.16
$v/\sqrt{6}$	$1/\sqrt{2}$	2	1.16	4.64

Table: Total cross sections for the production of the V^+V^- and V^0V^0 final states by gluon fusion at the LHC for $\sqrt{S} = 14$ TeV as functions of the different parameters for $M_V = 500$ GeV. Here $\alpha_S = 0.12$, $M_h = 125$ GeV and $m_t = 171.3$ GeV while the factorization scale is taken to be equal to $2M_V$.

G_V	a	d	V^+V^- (fb)	V^0V^0 (fb)
$\sqrt{5}v/4$	$1/4$	0	0	0
$\sqrt{5}v/4$	$1/4$	1	0.02	0.08
$\sqrt{5}v/4$	$1/4$	2	0.08	0.32
$v/2$	$1/2$	0	0	0
$v/2$	$1/2$	1	0.01	0.04
$v/2$	$1/2$	2	0.04	0.16
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.01	0.04
$v/\sqrt{6}$	$1/\sqrt{2}$	2	0.04	0.16

Table: Total cross sections for the production of the V^+V^- and V^0V^0 final states by gluon fusion at the LHC for $\sqrt{S} = 14$ TeV as functions of the different parameters for $M_V = 1$ TeV. Here $\alpha_S = 0.12$, $M_h = 125$ GeV and $m_t = 171.3$ GeV while the factorization scale is taken to be equal to $2M_V$.

Composite versus gauge models

Considering the following $SU(2)_L \times SU(2)_C \times SU(2)_R$ Lagrangian [3]:

$$\mathcal{L}_V^{\text{gauge}} = \mathcal{L}_\chi^{\text{gauge}} - \frac{1}{2g_C^2}\left\langle v_{\mu\nu} v^{\mu\nu} \right\rangle - \frac{1}{2g^2}\left\langle W_{\mu\nu} W^{\mu\nu} \right\rangle - \frac{1}{2g'^2}\left\langle B_{\mu\nu} B^{\mu\nu} \right\rangle , \tag{2}$$

where

$$v_\mu = \frac{g_C}{2} v_\mu^a \tau^a \tag{3}$$

is the $SU(2)_C$-gauge vector and the symmetry breaking Lagrangian is described by

$$\mathcal{L}_\chi^{\text{gauge}} = \frac{v^2}{2}\left\langle D_\mu \Sigma_{RC} \left(D^\mu \Sigma_{RC}\right)^\dagger \right\rangle + \frac{v^2}{2}\left\langle D_\mu \Sigma_{CL} \left(D^\mu \Sigma_{CL}\right)^\dagger \right\rangle . \tag{4}$$

Denoting collectively the three gauge vectors by

$$v_\mu^I = (W_\mu, v_\mu, B_\mu), \quad I = (L, C, R), \tag{5}$$

one has for the two bi-fundamental scalars Σ_{IJ}

$$D_\mu \Sigma_{IJ} = \partial_\mu \Sigma_{IJ} - i v_\mu^I \Sigma_{IJ} + i \Sigma_{IJ} v_\mu^J . \tag{6}$$

$\Sigma_{IJ} = \sigma_I \sigma_J^\dagger$, where σ_I are the elements of $SU(2)_I/H$. As the result of a gauge transformation

$$v_\mu^I \to \sigma_I^\dagger v_\mu^I \sigma_I + i\sigma_I^\dagger \partial_\mu \sigma_I \equiv \Omega_\mu^I, \quad \Sigma_{IJ} \to \sigma_I^\dagger \Sigma_{IJ} \sigma_J = 1, \tag{7}$$

and after the gauge fixing $\sigma_R = \sigma_L^+ \equiv u$ and $\sigma_C = 1$, one has

$$\mathcal{L}_\chi^{\text{gauge}} = v^2 \left\langle (v_\mu - i\Gamma_\mu)^2 \right\rangle + \frac{v^2}{4} \left\langle u_\mu^2 \right\rangle, \tag{8}$$

where

$$u_\mu = \Omega_\mu^R - \Omega_\mu^L, \quad \Gamma_\mu = \frac{1}{2i}(\Omega_\mu^R + \Omega_\mu^L), \quad v_\mu = V_\mu + i\Gamma_\mu \tag{9}$$

by use of the identity:

$$v_{\mu\nu} = \hat{V}_{\mu\nu} - i[V_\mu, V_\nu] + \frac{i}{4}[u_\mu, u_\nu] + \frac{1}{2}(uW_{\mu\nu}u^\dagger + u^\dagger B_{\mu\nu} u). \tag{10}$$

With the replacement $V_\mu \to \frac{g_C}{\sqrt{2}} V_\mu$, ${}^{gauge}_{V}$ coincides with $\mathcal{L}^V$ for

$$g_V = \frac{1}{2g_C} = \frac{1}{g_K}, \quad g_3 = -\frac{1}{4}, \quad g_6 = \frac{1}{2}, \quad f_V = 2g_V \quad M_V = g_C v \tag{11}$$

with $G_V = g_V M_V$, $\nabla_\mu V = \partial_\mu V + [\Gamma_\mu, V]$,

A well behaved theory at all energies

Let us consider the following $SU(2)_L \times SU(2)_C \times U(1)_Y$ invariant non-linear sigma model Lagrangian:

$$\mathcal{L}_V^{\text{gauge}} = \mathcal{L}_\chi^{\text{gauge}} - \frac{1}{2g_C^2}\left\langle v_{\mu\nu} v^{\mu\nu}\right\rangle - \frac{1}{2g^2}\left\langle W_{\mu\nu} W^{\mu\nu}\right\rangle - \frac{1}{2g'^2}\left\langle B_{\mu\nu} B^{\mu\nu}\right\rangle , \tag{12}$$

where

$$v_\mu = \frac{g_C}{2} v_\mu^a \tau^a \tag{13}$$

is the $SU(2)_C$-gauge vector and the symmetry breaking Lagrangian is described by

$$\mathcal{L}_\chi^{\text{gauge}} = \frac{v^2}{2}\left\langle D_\mu \Sigma_{YC}\left(D^\mu \Sigma_{YC}\right)^\dagger\right\rangle + \frac{v^2}{2}\left\langle D_\mu \Sigma_{CL}\left(D^\mu \Sigma_{CL}\right)^\dagger\right\rangle \tag{14}$$

$$\Sigma_{RC} = \left(1 + \frac{h+H}{2v}\right) U_{YC} , \qquad U_{RC} = \exp\left[\frac{i}{2v}\left(\pi + \sigma\right)\right] , \tag{15}$$

$$\Sigma_{CL} = \left(1 + \frac{h-H}{2v}\right) U_{CL} , \qquad U_{CL} = \exp\left[\frac{i}{2v}\left(\pi - \sigma\right)\right] , \tag{16}$$

$V\left(\Sigma_{YC},\Sigma_{CL}\right)$ is the scalar potential, which has the form

$$V\left(\Sigma_{YC},\Sigma_{CL}\right)=\frac{\mu^2 v^2}{2}\left\langle\Sigma_{YC}\Sigma_{YC}^{\dagger}\right\rangle+\frac{\mu^2 v^2}{2}\left\langle\Sigma_{CL}\Sigma_{CL}^{\dagger}\right\rangle-\frac{\lambda v^4}{4}\left(\left\langle\Sigma_{YC}\Sigma_{YC}^{\dagger}\right\rangle\right)$$
$$-\frac{\lambda v^4}{4}\left(\left\langle\Sigma_{CL}\Sigma_{CL}^{\dagger}\right\rangle\right)^2-\kappa v^4\left\langle\Sigma_{YC}\Sigma_{CL}^{\dagger}\Sigma_{CL}\Sigma_{YC}^{\dagger}\right\rangle . \qquad (17)$$

where $\pi=\pi^a\tau^a$ and $\sigma=\sigma^a\tau^a$, with:

$$m_h^2=4v^2\left(\lambda+\kappa\right) , \qquad m_H^2=4v^2\left(\lambda-\kappa\right) . \qquad (18)$$

The covariant derivatives appearing in (14) are given by

$$\begin{aligned} D_\mu U_{YC} &= \partial_\mu U_{YC}-iB_\mu U_{YC}+iU_{YC}v_\mu , \\ D_\mu U_{CL} &= \partial_\mu U_{CL}-iv_\mu U_{CL}+iU_{CL}W_\mu . \end{aligned} \qquad (19)$$

The U fields can be written as $U_{YC}=\sigma_Y\sigma_C^{\dagger}$ and $U_{CL}=\sigma_C\sigma_L^{\dagger}$ where the $\sigma_{L,C,Y}$ are elements of $SU\left(2\right)_{L,C,R}/H$ respectively. These σ_I with $I=L,C,Y$ transform under the full $SU\left(2\right)_L\times SU\left(2\right)_C\times U\left(1\right)_Y$ as $\sigma_I\rightarrow g_I\sigma_I h^{\dagger}$.

By applying the gauge transformation

$$v_\mu^I \to \sigma_I^\dagger v_\mu^I \sigma_I + i\sigma_I^\dagger \partial_\mu \sigma_I = \Omega_\mu^I, \qquad U_{IJ} \to \sigma_I^\dagger U_{IJ} \sigma_J = 1\,,$$

and after the gauge fixing $\sigma_Y = \sigma_L^\dagger = u^2 = U = e^{\frac{i\hat{\pi}}{v}}$ and $\sigma_C = 1$, which implies that $U_{YC} = U_{CL}$ (i.e. $\hat{\sigma} = 0$), so we have:

$$\begin{aligned} \mathcal{L}_\chi^{\text{gauge}} &= v^2\left(1 + \frac{h^2 + H^2}{4v^2} + \frac{h}{v}\right)\left(\left\langle \left(v_\mu - i\Gamma_\mu\right)^2 \right\rangle + \frac{1}{4}\left\langle u_\mu u^\mu \right\rangle\right) \\ &- \frac{1}{2}\left(2vH + hH\right)\left\langle u^\mu \left(v_\mu - i\Gamma_\mu\right)\right\rangle\,, \end{aligned} \tag{20}$$

where

$$\begin{aligned} u_\mu &= \Omega_\mu^Y - \Omega_\mu^L = iu^\dagger D_\mu U u^\dagger, \\ \Gamma_\mu &= \frac{1}{2i}\left(\Omega_\mu^Y + \Omega_\mu^L\right) = \frac{1}{2}\left[u^\dagger\left(\partial_\mu - iB_\mu\right)u + u\left(\partial_\mu - iW_\mu\right)u^\dagger\right] \end{aligned} \tag{21}$$

Now by setting

$$v_\mu = V_\mu + i\Gamma_\mu\,, \tag{22}$$

by using the identity

$$v_{\mu\nu} = V_{\mu\nu} - i\left[V_\mu, V_\nu\right] + \frac{i}{4}\left[u_\mu, u_\nu\right] + \frac{1}{2} f^+_{\mu\nu}\,, \tag{23}$$

where $f^+_{\mu\nu} = uW_{\mu\nu}u^\dagger + u^\dagger B_{\mu\nu} u$, by redefining $V_\mu \to \frac{g_C}{\sqrt{2}} V_\mu$, and taking the mass of the L-R-parity odd H given in (18) infinitely large, $\mathcal{L}^{\text{gauge}}$ coincides with $\mathcal{L}_{\text{eff}}$ up to operators irrelevant for the processes under consideration, only for the values of the parameters:

$$\begin{aligned} &g_V = \frac{1}{2g_C} = \frac{1}{g_K} = \frac{v}{2M_V}, \qquad f_V = 2g_V\,, \\ &a = \frac{1}{2}, \qquad b = \frac{1}{4}, \qquad d = 1, \qquad G_V = \frac{v}{2}\,, \\ &M_V = g_C v = \tfrac{1}{2} g_K v = \tfrac{v}{2g_V} \end{aligned} \tag{24}$$