Fermions on Thick D-branes

Localisation and physical implications

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Outline

Motivation

- D-branes appear in the AdS/CFT correspondence.
- May 'solve' the hierarchy problem.
- May be used as a new way of compactification.
- Different behaviour of chiralities in brane-world scenarios.
- May be thought as realization of 'string' phenomenology.

Randall-Sundrum 1

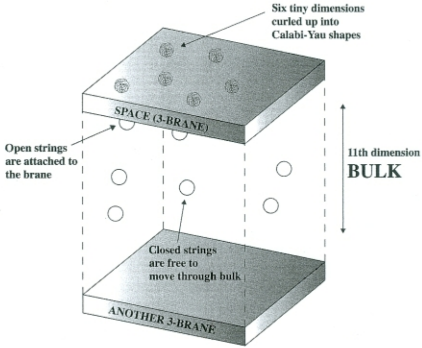
- Setup: Couple of $D3$-branes.
- Interaction: 5-dimensional Gravity.

Hierarchy Problem

$$\left(M_p^{(4)}\right)^2 \propto \frac{\left(M_p^{(5)}\right)^3}{k}\left(1 - e^{-2\pi k r_c}\right)$$

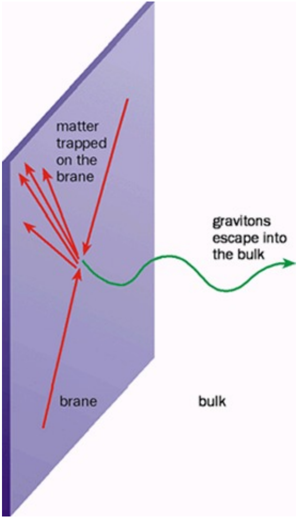
but

$$M_p^{(5)} e^{-2\pi k r_c} \sim M_W.$$



Randall-Sundrum 2

- Setup: Single $D3$-branes.
- Interaction: 5-dimensional Gravity.

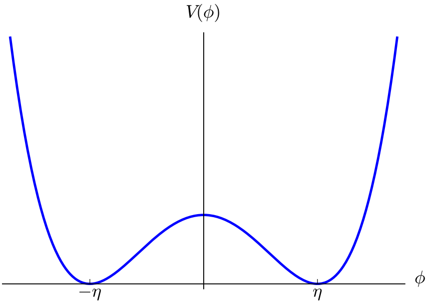


4-dimensional Gravity

The induced gravity on the brane is Newton's plus consistent corrections with GR.

Domain Wall Scenarios

- Coupled Einstein-Scalar field.
- Self-interacting scalar field.
- $V(\phi)$ with degenerated minima.
- Types:
  - Static DW's.
  - Dynamical DW's.



Localisation of Gravity

Most of the Domain Walls share the property of gravity localisation, depending on the Z_2 symmetry of the spacetime.

Early Stages

- Imposition that matter is localised. [ADD, 1998]
- Matter is not localised on RS scenarios. [Bajc and Gabadadze, 2000]
- Move to generalised models...
- Add new interactions. [BG, 2000]

General Set Up

- Metric:

$$\hat{g}_{\hat{\mu}\hat{\nu}} = e^{2A(\xi)}\left(-\mathbf{d}t_{\hat{\mu}}\mathbf{d}t_{\hat{\nu}} + e^{2B(\xi,t)}\mathbf{d}\vec{x}_{\hat{\mu}}\mathbf{d}\vec{x}_{\hat{\nu}}\right) + e^{2C(\xi)}\mathbf{d}\xi_{\hat{\mu}}\mathbf{d}\xi_{\hat{\nu}}.$$

- Fermions in curved spacetime,

$$\mathscr{L} = -\sqrt{\hat{g}}\bar{\Psi}\ \hat{\nabla}\!\!\!/\ \Psi$$

- Yukawa-like coupling with the scalar of the DW,

$$\mathscr{L}_I = \sqrt{\hat{g}}\bar{\Psi}\left(\lambda\phi^n - M\,\mathrm{sgn}(\phi)\right)\Psi.$$

- $n \in 2\mathbb{N}+1$.

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Procedure

1. Find the e.o.m. for the 5-dimensional spinor.
2. Separate the diff. op. in 4-dimensional part and extra terms.
3. Decompose,

$$\Psi = f_+ \psi_+ + f_- \psi_- .$$

4. Find the e.o.m. for $f_\pm$

$$\hat{\not\nabla}\Psi + M\Psi = \lambda\Phi\Psi$$

Vielbein Formalism

When spinors in curve spacetimes are considered, the vielbein formalism is the right way to follow.

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$$\hat{\not\nabla} = e^{A(\xi)} \not\nabla + e^{C(\xi)}\gamma^*\partial_\xi$$

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4. Find the e.o.m. for $f_\pm$

$$\nabla\!\!\!/^{\,(4)}\psi_\pm \sim m\psi_\mp$$

$$\gamma^*\psi_\pm = \pm\psi_\pm.$$

Factorise them

Factorise the ψ_+ and ψ_-

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E.O.M.

Decouple and transform to Schödinger-like equation.

Generalised Coupling

If general polynomial coupling is considered,

$$\mathscr{L}_I = \sqrt{\hat{g}}\lambda\bar{\Psi}\Phi^n\Psi,$$

the e.o.m. might be integrated for $n \in \mathbb{Z}$.

Most Interesting Cases

For $n \in 2\mathbb{N}+1$ the profile potential gives a condition on the coupling, λ, for localisation to occur.
For $n \in 2\mathbb{N}$ the profile potential gives a condition on the dimension of the extra dimension for localisation to occur.

Curiosity

The e.o.m. might be solved formally for $n \in -\mathbb{N}$.

Possible Choices of E.O.M.

- $B(\xi = 0) = 0$ and $\not{\partial}^{(4)} \psi_\pm = 0$.
- $B(\xi = 0) = 0$ and $\not{\partial}^{(4)} \psi_\pm = -m\psi_\mp$.
- $\not{\nabla}^{(4)} \psi_\pm = 0$.
- $\not{\nabla}^{(4)} \psi_\pm = -m\psi_\mp$.

Profile solutions

$$f_\pm(\xi) = e^{-\frac{1}{4}(4A' + 3B') \pm \lambda \int d\xi \mathcal{P}(\phi) e^{C}}.$$

$$f_\pm(\chi) = e^{-A \pm \lambda \int d\chi \mathcal{P}(\phi) e^{A}}.$$

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$$f_{\pm}(\xi) = e^{-\frac{1}{4}(4A'+3B')\pm\lambda\int d\xi\mathcal{P}(\phi)e^{C}}.$$

$$f_{\pm}(\chi) = e^{-A\pm\lambda\int d\chi\mathcal{P}(\phi)e^{A}}.$$

For massive cases

The initial e.o.m. is,

$$f'_\pm + A' f_\pm \mp \lambda \mathcal{P}(\phi) e^A f_\pm = \pm m f_\mp .$$

Decoupling the profiles equations, and changing,

$$f_\pm = e^{-A} u_\pm ,$$

these equations are rewritten in a Schrödinger-like form,

$$\left[-\partial_\chi^2 + V_{qm}^\pm \right] u_\pm = m^2 u_\pm ,$$

with

$$V_{qm}^\pm = \left(\lambda \mathcal{P}(\phi) e^A \right)^2 \pm \partial_\chi \left(\lambda \mathcal{P}(\phi) e^A \right) .$$

Numerov's Method

Numerov's method is a numerical method to solve ordinary differential equations of second order in which the first-order term does not appear.

The algorithm

For a ODE.,

$$\left(\frac{\partial^2}{\partial x^2} + f(x)\right) y(x) = 0,$$

Numerov's solution is,

$$y_{n+1} = \frac{\left(2 - \frac{5h^2}{6} f_n\right) y_n - \left(1 + \frac{h^2}{12} f_{n-1}\right) y_{n-1}}{\left(1 + \frac{h^2}{12} f_{n+1}\right)}$$

Implementation

- Numerov algorithm: for solving the ODE.
- Swept of energy: for finding the approx. eigen-energy.
- Bisection algorithm: For refining the energy eigenvalue.

Moduli Space

The five dimensional mass takes values from 0.00 to 1.00 in steps of 0.02, while $\lambda \in [1.45, 5.00]$ in steps of 0.05.
Additionally, the computations have been restricted to values of $s = 1, 3, 5$.

Double Domain Walls

- Solution:

$$\phi(\xi) = \frac{\sqrt{6s-3}}{s} \arctan\left(\alpha\xi\right)^{s},$$

$$A(\xi) = C(\xi) = -\frac{1}{2s} \ln\left(1 + (\alpha\xi)\right.$$

- Parameters: $n, s \in 2\mathbb{N}+1$.

Results

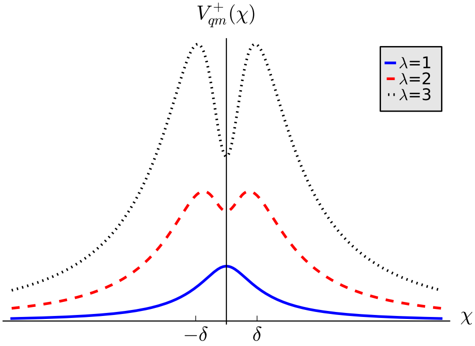
- Existence of a λ_* below which no positive chirality is localised. $(n = 1)$
- Existence of a λ_c above which no negative chirality is localised.
- $\lambda_* < \lambda_c$.

Physical Implications

Case $n = s = 1$:

Interpretation

- Lightest particles have no detectable right-handed chirality. (Neutrinos)
- Not so light particles might localise both chiralities. (Electrons)

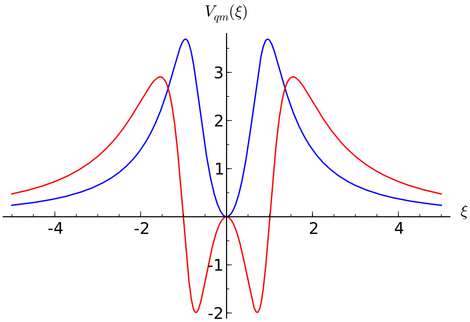


Physical Implications

Case $n, s \neq 1$:

Interpretation

- Not so light particles might localise both chiralities.
- Mass difference could favour the See-Saw mechanism.



Additional Configurations

- Kink Domain Wall:

$$A(\xi) = -\frac{2}{3}\delta\left[\ln\left(\cosh\left(\frac{\alpha\xi}{\delta}\right)\right) + \frac{1}{4}\tanh^2\left(\frac{\alpha\xi}{\delta}\right)\right],$$

$$B(\xi) = C(\xi) = 0,$$

$$\phi(\xi) = \sqrt{3\delta}\tanh\left(\frac{\alpha\xi}{\delta}\right).$$

Additional Configurations

- Sine-Gordon Wall:

$$A(\xi) = -\delta \ln\left(\cosh\left(\frac{\alpha\xi}{\delta}\right)\right),$$
$$B(\xi) = C(\xi) = 0,$$
$$\phi(\xi) = \sqrt{3\delta}\,\arctan\left(\sinh\left(\frac{\alpha\xi}{\delta}\right)\right).$$

Additional Configurations

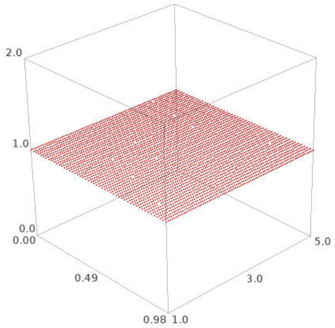
- Asymmetric Domain Wall:

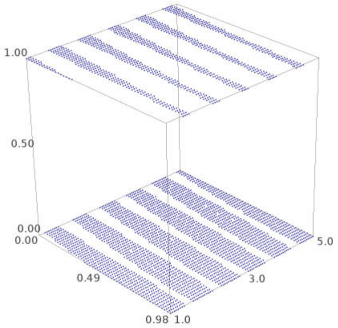
$$A(\xi) = \alpha\xi - \delta \exp\left(-2e^{-\frac{\beta\xi}{\delta}}\right) + \delta\,\mathrm{Ei}\left(-2e^{-\frac{\beta\xi}{\delta}}\right),$$
$$B(\xi) = C(\xi) = 0,$$
$$\phi(\xi) = 2\sqrt{3\delta}\left(\exp\left(-e^{-\frac{\beta\xi}{\delta}}\right) - \varepsilon\right).$$

with

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} dt \frac{e^t}{t}.$$

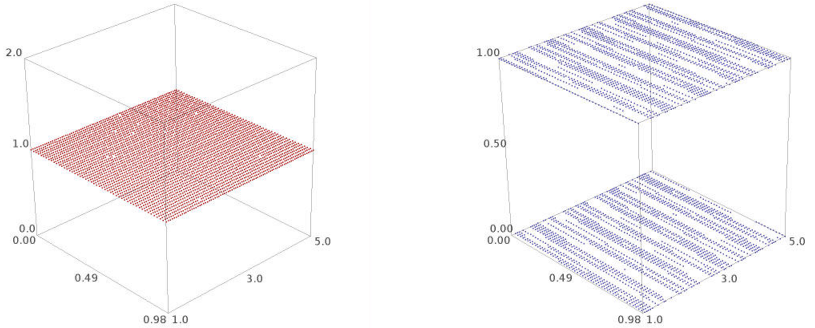
Results





Moduli space for the asymmetric domain wall, with $n = 1, 3, 5$.
Negative and positive chirality respectively.

Results



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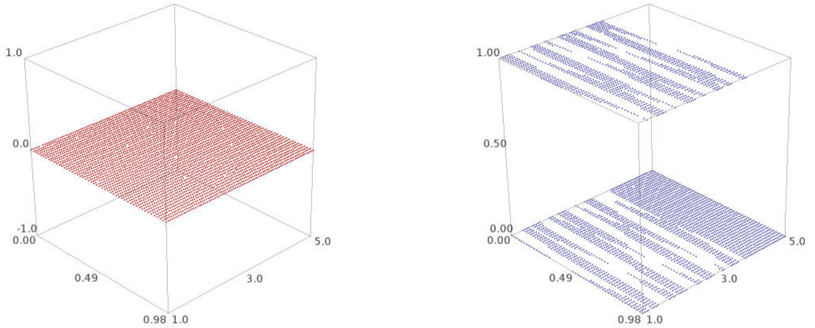

Moduli space for the asymmetric domain wall, with $n = 1, 3, 5$.
Negative and positive chirality respectively.

Conclusions

- More general profile equations were found. These are useful when more complex systems are considered.
- Restrictions to the moduli space of parameter where found, by restricting the lower dimensional theory to be tachyon free.
- Due to the mass difference between localised chiralities, it is interesting to consider the See-Saw mechanism.
- The mass difference between neutrinos and electrons makes natural the absence of right-handed part of the former, while the analog of the later is present in the low-dimensional theory.
- Similar behaviour for diverse DW's. Except the appearance of a sort of band structure of the moduli space.