

# *R*-parity breaking and baryon number violation in anomalous $U(1)_H$ models

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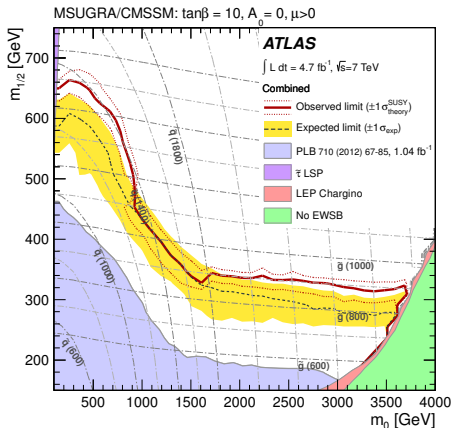
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<sup>1</sup>Work in progress with A. Florez, M. Velasquez and D. Restrepo

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# Motivation

- LHC searches for susy:  $m_{\tilde{q}} \approx m_{\tilde{g}} \gtrsim 1.4\text{TeV}$  in cMSSM.
- Searches based on missing transverse momentum carried by LSP, which is stable because R-parity conservation assumption.
- A high mass scale for  $m_{\tilde{q}}$  represents a potential chink in the initial proposal of the MSSM as a possible solution to the hierarchy problem.



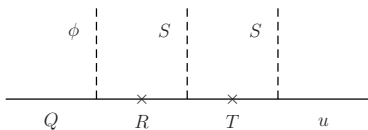
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- In  $R_p$  scenarios such results have a lesser impact because the LSP can decay inside the detector, and the missing energy signal is degraded.
- $R_p$  with  $B$  lead to the most difficult signal to be searched at hadron colliders.
- If lepton parity is imposed several issues arise. The size of  $R$ -parity breaking couplings must be precisely chosen by hand in order to avoid constraints from flavor physics observables.
- It will be desirable to build a self-consistent supersymmetric framework with baryon number violation, where the operators and size of their couplings can be generated.

# Froggat-Nielsen mechanism

The approach to account for the fermion mass hierarchy is based on an hypothetical  $U(1)_H$  symmetry which is spontaneously broken by vev of one flavon field  $S$  of horizontal charge  $H[S] = -1$ .

At high energies:  $\mathcal{L} = \bar{Q}\phi_{(0)}R + \bar{R}S_{(-1)}T + \bar{T}S_{(-1)}u$



At low energies:  $\mathcal{L}_{\text{eff}} \sim \left(\frac{\langle S \rangle}{M_P}\right)^n \bar{Q}\phi u = \theta^n \bar{Q}\phi u$ ,  
 $n = n_Q + n_\phi + n_u$ .

## Charged fermion masses hierarchy

$$m_u : m_c : m_t \simeq \theta^8 : \theta^4 : 1,$$

$$m_d : m_s : m_b \simeq \theta^4 : \theta^2 : 1,$$

$$m_e : m_\mu : m_\tau \simeq \theta^5 : \theta^2 : 1,$$

$$V_{us} \simeq \theta, \quad V_{cb} \simeq \theta^2,$$

with  $\theta \approx 0.22$ .

## The superpotential

$$\begin{aligned}
 W = & h_{ij}^u \widehat{H}_u \widehat{Q}_i \widehat{u}_j + h_{ij}^l \widehat{H}_d \widehat{L}_j \widehat{l}_k + h_{ij}^d \widehat{H}_d \widehat{Q}_j \widehat{d}_k \\
 & + \mu_0 \widehat{H}_d \widehat{H}_u \\
 & + \mu_i \widehat{L}_i \widehat{H}_u \\
 & + \lambda_{ijk} \widehat{L}_i \widehat{L}_j \widehat{l}_k + \lambda'_{ijk} \widehat{L}_i \widehat{Q}_j \widehat{d}_k \\
 & + \lambda''_{ijk} \widehat{u}_i \widehat{d}_j \widehat{d}_k,
 \end{aligned}$$

- $W \Rightarrow \mathcal{B}$  and  $\mathcal{L} \Rightarrow$  fast proton decay.
- To avoid it, it is imposed R-parity, obtaining the MSSM.  
 $\mu_i$ ,  $\lambda_{ijk}$ ,  $\lambda'_{ijk}$  and  $\lambda''_{ijk}$  are forbidden.
- Because  $R_P$  conservation LSP is stable and DM candidate.
- However, either  $\mathcal{B}$  or  $\mathcal{L}$  can exist.

Using a supersymmetric model extended by a anomalous  $U(1)_H$  flavor symmetry, it is possible to obtain either  $\cancel{L}$  or  $\cancel{B}$ , in addition to the fermion mass hierarchy.

The effective bilinear and trilinear  $\cancel{RP}$  terms are given by

$$\mu_\alpha \sim \begin{cases} M_P \theta^{n_\alpha} & n_\alpha \geq 0 \\ m_{3/2} \theta^{|n_\alpha|} & n_\alpha < 0 \\ 0 & n_\alpha \text{ fractional} \end{cases}$$

$$\lambda_T \sim \begin{cases} \theta^{n_\lambda} & n_\lambda \geq 0 \\ (m_{3/2}/M_P) \theta^{|n_\lambda|} & n_\lambda < 0 \\ 0 & n_\lambda \text{ fractional} \end{cases}$$

$$n_\alpha = L_\alpha + H_u, \quad n_\lambda = L_i + L_j + \ell_k.$$

In order to obtain a viable flavor model, the  $U(1)_H$  charges must satisfy several phenomenological and theoretical constraints.

- 8 phenomenological constraints corresponding to six mass ratios for the charged fermions and two quarks mixing angles.
- Reproduce the third generation fermion masses

$$m_t \sim \langle \phi_u \rangle \Rightarrow \phi_u + u_3 + Q_3 = 0,$$

$$m_b \sim m_\tau \Rightarrow \phi_d + d_3 + Q_3 = \phi_d + \ell_3 + L_3 = x,$$

- 3 conditions from anomaly cancellation.

We are left with 4 parameters that we choose to be  $n_i$  and  $x$  (Mira2000).

- More additional constraints must be included.
  - Neutrino oscillation data (Dreiner2003)
  - Neutralino as decaying dark matter (Sierra2009).



$$H[\mu_i] = n_i$$

$$H \begin{pmatrix} \lambda_{121} & \lambda_{122} & \lambda_{123} \\ \lambda_{131} & \lambda_{132} & \lambda_{133} \\ \lambda_{231} & \lambda_{232} & \lambda_{233} \end{pmatrix} = \begin{pmatrix} 5 + n_2 & 2 + n_1 & n_1 + n_2 - n_3 \\ 5 + n_3 & 2 + n_1 - n_2 + n_3 & n_1 \\ 5 - n_1 + n_2 + n_3 & 2 + n_3 & n_2 \end{pmatrix} \\ + (1 + x)\mathbf{1}_3,$$

$$H(\lambda'_{ijk}) = \begin{pmatrix} 4 & 3 & 3 \\ 3 & 2 & 2 \\ 1 & 0 & 0 \end{pmatrix} + (1 + n_i + x)\mathbf{1}_3,$$

$$H \begin{pmatrix} \lambda''_{112} & \lambda''_{212} & \lambda''_{312} \\ \lambda''_{113} & \lambda''_{213} & \lambda''_{313} \\ \lambda''_{123} & \lambda''_{223} & \lambda''_{323} \end{pmatrix} = \begin{pmatrix} 6 & 3 & 1 \\ 6 & 3 & 1 \\ 5 & 2 & 0 \end{pmatrix} + n_{\lambda''}\mathbf{1}_3,$$

where  $n_{\lambda''} = \frac{1}{3}(3x + n_1 + n_2 + n_3 - 1)$ .

Once  $n_i$  are fractional it is obtained

- All  $\lambda'_{ijk}$  are forbidden.
- All  $\lambda_{ijk}$  with repeated indexes are also forbidden.

Several possibilities

- Choose the fractional  $n_i$  charges such that all the  $L$  and  $B$  violating terms in the superpotential also have fractional charges.  $R_p$  and  $P_6$  discrete symmetries respectively, are obtained as remnants of a spontaneously broken  $U(1)_H$ .
- If  $n_{1,2,3}$  not half-integers and  $n_{\lambda''} = (n_1 + n_2 + n_3 - 1)/3$  integer, we have only trilinear  $B$  violating terms  $\lambda''$  in the supersymmetric Lagrangian.

# Baryon number violation

Once fixed  $n_i$  charges such that  $\mu_i$ ,  $\lambda_{ijk}$  and  $\lambda'_{ijk}$  are forbidden, we obtain

$$\begin{pmatrix} \lambda''_{112} & \lambda''_{212} & \lambda''_{312} \\ \lambda''_{113} & \lambda''_{213} & \lambda''_{313} \\ \lambda''_{123} & \lambda''_{223} & \lambda''_{323} \end{pmatrix} \sim \theta^{n_{\lambda''}} \begin{pmatrix} \theta^6 & \theta^3 & \theta \\ \theta^6 & \theta^3 & \theta \\ \theta^5 & \theta^2 & 1 \end{pmatrix}$$

The most important constraint on  $\lambda''_{ijk}$  are from neutron-antineutron oscillations and double nucleon decay, requiring  $\lambda''_{112} \lesssim 10^{-8}$  and  $\lambda''_{113} \lesssim 10^{-4}$  for  $\tilde{m} \sim 200$  GeV.

$$\begin{aligned} \lambda''_{112} \lesssim 10^{-8} &\Rightarrow \lambda''_{112} \lesssim \theta^{12} \\ &\Rightarrow n_{\lambda} \geq 6, n_{\lambda} \leq -7 \\ &\Rightarrow \lambda''_{323} \lesssim \theta^4 \approx 10^{-4}. \end{aligned}$$

LSP decays with displaced vertices are expected.

## Dimension-5 operators and proton decay

$$\begin{aligned}W_{D5} &= \frac{(\kappa_1)_{ijkl}}{M_P} \widehat{Q}_i \widehat{Q}_j \widehat{Q}_k \widehat{L}_l + \frac{(\kappa_2)_{ijkl}}{M_P} \widehat{u}_i \widehat{u}_j \widehat{d}_k \widehat{e}_l + \frac{(\kappa_3)_{ijk}}{M_P} \widehat{Q}_i \widehat{Q}_j \widehat{Q}_k \widehat{H}_d \\ &+ \frac{(\kappa_4)_{ijk}}{M_P} \widehat{Q}_i \widehat{H}_d \widehat{u}_j \widehat{e}_k + \frac{(\kappa_5)_{ij}}{M_P} \widehat{L}_i \widehat{H}_u \widehat{L}_j \widehat{H}_u + \frac{(\kappa_6)_i}{M_P} \widehat{L}_i \widehat{H}_u \widehat{H}_d \widehat{H}_d, \\ V_{5D} &= \frac{(\kappa_7)_{ijk}}{M_P} \widehat{u}_i \widehat{d}_j^* \widehat{e}_k + \frac{(\kappa_8)_i}{M_P} \widehat{H}_u^* \widehat{H}_d \widehat{e}_i + \frac{(\kappa_9)_{ijk}}{M_P} \widehat{Q}_i \widehat{L}_j^* \widehat{u}_k + \frac{(\kappa_{10})_{ijk}}{M_P} \widehat{Q}_i \widehat{Q}_j \widehat{d}_k^*.\end{aligned}$$

- $\kappa_{3,10}$  couplings induce  $\mathcal{B}$  processes and do not pose threat to the proton decay as long as there is no major  $\mathcal{L}$ .
- $\kappa_{4,\dots,9}$  couplings induce  $\mathcal{L}$  processes  $\rightarrow$  must be suppressed enough.
- $QQQL$  and  $UUDE$  operators violate both  $B$  and  $L$ , being the most dangerous for the stability of the proton as they require no other terms.

## Dimension-5 operators and proton decay

The horizontal charges for those dimension-5 operators that violate only baryon number are given by

$$H \left[ (\kappa_3)_{1jk} \widehat{Q}_1 \widehat{Q}_j \widehat{Q}_k \widehat{H}_d \right] = A_3 + (2x + 4 - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_3)_{2jk} \widehat{Q}_2 \widehat{Q}_j \widehat{Q}_k \widehat{H}_d \right] = A_3 + (2x + 3 - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_3)_{3jk} \widehat{Q}_3 \widehat{Q}_j \widehat{Q}_k \widehat{H}_d \right] = A_3 + (2x + 1 - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_{10})_{ij1} \widehat{Q}_i \widehat{Q}_j \widehat{d}_1^* \right] = A_3 + (x - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_{10})_{ij2} \widehat{Q}_i \widehat{Q}_j \widehat{d}_2^* \right] = A_3 + (x + 1 - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_{10})_{ij2} \widehat{Q}_i \widehat{Q}_j \widehat{d}_3^* \right] = H \left[ (\kappa_{10})_{ij3} \widehat{Q}_i \widehat{Q}_j \widehat{d}_2^* \right].$$

It follows that if the  $UDD$  operator is allowed by the horizontal symmetry, then the  $QQQH_d$  and  $QQd^*$  are also allowed.

For the violating baryon and lepton number operators we have that

$$H \left[ (\kappa_1)_{1jkl} \widehat{Q}_1 \widehat{Q}_j \widehat{Q}_k \widehat{L}_l \right] = A_1 + (5 + 2x + n_l - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_1)_{2jkl} \widehat{Q}_2 \widehat{Q}_j \widehat{Q}_k \widehat{L}_l \right] = A_1 + (4 + 2x + n_l - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_1)_{3jkl} \widehat{Q}_3 \widehat{Q}_j \widehat{Q}_k \widehat{L}_l \right] = A_1 + (2 + 2x + n_l - n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_2)_{ij11} \widehat{u}_i \widehat{u}_j \widehat{d}_1 \widehat{e}_1 \right] = A_2 + (6 - n_1 + n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_2)_{ij21} \widehat{u}_i \widehat{u}_j \widehat{d}_2 \widehat{e}_1 \right] = A_2 + (5 - n_1 + n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_2)_{ij31} \widehat{u}_i \widehat{u}_j \widehat{d}_3 \widehat{e}_1 \right] = H \left[ (\kappa_2)_{ij21} \widehat{u}_i \widehat{u}_j \widehat{d}_2 \widehat{e}_1 \right],$$

$$H \left[ (\kappa_2)_{ij12} \widehat{u}_i \widehat{u}_j \widehat{d}_1 \widehat{e}_2 \right] = A_2 + (3 - n_2 + n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_2)_{ij22} \widehat{u}_i \widehat{u}_j \widehat{d}_2 \widehat{e}_2 \right] = A_2 + (2 - n_2 + n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_2)_{ij32} \widehat{u}_i \widehat{u}_j \widehat{d}_2 \widehat{e}_2 \right] = H \left[ (\kappa_2)_{ij22} \widehat{u}_i \widehat{u}_j \widehat{d}_3 \widehat{e}_2 \right]$$

$$H \left[ (\kappa_2)_{ij13} \widehat{u}_i \widehat{u}_j \widehat{d}_1 \widehat{e}_3 \right] = A_2 + (1 - n_3 + n_{\lambda''}) \mathbf{1}_3,$$

$$H \left[ (\kappa_2)_{ij23} \widehat{u}_i \widehat{u}_j \widehat{d}_2 \widehat{e}_3 \right] = A_2 + (-n_3 + n_{\lambda''}) \mathbf{1}_3,$$

$$\left[ (\kappa_2)_{ij33} \widehat{u}_i \widehat{u}_j \widehat{d}_2 \widehat{e}_3 \right] = H \left[ (\kappa_2)_{ij23} \widehat{u}_i \widehat{u}_j \widehat{d}_3 \widehat{e}_3 \right].$$

Finally, for those couplings that violate only the lepton number we found

$$\begin{aligned}
H \left[ (\kappa_4)_{ij1} \widehat{Q}_i \widehat{H}_d \widehat{U}_j \widehat{e}_1 \right] &= A_4 + (5 - n_1 + x) \mathbf{1}_3, \\
H \left[ (\kappa_4)_{ij2} \widehat{Q}_i \widehat{H}_d \widehat{U}_j \widehat{e}_2 \right] &= A_4 + (2 - n_2 + x) \mathbf{1}_3, \\
H \left[ (\kappa_4)_{ij3} \widehat{Q}_i \widehat{H}_d \widehat{U}_j \widehat{e}_3 \right] &= A_4 + (-n_3 + x) \mathbf{1}_3, \\
H \left[ (\kappa_5)_{ij} \widehat{L}_i \widehat{H}_u \widehat{L}_j \widehat{H}_u \right] &= \begin{pmatrix} 2n_1 & n_1 + n_2 & n_1 + n_3 \\ n_1 + n_2 & 2n_2 & n_2 + n_3 \\ n_1 + n_3 & n_2 + n_3 & 2n_3 \end{pmatrix}, \\
H \left[ (\kappa_6)_i \widehat{L}_i \widehat{H}_u \widehat{H}_d \widehat{H}_u \right] &= -1 + n_i, \\
H \left[ (\kappa_7)_{ij1} \widehat{U}_i \widehat{d}_j^* \widehat{e}_1 \right] &= A_7 + (4 - n_1) \mathbf{1}_3, \\
H \left[ (\kappa_7)_{ij2} \widehat{U}_i \widehat{d}_j^* \widehat{e}_2 \right] &= A_7 + (1 - n_2) \mathbf{1}_3, \\
H \left[ (\kappa_7)_{ij3} \widehat{U}_i \widehat{d}_j^* \widehat{e}_3 \right] &= A_7 + (-1 - n_3) \mathbf{1}_3, \\
H \left[ (\kappa_8)_1 \widehat{H}_u^* \widehat{H}_d \widehat{e}_1 \right] &= 5 - n_1 + x, \\
H \left[ (\kappa_8)_2 \widehat{H}_u^* \widehat{H}_d \widehat{e}_2 \right] &= 2 - n_2 + x, \\
H \left[ (\kappa_8)_3 \widehat{H}_u^* \widehat{H}_d \widehat{e}_3 \right] &= -n_3 + x, \\
H \left[ (\kappa_9)_{i1k} \widehat{Q}_i \widehat{L}_j^* \widehat{U}_k \right] &= A_9 + (-n_j) \mathbf{1}_3.
\end{aligned}$$

For  $n_i$  fractional all  $\not\propto$  D-5 operators are also automatically forbidden by the  $U(1)_H$ .

- The proton decay mediated only by  $\lambda''$  couplings occurs in scenarios with a gravitino lighter than proton (Choi:1996), leading to strong bounds on these couplings.
- By ensuring gravitino masses greater than 1 GeV in these scenarios there will be no contribution to the proton decay coming from gravitino.
- Because Planck mass suppression, a gravitino LSP can be also a dark matter candidate (Takayama:2000, Buchmuller:2007).
- For  $\lambda'' \lesssim 10^{-5} \Rightarrow \tau_{\tilde{G}} \gtrsim 10^{26} s$  (Lola2008).



## Dirac neutrinos

- $LH_u LH_u$  is forbidden by  $U(1)_H$  because  $n_i$  are not half-integers  $\Rightarrow \underline{m_L \nu_L \nu_L}$
- Introducing  $N_R$ : as  $n_i$  are non half-integers  $\Rightarrow \underline{M_N N_R N_R}$  if  $L_i H_u N_j = n_i + N_j = \text{integer}$ .
- Dirac mass terms can only be generated in this scenario.

## Majorana neutrinos

- By adding a new flavon  $S'$  with fractional  $H$ -charge Majorana neutrinos can be obtained.
- The  $H$ -charge of  $S'$  is such that it does not get coupled to  $L$  violating operators because the respective total  $H$ -charge are fractional (negative) and therefore forbidden (suppressed enough).
- The introduction of an additional flavon field do not spoil the proton stability via D5 operators.

The neutrino mass matrix is given by

$$M_\nu \sim m_{\text{eff}} \theta^n \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

- A large  $\theta_{13}$  supports models based on an anarchical texture (deGouvea2012, Altarelli2012).
- An immediate consequence of the anarchy assumption is that the bilinear charges are equal and are set to  $n_i = n_{\chi''} - x + \frac{1}{3}$ , being clearly non-integer numbers.
- Pseudo  $\mu\tau$ -anarchy or hierarchical textures are also possible.

# Implications on collider searches

When R-parity conservation is assumed, the production of sparticles is in pairs and the LSP is stable.

R-parity violation allows for the single production of supersymmetric particle and decay of the LSP.

From  $\mathcal{B}$  terms, the LSP can decay directly or indirectly to quarks, and depending on whether  $m_{LSP} > m_t$  or  $m_{LSP} < m_t$ , top quark can be present on the final states.

It is clear that  $\lambda''_{323}$  coupling dominates over the other couplings, and heavy quarks are to be present in the final states.

LSP can be any supersymmetric particle: neutralino, chargino, squark, slepton or gluino.

If  $\tilde{G}$  is the real LSP  $\Rightarrow \chi, \tilde{t} \rightarrow \text{NLSP}$ .

- $\tilde{t}$  can decay directly into a two down quarks of different generations through the  $\lambda''_{3jk}$  coupling.
- Hence, final states with at least two  $b$ -jets are also expected, which could be observed at the LHC.

$$\frac{\text{Br}(\tilde{t} \rightarrow bd)}{\text{Br}(\tilde{t} \rightarrow bs)} \sim \frac{\text{Br}(\tilde{t} \rightarrow sd)}{\text{Br}(\tilde{t} \rightarrow bs)} \sim \theta^2 \approx 0.05.$$

- $\text{Br}(\tilde{t} \rightarrow bd) \sim 0.9$ .
- Plausible in natural SUSY.

- Through direct decays, the sbottom can decay into an up quark and a down quark, *i.e.* no  $b$ -jets).
- Approximately more than 99% of the decays involve a top quark, completely opposite to the case when  $m_{\tilde{b}} < m_t$ , where no top quarks are produced.
- $m_{\tilde{b}}^0 > m_t$ .

$$\frac{\text{Br}(\tilde{b} \rightarrow td)}{\text{Br}(\tilde{b} \rightarrow ts)} \sim \theta^2 \approx 0.05.$$

- $m_{\tilde{b}} < m_t$ . In this point we have that  $\tilde{b} \rightarrow cs$  is the dominant decay.

$$\frac{\text{Br}(\tilde{b} \rightarrow cd)}{\text{Br}(\tilde{b} \rightarrow cs)} \sim \theta^2 \approx 0.05.$$

The lightest neutralino  $\chi$  can decay to three quarks through virtual scalar exchange and the dominant decay mode will be  $\chi \rightarrow tbs$ . If the neutralino is lighter than top quark, then the dominant mode is  $\chi \rightarrow cbs$ . Therefore, horizontal symmetry allows for estimating relations between different ratios of branching ratios.

- $m_\chi > m_t$ . Dominated by  $\lambda''_{323}$ , involving a top quark in the final state.

$$\frac{\text{Br}(\chi \rightarrow tdb)}{\text{Br}(\chi \rightarrow tsb)} \sim \frac{\text{Br}(\chi \rightarrow tds)}{\text{Br}(\chi \rightarrow tsb)} \sim \theta^2 \approx 0.05.$$

- $m_\chi < m_t$ . Dominated by  $\lambda''_{223}$ .

$$\frac{\text{Br}(\chi \rightarrow cdb)}{\text{Br}(\chi \rightarrow csb)} \sim \frac{\text{Br}(\chi \rightarrow cds)}{\text{Br}(\chi \rightarrow csb)} \sim \theta^2 \approx 0.05.$$

In either case, neutralino decays are dominated by heavy flavors, and should contain displaced vertices.

- We have studied the question about if it is possible to obtain a framework with baryon number violation. By using a  $U(1)_H$  horizontal gauge symmetry, we have found charge assignments that allow all  $\lambda''$  couplings, being  $\lambda''_{3jk}$  the dominant ones.
- All  $\mathcal{B}$  terms satisfy current experimental bounds.
- Gravitino as a dark matter candidate.
- It is possible to obtain a neutrino matrix with a acceptable phenomenological texture with Majorana or Dirac neutrinos (with or without  $S'$ ).
- The ratio of branching ratios for the LSP have been analyzed. Hence, it is possible to infer the main decay channels of the LSP. Displaced vertices are expected.

Set of  $H$ -charges allowing a  $\mathcal{B}$  self-consistent framework

$x$	0	0	1	0	1	2
$n_{\lambda''}$	8	9	9	10	10	10
$n_i$	25/3	28/3	25/3	31/3	28/3	25/3
$N_R$	29/3	26/3	29/3	23/3	26/3	29/3

$x$	1	1	1	2	2	2	3	3
$n_{\lambda''}$	5	6	7	6	7	8	8	9
$n_i$	13/3	16/3	19/3	13/3	16/3	19/3	16/3	19/3
$S'$	-47/6	-53/6	-59/6	-47/6	-53/6	-59/6	-53/6	-59/6