On the QCD of a Massive Vector Field in the Adjoint Representation

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December 9, 2012

Outlook

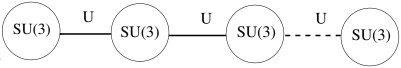
Models Beyond the Standard Model

Many New Physics models predict new spin-1 color octet fields:

Model	Field
Top Color	Coloron
One-Family Technicolor	Color-Octet Technirho
Extra-dimensions	Kaluza-Klein excitations of the gluon
Chiral Color	Axigluon

Table: Spin-1 Color-Octet Fields predicted by some New Physics models

How can we describe them ?

- Extending the gauge symmetry (Deconstruction Theory)
  - $SU(3)_c \Rightarrow$ 

- Coloron as a matter field using Vector-Meson-Dominance (VMD)-like Lagrangians (for example: Technicolor)
  - Kroll-Lee-Zumino-like Lagrangians (based on kinetic mixing)
  - Sakurai-like Lagrangian (based on mass mixing)
  - OBS: these two versions of VMD are completely equivalent
  - This description quickly violate unitarity and is not renormalizable
- All of them are effective descriptions

Why not a renormalizable theory ?

- We know how to construct a renormalizable QCD with scalars and spin-$\frac{1}{2}$ fields as matter fields.
- Is it possible to construct a consistent and renormalizable theory with massive spin-1 fields without introducing additional structures ("Higgses", extended local symmetries, etc) ?
- Common lore says it's not possible because:
  1. Scattering of longitudinally polarized massive vector violates unitarity
  2. The propagator of a massive spin-1 field has a bad UV behavior and spoil the "power counting" in loops diagrams

$$\Delta_{\mu\nu} = \frac{-i}{q^2 - M^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)$$

- The aim of this talk is to give an example of construction of a reasonable candidate.

The very beginning: Proca Lagrangian

- Let's consider a generalization of Proca theory for a non-Abelian *global* symmetry:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}M^2 V^a_\mu V^{a\mu} - V^a_\mu J^{a\mu}(+\mathcal{L}_{int})$$

with $F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu$ and V^a_μ transforms homogeneously under the global symmetry ($V_\mu \to U^\dagger V_\mu U$).

- The equation of motion is

$$\partial_\rho F^{a\rho\nu} + M^2 V^{a\nu} = J^{a\nu}$$

- The anti-symmetry of $F^a_{\mu\nu}$ implies the Lorentz $\partial_\mu V^{a\mu} = 0$, assuming that $J^{a\mu}$ is a conserved current.

Propagator

- From the Proca Lagrangian we can obtain the propagator and we get:

$$\Delta_{\mu\nu} = \frac{-i}{q^2 - M^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)$$

- We have already comment that this propagator has a bad UV behavior and spoil renormalizability
- The origin of this propagator and all its problems is the anti-symmetric structure of the $F^a_{\mu\nu}$
  - That means that $\partial_\mu V_\nu \partial^\mu V^\nu$ and $\partial_\mu V_\nu \partial^\nu V^\mu$ enter in the Lagrangian with the same weight

Recipe for a local gauge theory

- A direct way to turn the Proca-Stueckelberg Lagrangian into a gauge theory is:
  - replace all the derivatives by covariant derivatives ($\partial_\mu \to D_\mu$)
  - Include in $\mathcal{L}_{int}$ all the gauge invariant and renormalizable terms we can form with V_μ, $D_\mu V_\nu$ and $G_{\mu\nu}$ (strength field of gluons) with arbitrary coefficients.
  - You may include a kinetic mixing term $G_{\mu\nu}F^{\mu\nu}$ (KLZ-like Lagrangian) but this term can be removed by a simple field redefinition.

$$\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2} Tr\left\{G_{\mu\nu}G^{\mu\nu}\right\} - Tr\left\{D_\mu V_\nu D^\mu V^\nu\right\} + (1+a)\, Tr\left\{D_\mu V_\nu D^\nu V^\mu\right\} \\
& + a_{11}\, Tr\left\{D_\mu V_\nu V^\mu V^\nu\right\} + a_{12}\, Tr\left\{D_\mu V_\nu V^\nu V^\mu\right\} \\
& + a_{21}\, Tr\left\{V_\mu V_\nu V^\mu V^\nu\right\} + a_{22}\, Tr\left\{V_\mu V_\nu V^\mu V^\nu\right\} \\
& + a_3\, Tr\left\{G_{\mu\nu}\left[V^\mu, V^\nu\right]\right\} + M^2\, Tr\{V_\nu V^\nu\}
\end{aligned}$$

Yang-Mills Theory with Two Connections

- Consider two color-octet vector fields, $A^a_{1\mu}$ and $A^a_{2\mu}$, which transform like connections of the same group $SU(3)$:

$$
\begin{aligned}
A^a_{1\mu} &\rightarrow UA^a_{1\mu}U^\dagger + \frac{1}{g_1}\left(\partial_\mu U\right)U^\dagger \\
A^a_{2\mu} &\rightarrow UA^a_{2\mu}U^\dagger + \frac{1}{g_2}\left(\partial_\mu U\right)U^\dagger
\end{aligned}
$$

- Key observation: $\mathfrak{V}^a_\mu = gA^a_{1\mu} - g_2 A^a_{2\mu}$ transforms homogeneously (it is covariant under gauge transformations).

Yang-Mills Theory with Two Connections

- Now we can write the following Lagrangian:

$$\begin{aligned}\mathcal{L}_{2CYM} &= -\frac{1}{2}Tr\left[F_{1\mu\nu}F_1^{\mu\nu}\right]-\frac{1}{2}Tr\left[F_{2\mu\nu}F_2^{\mu\nu}\right]\\ &+\frac{M^2}{g_1^2+g_2^2}Tr\left[\left(g_1A_{1\mu}-g_2A_{2\mu}\right)^2\right]\\ &+\frac{a}{g_1^2+g_2^2}Tr\left[\left(g_1D_\mu A_{1\nu}-g_2D_\mu A_{2\nu}\right)\left(g_1D^\nu A_1^\mu-g_2D^\nu A_2^\mu\right)\right]\\ &+\mathcal{L}_{NM}\end{aligned}$$

where $\mathcal{L}_{NM}$ contains all the gauge invariant and renormalizable terms we can form with $\mathfrak{V}_\mu$, $D_\mu\mathfrak{V}_\nu$, $F_{1\mu\nu}$ and $F_{2\mu\nu}$, with arbitrary coefficients.

General Lagrangian

The physical fields (mass eigenstates) are:

$$
\begin{aligned}
G_\mu &= \frac{g_2}{\sqrt{g_1^2+g_2^2}} A_{1\mu} + \frac{g_1}{\sqrt{g_1^2+g_2^2}} A_{2\mu} \\
V_\mu &= \frac{g_1}{\sqrt{g_1^2+g_2^2}} A_{1\mu} - \frac{g_2}{\sqrt{g_1^2+g_2^2}} A_{2\mu}
\end{aligned}
$$

In terms of them, the previous Lagrangian can be written as

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{2} Tr\left\{G_{\mu\nu} G^{\mu\nu}\right\} - Tr\left\{D_\mu V_\nu D^\mu V^\nu\right\} + (1+a)\, Tr\left\{D_\mu V_\nu D^\nu V^\mu\right\} \\
&+ a_{11}\, Tr\left\{D_\mu V_\nu V^\mu V^\nu\right\} + a_{12}\, Tr\left\{D_\mu V_\nu V^\nu V^\mu\right\} \\
&+ a_{21}\, Tr\left\{V_\mu V_\nu V^\mu V^\nu\right\} + a_{22}\, Tr\left\{V_\mu V_\nu V^\mu V^\nu\right\} \\
&+ a_3\, Tr\left\{G_{\mu\nu}\left[V^\mu, V^\nu\right]\right\} + M^2\, Tr\{V_\nu V^\nu\}
\end{aligned}
$$

BRST Symmetry

Because we have two connections ($A_{1\mu}$ and $A_{2\mu}$) it is natural to write the gauge-fixing and ghost sector as:

$$\begin{aligned}\mathcal{L}_{BRST} &= \frac{1}{2}\xi_1 B_1^a B_1^a - B_1^a \partial^\mu A_{1\mu}^a + \bar{c}_1^a \partial^\mu D_{1\mu}^{ab} c^b \\ & \frac{1}{2}\xi_2 B_2^a B_2^a - B_2^a \partial^\mu A_{2\mu}^a + \bar{c}_2^a \partial^\mu D_{2\mu}^{ab} c^b\end{aligned}$$

where $D_{i\mu}^{ab} = \delta^{ab}\partial_\mu - g_i f^{abc} A_{i\mu}^c$. The whole Lagrangian is invariant under:

BRST transformations

$$\begin{aligned}\delta_B A_{i\mu}^a &= \frac{1}{g_i} D_{i\mu}^{ab} c^b \\ \delta_B c^a &= -\frac{1}{2} f^{abc} c^b c^c\end{aligned}$$

$$\begin{aligned}\delta_B \bar{c}_i^a &= B_i^a \\ \delta_B B_i^a &= 0\end{aligned}$$

In order to avoid kinetic mixing terms, we impose $\xi_1 = \xi_2 = \xi$

BRST Symmetry

In terms of the physical fields, the gauge fixing + ghost Lagrangian reads:

$$\begin{aligned}\mathcal{L}_{BRST} &= -\frac{1}{2\xi}\left(\partial^\mu G_\mu^a\right)^2 - \frac{1}{2\xi}\left(\partial^\mu V_\mu^a\right)^2 + +\bar{c}^a \partial^\mu D_\mu^{ab} c^b \\ &\quad +\alpha f^{abc}\left(\partial^\mu \bar{c}^a\right) V_\mu^c c^b + \beta f^{abc}\left(\partial^\mu \bar{\eta}^a\right) V_\mu^c c^b\end{aligned}$$

where $\bar{c} = \bar{c}_1 + \bar{c}_2$ and $\bar{\eta} = \bar{c}_2 - \bar{c}_1$. Notice that $\bar{\eta}$ doesn't have a kinetic term. Indeed its equation of motion produce the constrain:

$$f^{abc}\partial^\mu \left(V_\mu^c c^b\right) = 0$$

Putting it back to the Lagrangian, we get:

$$\mathcal{L}_{BRST} = -\frac{1}{2\xi}\left(\partial^\mu G_\mu^a\right)^2 - \frac{1}{2\xi}\left(\partial^\mu V_\mu^a\right)^2 + \bar{c}^a \partial^\mu D_\mu^{ab} c^b$$

Propagator Again

With these gauge-fixing terms the propagators of the gluon and the massive vector boson are

$$\Delta_G = \frac{-i\delta^{ab}}{q^2}\left(g^{\mu\nu} + (\xi - 1)\frac{q^\mu q^\nu}{q^2}\right)$$

$$\Delta_V = \frac{-i\delta^{ab}}{q^2 - M^2}\left(g^{\mu\nu} + (\xi + \xi a - 1)\frac{q^\mu q^\nu}{(1 - \xi a)q^2 - \xi M^2}\right)$$

$V_L V_L \to V_L V_L$

In order to study the constraints on the theory imposed by the unitarity of the S-matrix, we will use a simpler version of
the previous Lagrangian. The commutator where introduced in order to cancel terms proportional to d^{abc}

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2} Tr\left\{G_{\mu\nu}G^{\mu\nu}\right\} - Tr\left\{D_\mu V_\nu D^\mu V^\nu\right\} + (1+a)\, Tr\left\{D_\mu V_\nu D^\nu V^\mu\right\} \\
& + a_1\, Tr\left\{\left(D_\mu V_\nu - D_\nu V_\mu\right)\left[V^\mu, V^\nu\right]\right\} \\
& + a_2\, Tr\left\{\left[V_\mu, V_\nu\right]\left[V^\mu, V^\nu\right]\right\} \\
& + a_3\, Tr\left\{G_{\mu\nu}\left[V^\mu, V^\nu\right]\right\} + M^2\, Tr\{V_\nu V^\nu\} \\
& + \mathcal{L}_{\mathcal{BRST}}
\end{aligned}
$$

The results for $V_L V_L \to V_L V_L$ scattering are too long to be shown (they were computed with REDUCE), but in order to cancel the unitarity violating terms, it should happen that: $a = 0$, $a_1 = 0$ $a_2 = -g^2$

$GG \to V_L V_L$

In order to cancel the unitarity violating terms in $GG \to V_L V_L$, we need $a_3 = -g$. So, the Lagrangian now reads:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2} Tr\left\{G_{\mu\nu} G^{\mu\nu}\right\} - Tr\left\{D_\mu V_\nu D^\mu V^\nu\right\} + Tr\left\{D_\mu V_\nu D^\nu V^\mu\right\} \\ &\quad -g^2\, Tr\left\{\left[V_\mu, V_\nu\right]\left[V^\mu, V^\nu\right]\right\} \\ &\quad -g\, Tr\left\{G_{\mu\nu}\left[V^\mu, V^\nu\right]\right\} + M^2\, Tr\{V_\nu V^\nu\} \\ &\quad +\mathcal{L}_{\mathcal{BRST}}\end{aligned}$$

Which is exactly the Lagrangian obtained from the minimal Yang-Mills theory with two connections with $g_1 = g_2 = \sqrt{2}g$

$$\begin{aligned}\mathcal{L}_{m2CYM} &= -\frac{1}{2} Tr\left[F_{1\mu\nu} F_1^{\mu\nu}\right] - \frac{1}{2} Tr\left[F_{2\mu\nu} F_2^{\mu\nu}\right] + \frac{M^2}{2} Tr\left[\left(A_{1\mu} - A_{2\mu}\right)^2\right] \\ &\quad +\mathcal{L}_{BRST}\end{aligned}$$

Consequences

- The $a = a_1 = 0$, $a_2 = -g^2$ and $a_3 = -g$ assignment makes the Lagrangian invariant under the Z_2 transformation $G \to G$ and $V \to -V$
- This correspond to a symmetry of $\mathcal{L}_{m2CYM}$ under $A_1 \longleftrightarrow A_2$ interchange
- This symmetry implies that V doesn't couple to quarks
- V can not decay in two gluon.
- The Z_2 symmetry makes V stable
- V conveniently dressed by gluons should originate a new neutral and stable hadron which may be a dark matter candidate
- The Z_2 symmetry makes the observation of a V very difficult.
- Now we have a theory BRST invariant, with a good propagator and unitarity protected by a Z_2 symmetry. I expect the theory be renormalizable

Conclusions

- We have analyzed the construction of the QCD of a massive spin-1 color-octet particle
- In the general case, the *GVV* coupling is modified by the Stueckelberg term and the expectation that a massive spin-1 color-octet particle be produced with "QCD intensity" may not be true in general
- Unitarity put constraints on the couplings and implies:
  - A new discrete symmetry
  - A dark matter candidate
  - Probably a renormalizable theory without introducing higher structures
- We gave a meaning to a Yang-Mills theory with 2 connections and we have shown that it is consistent and maybe renormalizable.