Background

- Guild: species with common resource
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- **Guild**: species with common resource
- **Focus on competition/predation**
Background

- Guild: species with common resource
- Focus on competition/predation
- Intraguild mutualism
Background

- **Guild**: species with common resource
- Focus on competition/predation
- **Intraguild mutualism**
- Consequences for biodiversity and stability
e.g. Gross 2008 *Ecol. Lett.* 11:929–936
Intraguild mutualism

Crowley & Cox 2011

Crowley & Cox 2011
Groupers and moray eels

Bshary et al. 2006 *PLoS Biol.* 4:e431

**Table 1.** Groupers Benefit from Joint Hunting

<table>
<thead>
<tr>
<th>Moray Status</th>
<th>Observed</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>With moray</td>
<td>6</td>
<td>1.6</td>
</tr>
<tr>
<td>Without moray</td>
<td>10</td>
<td>14.4</td>
</tr>
</tbody>
</table>

Number of hunts in which groupers were observed to eat a fish in association with a moray and in absence of a moray, and expected successful hunts based on association rates (~10%, see Methods)
Objectives

- Does intraguild mutualism promote consumer coexistence?
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- General dynamical model (any mutualistic functional form)
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- Does intraguild mutualism promote consumer coexistence?
- General dynamical model (any mutualistic functional form)
- Stability analysis and simulation
Particular model 1

\[ \dot{X} = X(a_{21}YZ + a_1Z - d_1) \]
\[ \dot{Y} = Y(a_{12}XZ + a_2Z - d_2) \]
\[ \dot{Z} = r(S - Z) - (e_1YZ + e_2XZ + e_{12}YZX) \]
Simulation
Particular model 2

\[
\begin{align*}
\dot{X} & = X(a_{12}YZ + a_1 Z - d_1) \\
\dot{Y} & = Y(a_{21}XZ + a_2 Z - d_2) \\
\dot{Z} & = Z(r - e_1 X - e_2 Y - e_{12} Y X)
\end{align*}
\]
General model

\[
\begin{align*}
\dot{X} &= X [Z f(Y, \bar{\alpha}_1) - d_1] \\
\dot{Y} &= Y [Z f(X, \bar{\alpha}_2) - d_2] \\
\dot{Z} &= Z [r - e_1 X f(Y, \bar{\alpha}_1) - e_2 Y f(X, \bar{\alpha}_2)]
\end{align*}
\]
General model

\[ \dot{X} = X \left[ Zf(Y, \vec{\alpha}_1) - d_1 \right] \]

\[ \dot{Y} = Y \left[ Zf(X, \vec{\alpha}_2) - d_2 \right] \]

\[ \dot{Z} = Z \left[ r - e_1 X f(Y, \vec{\alpha}_1) - e_2 Y f(X, \vec{\alpha}_2) \right] \]

Conditions for mutualism

\[ \frac{\partial f}{\partial X} = g(X, \vec{\alpha}_2) > 0 \]

\[ \frac{\partial f}{\partial Y} = g(Y, \vec{\alpha}_1) > 0 \]
Fixed points

\[ Z^* f(Y^*, \vec{\alpha}_1) - d_1 = 0 \]
\[ Z^* f(X^*, \vec{\alpha}_2) - d_2 = 0 \]
\[ e_1 X^* f(Y^*, \vec{\alpha}_1) + e_2 Y^* f(X^*, \vec{\alpha}_2) = r \]
Fixed points

\[ Z^* f(Y^*, \alpha_1) - d_1 = 0 \]
\[ Z^* f(X^*, \alpha_2) - d_2 = 0 \]
\[ e_1 X^* f(Y^*, \alpha_1) + e_2 Y^* f(X^*, \alpha_2) = r \]

\[ (0, 0, 0) \quad (0, Y^*, Z^*) \quad (X^*, 0, Z^*) \quad (X^*, Y^*, Z^*) \]
Fixed point \((X^*, Y^*, Z^*)\)

\[
\begin{bmatrix}
0 & X^*Z^*g(Y^*, \bar{\alpha}_1) & X^*f(Y^*, \bar{\alpha}_1) \\
Y^*Z^*g(X^*, \bar{\alpha}_2) & 0 & Y^*f(X^*, \bar{\alpha}_2) \\
Z^*[−e_1f(Y^*, \bar{\alpha}_1)]−e_2Y^*g(X^*, \bar{\alpha}_2) & Z^*[−e_1X^*g(Y^*, \bar{\alpha}_1)]−e_2f(X^*, \bar{\alpha}_2) & 0
\end{bmatrix}
\]
Fixed point \((X^*, Y^*, Z^*)\)

\[
\begin{bmatrix}
0 & X^*Z^*g(Y^*, \vec{\alpha}_1) & X^*f(Y^*, \vec{\alpha}_1) \\
Y^*Z^*g(X^*, \vec{\alpha}_2) & 0 & Y^*f(X^*, \vec{\alpha}_2) \\
Z^*[-e_1f(Y^*, \vec{\alpha}_1) - e_2Y^*g(X^*, \vec{\alpha}_2)] & Z^*[-e_1X^*g(Y^*, \vec{\alpha}_1) - e_2f(X^*, \vec{\alpha}_2)] & 0
\end{bmatrix}
\]

\[-\lambda_3 = m\lambda - b\]

\[\lambda_0 < 0 \Rightarrow \mathbb{R}(\lambda_1) = \mathbb{R}(\lambda_2) > 0\]
Fixed point \((X^*, 0, Z^*)\)

\[
\begin{bmatrix}
0 & X^* Z^* g(0, \bar{\alpha}_1) & \frac{X^* d_1}{Z} \\
0 & Z^* f(X^*, \bar{\alpha}_2) - d_2 & 0 \\
-Z^* e_1 f(0, \bar{\alpha}_1) & -e_1 Z^* X^* g(0, \bar{\alpha}_1) - e_2 Z^* f(X^*, \bar{\alpha}_2) & 0
\end{bmatrix}
\]
Fixed point \((X^*, 0, Z^*)\)

\[
\begin{bmatrix}
0 & X^*Z^*g(0, \bar{\alpha}_1) & \frac{X^*d_1}{Z} \\
0 & Z^*f(X^*, \bar{\alpha}_2) - d_2 & 0 \\
-Z^*e_1f(0, \bar{\alpha}_1) & -e_1X^*Z^*g(0, \bar{\alpha}_1) - e_2Z^*f(X^*, \bar{\alpha}_2) & 0
\end{bmatrix}
\]

\[\lambda = Z^*f(X^*, \bar{\alpha}_2) - d_2\]

\[
\frac{f(X^*, \bar{\alpha}_2)}{d_2} > \frac{f(0, \bar{\alpha}_1)}{d_1}
\]

\[
\frac{f(Y^*, \bar{\alpha}_1)}{d_1} > \frac{f(0, \bar{\alpha}_2)}{d_2}
\]
Stability characteristics

\[ \lambda_1 > 0 \land \lambda_2 > 0 \Rightarrow \text{coexistence} \]

\[ \lambda_1 < 0 \land \lambda_2 > 0 \Rightarrow \text{competitive exclusion} \]

\[ \lambda_1 < 0 \land \lambda_2 < 0 \Rightarrow \text{bistability} \]
Simulations

\[ f(Y^*, \bar{\alpha}_1) = \alpha_{12}Y + \alpha_1 \]
\[ f(X^*, \bar{\alpha}_2) = \alpha_{21}X + \alpha_2 \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td>0.5</td>
</tr>
<tr>
<td>(d_1)</td>
<td>0.1</td>
</tr>
<tr>
<td>(d_2)</td>
<td>0.08</td>
</tr>
<tr>
<td>(e_1)</td>
<td>1.002</td>
</tr>
<tr>
<td>(e_2)</td>
<td>1.001</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>0.01</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>0.02</td>
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<tr>
<td>(\alpha_{12})</td>
<td>0.0009</td>
</tr>
<tr>
<td>(\alpha_{21})</td>
<td>0.00026</td>
</tr>
</tbody>
</table>
Simulations

Intraguild mutualism

January 22, 2012
Behaviour

Intraguild mutualism

January 22, 2012
Conclusion

- Parameters exist for mutualistic coexistence
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- Resultant dynamics are oscillatory
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- Works for any mutualistic functional form
Conclusion

- Parameters exist for mutualistic coexistence
- Resultant dynamics are oscillatory
- Works for any mutualistic functional form
- Intraguild mutualism may enhance stability
Directions

- Empirical data
- More species
Directions

- Empirical data
- More species
- Realistic network structure
Directions

- Empirical data
- More species
- Realistic network structure
- Stochasticity
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