PONTIFICIA UNIVERSIDAD CATOLICA DE VALPARAISO

CECS

Gravity around a spherically symmetric Spinorial Source

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Outline

- Motivation
- Einstein-Cartan-Holts Theory
- Stationary & Spherically Symmetric Solutions
- Conclusions and further work

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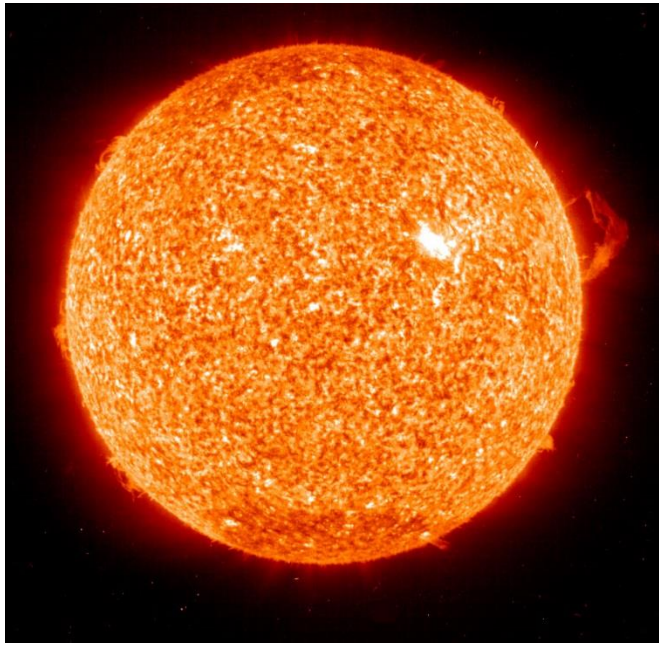
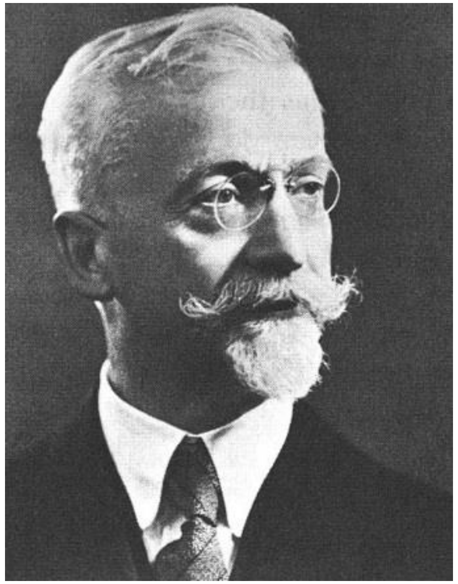
Motivation

- Cartan formalism (1922)
- Poincaré Gauge Theory (Kibble-Sciama 1961-1962)





Èlie Cartan (1869-1951)

¿What is Matter made of?

First Order Formalism

	Traslations	Rotations
Field	$e^a = e^a{}_\mu \mathrm{d}x^\mu$	$\omega^{ab} = \omega^{ab}{}_\mu \mathrm{d}x^\mu$
Structure equations	$T^a = \mathrm{d}e^a + \omega^a{}_b \wedge e^b$	$R^{ab} = \mathrm{d}\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}$
Bianchi Identities	$\mathrm{D}T^a = R^a{}_b e^b$	$\mathrm{D}R^{ab} = 0$

Einstein-Cartan-Holts Theory

Action Principle $(\kappa = 8\pi G)$

$$I[e,\omega,\psi] = \frac{1}{4\kappa}\int\left(\epsilon_{abcd}R^{ab}\wedge e^c\wedge e^d + 2\gamma R_{ab}\wedge e^a\wedge e^b\right) + \int L_M[\psi]$$

Equations of Motion

$$\begin{aligned}
\delta e &: \quad -\frac{1}{2}\epsilon_{abcd}R^{ab}\wedge e^c + \gamma R_{da}\wedge e^a = \kappa\tau_d \\
\delta\omega &: \quad \epsilon_{abcd}T^c\wedge e^d + 2\gamma T_{[a}\wedge e_{b]} = \kappa\sigma_{ab} \\
\delta\psi &: \quad \frac{\delta L_M}{\delta\psi} = 0
\end{aligned}$$

where

$$\delta L_M = -\delta e^a\wedge\tau_a - \frac{1}{2}\delta\omega^{ab}\wedge\sigma_{ab} + \frac{\delta L_M}{\delta\psi}\delta\psi$$

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(non-Grassmann) Dirac Field

Consider

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \quad \bar{\psi} = -i\psi^\dagger \Gamma^0$$

$$\Gamma = \Gamma_a e^a \quad , \quad \{\Gamma_a, \Gamma_b\} = 2\eta_{ab}\mathbb{I} \quad , \quad \Gamma_{ab} = \frac{1}{2}[\Gamma_a, \Gamma_b]$$

Quiral Representation

$$\Gamma_0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad , \quad \Gamma_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \quad , \quad \Gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Gamma_{0i} = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \quad , \quad \Gamma_{ij} = i\epsilon_{ijk}\Sigma_i \quad , \quad \Sigma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}$$

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(non-Grassmann) Dirac Field

Covariant Derivative $\left(\Gamma_{ab} = \frac{1}{2}[\Gamma_a, \Gamma_b]\right)$

$$\begin{aligned} \mathrm{D}\psi &= \mathrm{d}\psi + \frac{1}{4}\omega^{ab}\Gamma_{ab}\psi \\ \mathrm{D}\bar{\psi} &= \mathrm{d}\bar{\psi} - \frac{1}{4}\omega^{ab}\bar{\psi}\Gamma_{ab} \\ \mathrm{D}\Gamma^a &= 0 \end{aligned}$$

define

(vector current) $\quad J^a = i\bar{\psi}\Gamma^a\psi$

(quiral current) $\quad J^{5a} = i\bar{\psi}\Gamma^5\Gamma^a\psi$

(non-Grassmann) **Dirac Field**

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Matter Lagrangian

Impose conditions:

- Lorentz & diffeomorphism invariance
- First order equations
- Minimal Coupling
- Real Action

$$I_M[\psi] = \int \left(\frac{1}{2}\bar{\psi} \star \Gamma \wedge \mathrm{D}\psi + \frac{1}{2}\mathrm{D}\bar{\psi} \wedge\star \Gamma\psi - \frac{1}{2} m\bar{\psi}\psi e^4 \right)$$

$$+ in \left(\frac{1}{2}\bar{\psi} \star \Gamma \wedge \mathrm{D}\psi - \frac{1}{2}\mathrm{D}\bar{\psi} \wedge\star \Gamma\psi \right)$$

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$\star$: Hodge dual

$e^4 = |e| \mathrm{d}^4 x$: Volume 4-form

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Additional term

$$
\begin{aligned}
in\left(\frac{1}{2}\bar{\psi}\star\Gamma\wedge \mathrm{D}\psi-\frac{1}{2}\mathrm{D}\bar{\psi}\wedge\star\,\Gamma\psi\right) &= -\frac{in}{2}\mathrm{d}(\bar{\psi}\star\Gamma\psi)+\frac{in}{2}\bar{\psi}\mathrm{D}\star\Gamma\psi \\
&= \frac{n}{2}T^{a}\bar{\psi}\star(\Gamma\mathrm{e}_{a})\psi+b.t. \\
&= \frac{n}{2}i(\bar{\psi}\Gamma^{a}\psi)T^{b}{}_{ab}|e|d^{4}x+b.t.
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where $\alpha = 1 + in$

Equations of Motion

(Einstein) $$-\frac{1}{2}\epsilon_{abcd}R^{ab}\wedge e^{c}+\gamma R_{da}\wedge e^{a}=\kappa\tau_{d}$$

(Cartan) $$\epsilon_{abcd}T^{c}\wedge e^{d}+2\gamma T_{[a}\wedge e_{b]}=\kappa\sigma_{ab}$$

$$\left(\begin{array}{ll} \text{(Stress)} & \tau_{d}=-\frac{\alpha}{2}\bar{\psi}\star(\Gamma e_{d})\wedge \mathrm{D}\psi+\frac{\bar{\alpha}}{2}\mathrm{D}\bar{\psi}\wedge\star(\Gamma e_{d})\psi+m\bar{\psi}\psi\star e_{d} \\ \text{(Spin)} & \sigma_{ab}=\frac{1}{2}\epsilon_{abcd}J^{5c}\star e^{d}+n\star e_{[a}J_{b]} \end{array}\right)$$

and

(Dirac) $$\bar{\not{D}}\psi-m\psi-\frac{3i}{4}A_{a}\Gamma^{5}\Gamma^{a}\psi+\frac{3ni}{4}V_{a}\Gamma^{a}\psi=0$$

were $(\bar{\mathrm{D}}=\mathrm{d}+\bar{\omega}\ \Rightarrow \mathrm{d}e^{a}+\bar{\omega}^{a}{}_{b}e^{b}=0)$

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Splitting Torsion

$$T_{abc} = \epsilon_{abcd} A^d + \eta_{a[b} V_{c]} + M_{abc}$$

$$\eta^{ab} M_{abc} = \epsilon^{abcd} M_{abc} = 0$$

From Cartan Equation

$$A_a = \frac{\kappa}{2(1+\gamma^2)} \left(-J_a^5 + n\gamma J_a\right) \qquad , \qquad M^a{}_{bc} = 0$$

$$V_a = -\frac{\kappa}{2(1+\gamma^2)} \left(n\, J_a + \gamma\, J_a^5\right)$$

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From Cartan Equation

$$A_a = \frac{\kappa}{2(1+\gamma^2)} \left(-J_a^5 + n\gamma J_a\right) \qquad , \qquad M^a{}_{bc} = 0$$

$$V_a = -\frac{\kappa}{2(1+\gamma^2)} \left(n\, J_a + \gamma\, J_a^5\right)$$

$$\boxed{T^a{}_{bc} = \frac{\kappa}{2(1+\gamma^2)} \left(\epsilon^a{}_{bcd} \left(-J^{5d} + n\gamma J^d\right) - n\delta^a_{[b}\, J_{c]} - \gamma\delta^a_{[b}\, J^5_{c]}\right)}$$

Stationary & Spherically Symmetric Solutions

Metric:

$$ds^2 = -f(r)\mathrm{d}t^2 + h(r)\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

Torsion:

$$T^a{}_{bc} = T^a{}_{bc}(r)$$

$$A_2 = A_3 = V_2 = V_3 = 0$$

first constraints

$$\bar{\psi}\Sigma_2\psi = \bar{\psi}\Sigma_3\psi = 0$$

$$n\bar{\psi}\Gamma_{02}\psi = n\bar{\psi}\Gamma_{03}\psi = 0$$

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$$e^0 = f^{1/2}\mathrm{d}t \;,\; e^1 = h^{1/2}\mathrm{d}r \;,\; e^2 = r\mathrm{d}\theta \;,\; e^3 = \sin\theta\,\mathrm{d}\phi^2$$

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$$V_0 = \frac{\kappa}{2(1+\gamma^2)}\left(-n\,\psi^\dagger\psi + \gamma\,\psi^\dagger\Gamma^5\psi\right)$$

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Stationary & Spherically Symmetric Solutions

Spin Connection: from first structure equation $T^a = \mathrm{d}e^a + \omega^a{}_b \wedge e^b$

$$\omega^{01} = \frac{1}{2}\left(\frac{f'}{fh^{1/2}} + V_1\right)e^0 + \frac{1}{2}V_0 e^1$$

$$\omega^{02} = \frac{1}{2}V_0 e^2 + \frac{1}{2}A_1 e^3$$

$$\omega^{03} = \frac{1}{2}V_0 e^3 - \frac{1}{2}A_1 e^2$$

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Einstein Equations

$$\frac{h'}{rh^2}+\frac{1}{r^2}\left(1-\frac{1}{h}\right)=\kappa\left(\frac{i}{2h^{1/2}}\psi^\dagger\Gamma_{01}\overleftrightarrow{\partial}_r\psi+\frac{i}{2r}\psi^\dagger\Gamma_{02}\overleftrightarrow{\partial}_\theta\psi+\frac{i}{2r\sin\theta}\psi^\dagger\Gamma_{03}\overleftrightarrow{\partial}_\phi\psi+\Delta+m\bar{\psi}\psi\right)$$

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$$-\frac{\left(\frac{f'}{(fh)^{1/2}}\right)'}{2(fh)^{1/2}}+\frac{1}{2rf}\left(\frac{f}{h}\right)'=\kappa\left(\frac{i}{2f^{1/2}}\psi^\dagger\overleftrightarrow{\partial}_r\psi+\frac{i}{2h^{1/2}}\psi^\dagger\Gamma_{01}\overleftrightarrow{\partial}_r\psi+\frac{i}{2r}\psi^\dagger\Gamma_{02}\overleftrightarrow{\partial}_\theta\psi+\Delta+m\bar{\psi}\psi\right)$$

where

$$\Delta=-\frac{3n}{8}V_0\psi^\dagger\psi-\frac{3n}{8}V_1\psi^\dagger\Gamma_{01}\psi-\frac{3}{8}A_0\psi^\dagger\Gamma^5\psi+\frac{3}{8}A_1\psi^\dagger\Sigma_1$$

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Dirac Equation

$$\frac{1}{f^{1/2}}\partial_t\psi + \frac{1}{h^{1/2}}\Gamma_{01}\partial_r\psi + \frac{1}{r}\Gamma_{02}\partial_\theta\psi + \frac{1}{r\sin\theta}\Gamma_{03}\partial_\phi\psi + \frac{1}{h^{1/2}}\left(\frac{f'}{4f} + \frac{1}{r}\right)\Gamma_{01}\psi$$
$$+ \frac{\cot\theta}{2r}\Gamma_{02}\psi - m\Gamma_0\psi + \frac{3i}{4}(nV_0 + nV_1\Gamma_{01} + A_0\Gamma^5 - A_1\Sigma_1)\psi = 0$$

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additional constraints

plus additional constraints

- $\psi^\dagger \Gamma_{01} \overleftrightarrow{\partial}_t \psi = \psi^\dagger \Gamma_{02} \overleftrightarrow{\partial}_t \psi = \psi^\dagger \Gamma_{03} \overleftrightarrow{\partial}_t \psi = 0$
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- $\psi^\dagger \Gamma_{03} \overleftrightarrow{\partial}_\theta \psi = -\frac{1}{\sin\theta} \psi^\dagger \Gamma_{02} \overleftrightarrow{\partial}_\phi \psi = -\frac{i}{2r(fh)^{1/2}} \partial_r \left(f^{1/2} r^2 \psi \Gamma^5 \psi\right)$
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Case $n \neq 0$

Solution of constraints

$$\psi_L = \Phi(\mathrm{r})e^{i\lambda(x)}\begin{pmatrix}1\\ \pm 1\end{pmatrix} \quad , \quad \psi_R = \Phi(\mathrm{r})e^{-i\lambda(x)}\begin{pmatrix}1\\ \mp 1\end{pmatrix}$$

here

$$\psi^\dagger\Gamma^5\psi = \psi^\dagger\Sigma_1\psi = 0 \quad \text{but} \quad \psi^\dagger\psi = 4\Phi^2 \;\;\&\;\; \psi^\dagger\Gamma_{01}\psi = \mp 4\Phi^2$$

so

$$j_a = 4\Phi^2(1, \mp 1,0,0\,) \quad \& \quad j_a^5 = \left(0, \vec{0}\right)$$

$$V_a = -\frac{2n\kappa}{(1+\gamma^2)}\Phi^2(1, \pm 1,0,0\,)$$

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Final Equations for $n \neq 0$

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$$\frac{h'}{rh^2} + \frac{1}{r^2}\left(1 - \frac{1}{h}\right) = 0$$

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$$-\frac{1}{2(fh)^{1/2}}\left(\frac{f'}{(fh)^{1/2}}\right)' + \frac{1}{2rf}\left(\frac{f}{h}\right)' = 0$$

Dirac Equation

$$\Phi' + \left(\frac{f'}{4f} + \frac{1}{r}\right)\Phi = 0$$

$$\Phi\left(\partial_t\lambda \mp \left(\frac{f}{h}\right)^{\frac{1}{2}}\partial_r\lambda\right) = 0$$

$$\Phi(\partial_\theta\lambda \pm mr\cos 2\lambda) = 0$$

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Solutions for $n \neq 0$

Metric

$$f = \frac{1}{h} = 1 - \frac{2GM}{r}$$

Spinor

$$\Phi = \frac{\Phi_0}{r f^{1/4}}$$

Torsion

$$V_a = -\frac{2n\kappa}{(1+\gamma^2)} \frac{\Phi_0^2}{r^2 f^{1/2}} (1, \pm 1, 0, 0)$$

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Spinor Density

non trivial spinor density

$$j^{\mu} = \rho(r)(1, \mp 1, 0, 0)$$

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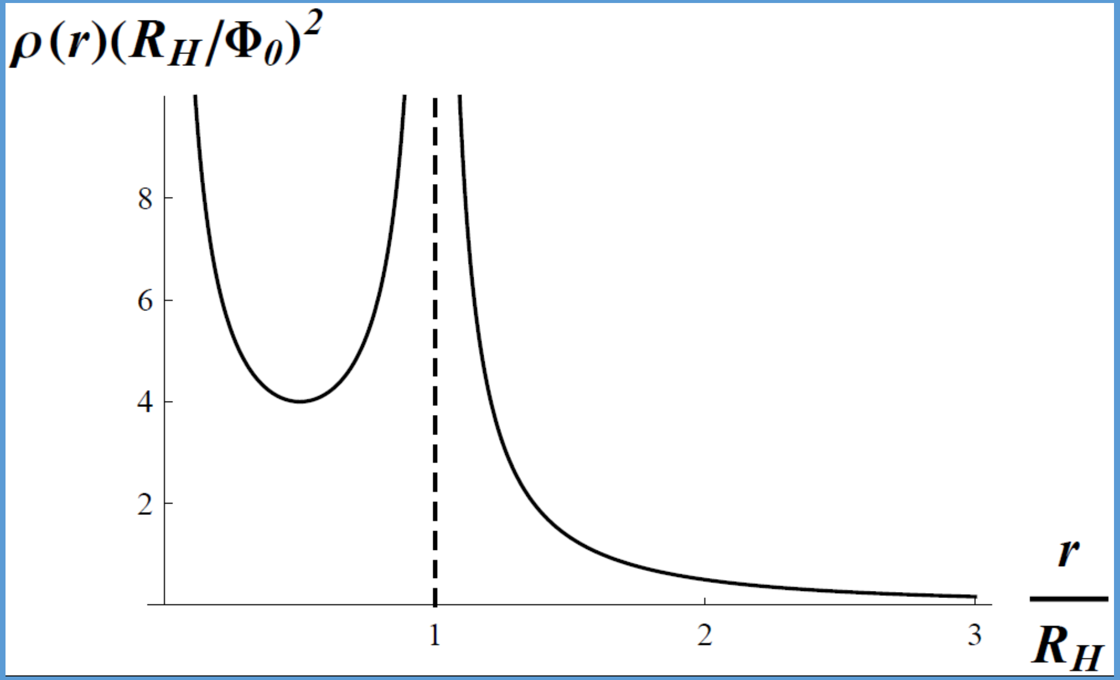

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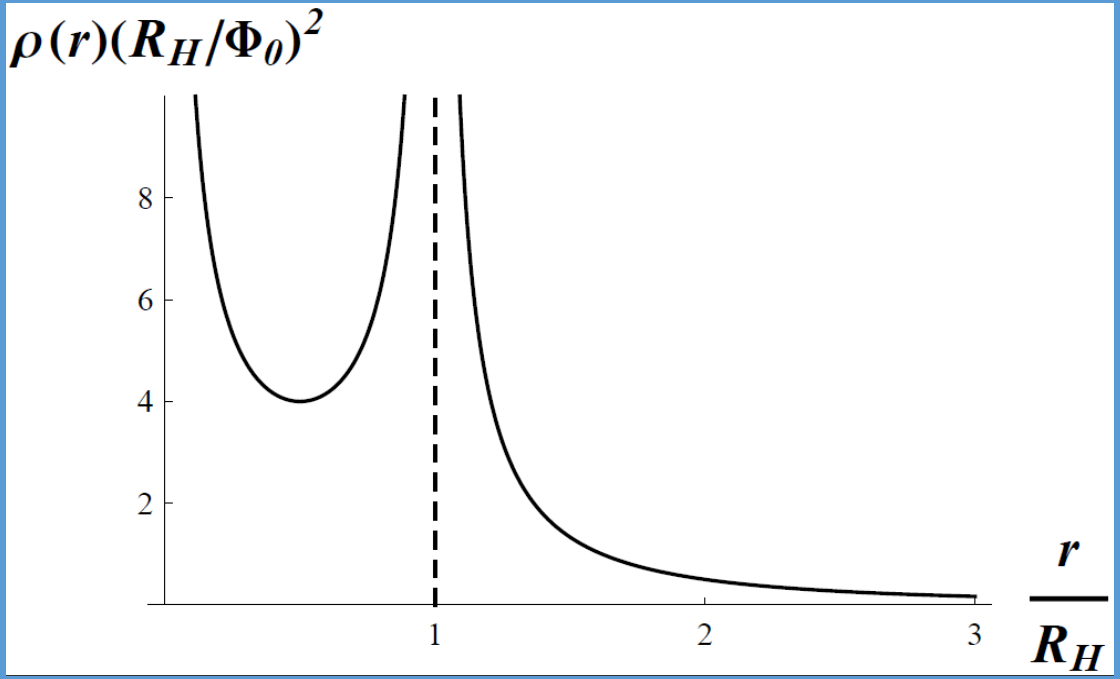

Torsion

Effect of Immirzi parameter

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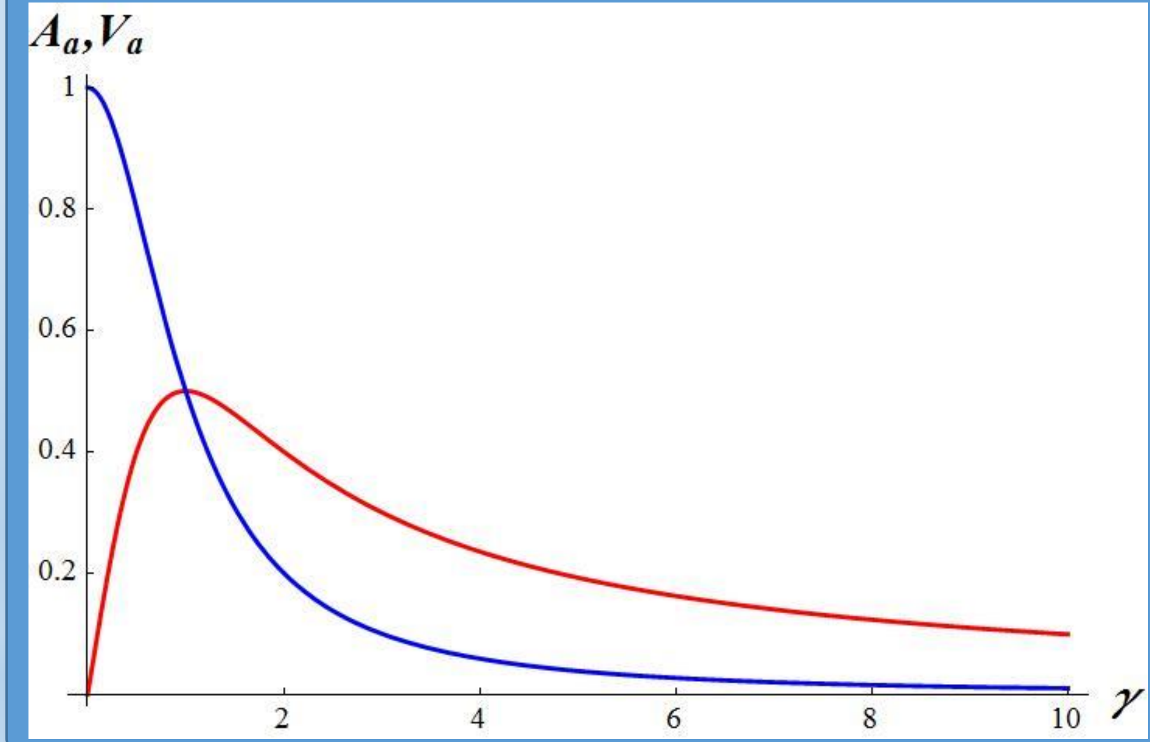
A and *V* vectors



Conclusions and further work

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- Study the $n = 0$ case
- Reconsider Grassmann Dirac Spinor
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Thank You