Black holes and phase transitions in higher curvature gravity

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Quantum Gravity in the Southern Cone VI

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Based on joint work with:

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Higher curvature corrections and quantum gravity

Classical gravity seems well-described by the Einstein-Hilbert action.

Quantum corrections generically involve higher curvature corrections:

- Wilsonian approaches.
- α' corrections in string theory.
- Higher dimensional scenarios.
- Relevant when studying generic strongly coupled CFTs under the light of the gauge/gravity correspondence (*e.g.*, 4d CFTs with $a \neq c$).

They are typically argued to be plagued of ghosts.

Lovelock gravities are the most general second order theories free of ghosts when expanding about flat space. Lovelock (1971)

Lovelock theory

The action is compactly expressed in terms of differential forms

$$\mathcal{I} = \sum_{k=0}^{K} \frac{c_k}{d-2k} \left(\int_{\mathcal{M}} \mathcal{I}_k - \int_{\partial\mathcal{M}} \mathcal{Q}_k \right)$$

where $K \leq [\frac{d-1}{2}]$ and c_k is a set of couplings with length dimensions $L^{2(k-1)}$.

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- $\mathcal{I}_k$ is the extension of the Euler characteristic in $2k$ dimensions

$$\mathcal{I}_k = \epsilon_{a_1 \cdots a_d} \, R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2k-1} a_{2k}} \wedge e^{a_{2k+1}} \wedge \cdots \wedge e^{a_d}$$

with $R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb} = \frac{1}{2} R^{ab}{}_{\mu\nu} \, dx^\mu \wedge dx^\nu$.

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- $\mathcal{Q}_k$ comes from the GB theorem in manifolds with boundaries Myers (1987)

$$\mathcal{Q}_k = k \int_0^1 d\xi \, \epsilon_{a_1 \cdots a_d} \, \theta^{a_1 a_2} \wedge \mathfrak{F}^{a_3 a_4}(\xi) \wedge \cdots \wedge \mathfrak{F}^{a_{2k-1} a_{2k}}(\xi) \wedge e^{a_{2k+1}} \wedge \cdots \wedge e^{a_d}$$

where $\theta^{ab} = n^a K^b - n^b K^a$ and $\mathfrak{F}^{ab}(\xi) \equiv R^{ab} + (\xi^2 - 1)\, \theta^a{}_e \wedge \theta^{eb}$.

Lovelock theory: lowest order examples

The first two contributions (most general up to $d = 4$) correspond to:

- The cosmological term: we set $2\Lambda = -\frac{(d-1)(d-2)}{L^2}$ $\qquad c_0 = \frac{1}{L^2}$
- The EH action (with GH term): we set $16\pi(d-3)!G_N = 1$ $\qquad c_1 = 1$

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For $d \geq 5$, we have the Lanczos-Gauss-Bonnet (LGB) term ($c_2 = \lambda\, L^2$),

$$\mathcal{I}_2 \simeq d^d x \sqrt{-g}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) \qquad \mathcal{Q}_2 \sim \sqrt{-h}\,(KR + \ldots)$$

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while for $d \geq 7$, the cubic Lovelock Lagrangian ($c_3 = \mu\, L^4$),

$$\begin{aligned} \mathcal{I}_3 \quad \simeq \quad & d^d x \sqrt{-g} \Big(R^3 + 3RR^{\mu\nu\alpha\beta}R_{\alpha\beta\mu\nu} - 12RR^{\mu\nu}R_{\mu\nu} + 24R^{\mu\nu\alpha\beta}R_{\alpha\mu}R_{\beta\nu} + \\ & 16R^{\mu\nu}R_{\nu\alpha}R_{\mu}^{\ \alpha} + 24R^{\mu\nu\alpha\beta}R_{\alpha\beta\nu\rho}R_{\mu}^{\ \rho} + 8R^{\mu\nu}_{\ \ \ \alpha\rho}R^{\alpha\beta}_{\ \ \ \nu\sigma}R^{\rho\sigma}_{\ \ \ \mu\beta} + 2R_{\alpha\beta\rho\sigma}R^{\mu\nu\alpha\beta}R^{\rho\sigma}_{\ \ \ \mu\nu} \Big) \end{aligned}$$

AdS/dS/flat vacua

Varying the action with respect to the connection,

$$\epsilon_{aba_3\cdots a_d} \sum_{k=1}^{K} \frac{k\, c_k}{d-2k} \left(R^{a_3 a_4} \wedge \cdots \wedge R^{a_{2k-1} a_{2k}} \wedge e^{a_{2k+1}} \wedge \ldots \wedge e^{a_{d-1}}\right) \wedge T^{a_d} = 0$$

we can safely impose $T^a = 0$ as in standard Einstein's gravity.

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The equations of motion, when varying with respect to the vierbein,

$$\epsilon_{aa_1\cdots a_{d-1}}\, \mathcal{F}_{(1)}^{a_1 a_2} \wedge \cdots \wedge \mathcal{F}_{(K)}^{a_{2K-1} a_{2K}} \wedge e^{a_{2K+1}} \wedge \ldots \wedge e^{a_{d-1}} = 0$$

admit K constant curvature *vacua*,

$$\mathcal{F}_{(i)}^{ab} := R^{ab} - \Lambda_i\, e^a \wedge e^b = 0$$

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The cosmological constants being the roots of the polynomial $\Upsilon[\Lambda]$:

$$\Upsilon[\Lambda]:=\sum_{k=0}^{K}c_k\,\Lambda^k=c_K\prod_{i=1}^{K}(\Lambda-\Lambda_i)=0$$

Degeneracies arise when $\Delta:=\prod_{i<j}(\Lambda_i-\Lambda_j)^2=0$

Lovelock black holes

The black hole solution can be obtained via the ansatz Wheeler (1986)

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}\,d\Sigma^2_{\sigma,d-2}$$

where $d\Sigma_{\sigma,d-2}$ is the metric of a maximally symmetric space.

The equations of motion can be nicely rewritten as

$$\left[\frac{d}{d\log r} + (d-1)\right]\left(\sum_{k=0}^{K} c_k\, g^k\right) = 0$$

where $g(r) = \frac{\sigma - f(r)}{r^2}$, and easily solved as Kastor, Ray, Traschen (2010)

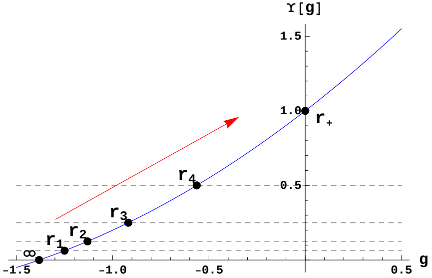
$$\Upsilon[g] = \sum_{k=0}^{K} c_k\, g^k = \mathrm{V}_{d-2}\,\frac{M}{r^{d-1}}$$

The black hole solution is implicitly given by this polynomial equation.

Each branch, $g_i(r)$, corresponds to a monotonous part of the polynomial,

$$\Upsilon[g] = \sum_{k=0}^{K} c_k\, g^k = \mathrm{V}_{d-2}\, \frac{M}{r^{d-1}}$$

The variation of r translates the curve (y-intercept) rigidly, upwards,

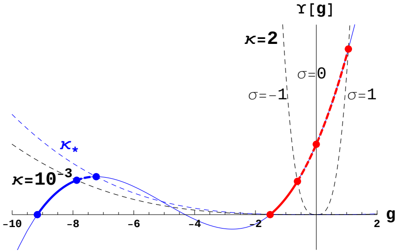


This leads to K branches, $g_i(r)$, associated with each Λ_i: $g_i(r \to \infty) = \Lambda_i$

Lovelock black holes (and naked singularities)

The existence of a black hole horizon requires $g_+ = 0$ for planar black holes (recall $g(r) = \frac{\sigma - f(r)}{r^2}$), and

$$\Upsilon[g_+] = \mathrm{V}_{d-2}\,\frac{M}{r_+^{d-1}} = \mathrm{V}_{d-2}\,M\,|g_+|^{(d-1)/2} \qquad \text{since} \quad g_+ = \frac{\sigma}{r_+^2}$$



- Planar case, only the EH-branch has an event horizon.
- Non-planar case, $\sigma = \pm 1$, several branches can have the same mass or temperature.

Features of Lovelock theory

Some of the new features seemingly unnatural or pathological

Additional couplings

new scales

Naked singularities

mass gap

Branches

multivaluedness

Features of Lovelock theory

Some of the new features seemingly unnatural or pathological

Additional couplings new scales	$\xrightarrow[\textit{instabilities}]{AdS/CFT}$	constraints compact domain
Naked singularities mass gap	$\xrightarrow{\textit{instabilities}}$	cosmic censor new phases?
Branches multivaluedness	$\longrightarrow$	phase transitions EH unambiguous

Bold statement: Maldacena (1997)

Quantum gravity in AdS_d space is equal to a CFT_{d-1} living at the boundary

The generating function reads

Gubser, Klebanov, Polyakov (1998)
Witten (1998)

$$\left\langle \exp\left(\int d\mathbf{x}\, \eta^{ab}(\mathbf{x}) T_{ab}(\mathbf{x}) \right) \right\rangle_{\mathrm{SYM}} = \mathcal{Z}_{\mathrm{QG}}\left[g_{\mu\nu}\right] \approx \exp\left(-\mathcal{I}_G[g_{\mu\nu}]\right)$$

where $\quad g_{\mu\nu} = g_{\mu\nu}(z, \mathbf{x}) \quad$ such that $\quad g_{ab}(0, \mathbf{x}) = \eta_{ab}(\mathbf{x}) \quad .$

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5d EH gravity describes 4d CFTs with $a = c$.

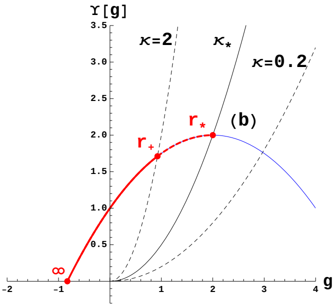
Higher curvature corrections are relevant when studying “more general” strongly coupled CFTs

Warming up: the LGB case

When $K = 2$:

$$\Upsilon[\Lambda] = \frac{1}{L^2} + \Lambda + \lambda L^2 \Lambda^2 = 0$$

$$\Lambda_{\pm} = -\frac{1 \pm \sqrt{1 - 4\lambda}}{2\lambda L^2}$$



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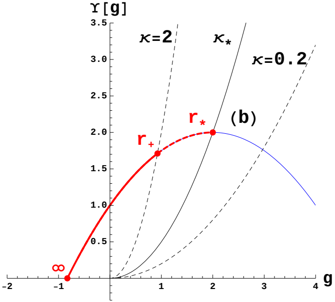

Each black hole solution *climbs up* a monotonous part of the polynomial.

In the planar case ($\sigma = 0$), just the EH branch (Λ_-) has a horizon ($g = 0$).

The EH-branch has $\Upsilon'[\Lambda_-] > 0$, a positive effective Newton constant.

Every branch *ends up* at a singularity: either $r = 0$ or $\Upsilon'[g] = 0$.

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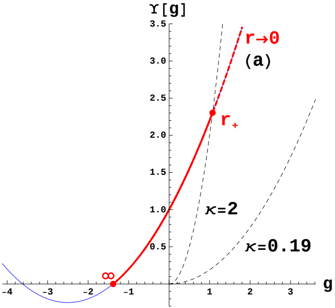

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Graviton potentials: unitarity & causality

EOM for perturbations are two derivative.

VACUUM: Coefficient of the kinetic term:

Unitarity

Boulware, Deser (1985)

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BLACK HOLE: at high momentum, EOM in Schrödinger form: Takahashi, Soda (2010)

$$-\hbar^2 \partial_y \Psi_i + c_i^2(y) \Psi_i = \frac{\omega^2}{q^2} \Psi_i \quad , \qquad \hbar \equiv \frac{1}{q} \to 0$$

for c_i speed of gravitons on radial slices. de Boer, Kulaxizi, Parnachev (2009) Camanho, Edelstein (2009)

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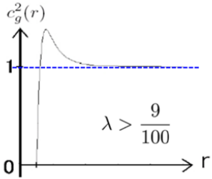
Causality **Brigante, Liu, Myers, Shenker, Yaida (2008)**

$$c_i^2 < 1$$



Causality violation, $c_i^2 > 1$

The potentials close to the boundary of AdS

de Boer, Kulaxizi, Parnachev (2009)
Camanho, Edelstein (2009)

$$c_2^2 \approx 1 + \frac{1}{L^2\Lambda}\left(\frac{r_+}{r}\right)^{d-1}\left[1 + \frac{2(d-1)}{(d-3)(d-4)}\frac{\Lambda\Upsilon''[\Lambda]}{\Upsilon'[\Lambda]}\right]$$

$$c_1^2 \approx 1 + \frac{1}{L^2\Lambda}\left(\frac{r_+}{r}\right)^{d-1}\left[1 - \frac{d-1}{d-3}\frac{\Lambda\Upsilon''[\Lambda]}{\Upsilon'[\Lambda]}\right]$$

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\end{aligned}
$$

Causality imposes

$$
-\frac{d-2}{d-4} \leq -\frac{2(d-1)(d-2)}{(d-3)(d-4)}\,\frac{\Lambda\Upsilon''[\Lambda]}{\Upsilon'[\Lambda]} \leq d-2
$$

Causality violations may also occur in the interior of geometry.

Camanho, Edelstein, Paulos (2010)

Consider a CFT_{d-1}. The leading singularity of the 2-point function is fully characterized by the central charge C_T Osborn, Petkou (1994)

$$\langle T_{ab}(\mathbf{x})\, T_{cd}(\mathbf{0})\rangle \sim \frac{C_T}{2\,\mathbf{x}^{2(d-1)}}(\ldots)$$

$$C_T = \frac{d}{d-2}\,\frac{\Gamma[d]}{\pi^{\frac{d-1}{2}}\,\Gamma\left[\frac{d-1}{2}\right]}\,\frac{\Upsilon'[\Lambda]}{(-\Lambda)^{d/2}}$$

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A good parametrization of 3-point functions

Hofman, Maldacena (2008)

$$\langle \mathcal{E}(\mathbf{n})\rangle_{\mathcal{O}} = \frac{\langle 0|\mathcal{O}^\dagger \mathcal{E}(\mathbf{n})\mathcal{O}|0\rangle}{\langle 0|\mathcal{O}^\dagger \mathcal{O}|0\rangle}\,, \qquad \mathcal{E}(\mathbf{n}) = \lim_{r\to\infty} r^{d-2}\int_{-\infty}^{\infty} dt\ \mathbf{n}^i\, T^0{}_i(t, r\,\mathbf{n})$$

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This is the expectation value for the total energy flux per unit angle measured in a state created by a local gauge invariant operator $\mathcal{O}$

For $\mathcal{O} = \epsilon_{ij}\, T_{ij}$ determined by 2 parameters (t_2 and t_4) in any CFT.

$$\langle \mathcal{E}(\mathbf{n}) \rangle_{\epsilon_{ij} T_{ij}} = \frac{E}{\omega_{d-3}} \left[1 + t_2 \left(\frac{\mathbf{n_i}\, \epsilon^*_{il}\, \epsilon_{lj}\, \mathbf{n_j}}{\epsilon^*_{ij}\, \epsilon_{ij}} - \frac{1}{d-2} \right) + t_4 \left(\frac{|\epsilon_{ij}\, \mathbf{n_i}\, \mathbf{n_j}|^2}{\epsilon^*_{ij}\, \epsilon_{ij}} - \frac{2}{d(d-2)} \right) \right]$$

since ϵ_{ij} is a symmetric and traceless polarization tensor.

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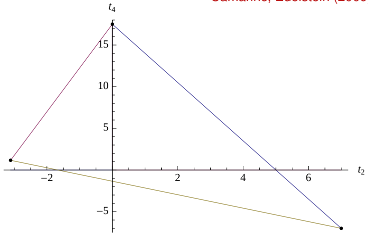
Demanding positivity of the different components gives

Hofman (2009)
Camanho, Edelstein (2009)

$$1 - \frac{1}{d-2}\, t_2 - \frac{2}{d(d-2)}\, t_4 \geq 0 \, ,$$

$$1 - \frac{1}{d-2}\, t_2 - \frac{2}{d(d-2)}\, t_4 + \frac{1}{2}\, t_2 \geq 0 \, ,$$

$$1 - \frac{1}{d-2}\, t_2 - \frac{2}{d(d-2)}\, t_4 + \frac{d-3}{d-2}\, (t_2 + t_4) \geq 0 \, .$$



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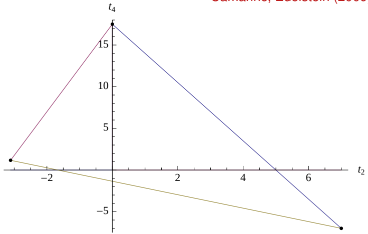
$$1 - \frac{1}{d-2}\, t_2 - \frac{2}{d(d-2)}\, t_4 \geq 0\,,$$

$$1 - \frac{1}{d-2}\, t_2 - \frac{2}{d(d-2)}\, t_4 + \frac{1}{2}\, t_2 \geq 0\,,$$

$$1 - \frac{1}{d-2}\, t_2 - \frac{2}{d(d-2)}\, t_4 + \frac{d-3}{d-2}\, (t_2 + t_4) \geq 0\,.$$



t_2 and t_4 may be calculated holographically,

de Boer, Kulaxizi, Parnachev (2009)
Camanho, Edelstein (2009)

$$t_2 = -\frac{2(d-1)(d-2)}{(d-3)(d-4)}\, \frac{\Lambda \Upsilon''[\Lambda]}{\Upsilon'[\Lambda]} \qquad ; \qquad t_4 = 0$$

For $\mathcal{O} = \epsilon_{ij}\, T_{ij}$ determined by 2 parameters (t_2 and t_4) in any CFT.

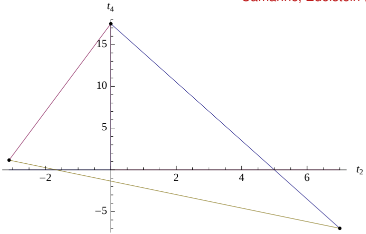
$$\langle \mathcal{E}(\mathbf{n}) \rangle_{\epsilon_{ij} T_{ij}} = \frac{E}{\omega_{d-3}} \left[1 + t_2 \left(\frac{\mathbf{n_i}\, \epsilon^*_{il}\, \epsilon_{lj}\, \mathbf{n_j}}{\epsilon^*_{ij}\, \epsilon_{ij}} - \frac{1}{d-2} \right) + t_4 \left(\frac{|\epsilon_{ij}\, \mathbf{n_i}\, \mathbf{n_j}|^2}{\epsilon^*_{ij}\, \epsilon_{ij}} - \frac{2}{d(d-2)} \right) \right]$$

since ϵ_{ij} is a symmetric and traceless polarization tensor.

Demanding positivity of the different components gives

Hofman (2009)
Camanho, Edelstein (2009)

$$-\frac{d-2}{d-4} \leq t_2 \leq d-2$$



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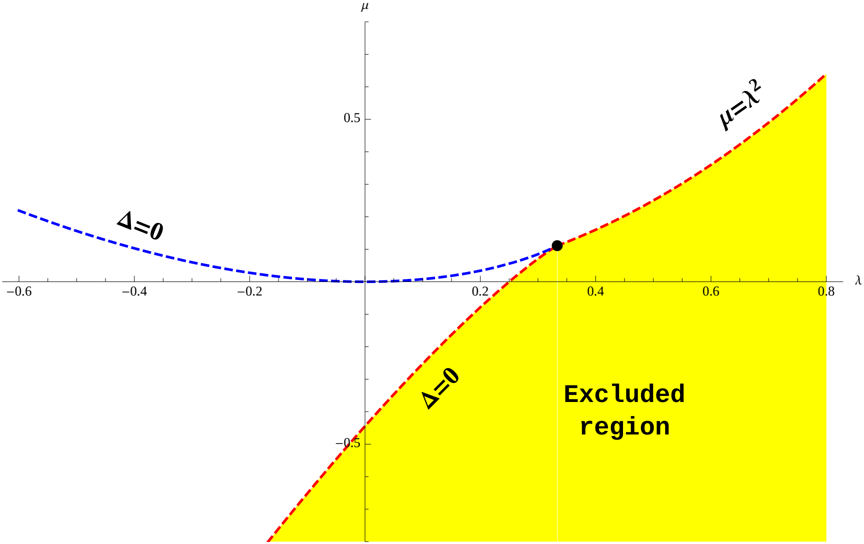
Restrictions in the Lovelock couplings



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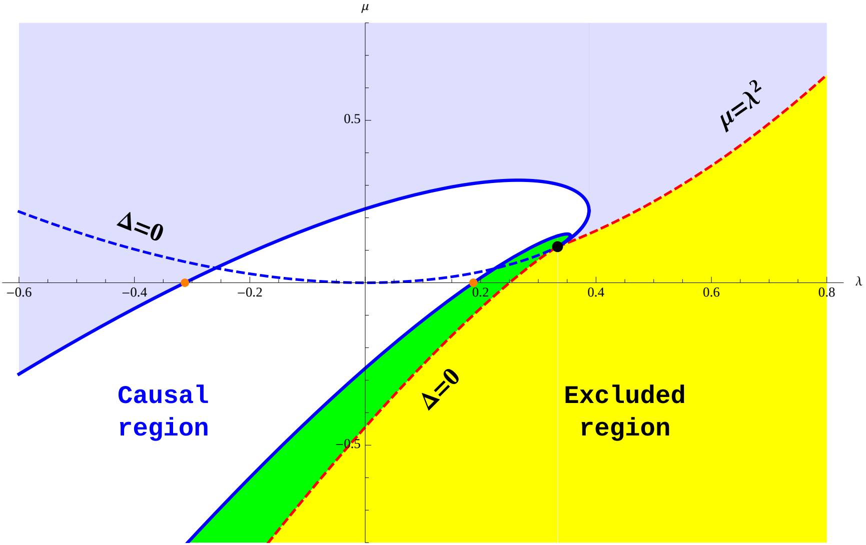
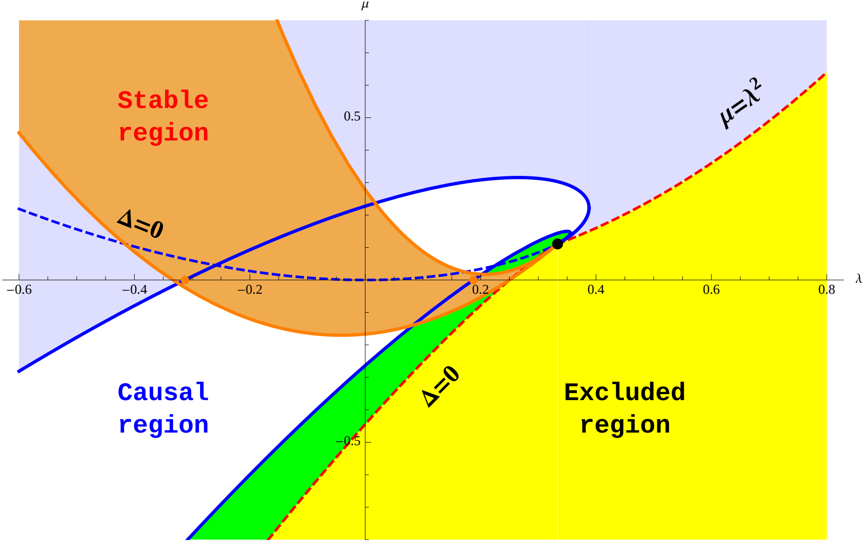
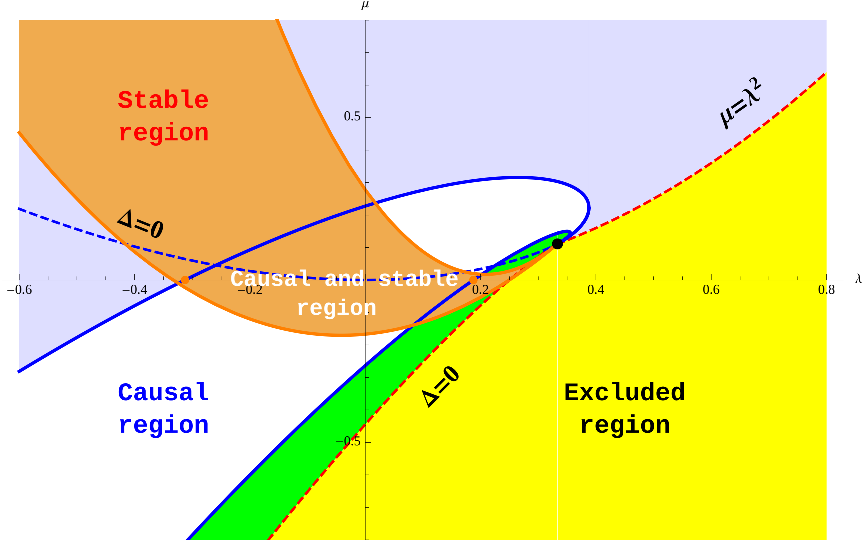



μ
λ
0.5
−0.5
−0.6
−0.4
−0.2
0.2
0.4
0.6
0.8
Stable
region
Causal
region
Excluded
region
Δ=0
Δ=0
μ=λ²



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Lovelock terms lead to a violation of the KSS bound

Kovtun, Son, Starinets (2004)
Shu (2009)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 2\,\frac{d-1}{d-3}\,\lambda\right)\frac{\hbar}{k_B}$$

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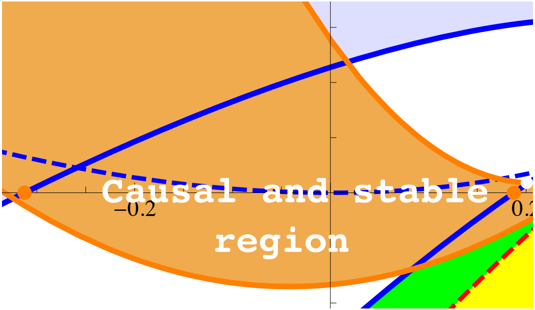
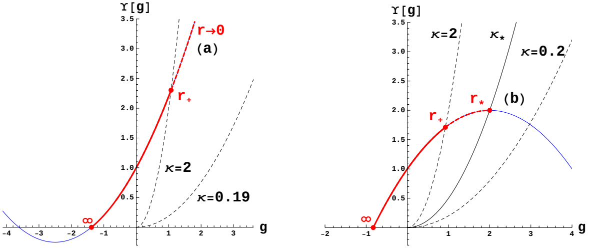
- They **do** contribute to the lower bound of η/s!

Camanho, Edelstein, Paulos (2010)



The existence of a black hole horizon requires

$$\Upsilon[g_+] = \frac{\kappa}{r_+^{d-1}} = \kappa\left(\sqrt{\frac{g_+}{\sigma}}\right)^{d-1} \qquad \text{since} \qquad g_+ = \frac{\sigma}{r_+^2}$$



Lovelock black holes: the cosmic censor

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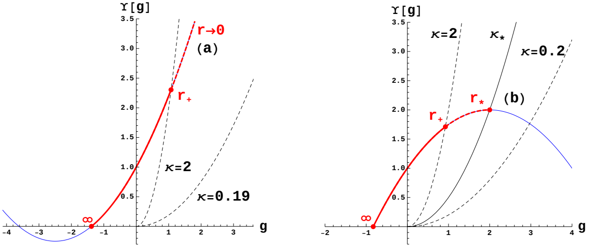

The singularity becomes naked (mass gap)

- $\lambda > 0$, for $\kappa \leq \lambda$ in 5D.
- $\lambda < 0$, for $\kappa \leq \kappa_\star$ in arbitrary dimension.

Instabilities and the cosmic censor

Camanho, Edelstein (2013)

The singular solutions are in all cases unstable. Stability imposes a more constraining mass gap.

Naked singularities cannot be reached as the final state of the evolution of generic initial conditions, *e.g.* collapse

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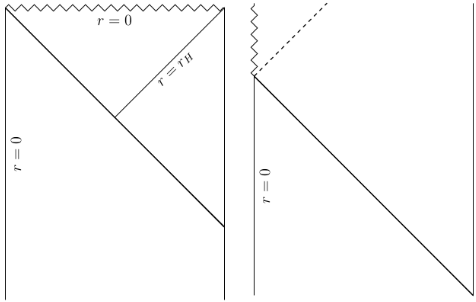
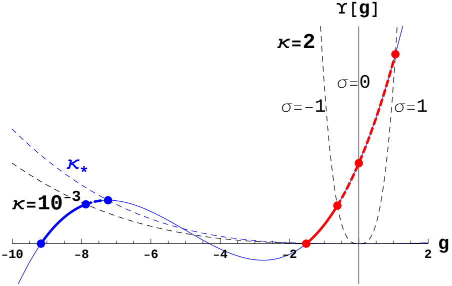


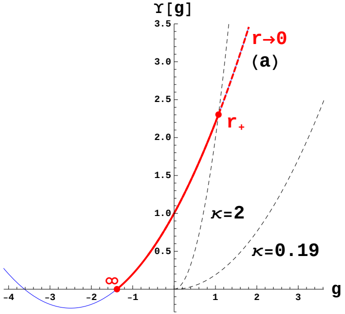
Figure: Collapse of a shell of radiation (thick line) to a black hole (left) and a naked singularity (right). In the latter case, radiation has no obstacle to escape *across* (or bouncing on) the singularity.

A new type of (branch) phase transitions

For this talk, we consider $\lambda > 0$ in LGB theory

Camanho, Edelstein, Giribet, Gomberoff (2012)





In the canonical ensemble, we study processes where the system undergoes a phase transition between thermal AdS$_+$ (Λ_+, β_+) and a given BH$_-$ (Λ_-, β_-).

How to deal with solutions that differ in the asymptotics?

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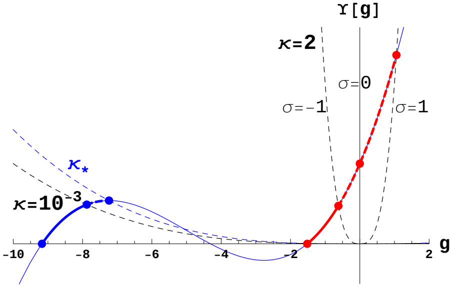

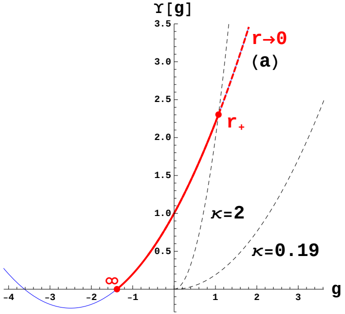

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How to deal with solutions that differ in the asymptotics?

Likely mechanism: *thermalon* mediated transition.

Gomberoff, Henneaux, Teitelboim, Wilczek (2004)

The two phases and the thermalon

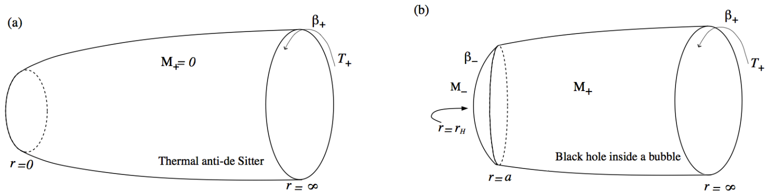


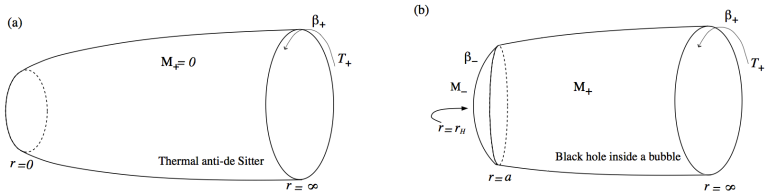
Figure: Euclidean sections for (a) empty AdS and (b) bubble hosting a black hole.

The thermalon

Inner region: black hole with mass M_-, corresponding to the EH branch (Λ_-).

Outer region: asymptotes AdS space with Λ_+ (and total mass M_+).

The two phases and the thermalon



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The thermalon

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Outer region: asymptotes AdS space with Λ_+ (and total mass M_+).

- Inner periodicity: demanding regularity at the black hole horizon.
- Outer periodicity: fully determined by continuity.

there is a unique free parameter.

The thermalon: periodicity, temperature and bubble dynamics

For *bubble* configurations, it is convenient to break the action into bulk and surface pieces, $\mathcal{M} = \mathcal{M}_- \cup \Sigma \cup \mathcal{M}_+$ Davis (2003) Gravanis, Willison (2003)

$$\mathcal{I} = \int_{\mathcal{M}_-} \mathcal{L}^- - \int_{\Sigma} \mathcal{Q}^- + \int_{\mathcal{M}_+} \mathcal{L}^+ + \int_{\Sigma} \mathcal{Q}^+ - \int_{\partial\mathcal{M}} \mathcal{Q}^+$$

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Varying with respect to the induced vierbein at the bubble, $a(\tau)$, gives the junction conditions (Israel conditions of GR).

$$\widetilde{\mathcal{Q}}_{ab} = \left.\frac{\delta(\mathcal{Q}^+ - \mathcal{Q}^-)}{\delta h^{ab}}\right|_{\Sigma} = 0 \qquad \Longleftrightarrow \qquad \dot{a} = \dot{a}(a; M_\pm)$$

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We may fix $M_\pm$ so that an equilibrium position exists at $a = a_\star > r_H$. Each of the two (Euclidean) bulk regions is characterized by $f_\pm$.

The periodicity of the inner solution is fixed by regularity of the black hole horizon, that of the outer solution gets fully determined by gluing conditions,

$$\sqrt{f_-(a)}\,\beta_- = \sqrt{f_+(a)}\,\beta_+$$

There is a unique free parameter, say, β_+.

The phase transition

The canonical ensemble at temperature $1/\beta$ is defined by the Euclidean path integral over all metrics which asymptote AdS identified with period β,

$$Z = \int \mathcal{D}g \; e^{-\hat{\mathcal{I}}[g]} \qquad \hat{\mathcal{I}} = \hat{\mathcal{I}}_{bubble} + \hat{\mathcal{I}}_{black\ hole}$$

Dominant contributions come from the saddle points, $\hat{\mathcal{I}}_{\rm cl} \simeq -\log Z = \beta F$

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Remarkably enough, once the junction conditions are imposed,

$$\hat{\mathcal{I}}_{bubble} = \beta_+ M_+ - \beta_- M_- \qquad \Rightarrow \qquad \hat{\mathcal{I}} = \beta_+ M_+ - S$$

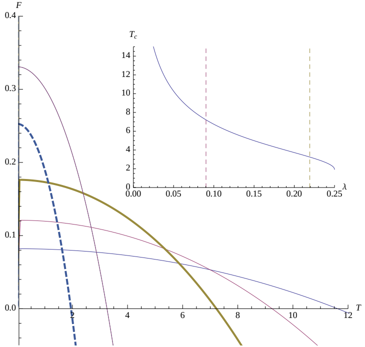
which is exactly needed to preserve the thermodynamic interpretation; also

$$\beta_+ dM_+ = \beta_- dM_- = dS$$

the first law of thermodynamics holds for both configurations.

Global thermodynamic stability: sign of the free energy

There is a critical temperature, $T_c(\lambda)$, above which F becomes negative triggering the phase transition.



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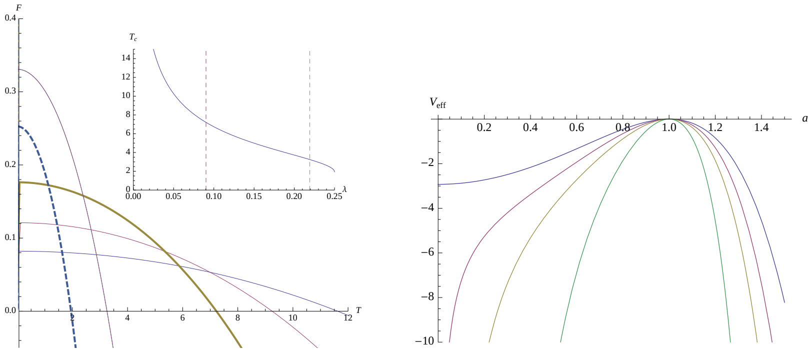

Figure: [LEFT] Free energy versus temperature in $d = 5$ for $\lambda = 0.04, 0.06, \ldots, 1/4$ (from right to left). [RIGHT] Bubble potential for $\lambda = 0.1$ and $d = 5, 6, 7, 10$.

The bubble may expand reaching the boundary at finite proper time changing asymptotics and charges: $\Lambda_+ \rightarrow \Lambda_-$ and $(M_+, T_+) \rightarrow (M_-, T_-)$

On the consistency of higher curvature gravities

Additional couplings new scales	$\xrightarrow[\textit{instabilities}]{AdS/CFT}$	constraints compact domain
Naked singularities mass gap	$\xrightarrow{\textit{instabilities}}$	cosmic censor new phases?
Branches multivaluedness	$\longrightarrow$	phase transitions EH unambiguous

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- **Lovelock theory** is a useful playground for AdS/CFT.
- A novel mechanism for phase transitions in higher curvature gravity.
- Are these **different phases of the dual field theory**?
- It deserves further exploration.

Thank you for your attention!