Hybrid quantization, inflation and cosmological perturbations

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Motivation

1) Inflationary scenario: (compact) flat FRW + massive scalar field
   a) Flatness and particle horizon problems.
   b) Natural conditions for the evolution of primordial inhomogeneities.
2) LQC effective dynamics
   a) The classical singularity is solved.
   b) No slow-roll fine tuning problem.
   c) Pre-inflationary era.
3) Inhomogeneities: scalar perturbations around FRW-like solutions.

Classical system: perturbations and gauge fixing

1) Homogeneous and isotropic model + perturbations

$$h_{ij}(t,\vec{\theta}) = \left(\sigma e^{\alpha(t)}\right)^2\, {}^0h_{ij}(\vec{\theta})\Big(1 + 2\sum_{\vec{n},\epsilon} a_{\vec{n},\epsilon}(t)\tilde{Q}_{\vec{n},\epsilon}(\vec{\theta})\Big)$$

$$+\,6\left(\sigma e^{\alpha(t)}\right)^2 \sum_{\vec{n},\epsilon} b_{\vec{n},\epsilon}(t)\left(\frac{1}{\omega_n^2}(\tilde{Q}_{\vec{n},\epsilon})_{|ij}(\vec{\theta}) + \frac{1}{3}\,{}^0h_{ij}(\vec{\theta})\tilde{Q}_{\vec{n},\epsilon}(\vec{\theta})\right),$$

$$\Phi(t,\vec{\theta}) = \frac{1}{l_0^{3/2}\sigma}\Big(\varphi(t) + \sum_{\vec{n},\epsilon} f_{\vec{n},\epsilon}(t)\tilde{Q}_{\vec{n},\epsilon}(\vec{\theta})\Big).$$

$$H = N_0\Big(H_0 + \sum_{\vec{n},\epsilon} H_2^{\vec{n},\epsilon}\Big) + \sum_{\vec{n},\epsilon} N_0 g_{\vec{n},\epsilon} H_1^{\vec{n},\epsilon} + \sum_{\vec{n},\epsilon} k_{\vec{n},\epsilon} D_1^{\vec{n},\epsilon},$$

a) Zeroth-order constraint

$$H_0 = \tfrac{1}{2}e^{-3\alpha}\left(-\pi_\alpha^2 + \pi_\varphi^2 + e^{6\alpha}\tilde{m}^2\varphi^2\right),$$

Classical system: perturbations and gauge fixing

b) Gauge fixing + canonical transformation

$$C_{\vec{n},\epsilon} \equiv \pi_{a_{\vec{n},\epsilon}} - \pi_\alpha a_{\vec{n},\epsilon} - 3\pi_\varphi f_{\vec{n},\epsilon} = 0, \quad b_{\vec{n},\epsilon} = 0; \quad H_1^{\vec{n},\epsilon} = 0 = D_1^{\vec{n},\epsilon}.$$

$$\tilde{f}_{\vec{n},\epsilon} = e^{\alpha} f_{\vec{n},\epsilon}, \quad \tilde{\pi}_{f_{\vec{n},\epsilon}} = e^{-\alpha}(\pi_{f_{\vec{n},\epsilon}} - 3\pi_\varphi a_{\vec{n},\epsilon} - \pi_\alpha f_{\vec{n},\epsilon}),$$

$$\bar{\alpha} = \alpha + \frac{1}{2}\sum_{\vec{n},\epsilon}\left(a^2_{\vec{n},\epsilon} + f^2_{\vec{n},\epsilon}\right), \quad \bar{\varphi} = \varphi + 3\sum_{\vec{n},\epsilon} a_{\vec{n},\epsilon} f_{\vec{n},\epsilon},$$

$$\pi_{\bar{\alpha}} = \pi_\alpha - \sum_{\vec{n},\epsilon}\left(f_{\vec{n},\epsilon}\pi_{f_{\vec{n},\epsilon}} - 3\pi_\varphi a_{\vec{n},\epsilon} f_{\vec{n},\epsilon} - \pi_\alpha f^2_{\vec{n},\epsilon}\right),$$

Remark: $\tilde{f}_{\vec{n},\epsilon}$ and $\tilde{\pi}_{f_{\vec{n},\epsilon}}$ + unitary dynamics + spatial symmetries $\rightarrow$ unique Fock representation.

Classical system: LQC reduced model

c) Reduced model in LQC: $H(N_0) = N_0\Big(C_0 + \sum_{\vec{n},\epsilon} C_2^{\vec{n},\epsilon}\Big),$

$$C_0 = -\frac{6}{\gamma^2}\frac{\Omega^2}{v} + 8\pi G\left(\frac{1}{v}\pi_\phi^2 + vm^2\phi^2\right); \quad \Omega := v\frac{\sin(\sqrt{\Delta}\beta)}{\sqrt{\Delta}},$$

$$C_2^{\vec{n},\epsilon} = \frac{8\pi G}{v^{1/3}}\left(E^n_{\pi\pi}\pi^2_{\tilde{f}_{\vec{n},\epsilon}} + 2E^n_{f\pi}\tilde{f}_{\vec{n},\epsilon}\tilde{\pi}_{\tilde{f}_{\vec{n},\epsilon}} + E^n_{ff}\tilde{f}^2_{\vec{n},\epsilon}\right),$$

$$E^n_{\pi\pi} = 1 - 4\pi G\frac{\pi_\phi^2}{v^{4/3}\tilde{\omega}_n^2}, \quad E^n_{f\pi} = -\frac{4\pi G}{\tilde{\omega}_n^2}\left(m^2\phi\pi_\phi + \frac{2\pi_\phi^2\Lambda}{\gamma v^2}\right),$$

$$E^n_{ff} = \tilde{\omega}_n^2 + m^2v^{2/3} - \frac{1}{2v^{4/3}}\left[\frac{\Omega^2}{\gamma^2} + 4\pi G(5\pi_\phi^2 + m^2v^2\phi^2)\right] - \frac{4\pi G}{\tilde{\omega}_n^2 v^{8/3}}\left(\frac{2\pi_\phi\Lambda}{\gamma} + m^2\phi v^2\right)^2; \quad \Lambda = v\frac{\sin(2\sqrt{\Delta}\beta)}{2\sqrt{\Delta}}.$$

Quantization

1) Geometry $\mathcal{H}_{\rm kin}^{\rm grav} = L^2(\mathbb{R}_{\rm B}, d\mu_{\rm B})$ and standard for matter.

2) Constraint equation: ϕ as an internal time

   a) Complete

$$-\hbar^2\partial_\phi^2\Psi = \left(\hat{\mathcal{H}}_0^2 + {}^{(0)}\hat{\Theta}_2 - i\hbar\,{}^{(1)}\hat{\Theta}_2\partial_\phi\right)\Psi,$$

   b) Born-Oppenheimer approximation: $\Psi(\phi, v, \tilde{f}_{\vec{n},\epsilon}) = \frac{\chi_0(\phi,v)\psi(\phi,\tilde{f}_{\vec{n},\epsilon})}{\langle\hat{\mathcal{H}}_0(\phi)\rangle_{\chi_0}}$,

$$-i\hbar\partial_\phi\psi = \frac{1}{2}\frac{\langle{}^{(0)}\hat{\Theta}_2 + {}^{(1)}\hat{\Theta}_2\hat{\mathcal{H}}_0\rangle_{\chi_0}}{\langle\hat{\mathcal{H}}_0\rangle_{\chi_0}}\psi.$$

3) Initial data at $v = \epsilon$: $(\psi| = (\psi|^{(0)} + (\psi|^{(2)}$ with

$$(\psi|^{(2)}\hat{C}_0^\dagger + (\psi|^{(0)}\left(\sum_n \hat{C}_2^n\right)^\dagger = 0.$$

Effective dynamics of the massless field

1) Hamilton Jacobi equations

$$\dot{v} = \frac{3}{2}\bar{N}_0|v|\frac{\sin(2\sqrt{\Delta}\beta)}{\sqrt{\Delta}\gamma} + O(\epsilon^2), \quad \dot{\phi} = \bar{N}_0\frac{\pi_\phi}{v} + O(\epsilon^2),$$

$$\dot{\beta} = -\frac{3}{2}\bar{N}_0\frac{\sin^2(\sqrt{\Delta}\beta)}{\Delta\gamma} - \frac{2\pi G\gamma\bar{N}_0}{v^2}\pi_\phi^2 + O(\epsilon^2), \quad \dot{\pi}_\phi = O(\epsilon^4).$$

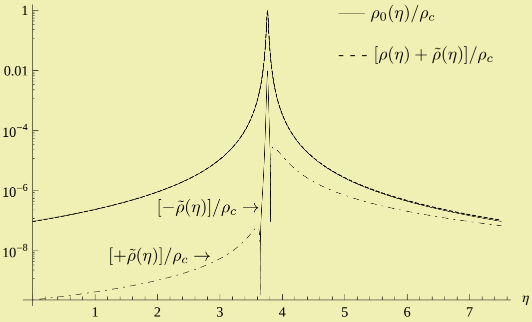
2) Perturbations:

$$\dot{\bar{f}}_{\vec{n},\epsilon} = \frac{\bar{N}_0}{v^{1/3}}(E^n_{\pi\pi}\bar{\pi}_{\bar{f}_{\vec{n},\epsilon}} + E^n_{f\pi}\bar{f}_{\vec{n},\epsilon}), \quad \dot{\bar{\pi}}_{\tilde{f}_{\vec{n},\epsilon}} = -\frac{\bar{N}_0}{v^{1/3}}(E^n_{ff}\bar{f}_{\vec{n},\epsilon} + E^n_{f\pi}\bar{\pi}_{\bar{f}_{\vec{n},\epsilon}}).$$

3) Constraint: if $\rho = \frac{p_\phi^2}{2v^2}$, and $\tilde{\rho} := \frac{1}{2v}\sum_n C_n^2$,

$$C_0 + \sum_{\vec{n},\epsilon} C_2^{\vec{n},\epsilon} = 0, \quad \Leftrightarrow \quad \rho_c \sin^2\left(\sqrt{\Delta}\beta\right) = \rho + \tilde{\rho}.$$

Numerical results: backreaction effects



$m = 0$, $\phi = 0.1$, $v = 10^5$ and $\pi_\phi = 700$ (β is determined by means of the constraint). The initial data for the perturbations is $a_{f_{\vec{n},\epsilon}} = \epsilon e^{i\alpha}$ and $a^*_{f_{\vec{n},\epsilon}} = \epsilon e^{-i\alpha}$, with ϵ and α randomly distributed (following respectively a normal distribution of zero mean and dispersion 10^{-2} and a flat distribution in $(0, 2\pi]$). The total number of modes is given by $\tilde{\omega}^2 \in [1, 6]$ including degeneration.

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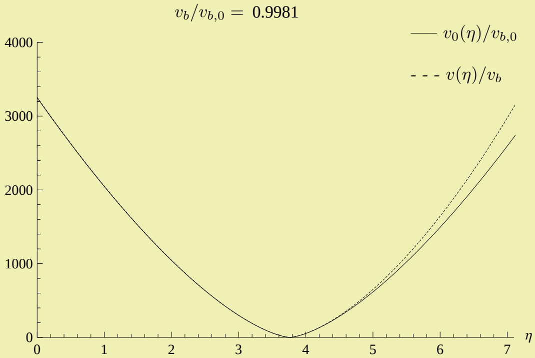

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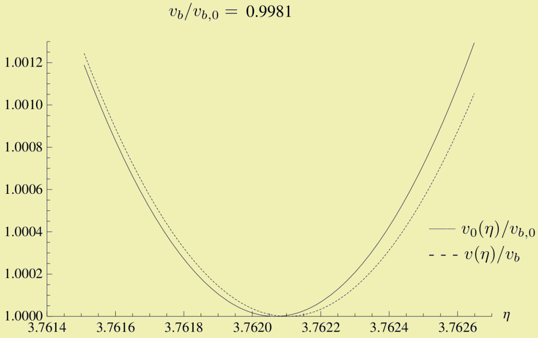

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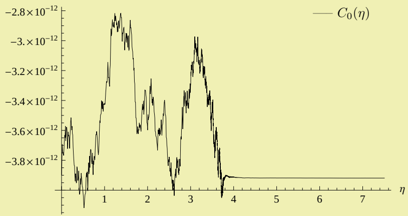

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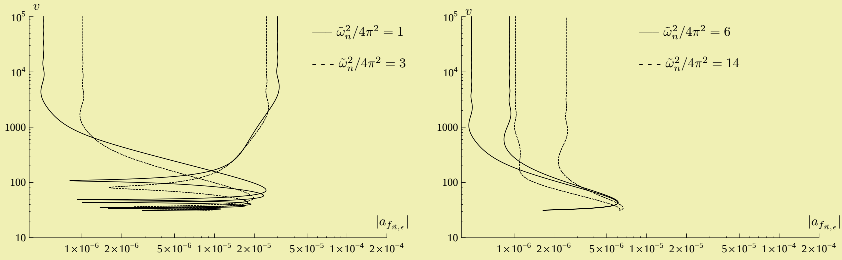
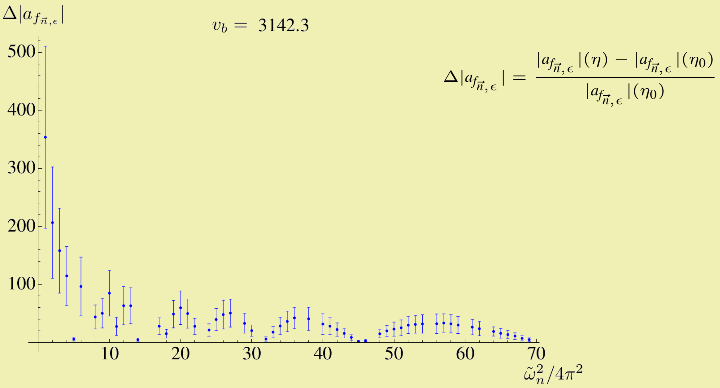
Numerical results: neglecting the backreaction



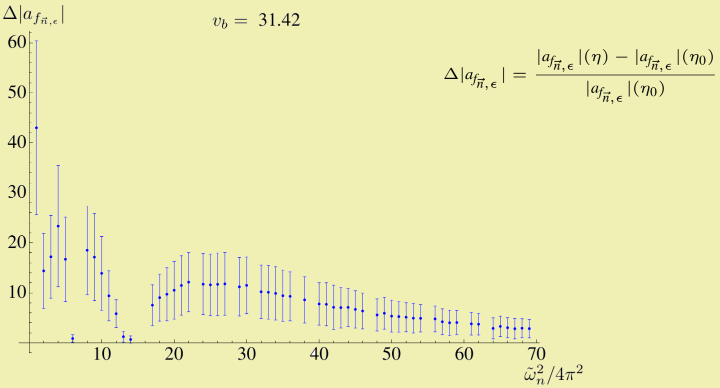
$m = 0$, $v = 10^5$, $\phi = 0.1$, $\beta = 0.9999\pi/\sqrt{\Delta}$ and π_ϕ is determined by means of the constraint. The initial data for the perturbations is $a_{f_{\vec{n},\epsilon}} = \epsilon e^{i\alpha}$ and $a^*_{f_{\vec{n},\epsilon}} = \epsilon e^{-i\alpha}$, with ϵ and α randomly distributed (following respectively a normal distribution of zero mean and dispersion 10^{-6} and a flat distribution in $(0, 2\pi]$).

Numerical results: neglecting the backreaction



$m = 0$, $\phi = 0.1$, $v = 10^8$ and $\sqrt{\Delta}\beta = 0.9999\pi$ (π_ϕ is determined by means of the constraint). The initial data for the perturbations is $a_{f_{\vec{n},\epsilon}} = \epsilon e^{i\alpha}$ and $a^*_{f_{\vec{n},\epsilon}} = \epsilon e^{-i\alpha}$ with ϵ and α randomly distributed (following respectively a normal distribution of zero mean and dispersion 10^{-10} and a flat distribution in $(0, 2\pi]$).

Numerical results: neglecting the backreaction



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Conclusions and outlook

1) Complete quantization of an inflationary model with small inhomogeneities
   a) Internal time characterizes physical states (1st and 2nd order evolution equations).
   b) Physical states characterized by their data on the minimum volume section.
2) Effective dynamics for the massless scalar field
   a) Backreaction breaks simultaneity of critical energy and minimum of the physical volume (as well as its value).
   b) Perturbations amplified at the strong quantum regime.
   c) Non-trivial amplification pattern with oscillations (quantum gravity phenomena).