Quantum Einstein Gravity: Asymptotic Safety and the physical mechanism behind it

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The fundamental problem:

Give a meaning to ("define", "renormalize", "take the continuum limit of", ...) a functional integral over all metrics on a spacetime $\mathcal{M}$:

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}_{\mu\nu}]}$$

S: diff $(\mathcal{M})$-invariant bare action,

e.g. S_{EH} + counter terms

$$\mathcal{D}\hat{g}_{\mu\nu} \equiv \prod_{x\in\mathcal{M}} \prod_{\mu,\nu} dg_{\mu\nu}(x)$$

requires regularization (UV Cutoff)

The Asymptotic Safety idea:

- Take the infinite-cutoff limit of an UV-regularized quantum theory of gravity at a non-trivial RG fixed point with a finite dimensional UV-critical hypersurface, assuming it exists.

- The resulting continuum theory is predictive and well behaved at arbitrarily short distances.

(S. Weinberg, 1979, 2009)

Our Approach:

M.R. (1996)

- Employ background field technique:

$$\hat{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \hat{h}_{\mu\nu}$$

Fluctuation: Quantize in the $\bar{g}_{\mu\nu}$-background "$\forall \bar{g}_{\mu\nu}$"

Background: fixed, but arbitrary

covariant Laplacian: $\bar{D}^2$

eigenmodes: $f^{\omega}_{\mu\nu}(x)$

- Expand fluctuation:

$$-\bar{D}^2 f^{\omega}_{\mu\nu} = \omega^2 f^{\omega}_{\mu\nu}$$

$$\hat{h}_{\mu\nu}(x) = \sum_{\omega} h_{\omega} f^{\omega}_{\mu\nu}(x)$$

- Background covariant gauge fixing:

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S} \longrightarrow \int \mathcal{D}\hat{h}_{\mu\nu}\, e^{-S'}$$

$S' \equiv S$ + gauge fixing + Faddeev-Popov

$$\int \mathcal{D}\hat{h}_{\mu\nu} \equiv \prod_{\omega} \int_{-\infty}^{\infty} dh_{\omega}$$

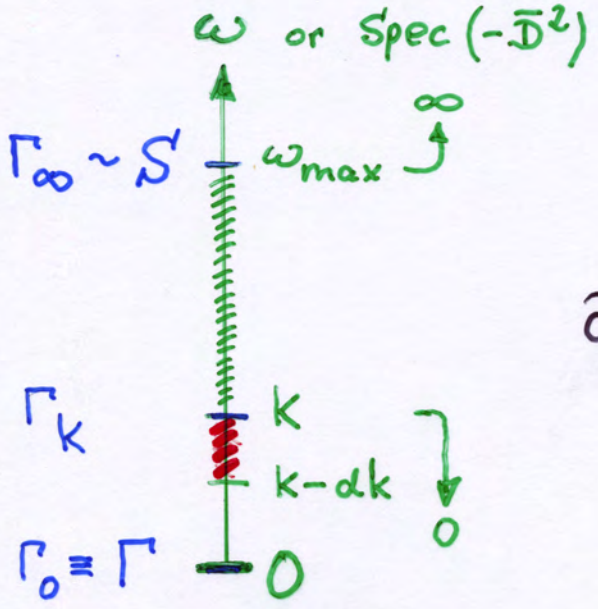
- **Regularize:**

$$\prod_{\omega = k}^{\omega = \omega_{max}} \int_{-\infty}^{\infty} dh_\omega \; e^{-S'\{h_\omega\}}$$

Dependence of the fctl. integral on IR cutoff is encoded in the **Effective Average Action:**

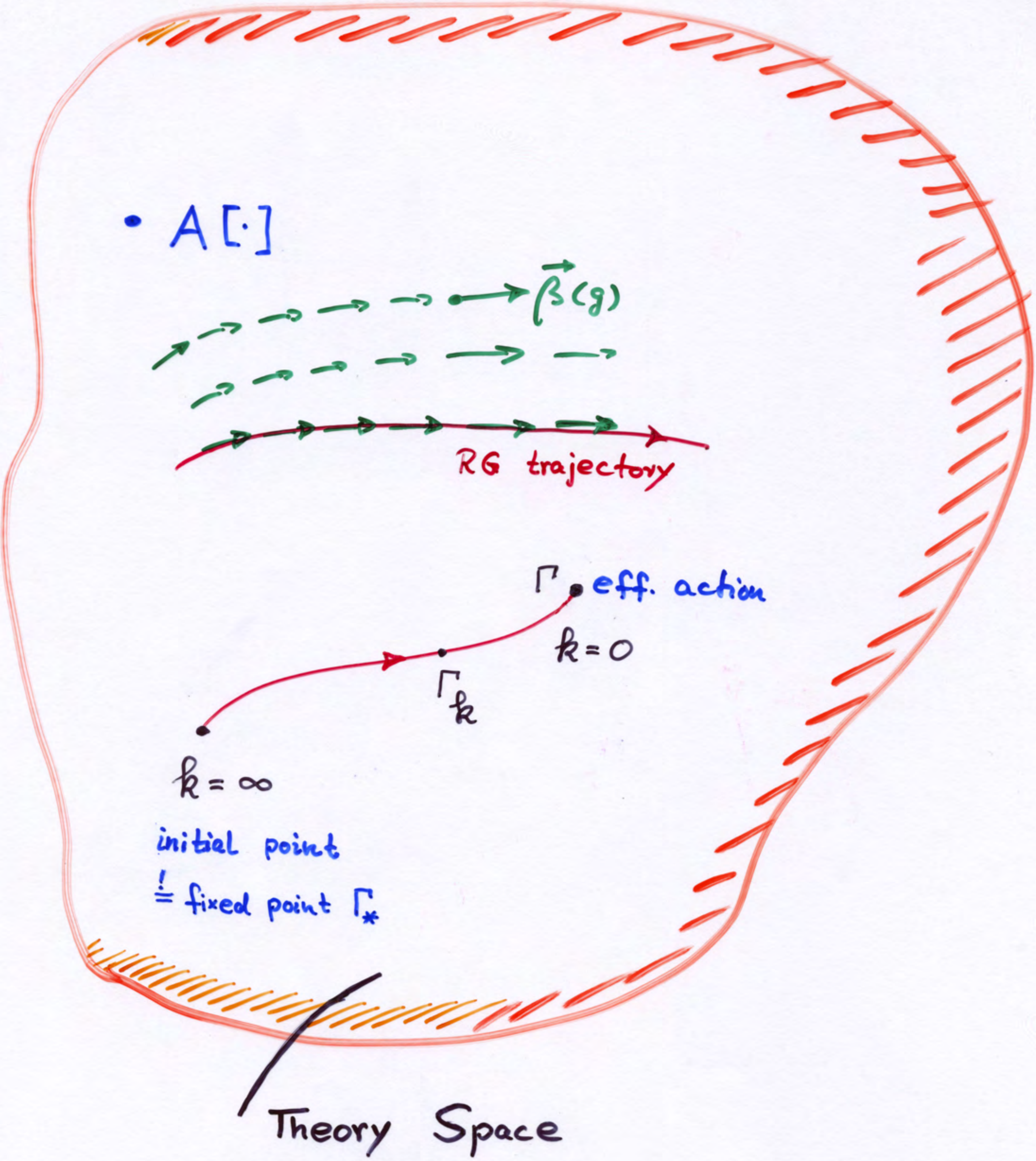
$$\Gamma_k[\underbrace{g_{\mu\nu}}_{=\langle \hat{g}_{\mu\nu}\rangle = \bar{g}_{\mu\nu} + \langle \hat{h}_{\mu\nu}\rangle}, \bar{g}_{\mu\nu}] \equiv \Gamma_k[\underbrace{h_{\mu\nu}}; \bar{g}_{\mu\nu}]$$

- **FRGE for the Eff. Average Action:**



$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr}\left[\left(\frac{\delta^2 \Gamma_k}{\delta g^2} + R_k\right)^{-1} \partial_k R\right]$$

- **Concrete implementation:** $\int \mathcal{D}\hat{h}\, e^{-S'}\, e^{-\int \hat{h} R_k(-\bar{D}^2)\hat{h}}$



$A[\cdot]$
$\vec{\beta}(g)$
RG trajectory
Γ eff. action
$k=0$
Γ_k
$k=\infty$
initial point
$\stackrel{!}{=}$ fixed point Γ_*
Theory Space

The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left\{ R - 2\Lambda_k \right\}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant G_k, dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k, dimensionless: $\lambda(k) = \Lambda_k / k^2$

insert ansatz into flow equation, expand

$$\mathrm{Tr}[\cdots] = (\cdots) \int \sqrt{g} + (\cdots) \int \sqrt{g} R + \cdots$$

$$\rightsquigarrow \quad \boxed{\begin{aligned} k\,\partial_k\, g(k) &= \beta_g(g,\lambda) \\ k\,\partial_k\, \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

The anomalous dimension η_N :

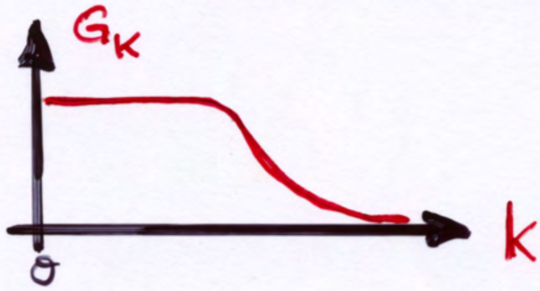
$$\begin{cases} \partial_t g = \beta_g \equiv \big[(d-2) + \underbrace{\eta_N(g,\lambda)}_{=O(\hbar)} \big] g \\ \partial_t \lambda = \beta_\lambda \end{cases}$$

$t \equiv \ln(k)$

with $\eta_N \equiv \frac{\partial_t G_k}{G_k}$

Explicit calculation : $\boxed{\eta_N < 0}$ $\Longrightarrow$

- Gravitational anti - screening:

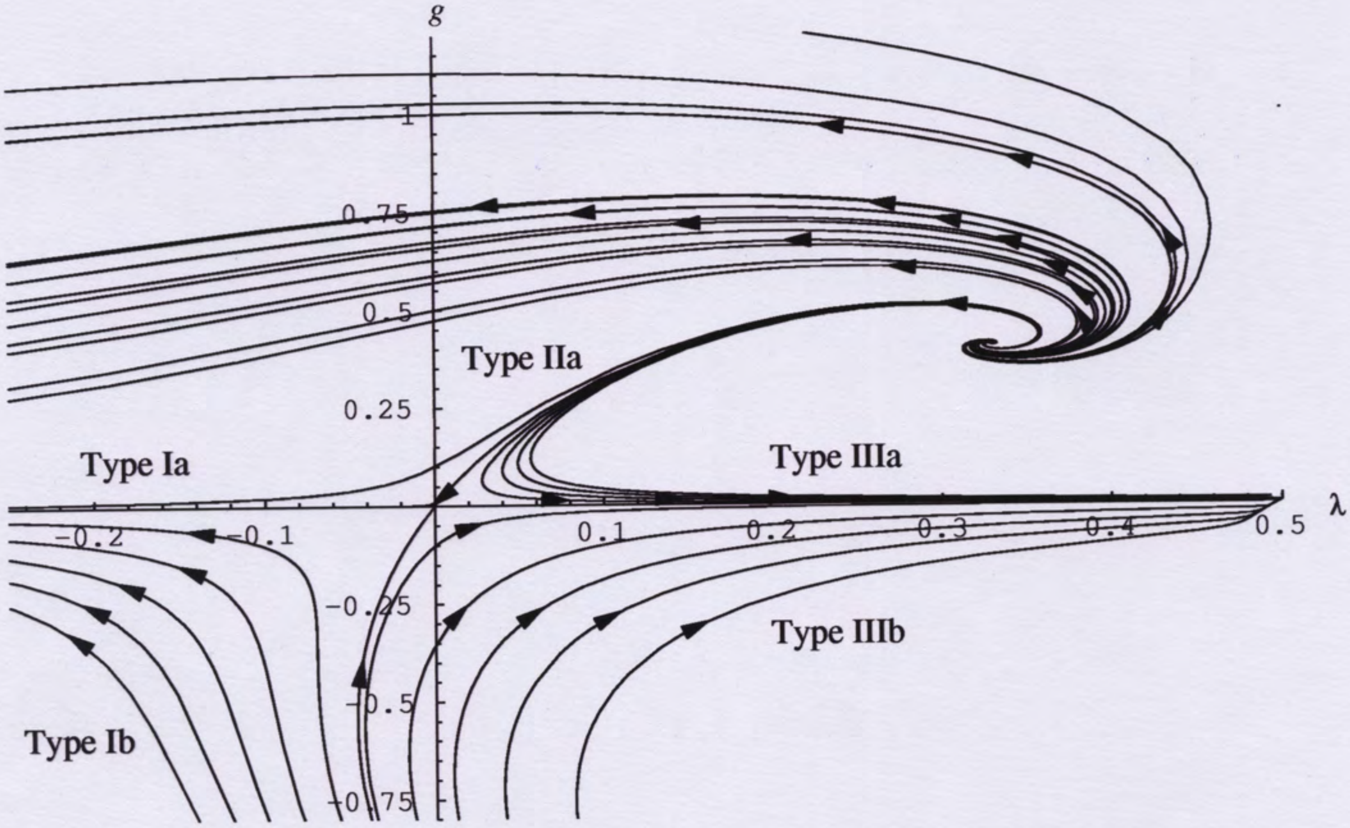


- Non-trivial fixed point:

$$\eta_N(g_*, \lambda_*) = -(d-2) \underset{d=4}{=} -2$$

Einstein - Hilbert Truncation :

RG Flow on the $g-\lambda$ plane



M.R., F. Saueressig, hep-th/0110054

Properties of QEG

- **Background-independent quantization scheme:**

No special metric plays any distinguished role!

The background field method:

a) Fix arbitrary $\bar{g}_{\mu\nu}$

b) Quantize (nonlinear) fluctuations $h_{\mu\nu} \equiv \gamma_{\mu\nu} - \bar{g}_{\mu\nu}$ in the backgrd. of $\bar{g}_{\mu\nu}$

c) Adjust $\bar{g}_{\mu\nu}$ such that $\langle h_{\mu\nu} \rangle = 0$

$$\rightsquigarrow \quad g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu}$$

- **Fundamental action $S \approx \Gamma_*$ is a prediction:**

No special action plays any distinguished role!

The only input: field contents + symmetries
$\hat{=}$ theory space

The output: $\Gamma_* = S_{\text{Einstein-Hilbert}} + \text{"more"}$

Einstein-Hilbert action is often a reliable approximation, but not distinguished conceptually.

- Combination average action + background method successfully tested in QED and Yang-Mills theory.

- QEG reproduces successes of classical General Relativity: $\exists$ trajectories with long classical regime ($G = \text{const}$, $\Lambda = \text{const}$)

- QEG reproduces results of "QFTh. in curved spacetimes" in the classical regime:
  Hawking radiation, cosmological particle creation, ...

- Coexistence Asymptotic Safety $\leftrightarrow$ perturbative non-renormalizability well understood.

- Consistent quantization of gravity seems not to require "fine tuning" of matter system, special symmetries (SUSY, etc.), or unification with the other fundamental forces of Nature.

- Coupling to gravity softens/cures matter divergences:

  $$\text{QEG} + \text{QED} \rightsquigarrow e^2(k^2 \to \infty) = 0$$

  U. Harst, MR 2011

  Potentially higher degree of predictivity

- QEG spacetimes have fractal microstructure of reduced dimensionality

  O. Lauscher, MR 2002, 2005
  MR, F. Saueressig 2012

- "Phenomenology" from RG-Improvement

  - <u>Cosmology:</u> automatic Λ-driven inflation, scale free perturbations from NGFP, entropy production, ...

    A. Bonanno, MR (2001, 2007)
    MR, F. Saueressig (2005)

  - <u>Black Holes:</u> modified horizons, causal structure, final state of Hawking evaporation, ...

    A. Bonanno, MR (1999, 2000, 2006)
    MR, E. Tuiran (2006, 2011)

The physical mechanism underlying Asymptotic Safety

A. Nink, MR (2012)

The Magnetic Analogy

- **Non-relativistic electrons, Pauli eq.**

$$H = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 + \mu_B \vec{B}\cdot\vec{\sigma}$$

orbital motion: Landau diamagnetism, $\chi_{dia} < 0$

spin alignment: Pauli spin paramagnetism, $\chi_{para} > 0$

total susceptibility: $$\chi_{tot} = [\underbrace{3}_{\text{para}} - \underbrace{1}_{\text{dia}}]\cdot\left(\begin{smallmatrix}\text{positive}\\ \text{constant}\end{smallmatrix}\right) > 0$$

- **Relativistic electrons, Dirac eq.**

$$\not{D}^2 = \underbrace{(i\partial_\mu - eA_\mu)^2}_{\text{"dia"}} - \underbrace{\tfrac{i}{2}e\,\gamma^\mu\gamma^\nu F_{\mu\nu}}_{\text{"para"}}$$

running electric charge in QED (at 1 loop):

$$\partial_t e^2 = \beta_{e^2} = +\frac{1}{12\pi^2}[\underbrace{3}_{\text{para}} - \underbrace{1}_{\text{dia}}]\,e^4 \quad > 0$$

- Charge screening is due to the electrons' predominantly paramagnetic interaction with A_μ.
- The diamagnetic interactions drive β_{e^2} in the opposite direction.

- ## Yang-Mills gauge field fluctuations

$$S_{YM}[A_\mu^b] = \frac{1}{4}\int d^4x\, F_{\mu\nu}^b F^{b\mu\nu} \quad + \text{g.f.}$$

Expand $S_{YM}[A = \bar{A} + a]$ to order a^2:

$$\frac{1}{2}\int d^4x\; a_\mu^b \left[\underbrace{(-\bar{D}^2)^{bc}\,\delta^\mu{}_\nu}_{\text{"dia"}} + \underbrace{2ig\,\bar{F}^{bc\mu}{}_\nu}_{\text{"para"}}\right] a^{c\nu}$$

Running gauge coupling:

$$\partial_t g^2 = \beta_{g^2} = -\frac{N}{24\pi^2}\left[\underbrace{12}_{\text{para}} \;\underbrace{-\;2}_{\text{dia}} \;\underbrace{+\;1}_{\text{ghosts}}\right] g^4 < 0$$

- Color anti-screening and Asymptotic Freedom are due to the fluctuations' predominantly paramagnetic interaction with the background.
- The diamagnetic interactions drive β_{g^2} in the opposite (screening) direction.

Fluctuations of the metric

Expand $S_{EH}[g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}]$ + g.f. to order h^2:

$$\rightsquigarrow \quad \int d^dx \sqrt{\bar{g}} \; h_{\mu\nu} \Big[\underbrace{-K^{\mu\nu}{}_{\rho\sigma} \bar{D}^2}_{\text{"dia"}} + \underbrace{\bar{U}^{\mu\nu}{}_{\rho\sigma}}_{\text{"para"}} \Big] h^{\rho\sigma}$$

with:

$$\bar{K}^{\mu\nu}{}_{\rho\sigma} = \frac{1}{4}\left[\delta^\mu_\rho \delta^\nu_\sigma + \delta^\mu_\sigma \delta^\nu_\rho - \bar{g}^{\mu\nu}\bar{g}_{\rho\sigma}\right]$$

$$\begin{aligned}\bar{U}^{\mu\nu}{}_{\rho\sigma} = & -\frac{1}{2}\left[\bar{R}^\nu{}_\rho{}^\mu{}_\sigma + \bar{R}^\nu{}_\sigma{}^\mu{}_\rho\right] \\ & + \frac{1}{2}\left[\bar{g}^{\mu\nu}\bar{R}_{\rho\sigma} + \bar{g}_{\rho\sigma}\bar{R}^{\mu\nu}\right] - \frac{1}{4}\left[\delta^\mu_\rho \bar{R}^\nu{}_\sigma + \dots\right] \\ & + \bar{R}\,\bar{K}^{\mu\nu}{}_{\rho\sigma}\end{aligned}$$

Fluctuations drive the RG flow:

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\underbrace{\frac{\delta^2\Gamma_k}{\delta h \delta h}}_{= -\bar{K}\bar{D}^2 + \bar{U}} + R_k\right)^{-1} \partial_t R_k\right] + \dots$$

$\rightsquigarrow$ clear separation of dia / para contributions

Anomalous dimension η_N (leading order):

$$\eta_N = -\underbrace{f}_{>0} g \Big[\underbrace{+12(d-1)}_{\text{para}} + \underbrace{\frac{48}{d}}_{\text{ghost-para}} \underbrace{- d(d+1)}_{\text{dia}} \underbrace{+ 4d}_{\text{ghost-dia}} \Big]$$

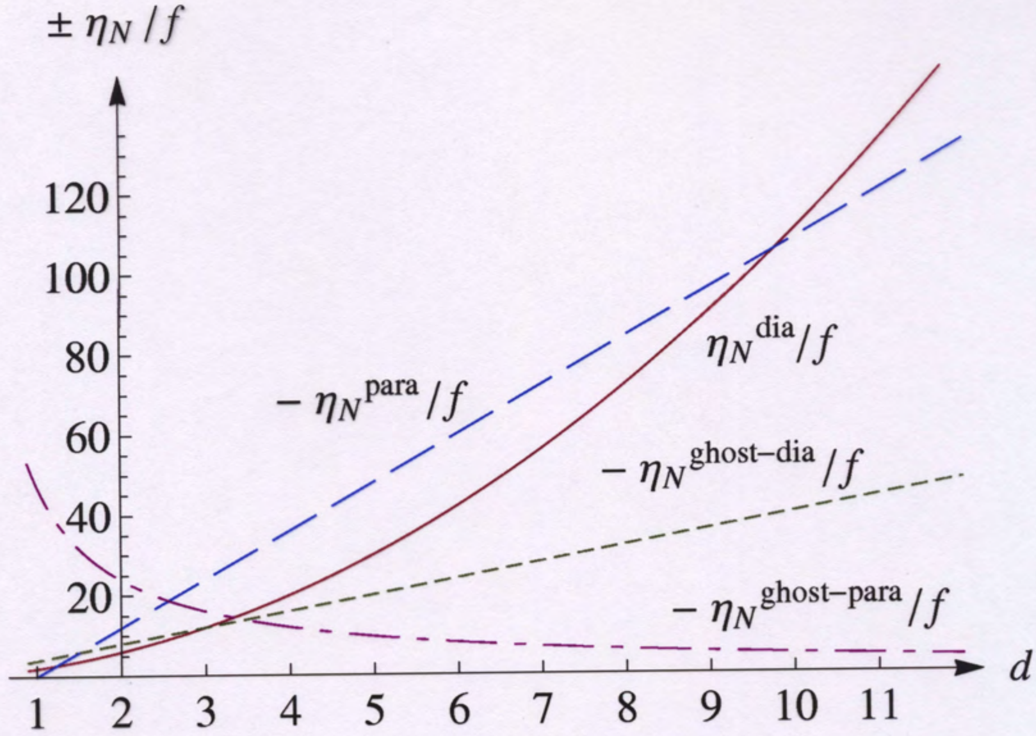
$$= -fg \Big[\underbrace{12(d-1) + \frac{48}{d}}_{\text{total para:}} \underbrace{- d(d-3)}_{\text{total dia:}} \Big]$$

	total para:	total dia:	
	positive $\forall d$	negative $\forall d > 3$ positive $\forall d < 3$	
$d=4$:	$+48$	-4	"para" is 12 times stronger!

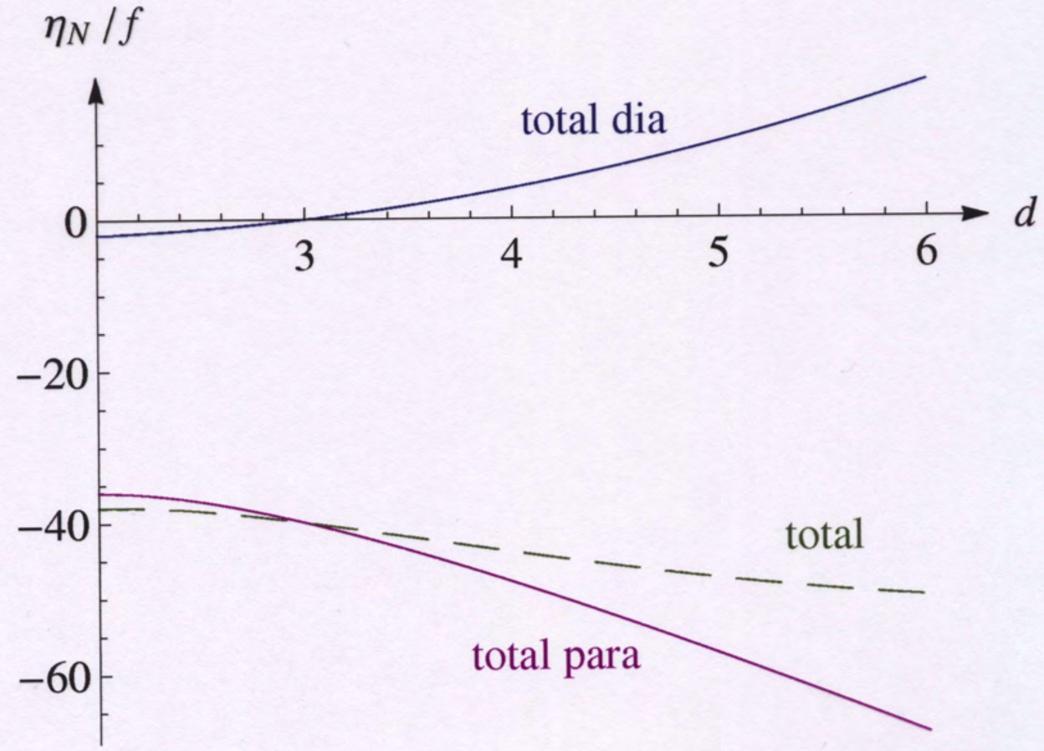
- Gravitational anti-screening and Asymptotic Safety in $d>3$ is due to the fluctuations' predominantly paramagnetic interaction with the background.

- In $d>3$, the diamag. interactions drive η_N in the opposite (screening) direction.

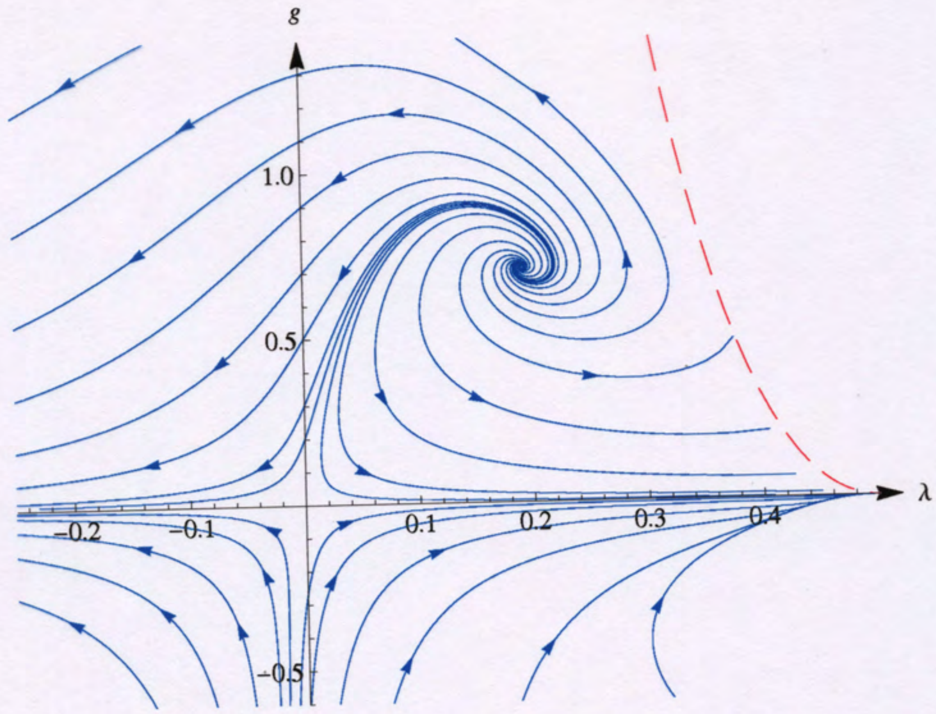
Very similar to Yang-Mills theory !



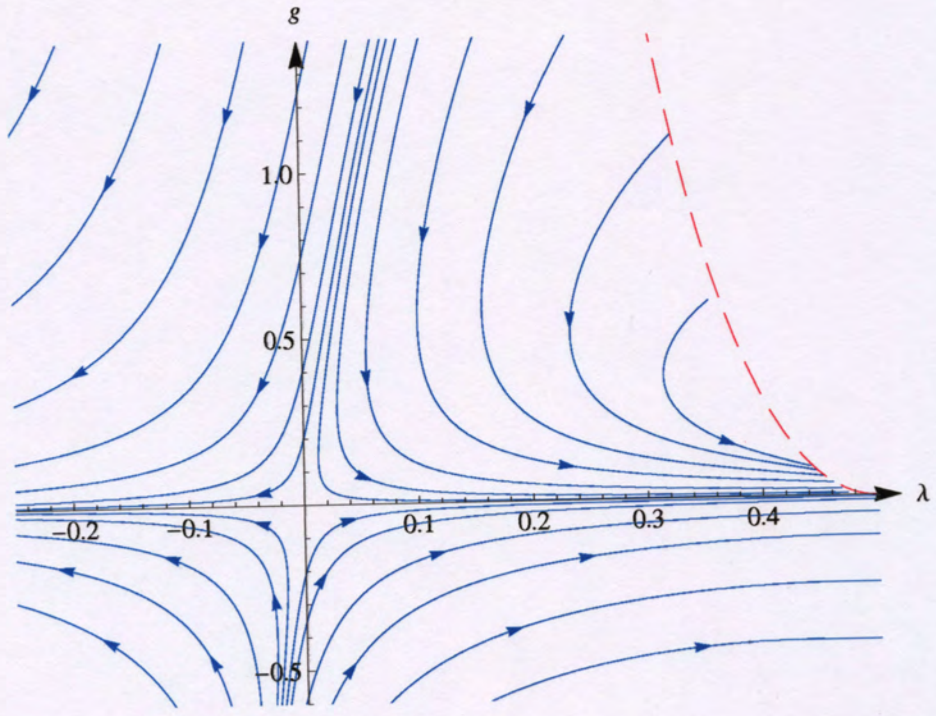
$\pm\,\eta_N\,/f$
120
100
80
60
40
20
1
2
3
4
5
6
7
8
9
10
11
d
$\eta_N{}^{\mathrm{dia}}/f$
$-\,\eta_N{}^{\mathrm{para}}/f$
$-\,\eta_N{}^{\mathrm{ghost-dia}}/f$
$-\,\eta_N{}^{\mathrm{ghost-para}}/f$



$\eta_N\,/f$
total dia
0
3
4
5
6
d
−20
−40
−60
total
total para



Flow diagram obtained from the *total paramagnetic contributions* to η_N alone.



Flow diagram taking into account the *total diamagnetic terms* in η_N only.

QEG spacetimes as a polarizable medium

Restrict $\Gamma_k[g_{\mu\nu}]$ to static (post) Newtonian metrics:

$$ds^2 = -(1+2\phi)\,dt^2 + 2\,\vec{\xi}\cdot d\vec{x}\,dt + (1-2\phi)\,d\vec{x}^{\,2}$$

$\rightsquigarrow$ "electric" field : $\vec{E}_{grav} \equiv -\nabla\phi$

"magnetic" field : $\vec{B}_{grav} \equiv -\frac{1}{2}\nabla\times\vec{\xi}$

Rewrite Γ_k in the form $\mathcal{L} = \frac{1}{2}\varepsilon\,\vec{E}^{\,2} - \frac{1}{2\mu}\vec{B}^{\,2}$:

$$\boxed{\Gamma_k = -\frac{1}{4\pi G_{bare}}\int d^4x\,\frac{1}{2}\left(\varepsilon_k^{grav}\,\vec{E}_{grav}^{\,2} - \frac{1}{\mu_k^{grav}}\vec{B}_{grav}^{\,2}\right)}$$

with $\quad \varepsilon_k^{grav} = \dfrac{1}{\mu_k^{grav}} = \dfrac{G_{bare}}{G_k} \leq 1$

$\Longrightarrow \quad \varepsilon_k^{grav} < 1 \quad$: charge (mass) anti-screening

$\mu_k^{grav} > 1 \quad$: medium is paramagnetic

Analogous to the color-dielectric properties of the YM, QCD vacuum!

Summary

Paramagnetic Dominance

- In a large class of well understood physical systems quantum fluctuations are governed by non-minimal differential operators $\Delta_A + \mathcal{F}(A)$ which give rise to antagonistic dia- and para-type interactions. The para-type interactions "win" and determine the qualitative properties of the system.
- QEG seems to belong to this class!
- The emerging picture of spacetime:
  - Paramagnetic coupling $\sim h\bar{R}h$ is ultra-local, analogous to $\bar{\Psi}\,(\vec{\sigma}\cdot\vec{B})\,\Psi$
  - Spin orientation effects dominate over orbital motion $\sim h\bar{D}^2 h$

$\Longrightarrow$ Analogy: Spin system with magnetic moments sitting at fixed lattice points, interacting with their mean field.

(Rather than a gas of itinerant electrons!)