Exact results for Wilson loops in $\mathcal{N} = 4$ SYM

Diego H. Correa

Universidad Nacional de La Plata - CONICET - Argentina

Based on arXives: 1202.4455, 1203.1019 and 1203.1913

In collaboration with: J. Henn, J. Maldacena and A. Sever

Quantum Gravity in the Southern Cone VI,

Maresias, Brazil, Sept. 13th, 2013

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- Wilson loops & physical interpretations

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- VEVs of Wilson loops in $\mathcal{N}=4$ SYM and AdS/CFT
- Cusp anomalous dimension in $\mathcal{N}=4$ SYM
- Exact computations of VEVs of $\mathcal{N}=4$ SYM Wilson loops
- Exact formula for the radiation of a quark in $\mathcal{N}=4$ SYM

Wilson loops

In any gauge theory:

$$W(\mathcal{R},\mathcal{C}) = \mathrm{tr}_{\mathcal{R}}\mathcal{P}\left(e^{i\oint_{\mathcal{C}} A_\mu \dot{x}^\mu d\tau}\right) := \mathrm{tr}_{\mathcal{R}}\prod_{\tau\in\mathcal{C}}(1 + iA_\mu \dot{x}^\mu d\tau)$$

measures the phase of an external particle

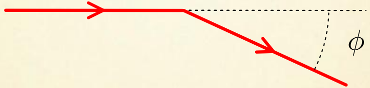
For a given theory a WL depends on 2 things:

- The trajectory of the particle, $\mathcal{C} : x^\mu(\tau)$
- The type/charge of the particle, $\mathcal{R}$: gauge group rep
  ($\mathcal{R}$ will be the fundamental in this talk)

Interesting physical interpretations $\cdots$

Cusp Anomalous dimension

- WL along a straight line with a **cusp**



- Its VEV develops logarithmic divergencies [Polyakov 80, Korchemsky 88]

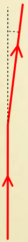
$$\langle W_{\text{cusp}} \rangle = e^{-\Gamma_{\text{cusp}}(\phi) \log(\frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}})} = \left(\frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}}\right)^{-\Gamma_{\text{cusp}}}$$

- Why to study the anomalous dimension $\Gamma_{\text{cusp}}(\phi)$?

  It has nice physical interpretations

Radiation of an accelerated charge

- Small deflection in a timelike curve



$$\langle W \rangle = 1 + \varphi^2 B(\lambda) \log\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right) + \mathcal{O}(\varphi^4)$$

φ: boost angle ($\varphi = i\phi$)

B: Bremsstrahlung function

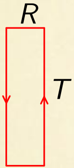
Energy emitted: $E = 2\pi B \int dt \dot{v}^2$

For a fundamental charge in $\mathcal{N} = 4$ SYM B is computed exactly

[D.C.,Henn,Maldacena,Sever]

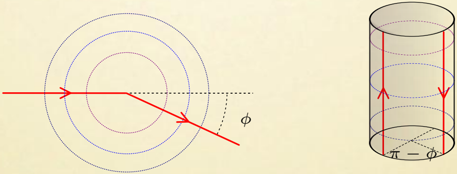
Γ_{cusp} as the $q\bar{q}$ potential in S^3

$T \gg R$



$$\langle \mathrm{tr}\mathcal{P}\left(e^{i\oint A.dx}\right)\rangle = e^{-V_{q\bar{q}}(R)T}$$

- Conformal theory: $\mathbb{R}^4$ can be mapped to $\mathbb{R}\times S^3$



Plane to cylinder map ($\log r = t$)

$$\langle W\rangle \simeq e^{-\log(\frac{\Lambda_{IR}}{\Lambda_{UV}})\Gamma_{\text{cusp}}} = e^{-T\Gamma_{\text{cusp}}} \quad \Rightarrow \quad \Gamma_{\text{cusp}} = V_{q\bar{q}}$$

- $\Gamma_{\text{cusp}}(\phi)$ is the **quark anti-quark potential** in the S^3 for a pair of charges separated by an angle $\delta = \pi - \phi$.

Wilson loops in $\mathcal{N} = 4$ SYM

- 'Simplest' interacting non-abelian 4-dimensional gauge theory

- Gauge theory in the prototypical AdS/CFT example

$$\begin{array}{ccc} \mathcal{N}=4 \text{ SYM in d} = 4 & & IIB \text{ string theory} \\ U(N) \text{ gauge group} & \longleftrightarrow & \text{on } AdS_5 \times S^5 \end{array}$$

$\lambda = g_{YM}^2 N \,,\ \frac{1}{N}$ $\qquad\qquad$ $\frac{R^2}{\alpha'} \,,\ g_s$

The parameters are related as: $\lambda = \left(\frac{R^2}{\alpha'}\right)^2$ $\qquad$ $\frac{\lambda}{N} = g_s$

Perturbative regimes: $\lambda \ll 1$ $\qquad$ $\lambda \gg 1$

- Exact gauge computations $\rightarrow$ Precision tests of AdS/CFT

  These results will be exact in λ and in N

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1/2 BPS Wilson loops in $\mathcal{N} = 4$ SYM

In $\mathcal{N} = 4$ SYM the external charge can also couple to other fields in addition to the gauge potential

In particular

$$W(\mathcal{R}, \mathcal{C}, \vec{n}) = \mathrm{tr}_{\mathcal{R}} \mathcal{P} \left(e^{i \oint_{\mathcal{C}} \left(A_\mu \dot{x}^\mu + \vec{n} \cdot \vec{\Phi} |\dot{x}^\mu| \right) d\tau} \right)$$

- For $\mathcal{C}$ a straight line
- For $\vec{n}$ constant

This is a **1/2 BPS Wilson loop** (invariant under Poincare supercharges) and

$$\langle W(\mathcal{R}, \mathcal{C}, \vec{n}) \rangle = 1$$

Gravity dual of Wilson loops

AdS/CFT intuition: Loop $\mathcal{C}$ provides a boundary condition

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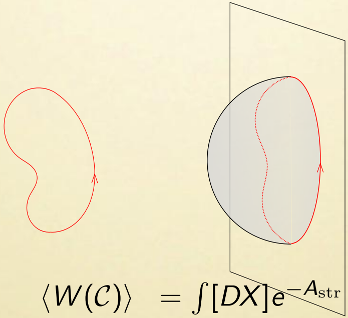
AdS/CFT intuition: Loop $\mathcal{C}$ provides a boundary condition

Maldacena 98: $\langle W \rangle$ given by the area of a fundamental string whose worldsheet describes the loop $\mathcal{C}$ at the boundary of AdS



$$\langle W(\mathcal{C}) \rangle = \int [DX] e^{-A_{\rm str}} \sim e^{-A^{clas}_{\rm str}}$$

Last approximation is valid for large N and large $\lambda = g^2_{YM} N$

1/2 BPS Circular Wilson Loop

$$W = \mathrm{tr}\mathcal{P}\left(e^{i\oint A\cdot dx + \oint |dx|\vec{n}\cdot\vec{\Phi}}\right)$$

$$x^\mu(\tau) = (0, 0, \cos\tau, \sin\tau)$$
$$\vec{n} = (0, 0, 0, 0, 0, 1)$$

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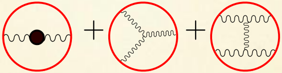
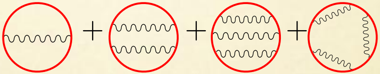
Non-ladders =  + ... = 0

Ladders =  + ...

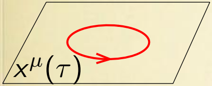
- Sum of non-ladders diagrams cancel for this 1/2 BPS loop
- Ladders diagrams can be resummated!

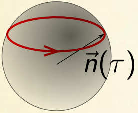
$$\langle W_{\bigcirc}\rangle = \sum_{n=0}^{\infty}\frac{\lambda^n}{4^n(n+1)!n!} = \frac{2}{\sqrt{\lambda}}I_1(\sqrt{\lambda})$$

[Erickson, Semenoff, Zarembo 00]
[Drukker, Gross 00]
[Pestun 07]

This all-loop field theory result has been tested with a strong coupling computation (w/dual string worldsheet area)

1/4 BPS Circular Wilson Loop





$$x^\mu(\tau) = (0, 0, \cos\tau, \sin\tau)$$
$$\vec{n} = (0, 0, 0, \sin\theta_0\cos\tau, \sin\theta_0\sin\tau, \cos\theta_0)$$

$$W = \mathrm{tr}\mathcal{P}\left(e^{i\oint A\cdot dx + \oint |dx|\vec{n}\cdot\vec{\Phi}}\right)$$

- 1-parameter deformation of the 1/2 BPS Circular Wilson Loop
- [Drukker 06]: Its VEV is the same as for the 1/2 BPS Circular Wilson Loop but with $\lambda \mapsto \lambda\cos^2\theta_0$

$$\langle W_{\bigcirc}^{\theta_0}\rangle = \frac{2}{\sqrt{\lambda}\cos\theta_0} I_1(\sqrt{\lambda}\cos\theta_0)$$

- This result allows the exact computations of more interesting Wilson loops: Γ_{cusp} in the small angle limit [D.C.,Henn,Maldacena,Sever]

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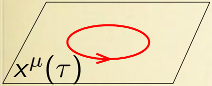

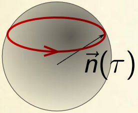

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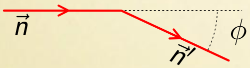
Exact formula for Bremsstrahlung function $B(\lambda)$

[D.C.,Henn,Maldacena,Sever]

- Recall, it's related to Γ_{cusp} in the small cusp angle(s) limit

$$\Gamma_{\text{cusp}}(\lambda, \phi, \theta) = (\theta^2 - \phi^2) B(\lambda) + \cdots$$

  where θ is an additional (internal) cusp angle



$$\langle W \rangle = e^{-\Gamma_{\text{cusp}}(\phi) \log(\frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}})}$$

  We can take $\vec{n}$ and $\vec{n}'$ for the 2 lines of the cusp. This introduces an internal cusp angle $\cos\theta = \vec{n} \cdot \vec{n}'$.

- For $\theta \approx \phi$ it's a near BPS Wilson loop
- We can compute it, by relating it the $\langle W_{\bigcirc}^{\theta_0} \rangle$, VEV of the circular Wilson loops.

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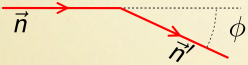

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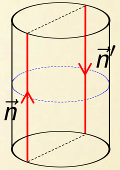
Case $\phi = 0$ with internal cusp angle θ

$$\vec{n}' = (0, 0, 0, 0, \sin\theta, \cos\theta)$$

$$\vec{n} = (0, 0, 0, 0, 0, 1)$$

$\Leftrightarrow$



$$\begin{aligned}
\langle W \rangle =& \langle \mathrm{tr}\mathcal{P}[e^{\int_{-\infty}^{\infty}(A_t + \vec{n}\cdot\vec{\Phi})dt} e^{\int_{\infty}^{-\infty}(A_t + \vec{n}'\cdot\vec{\Phi})dt}] \rangle \\
=& 1 + \theta \int_{\infty}^{-\infty} dt \langle\!\langle \Phi^5(t) \rangle\!\rangle - \tfrac{1}{2}\theta^2 \int_{\infty}^{-\infty} dt \langle\!\langle \Phi^6(t) \rangle\!\rangle \\
& + \tfrac{1}{2}\theta^2 \int_{\infty}^{-\infty} dt \int_{\infty}^{-\infty} dt' \langle\!\langle \Phi^5(t)\Phi^5(t') \rangle\!\rangle + \cdots
\end{aligned}$$

$\langle\!\langle \quad \rangle\!\rangle$ mean correlators of insertions along the loop

$$\langle\!\langle \mathcal{O}(t_1)\mathcal{O}(t_2) \rangle\!\rangle = \frac{\langle \mathrm{tr}\mathcal{P}[\mathcal{O}(t_1) e^{\int_{t_2}^{t_1} iA\cdot dx + |dx|\vec{n}\cdot\vec{\Phi}} \mathcal{O}(t_2) e^{\int_{t_1}^{t_2} iA\cdot dx + |dx|\vec{n}\cdot\vec{\Phi}}] \rangle}{\langle \mathrm{tr}\mathcal{P}[e^{i\oint A\cdot dx + \oint |dx|\vec{n}\cdot\vec{\Phi}}] \rangle}$$

Correlators $\langle\!\langle\ \ \rangle\!\rangle$ are constrained by conformal symmetry

$$\langle\!\langle \Phi^i(t_1) \rangle\!\rangle = 0 \qquad \langle\!\langle \Phi^i(t_1)\Phi^j(t_2) \rangle\!\rangle = -\frac{\gamma}{2}\frac{\delta^{ij}}{X(t_1)\cdot X(t_2)}$$

$X(t)$: curve's parametrization in embedding space.

$$X \in \mathbb{R}^{1,5} \qquad X^2 = 0 \qquad X \sim \lambda X$$

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For example, the t line in $R \times S^3$:

$$X(t) = (e^t, -e^{-t}, 1, 0, 0, 0) \quad \Rightarrow \quad X(t)\cdot X(t') = 1 - \cosh(t - t')$$

$$\langle W \rangle = e^{-\Gamma_{\text{cusp}} T} = 1 + \tfrac{\theta^2}{2}\frac{\gamma}{2}\int\!\!\int \frac{1}{\cosh(t-t') - 1} dt dt'$$

$$\Gamma_{\text{cusp}} = -\frac{\theta^2}{2}\frac{\gamma}{2}\lim_{\epsilon\to 0}\int_{-\infty}^{\infty}\frac{dt}{(\cosh t - 1)^{1-\epsilon}} = \gamma\frac{\theta^2}{2} \quad \Rightarrow \quad B(\lambda) = \frac{\gamma}{2}$$

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Relation between Γ_{cusp} and coefficient in 2-point $\langle\!\langle\ \rangle\!\rangle$ is general

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Relation between Γ_{cusp} and coefficient in 2-point $\langle\!\langle\ \rangle\!\rangle$ is general

In $\mathcal{N} = 4$ SYM we can get γ from the exact knowledge of $\langle W^{\theta_0}_{\bigcirc} \rangle$

$\gamma = 2B(\lambda)$ from 1/4 BPS Wilson loop VEV

For $\theta_0 \ll 1$, $\quad \lambda \cos^2\theta_0 \sim \lambda(1-\theta_0^2)$ and then

$$\frac{\langle W_{\bigcirc}^{\theta_0}\rangle - \langle W_{\bigcirc}\rangle}{\langle W_{\bigcirc}\rangle} \sim -\theta_0^2\, \lambda\partial_\lambda \log\langle W_{\bigcirc}\rangle$$

Expanding $\vec{n} = (0,0,0,\sin\theta_0\cos\tau, \sin\theta_0\sin\tau, \cos\theta_0)$ this VEV is reduced to insertions along the $\frac{1}{2}$-BPS circular Wilson loop

$$\frac{\langle W_{\bigcirc}^{\theta_0}\rangle - \langle W_{\bigcirc}\rangle}{\langle W_{\bigcirc}\rangle} \sim \frac{\theta_0^2}{2}\int_0^{2\pi} d\tau \int_0^{2\pi} d\tau'\, \hat{n}^i(\tau)\hat{n}^j(\tau')\, \langle\!\langle \Phi^i(\tau)\Phi^j(\tau')\rangle\!\rangle_{\circ}$$

$\hat{n}^1 + i\hat{n}^2 = e^{i\tau}$ and now $\langle\!\langle\ \rangle\!\rangle_{\circ}$ respect to the $\frac{1}{2}$-BPS circular loop

Circular loop in embedding formalism:
$(X^+, X^-, X^1, X^2, X^3, X^4) = (1, -1, \cos\tau, \sin\tau, 0, 0)$

$$\langle\langle \Phi^i(\tau)\Phi^j(\tau')\rangle\rangle_\circ = -\frac{\gamma_\circ}{2}\frac{\delta^{ij}}{X(t_1)\cdot X(t_2)} = \frac{\gamma_\circ}{2}\frac{\delta^{ij}}{[1-\cos(\tau-\tau')]}$$

- In general, the coefficient γ would change for $\langle\langle\ \rangle\rangle$ respect to different loops
- However, since the straight line and the circle are related by a conformal transformation, their $\langle\langle\ \rangle\rangle$ have the same γ

Then,

$$\lambda\partial_\lambda \log\langle W_\bigcirc\rangle = -\gamma\frac{\pi}{2}\lim_{\epsilon\to 0}\int_0^{2\pi} d\tau \frac{\cos\tau}{(1-\cos\tau)^{1-\epsilon}} \sim \pi^2\gamma$$

Finally,

$$B(\lambda) = \frac{\lambda}{2\pi^2}\partial_\lambda \log\langle W_\bigcirc\rangle = \frac{\sqrt{\lambda}}{4\pi^2}\frac{I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})}$$

Concluding remarks

- The relation between the 2-point correlator $\langle\!\langle\ \rangle\!\rangle$ coefficient γ and the Bremsstrahlung function B

$$\gamma = 2B(\lambda, N)$$

is valid in general for any conformal theory. In general γ refers to the coefficient in the 2-point correlator of displacement operators

$$\langle\!\langle \mathbb{D}_i(t_1)\mathbb{D}_j(t_2)\rangle\!\rangle = -\frac{\gamma}{2}\frac{\delta_{ij}}{X(t_1)\cdot X(t_2)}$$

for

$$\delta W \sim \int dt\ \delta x^i(t)\mathbb{D}_i(t)W$$

- Special about $\mathcal{N}=4$ SYM: coefficient γ can also be related to the VEV of a circular WL that is exactly known

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$$\gamma = \frac{1}{\pi^2}\lambda\partial_\lambda \log\langle W_{\bigcirc}\rangle$$

Concluding remarks

- The presented exact VEVs of circular Wilson Loops are the leading large N expressions. These VEVs are also known to all orders in the $1/N$ expansion:

$$\langle W_{\bigcirc} \rangle = \frac{1}{N} L^{1}_{N-1}\left(-\tfrac{\lambda}{4N}\right) e^{\frac{\lambda}{8N}}$$

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- For generic angles, not known the solution to the TBA system
- In the small angle limit, the TBA system is exactly solved [Gromov,Sever]. This completely independent computation repoduces the planar limit of the localization result

$$B(\lambda) = \frac{\sqrt{\lambda}}{4\pi^2} \frac{I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})}$$

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