Worldsheet dilatation operator for the *AdS* superstring

Israel Ramírez

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- AdS/CFT

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we find

$$\left\langle \frac{\delta S}{\delta\phi(z)} \mathcal{O}(y) \right\rangle = \left\langle \frac{\delta \mathcal{O}(y)}{\delta\phi(z)} \right\rangle .$$

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$$-\Box_z \langle \phi(z)\phi(y) \rangle = \delta^d(z - y)$$

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The algebra for this model is just

$$[T_a, T_b] = f_{ab}^c T_c$$

The 10-Steps rule

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$$J = J_0 + \nabla X + \frac{1}{2}[\nabla X, X] + \cdots$$

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$$\left\langle \frac{\delta S_{kin}}{\delta X^a(z)} X^b(y) \right\rangle + \int d^2w \frac{\delta^2 S_{int}}{\delta X^c(w) \delta X^a(z)} \langle X^c(w) X^b(y) \rangle = \delta_a^b \delta^2(z-y)$$

Working a little

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$$\begin{aligned}\Box_z G^{ab}(z,y) = &- \frac{1}{T}\eta^{ab}\delta^2(z-y) \\ &+ \frac{f^a_{ce}}{2}\left(\partial_z G^{cb}(z,y)\bar{J}^e_0 + \bar{\partial}_z G^{cb}(z,y) J^e_0\right)\end{aligned}$$

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$$G^{ab}(z,k) = \frac{\eta^{ab}}{T}\frac{1}{|k|^2} + \frac{\Box}{|k|^2}G^{ab} + i\left(\frac{\partial}{k} + \frac{\bar{\partial}}{\bar{k}}\right)G^{ab} - \frac{f^a_{ce}}{2}\left(\bar{J}^e_0\left[\frac{i}{\bar{k}} + \frac{\partial}{|k|^2}\right] + J^e_0\left[\frac{i}{k} + \frac{\bar{\partial}}{|k|^2}\right]\right)G^{cb}$$

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$$G^{ab}(z,k) = \frac{\eta^{ab}}{T}\frac{1}{|k|^2} - \frac{\eta^{bd}}{T}\frac{i}{2|k|^2} f^{a}_{de}\left(\frac{J_0^e}{k} + \frac{\bar{J}_0^e}{\bar{k}}\right) + \cdots$$

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$$\lim_{z\to y}\int \frac{d^d k}{(2\pi)^d}\frac{e^{ik(z-y)}}{|k|^2} = -\frac{1}{2\pi\epsilon}$$

where $d = 2 - \epsilon$.

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For a term with a derivative

$$\begin{aligned}\langle X\bar{\partial}X \rangle &\equiv \lim_{y \to z} \bar{\partial}_y \langle X(z)X(y) \rangle \\ &= \lim_{z \to y} \int \frac{d^d k}{(2\pi)^d} e^{ik(z-y)} \frac{-iC}{|k|^2} + \cdots = -iIC\end{aligned}$$

For our simple case

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$$\begin{aligned}\langle X^a X^b \rangle =& I\eta^{ab} \\ \langle X^a \partial X^b \rangle =& -\frac{I}{2}\eta^{bd} f^a_{de} J^e_0 \\ \langle X^a \bar{\partial} X^b \rangle =& -\frac{I}{2}\eta^{bd} f^a_{de} \bar{J}^e_0\end{aligned}$$

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$$\langle \mathcal{O}\mathcal{O}' \rangle = \langle \mathcal{O} \rangle \mathcal{O}' + \mathcal{O} \langle \mathcal{O}' \rangle + \langle \mathcal{O}, \mathcal{O}' \rangle$$

where

$$\langle \mathcal{O} \rangle = \frac{1}{2} \int d^2z d^2y \frac{\delta^2 \mathcal{O}}{\delta X^a(z) \delta X^b(y)} \left\langle X^a(x) X^b(y) \right\rangle$$

$$\langle \mathcal{O}, \mathcal{O}' \rangle = \int d^2z d^2y \frac{\delta \mathcal{O}}{\delta X^a(z)} \frac{\delta \mathcal{O}'}{\delta X^b(y)} \left\langle X^a(x) X^b(y) \right\rangle$$

Pure Spinors

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Pure Spinor model $\rightarrow$ a Super String model on a coset space.Equivalent to Green-Schwartz model.

Pure Spinor

The Pure Spinors action is

$$
\begin{aligned}
S =& \frac{R^2}{2\pi} \int d^2 z \, \mathrm{STr} \left[\frac{1}{2} J_2 \bar{J}_2 + \frac{1}{4} J_1 \bar{J}_3 + \frac{3}{4} \bar{J}_1 J_3 + \omega \bar{\nabla} \lambda + \hat{\omega} \nabla \hat{\lambda} - N \hat{N} \right] \\
N =& \{\omega, \lambda\} \\
\nabla =& \partial + [J_0,]
\end{aligned}
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$$

The $AdS_5 \times S^5$ is described by the group

$$
\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}
$$

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\begin{aligned}
J =& J_0 + J_1 + J_2 + J_3 = J_0 + K \\
=& J_0^i T_i + J_1^{\alpha} T_{\alpha} + J_2^m T_m + J_3^{\hat{\alpha}} T_{\hat{\alpha}}
\end{aligned}
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we can choose

$$X = X_1 + X_2 + X_3$$

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How does everything look like now?

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How does everything look like now?Matrix multiplication!

$$G^{AB} = I^{AB} + F^A_C G^{CB}$$

$$G = \begin{pmatrix} G^{\alpha\beta} & \cdots & & \\ \vdots & G^{mn} & & \\ & & G^{\hat{\alpha}\hat{\beta}} & \\ & & & \ddots \end{pmatrix}$$

Pure Spinor

Some results

$$\langle K \rangle = 0$$

$$\langle j \rangle = \left\langle g \left(J_2 + \frac{3}{2} J_3 + \frac{1}{2} J_1 + 2N \right) g^{-1} \right\rangle = 0$$

$$\langle b \rangle = \left\langle (\lambda \hat{\lambda})^{-1} \mathsf{STr} \left(\hat{\lambda} [J_2, J_3] + \{\omega, \hat{\lambda}\} [\lambda, J_1] \right) - \mathsf{STr} \left(\omega J_1 \right) \right\rangle = 0$$

Dilatation operator

$$\mathcal{O} = \langle J, J \rangle \frac{\delta}{\delta J}\frac{\delta}{\delta J} + \langle J, X \rangle \frac{\delta}{\delta J}\frac{\delta}{\delta X} + \langle X, X \rangle \frac{\delta}{\delta X}\frac{\delta}{\delta X} + \langle J \rangle \frac{\delta}{\delta J}$$

Conclusions

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- Can be extended to a 3-point function straightforward

Future Work

- Compute 3 and 4-point functions
- Do it with a computer program
- Look for signs of integrability in the string
- Find physical states of the string

Obrigado