A mini-course on:

The Very Early Universe

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Understanding emerged from the work of many researchers, especially:

Agullo, Barrau, Bojowald, Campiglia, Cailleatau, Corichi, Grain, Kaminski, Lewandowski, Mielczarek,Nelson, Pawlowski, Singh, Sloan, Thiemann, Velhinho, Wilson-Ewing ....

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Organization

The very early universe offers the best arena to probe both, the deep conceptual issues of quantum gravity and observational tests of theories.

Lecture 1

The Big Bang Singularity & Loop Quantum Cosmology

1. Introduction: The Big Bang Singularity
2. Singularity Resolution in LQC: Basic Results
3. Novel features at the Foundation

Lecture 2

Cosmological Perturbations

1. Classical Perturbation Theory
2. Quantum Fields on classical FLRW Space-times
3. Quantum fields on Quantum FLRW geometries

Lecture 3

An Extension of the Inflationary Scenario to the Planck Regime

1. Success and Limitations of Inflation
2. The LQC Strategy
3. Results: Pre-inflationary dynamics and its implications.

1. Introduction: The Big-Bang

• General Relativity: Gravity encoded in Geometry. Space-time geometry became a physical and dynamical entity. Spectacular consequences: Black holes, Gravitational Waves.

• But this fusion comes with a price: Now space-time itself ends at singularities. Big Bang: Absolute Beginning.

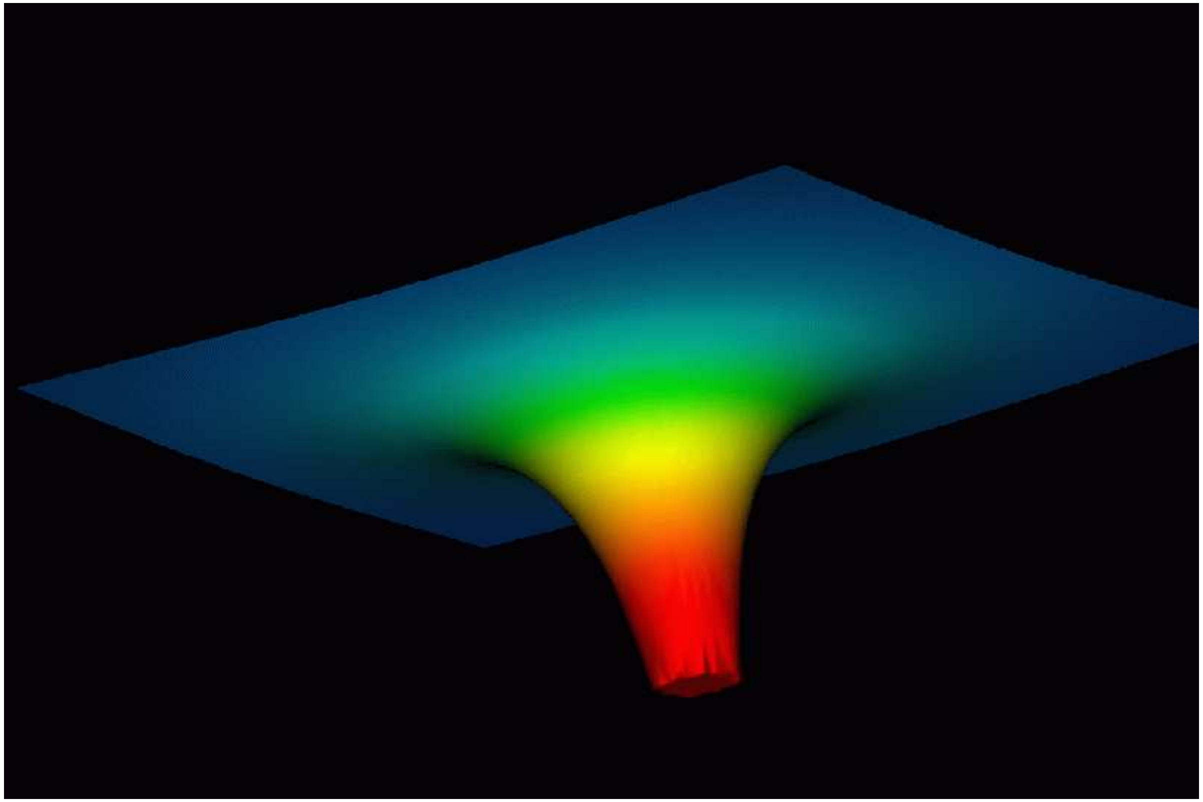
Friedmann (1921-1924): Primarily interested in mathematical aspects
Lemaître (1926-1965): understood the deep physical implications

The assumption of spatial homogeneity & isotropy implies that the metric has the FLRW form: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$
$a(t)$: Scale Factor; Volume $v(t) \sim [a(t)]^3$ Curvature $\sim [a(t)]^{-n}$
Einstein Equations $\Rightarrow$ volume $\rightarrow 0$ and Curvature $\rightarrow \infty$:

BIG BANG!! CLASSICAL PHYSICS STOPS!!

• This has become the standard paradigm for the Very Early Universe.
Λ CDM Cosmology: The homogeneous, isotropic FLW background space-time. Inhomogeneities described by linear quantum fields. Goal: To show that perturbations evolve precisely to the tiny inhomogeneities observed in the CMB.

The Big Bang in classical GR



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

Big Bang: Historical Twists and Turns

- Cosmology remained outside the main stream of physics and the steady state theory rivaled big bang cosmology until:

⋆ Gamow, Alpher, Herman (1948-1967) (Detailed Nucleosynthesis).
Gamow strongly disliked the emphasis on Big-Bang/Beginning. Preferred to emphasize *‘dynamical universe’* and thought the universe had a pre-big-bang branch.
⋆ Dicke, Peebles, Roll, Wilkinson (1965 $\rightarrow$) (CMB Background)
Dicke also disliked the Absolute Beginning; Preferred an "oscillating" universe.

- Einstein also did not take the Big-Bang/Beginning seriously. Suggested inhomogeneities may wash it away. This view persisted.

- The Khalatnikov-Lifshitz program: “General Solution” to Einstein’s equation will be singularity free (late 50’s - early 60’s).

- Paradigm Shift:

Penrose-Hawking Singularity Theorems (mid 60s): If matter satisfies ‘energy conditions’ then according to general relativity, cosmological space-times will necessarily have a singularity!
(Lemaître’s views vindicated.)

Beyond General Relativity

- But general expectation: theory is pushed beyond its domain of applicability. Must incorporate Quantum Physics. (Example: Instability of the Hydrogen atom in classical electrodynamics and $E_o = -me^4/2\hbar^2$ in quantum theory.)

- Big-bang is the prediction of General Relativity precisely in a domain in which it is inapplicable! Classical singularities are gates to Physics Beyond Einstein.

- Any viable quantum gravity theory should answer the questions: What *really* happened in the Planck regime? In the standard model, CMB occurs 400,000 years after the Big Bang. At the onset of inflation, matter density is less than $10^{-11}\,\rho_{\rm Pl}$. Far from 'proofs' that Big Bang occurred! Does quantum physics really stop if we went further back? Is there a finite Beginning? If not, what was really there before the GR era?

Conceptual Issues

• Some Long-Standing Questions expected to be answered by Quantum Gravity Theories from first principles:

⋆ How close to the big-bang does a smooth space-time of GR make sense? (Onset of inflation?)

⋆ Is the Big-Bang singularity naturally resolved by quantum gravity?
(answer is ‘No’ in the Wheeler-DeWitt theory)

⋆ Is a new principle/ boundary condition at the Big Bang essential?
(e.g. The Hartle-Hawking ‘no-boundary proposal’.)

⋆ Is the quantum evolution across the ‘singularity’ deterministic?
(So far the answer is ‘No’ e.g. in the Pre-Big-Bang and Ekpyrotic scenarios)

⋆ What is on the ‘other side’? A quantum foam? Another large, classical universe? ...

Singularity Resolution?

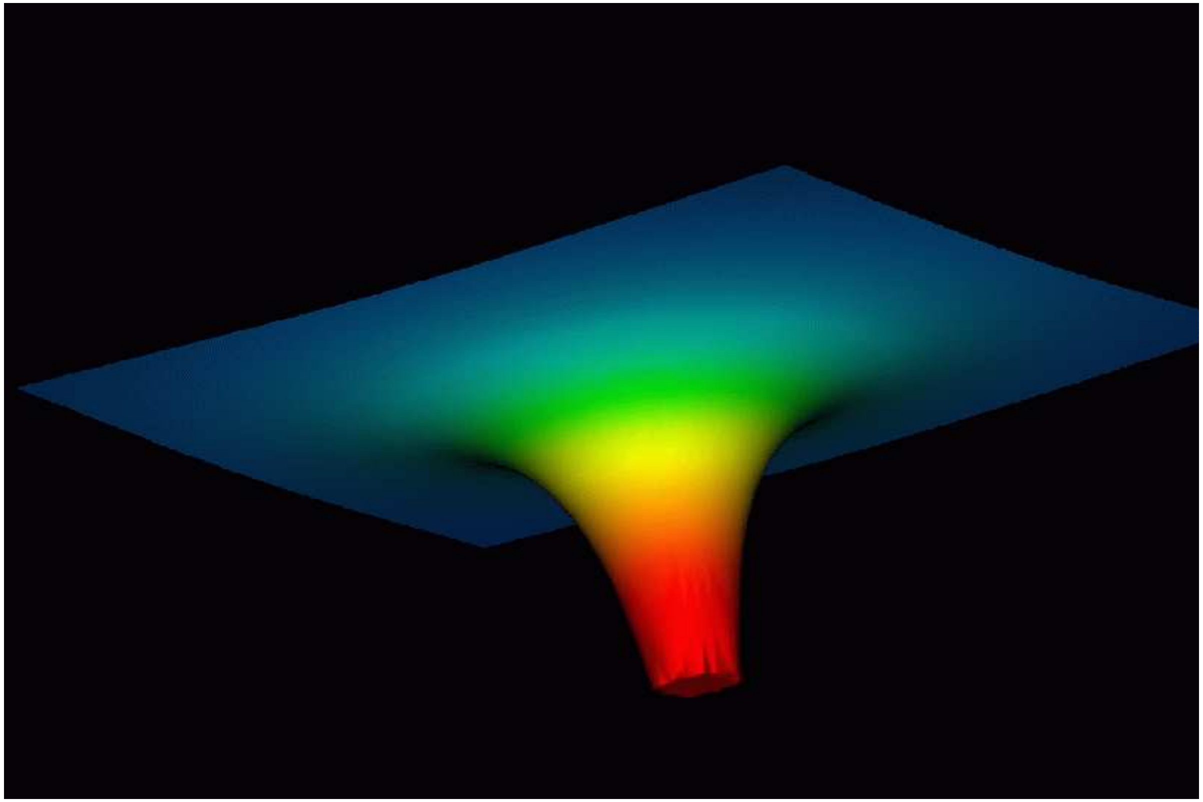
- Difficulty: UV - IR Tension. Can one have singularity resolution with ordinary matter and agreement with GR at low curvatures? e.g., recollpase in the closed (i.e., k=1) models? (Background dependent perturbative approaches have difficulty with the first while background independent approaches, with second.) (Green & Unruh; Brunnemann & Thiemann)

- These questions have been with us for 30-40 years since the pioneering work of DeWitt, Misner and Wheeler. WDW quantum cosmology is fine in the IR but not in the UV.

- In LQC, this issue has been resolved for a large class of cosmological models. Physical observables which are classically singular (eg matter density) at the big bang have a dynamically induced upper bound on the physical Hilbert space. Mathematically rigorous and conceptually complete framework.
(AA, Bojowald, Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

- Emerging Scenario: In simplest models, vast classical regions bridged deterministically by quantum geometry. No new principle needed to join the pre-big bang and post-big-bang branches.

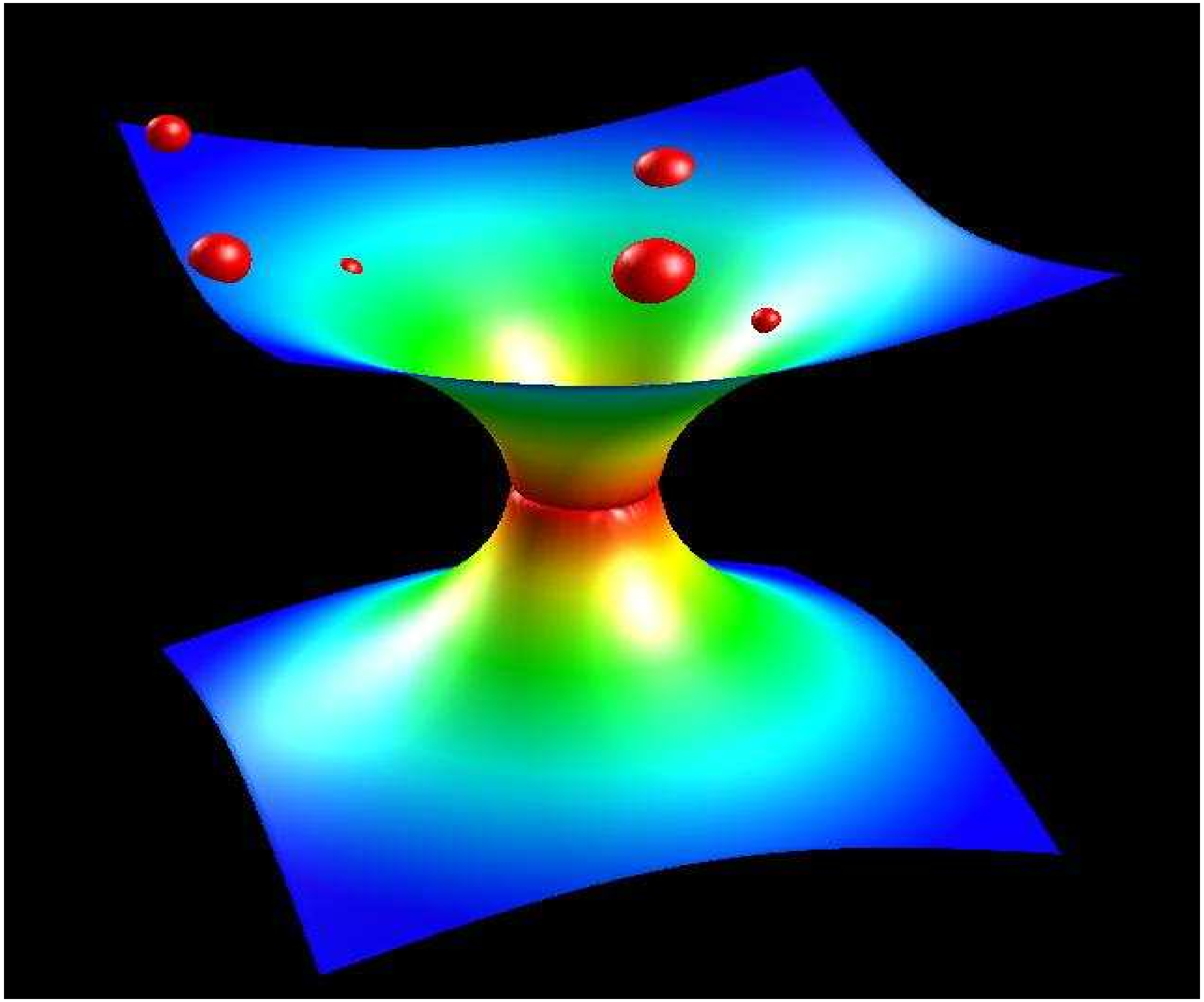
The Big Bang in classical GR: k=0 Model



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

The Big Bang in LQC: k= 0 Model



Artist's depiction of the Quantum Bounce Credits: Dr. Cliff Pickover.

In loop quantum cosmology, our post-big-bang branch of the universe is joined to a pre-big-bang branch by a quantum bridge: Gamow's bounce

Organization

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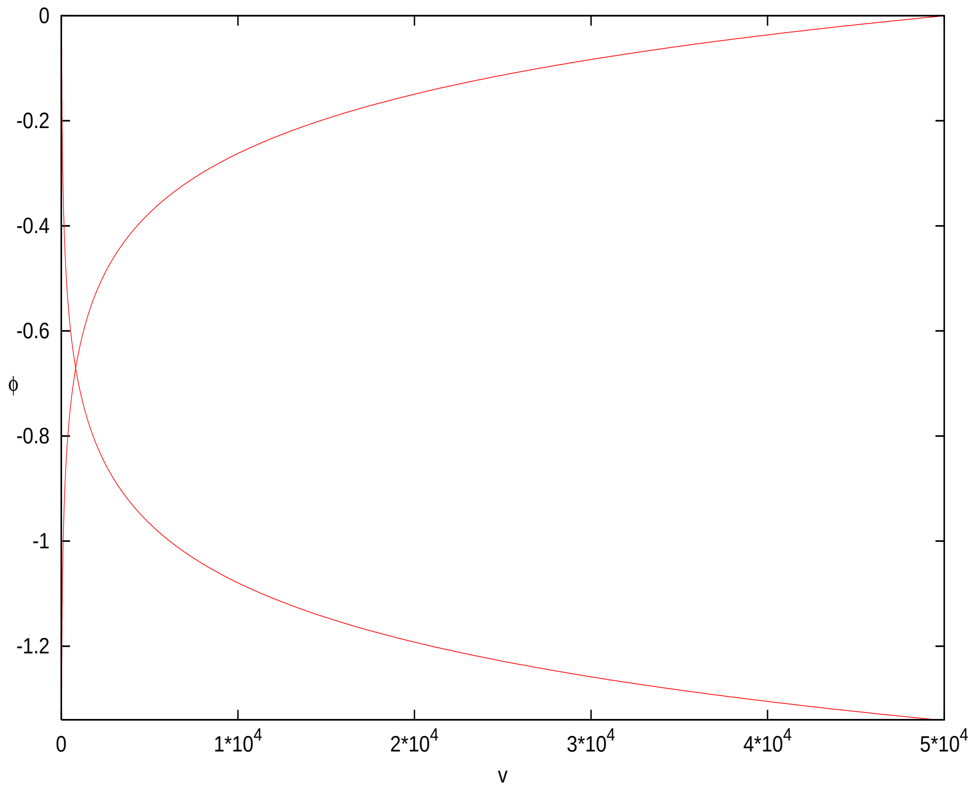
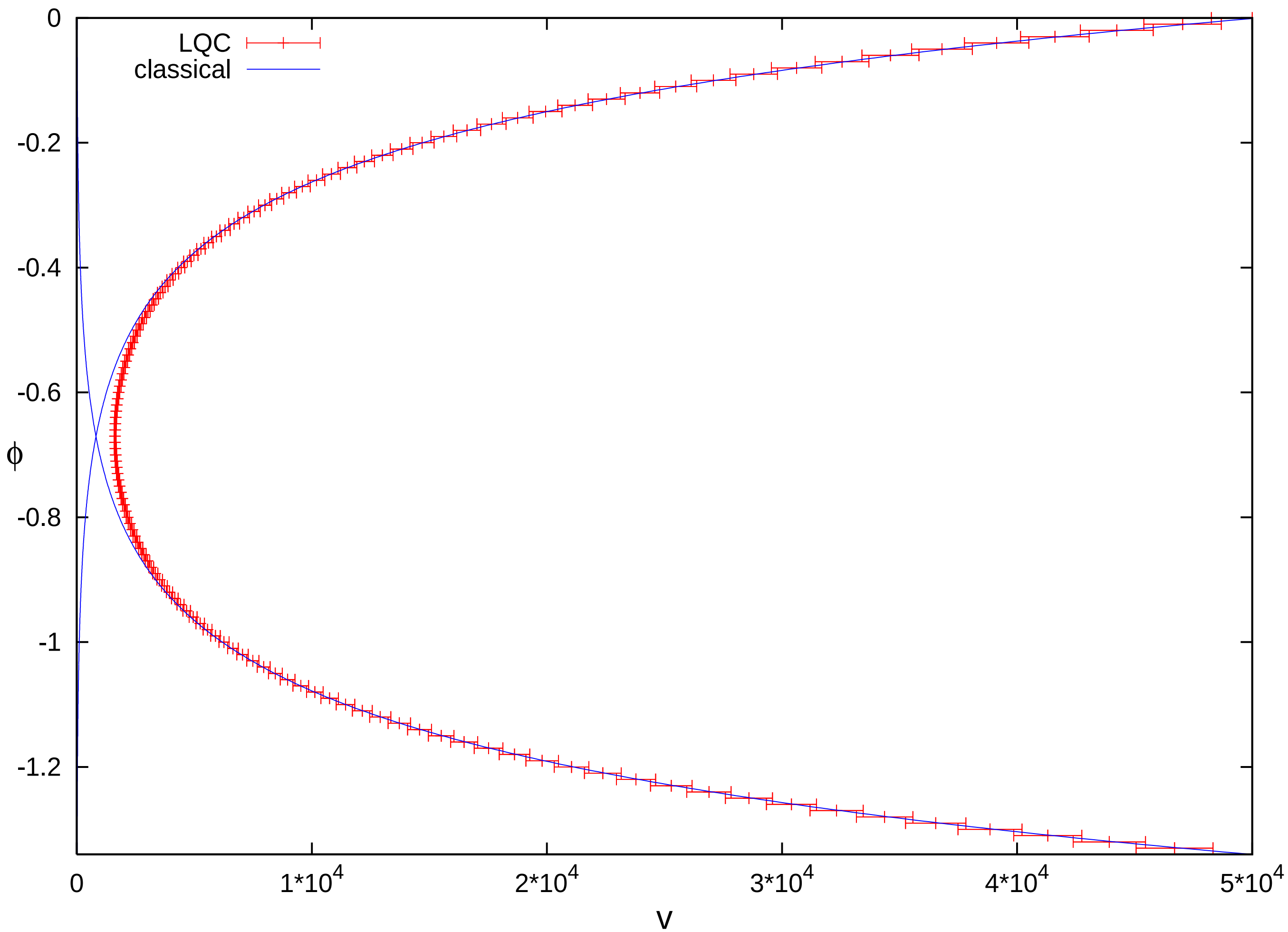
2. Singularity resolution in LQG: Basic Results

FLRW, k=0, $\Lambda = 0$ Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical trajectories

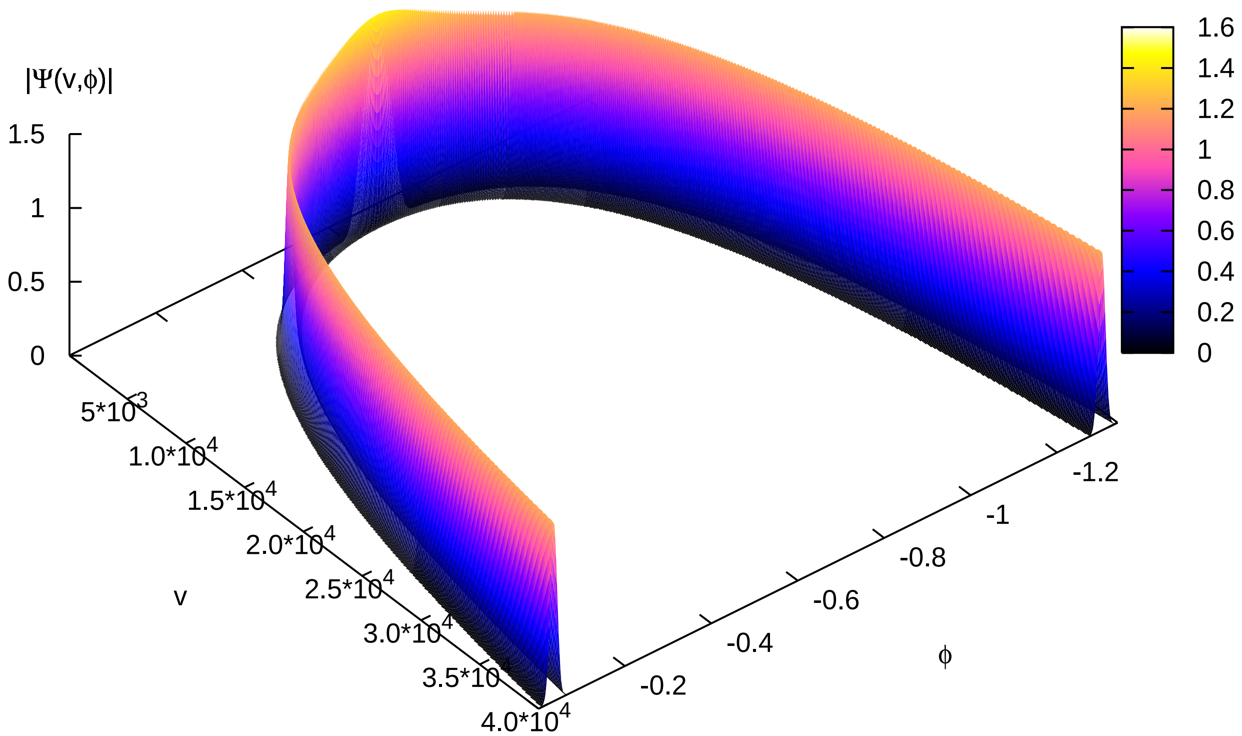
k=0 LQC



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh) Gamow's favorite paradigm realized.

k=0 LQC



Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

k=0 Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then: (AA, Pawlowski, Singh)

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \sim 10^{-2} m_{\rm Pl}^2$ or $\rho \sim 10^{-3} \rho_{\rm Pl}$. $\Rightarrow$ We know 'from first principles' that space-time can be taken to be classical during the inflationary era (since $\rho \sim 10^{-12} \rho_{\rm Pl}$ at the onset of inflation).

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic unlike in other approaches. No new principle needed. The final quantum space-time is vastly larger than what general relativity had us believe.

• No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects: Main difference from WDW theory. Finally, Effective equations surprisingly effective!

k=0 Results

- To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then the Quantum Corrected, Effective Friedmann Eq is:

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where } \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce.

- The matter density operator $\hat{\rho} = \frac{1}{2}\,(\hat{V}_\phi)^{-1}\,\hat{p}^2_{(\phi)}\,(\hat{V}_\phi)^{-1}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

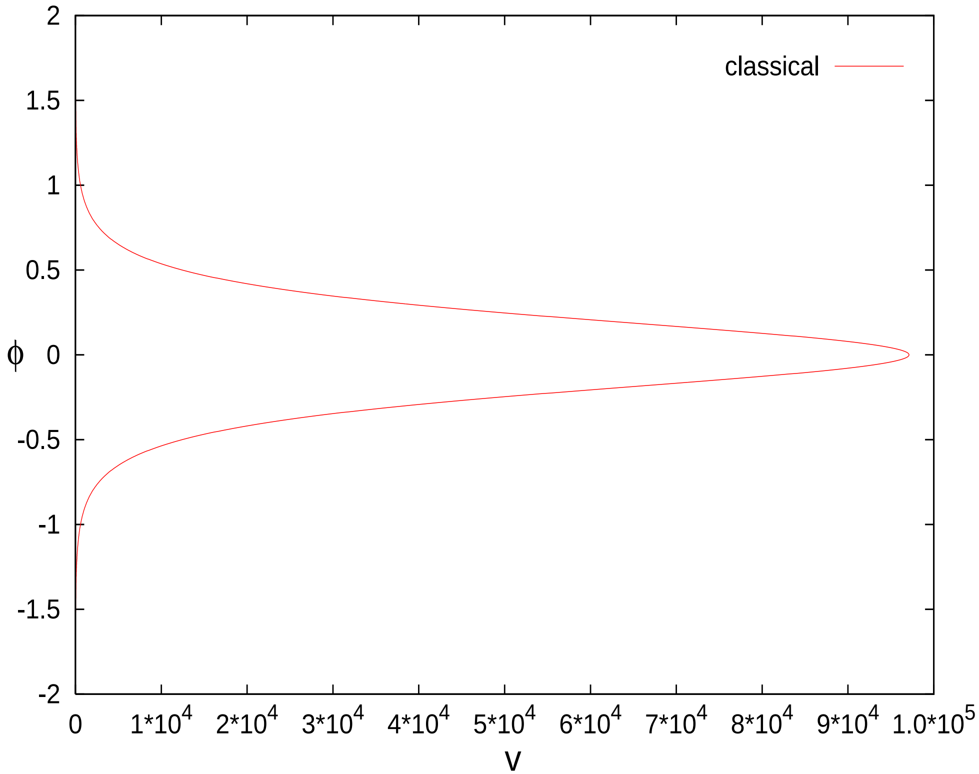
$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved.
(Brunnemann & Thiemann)

- Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive forces due to quantum matter are familiar: Fermi degeneracy pressure in Neutron stars. Difference: Quantum nature of geometry rather than matter. Rises and dies extremely rapidly but strong enough to resolve the singularity.

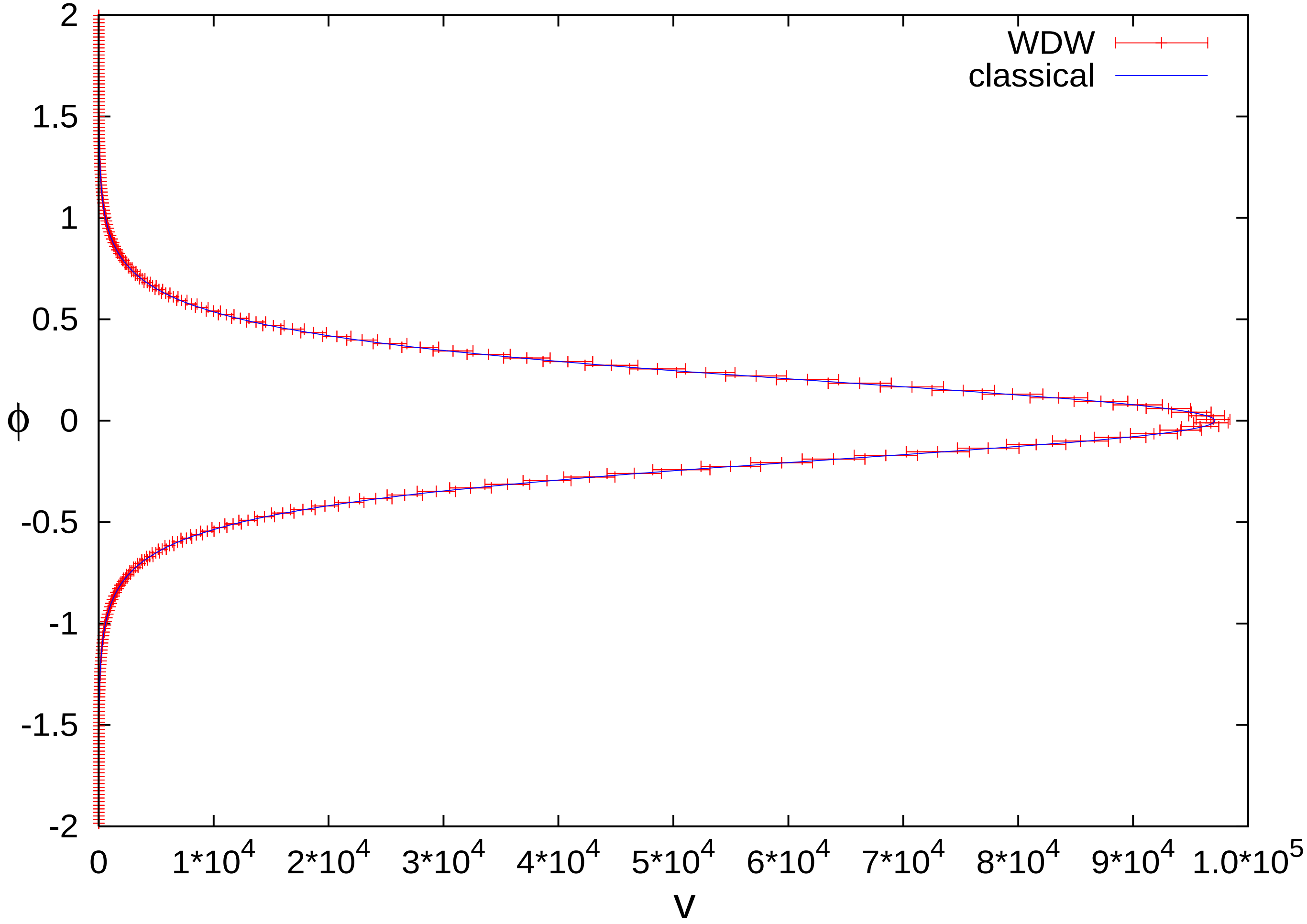
The Closed Model: Bouncing/Phoenix Universes.

Another Example: k=1 FLRW model with a massless scalar field ϕ. Instructive because again every classical solution is singular; scale factor not a good global clock; More stringent tests because of the classical re-collapse. (Tolman, Sakharov, Dicke,...)



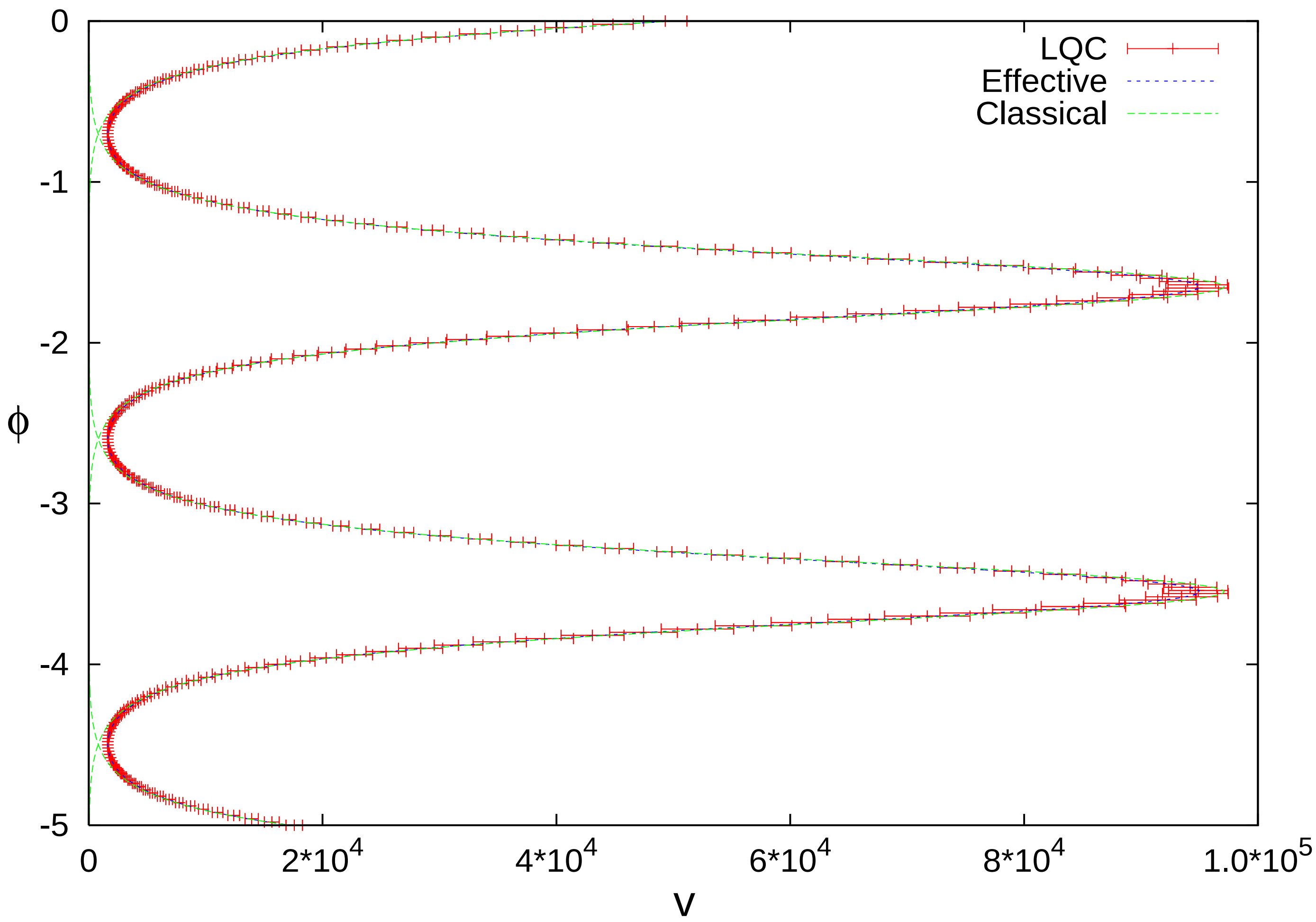
Classical Solutions

k=1 Model: WDW Theory



Expectations values and dispersions of $\hat{V}|_\phi$.

k=1 Model: LQC



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh, Vandersloot)

k=1: Domain of validity of classical GR

(AA, Pawlowski, Singh, Vandersloot)

- Classical Re-collapse: The infrared issue.

$$\rho_{\min} = (3/8\pi G a_{\max}^2)\left(1 + O(\ell_{\rm Pl}^4/a_{\max}^4)\right)$$

So, even for a very small universe, $a_{\max} \approx 23\ell_{\rm Pl}$, agreement with the classical Friedmann formula to one part in 10^5. Classical GR an excellent approximation for $a > 10\ell_{\rm Pl}$. For macroscopic universes, LQC prediction on recollapse indistinguishable from the classical Friedmann formula.

- Quantum Bounces: The ultra-violet issue

For a universe which attains $v_{\max} \approx 1\,{\rm Gpc}^3$,

$v_{\min} \approx 6 \times 10^{18}{\rm cm}^3 \approx 10^{117}\ell_{\rm Pl}^3$: $6km \times 18km \times 54km$ Mountain!

What matters is curvature, which enters Planck regime at this volume.

Generalizations

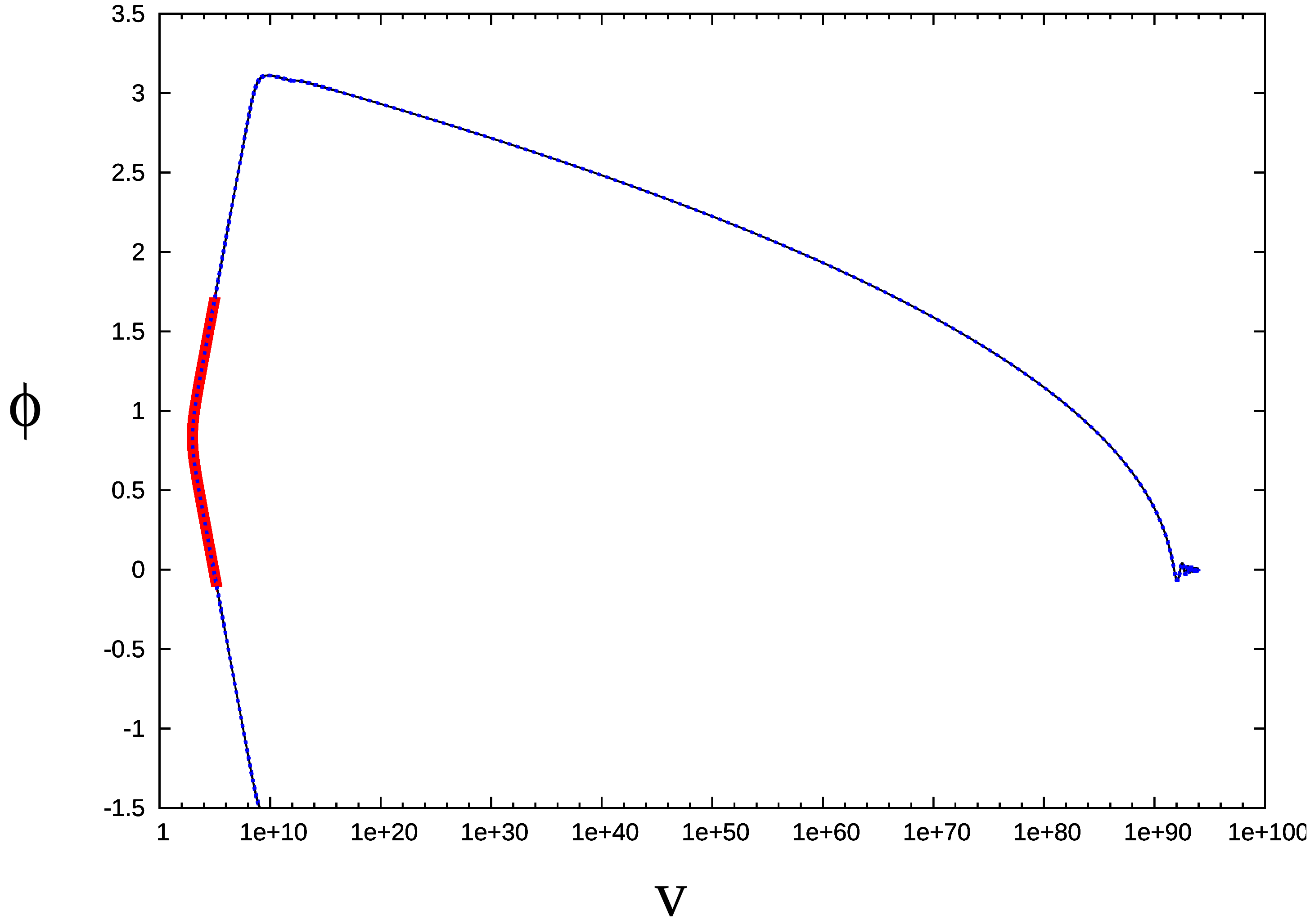
- Inclusion of Λ (A B P): $\surd$ (Infrared limit trickier)
  Inclusion of a $m^2\phi^2$ inflationary potential (A P S): $\surd$

- More general singularities: At finite proper time, scale factor may blow up, along with similar behavior of density or pressure (Big rip) or curvature or their derivatives diverge at finite values of scale factor (sudden death). Quantum geometry resolves all strong singularities in homogeneous isotropic models with $p = p(\rho)$ matter (Singh). $\surd$

- Beyond Isotropy and Homogeneity:
  Bianchi Models (A W-E): $\surd$ (Anisotropies & Grav Waves)
  The Gowdy model (G M-B M W-E): $\surd$ (Inhom and Grav Waves.)

These results by AA, Bentevigna, Garay, Martin-Benito, Mena, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ... show that the singularity resolution is quite robust. Anytime a physical observable reaches the Planck regime, the repulsive effect from quantum geometry effect becomes dominant and dilutes it.

Inflation



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

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3. Mathematical Foundations

- Basic idea of LQC (Misner, DeWitt, Wheeler; already in the 1970s!)

Since only finite number of DOF $a(t), \phi(t)$, field theoretical difficulties bypassed; analysis reduced to standard quantum mechanics.

- Quantum States: $\Psi(a,\phi)$; $\hat{a}\Psi(a,\phi) = a\Psi(a,\phi)$ etc.

Quantum evolution governed by the Wheeler-DeWitt differential equation

$$\ell_{\rm Pl}^4 \, \frac{\partial^2}{\partial a^2}(f(a)\Psi(a,\phi)) = {\rm const}\, G\, \hat{H}_\phi \, \Psi(a,\phi)$$

Hope: As the in H-atom, quantum effects would tame classical singularities. Unfortunately, without additional assumptions, e.g. matter violating energy conditions, hope not realized! Precise Statement provided by the consistent histories approach (Craig & Singh). However, there is also considerable work using Bohm's interpretation of quantum mechanics, not covered in these lectures.

General belief since the seventies: This is a real impasse because of the von-Neumman's uniqueness theorem.

von-Neumann Theorem & the WDW Theory

- von Neumann's uniqueness theorem:

There is a unique IRR of the Weyl operators $\hat{U}(\mu), \hat{V}(\alpha)$ by 1-parameter unitary groups on a Hilbert space satisfying:

i) $\hat{U}(\mu)\,\hat{V}(\alpha) = e^{i\alpha\mu}\,\hat{V}(\alpha)\,\hat{U}(\mu)$; and ii) Weak continuity in μ, α.

This is the standard Schrödinger representation: $\mathcal{H} = L^2(\mathbb{R}, \mathrm{d}x)$;

$\hat{x}\Psi(x) = x\Psi(x)$; $\hat{p}\Psi(x) = -i\hbar \mathrm{d}\Psi(x)/\mathrm{d}x$, and $U(\mu) = e^{i\mu\hat{x}}$, $V(\alpha) = e^{i\alpha\hat{p}}$

- In the WDW quantum cosmology, one did not have guidance from a full quantum gravity theory. Therefore, in quantum cosmology, WDW followed standard QM and constructed the Schrödinger representation of the fundamental Weyl algebra.

- Recall from Perez's lecture: In LQG, the conf variable is A_a^i $(\sim x)$ and the momentum variable is E_i^a $(\sim p)$. In LQG, the holonomies $h_e = \mathcal{P}\,\exp \int_e A.dl$ along edges e (analogs of $U(\mu) = e^{i\mu x}$) lead to well-defined operators. But there is no operator corresponding to $A_a^i \sim x$ itself. $\Rightarrow$ von Neumann theorem is by-passed. New reps of the Weyl algebra become available.

Novel features at the foundation of LQC

• Why was LQC able to resolve the Big Bang singularity when the WDW theory had failed in these models?

• In the WDW quantum cosmology, one did not have guidance from a full quantum gravity theory. Therefore, in quantum cosmology, one just followed standard QM and constructed the Schrödinger representation of the fundamental Weyl algebra.

• By contrast, quantum kinematics of LQG has been rigorously developed. Background independence $\Rightarrow$ unique representation of the kinematic algebra (Lewandowski, Okolow, Sahlmann, Thiemann; Fleishhack) Provides the arena to formulate quantum Einstein equations.

• In LQC we could mimic this framework step by step. Again (the remaining) diffeomorphism invariance leads to a unique representation of the quantum algebra constructed from LQC kinematics (AA, Campiglia). One of the assumptions of the von Neumann uniqueness theorem for quantum mechanics is bypassed. In LQC we are led to an inequivalent representation of the Weyl algebra; i.e., new quantum mechanics! WDW theory and LQC are distinct already kinematically.

LQC Kinematics from the uniqueness theorems

- The canonically conjugate variables of LQG:

A_a^i, SU(2) gravitational connections and, E_i^a, orthonormal triads.
Spatial homogeneity and isotropy implies

$\star \qquad A_a = c\,\underbrace{\mathring{\omega}_a^i \tau_i}_{\text{fixed}}, \quad E^a = p\,\underbrace{\mathring{e}_i^a \tau^i}_{\text{fixed}} \quad c \sim \dot{a}; \quad |p| = a^2$

$\star$ holonomy: $h_e(c) = \cos\mu c\, \mathbf{1} + \sin\mu c\; \mathring{e}^a \mathring{\omega}_a^i \tau_i$
(edge e of length μ in the ith direction. $h_e(A)$: Almost periodic in c)

$\star$ Canonically conjugate pairs:
c, p for gravity $\qquad\qquad \phi, p_\phi$ for matter

- The Hilbert space $\mathcal{H}$ is spanned by countable linear combinations of holonomies. $\Psi(c) = \sum_n c_n e^{i\mu_n c}$ with

$$||\Psi||^2 = \lim_{k\to\infty} \int_{-k}^{k} dc\, ||\Psi||^2$$

Now, the plane waves $e^{i\mu c}$ are normalizable!! with:

$$\langle e^{i\mu_1 c},\ e^{i\mu_2 c}\rangle = \delta_{\mu_1,\mu_2}.$$

On $\mathcal{H}$, holonomy operators $\hat{h}_e$ act by multiplication and $\hat{p}$ acts by $-i\hbar d/dc$.

LQC Dynamics

• The LQC kinematics cannot support the WDW dynamics. The Hamiltonian constraint involves the field strength F_{ab} of the gravitational connection $A_a = c\, \mathring{\omega}_a^i \tau_i$. In LQC, the corresponding operator $\hat{F}_{ab}$ is constructed from holonomies around closed loops (that enclose minimum non-zero area). Classical, local F_{ab} recovered only if we coarse grain to ignore the area gap.

• As a result, the dynamical WDW differential equation is replaced by a difference equation.

$$\partial_\phi^2 \Psi(v, \phi) \, = \, C^+(v)\, \Psi(v+4,\, \phi) + C^o(v)\, \Psi(v, \phi) + C^-(v) \Psi(v-4,\, \phi)$$

where the step size is governed by the 'area gap' of quantum geometry.

• Good agreement with the WDW equation at low curvatures **but drastic departures in the Planck regime** precisely because the WDW theory ignores quantum geometry. Non-triviality: LQC, based on the new kinematic arena and quantum geometry of LQG has good UV **as well as** good IR properties.

5. Summary

- Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive force rises and dies *very* quickly but makes dramatic changes to classical dynamics. (Origin: Planck scale non-locality of quantum Einstein's equations.)

New paradigm: Physics does not end at singularities.

Quantum space-times may be vastly larger than Einstein's.

- Long standing questions I began with have been answered. Challenge to background independent theories: Detailed recovery of classical GR at low curvatures/densities (Green and Unruh). Met in cosmological models. Singularities analyzed are of direct cosmological interest.

- Detailed analysis in specific models but taken together with the BKL conjecture on the nature of space-like strong curvature singularities in general relativity, the LQC results suggest that all these singularities may be resolved by the quantum geometry effects of LQG.

(Recall the history in classical GR).

Merits and Limitations of QC

One's first reaction to Quantum Cosmology is often: Symmetry reduction gives only toy models! Full theory much richer and much more complicated.

But examples can be powerful.

- Full QED versus Dirac's hydrogen atom.
- Singularity Theorems versus first discoveries in simple models.
- BKL behavior: homogeneous Bianchi models.

Do *not* imply that behavior found in examples is necessarily generic. Rather, they can reveal important aspects of the full theory and should not be dismissed a priori.

One can work one's way up by considering more and more complicated cases. (e.g. recent work of the Madrid group on Gowdy models which have infinite degrees of freedom). At each step, models provide important physical checks well beyond formal mathematics. Can have strong lessons for the full theory.