Lecture # 3.

I. Slow roll inflation with quadratic potential \( V(\phi) = \frac{1}{2} \kappa^2 \phi^2 \).

Observations provide the power spectrum \( P(k) = \frac{\kappa^2}{2 \pi^2} \frac{1}{H(\tau_0)} \frac{E(\tau_0)}{m_p^2} \) and spectral index \( n_s(\tau_0) = 0.968 \) at the time \( \tau_0 \) when the reference mode \( k^* \) exits the Hubble horizon. (All quantities in Planck units). For the quadratic potential, \( 4C = (1-n_s) \).

1. Show that (the Hubble parameter) \( H(\tau_0) = 7.83 \times 10^6 \, m_p \) and (the Hubble radius) \( R_0(\tau_0) = 1.28 \times 10^5 \, m_p \). Using the Friedmann equation \( H^2 = \frac{8 \pi G}{3} \rho \), find \( \rho(\tau_0) \). How likely is it that quantum gravity effects would be manifest during inflation?

2. Show that the Friedmann equation and the equation \( \ddot{\phi} + 3H\dot{\phi} + \nu \dot{\phi} = 0 \) imply the Raychaudhuri equation \( (3\dot{\phi}/\phi) + 4\pi G \rho m^2 = 0 \) (where \( \rho = \frac{1}{2} \dot{\phi}^2 + V \) and \( \rho = \frac{1}{2} \dot{\phi}^2 - V \)).

3. Using these equations and observational data, show that \( \phi(\tau_0) = \pm 3.15 \, m_p \), \( \dot{\phi}(\tau_0) = \mp 1.08 \times 10^7 \, m_p^2 \), \( m = 1.21 \times 10^5 \, m_p \).

Thus, initial conditions at \( \tau_0 \) and the inflaton mass are determined by observation.

II. In LQG, the scalar curvature at the bounce is universal, \( R_B = 62 \). Using problem 1 in HW# 2, show that this gives a new scale for propagation of tachyons modes: \( k_{loc} = (62) \frac{1}{16} \approx 3.81 \), or \( \lambda_{loc} = 0.997 \).

Argue that modes with \( \lambda > \lambda_{loc} \) at the bounce will be excited, during the pre-inflationary dynamics in LQG and would therefore not be in the BD vacuum at the onset of inflation.