PLASMA ASTROPHYSICS

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(references therein)

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Contents

- What is plasma?
- Why plasmas in astrophysics?
- Quasi neutrality
- Fluid approximation: MHD Eqs.
- Magnetic Force
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- Applications

What is PLASMA?

- Gas with sufficient number of **free charged particles** (positive + negative) → so that its behaviour dominated by **electromagnetic forces**.

- Even **low ionization** degree (~1%) :

  sufficient for gas to show electromagnetic properties (electrical conductivity ~ fully ionized gas).

Why Plasmas?

Plasma

- Most common (90%) state of matter in the universe.
- On earth exceptional, but obtained in laboratory thermonuclear fusion experiments at high temperatures ($T \sim 10^8\,\mathrm{K}$).
- Crude definition: *Plasma is a completely ionised gas, consisting of freely moving positively charged nuclei and negatively charged electrons.*

Applications

- Magnetic plasma confinement for (future) energy production by Controlled Thermonuclear Reactions.
- Dynamics of astrophysical plasmas (solar corona, planetary magnetospheres, pulsars, accretion disks, jets, etc.).

Why Plasmas in Astrophysics ?

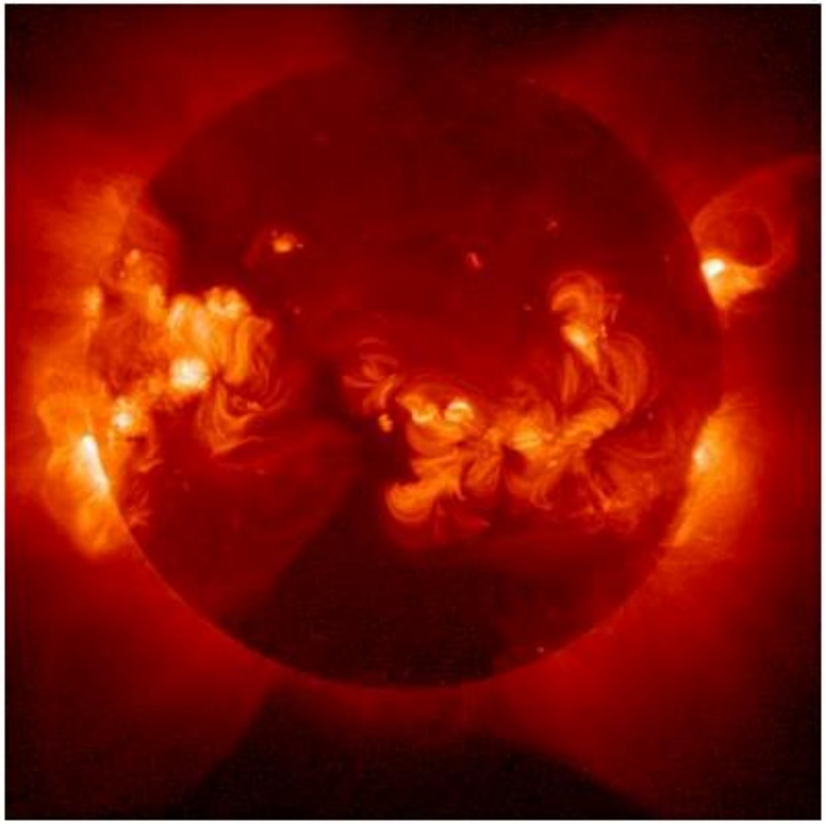
The universe does not consist of ordinary matter

- *> 90% is plasma:*

  electrically neutral, where the nuclei and electrons are not tied in atoms but *freely move as fluids.*

Exs. Astrophysical Plasmas

Example: The Sun

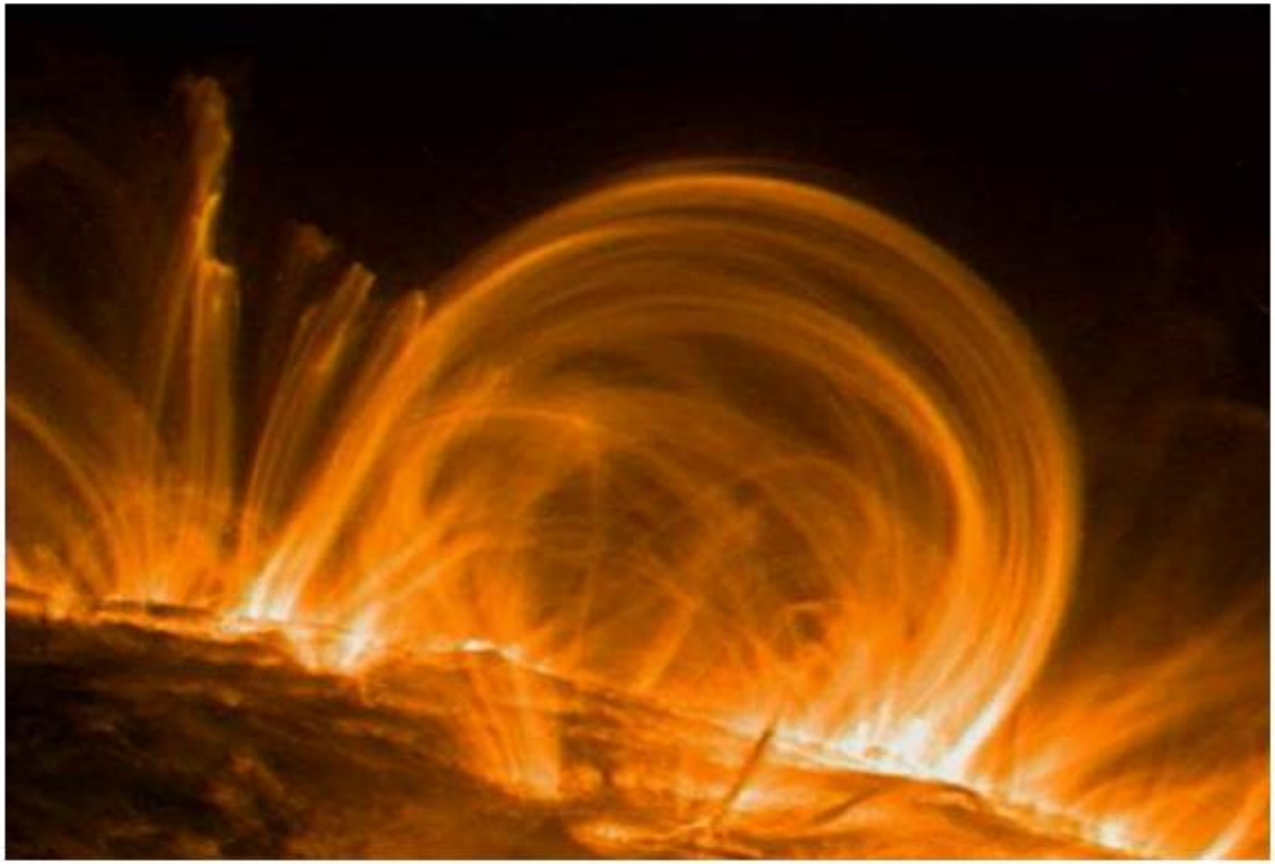


a magnetized plasma!

(sunatallwavelengths.mpeg)

Exs. Astrophysical Plasmas

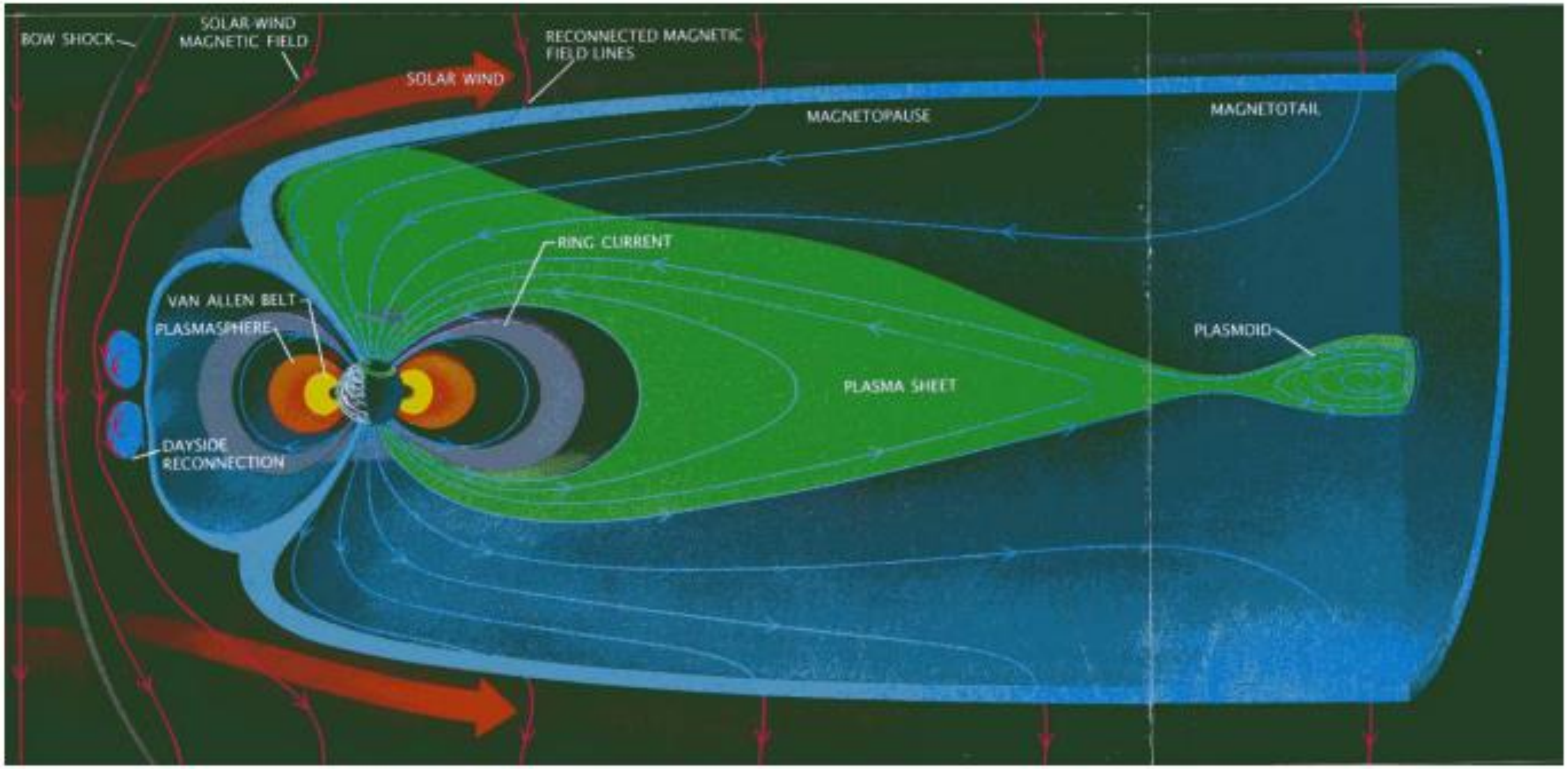
Example: Coronal loops (cont'd)



[from recent observations with TRACE spacecraft]

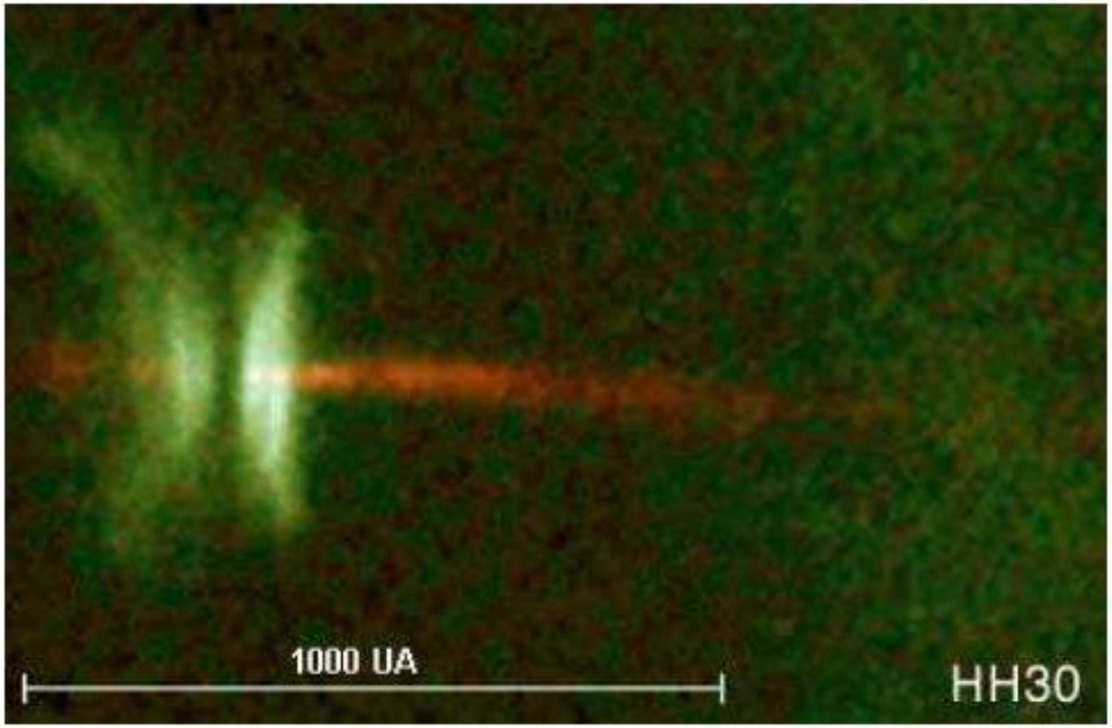
Exs. Astrophysical Plasmas

Example: Magnetosphere



Exs. Astrophysical Plasmas

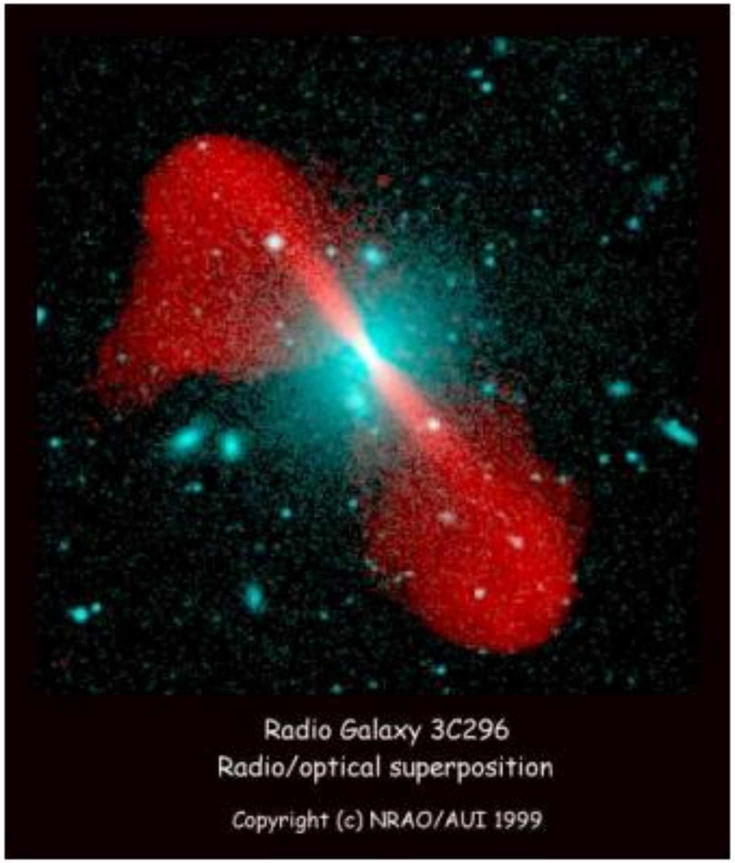
Example: Accretion disk and jets (YSO)



Young stellar object ($M_* \sim 1 M_\odot$): accretion disk 'seen' edge-on as dark strip, jets colored red.

Exs. Astrophysical Plasmas

Example: Accretion disk and jets (AGN)

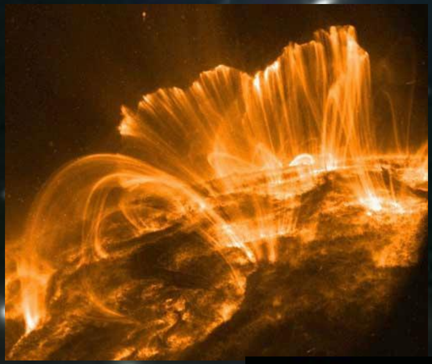


Active galactic nucleus ($M_* \sim 10^8 M_\odot$): optical emission (blue) centered on disk, radio emission (red) shows the jets.

Importance of Plasmas

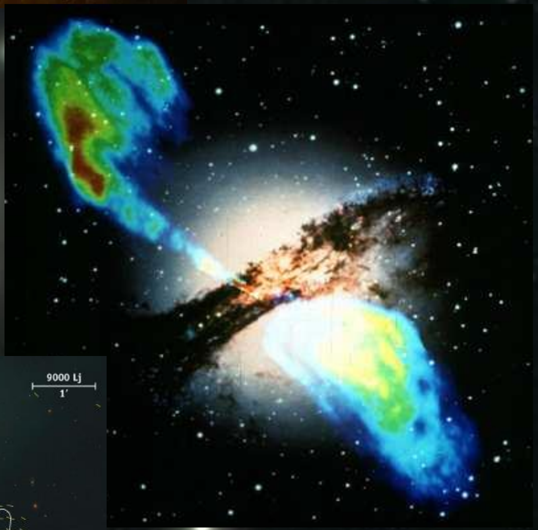
- **Magnetized plasmas are present in almost all astrophysical objects**

- They are ***crucial*** in:

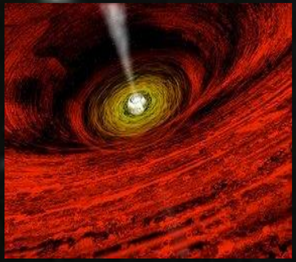


- star formation; late stages

- solar and stellar activity



- formation of jets and accretion disks



- formation and propagation of cosmic rays

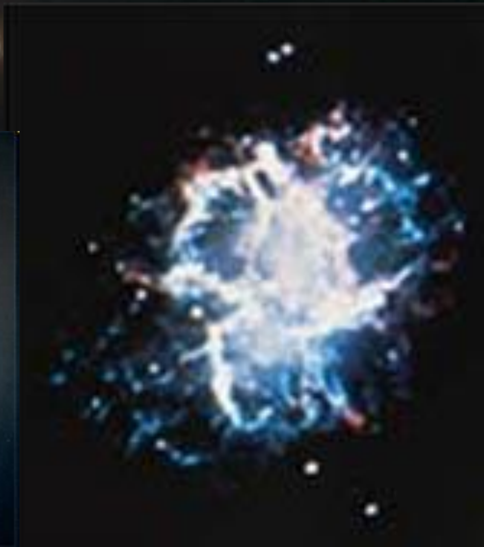
- galaxy structure

Importance of Plasmas

- They are also ***crucial*** in:

- ISM
- molecular clouds
- supernova remnants
- proto-planetary disks
- planetary nebulae

- GRBs



Importance of Plasmas

- **Their importance *not well understood* yet** in:

  - stellar evolution

  - galaxy evolution

  - structure formation in the early Universe

Exs. Plasmas in Earth

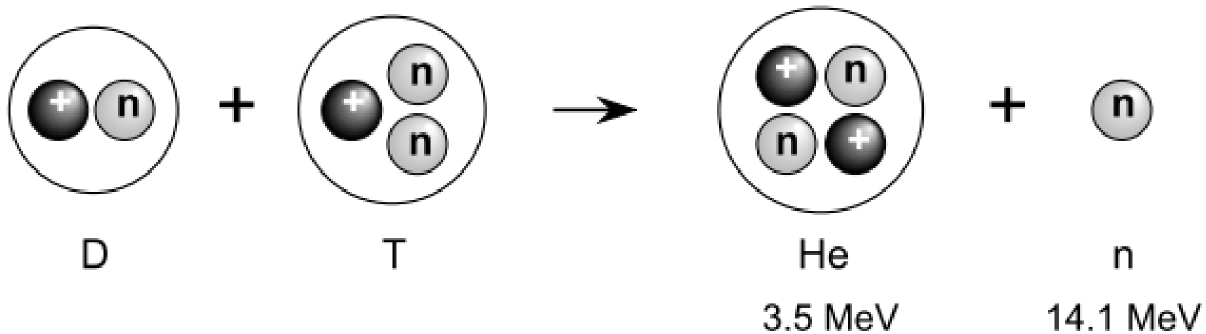
Example: Polar lights

Plasmas in Laboratory

Major goal:

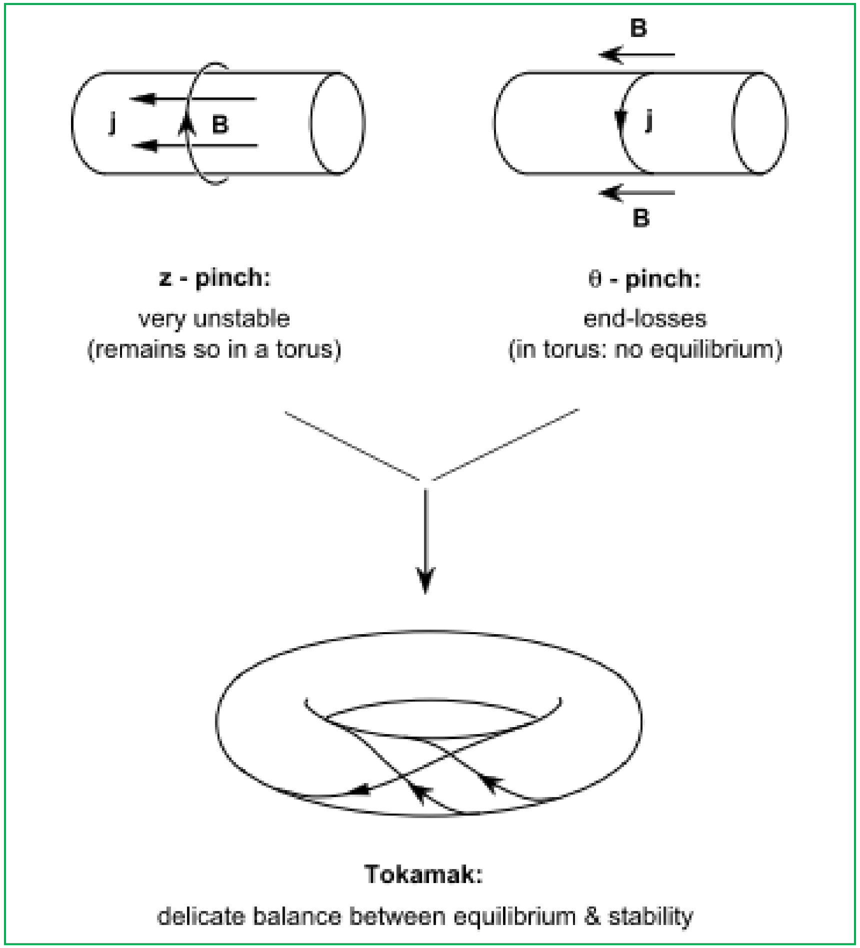
- Thermonuclear fusion (as in the interior of stars) $\rightarrow$ production of energy!

Reactions of hydrogen isotopes



$\Rightarrow$ Magnetic fields provide the only way to confine matter of high temperatures during long times.

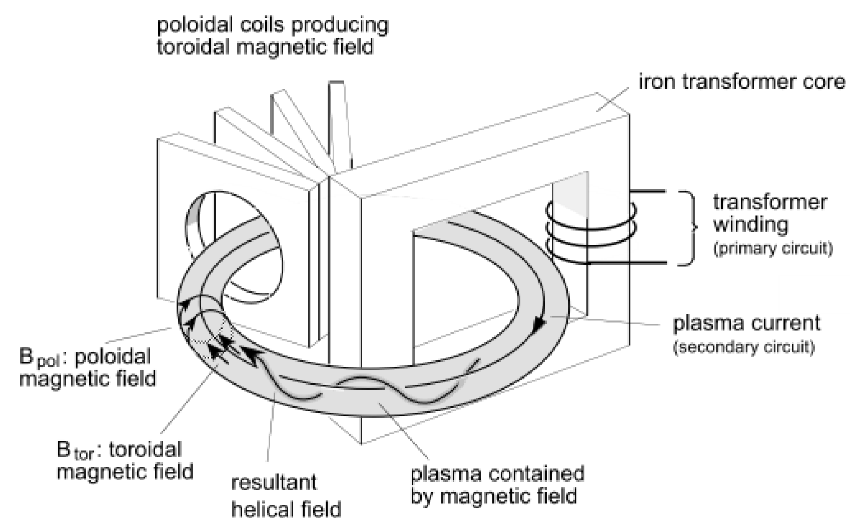
Plasmas in Laboratory: TOKAMAK



Controlled thermonuclear fusion experiments: still in progress…

Tokamak

- Magnetic confinement:



PLASMA: simple definition

Plasma is an ionized gas.

Rate of ionization: $$\frac{n_i}{n_n} = \left(\frac{2\pi m_e k}{h^2}\right)^{3/2} \frac{T^{3/2}}{n_i} e^{-U_i/kT}$$ (Saha equation)

– air: $T = 300\,\mathrm{K}\,,\, n_n = 3\times10^{25}\,\mathrm{m}^{-3}\,,\, U_i = 14.5\,\mathrm{eV} \quad \Rightarrow \quad n_i/n_n \approx 2\times10^{-122}$ (!)

– tokamak: $T = 10^{8}\,\mathrm{K}\,,\, n_i = 10^{20}\,\mathrm{m}^{-3}\,,\, U_i = 13.6\,\mathrm{eV} \quad \Rightarrow \quad n_i/n_n \approx 2.4\times10^{13}$

PLASMA: Microscopic Definition

Plasma is a quasi-neutral gas of charged and neutral particles which exhibits collective behaviour (Chen).

(a) Long-range collective interactions dominate over binary collisions with neutrals

(b) Length scales large enough that quasi-neutrality ($n_e \approx Zn_i$) holds

(c) Sufficiently many particles in a Debye sphere (statistics)

PLASMA: Quasi Neutrality

$$\boxed{Zen_i \approx en_e}$$

- Assume local charge concentration (δn e)

→ according to Coulomb law:

$$\vec{\nabla}.\vec{E} = \frac{\partial E_{//}}{\partial x} = 4\pi\delta\rho = -4\pi e\delta n_e$$

→ generates the electric field:

$$E_{//} = 4\pi e n_e \xi$$

→ $E_{//}$ provokes on the thermal random motion of electrons: flow with velocity $v_{//}$:

$$\frac{dv_{//}}{dt} = \frac{d^2\xi}{dt^2} = -\frac{e}{m_e}E_{//} = -\frac{4\pi n_e e^2}{m_e}\xi$$

- Solution: simple harmonic motion with plasma electron frequency that in the average neutralizes $E_{//}$:

$$\omega_{pe} = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \simeq 5.6 \times 10^4 n_e^{1/2} \quad s^{-1}$$

Quasi Neutrality - Plasma Frequency

- ω_{pe} **defines natural plasma frequency** (neutralizes E effect)
- ıons oscilate with much « frequency → (practically at rest in relation to els.)

$$\boxed{\omega_{pi} = \left(\frac{Zm_e}{m_i}\right)^{1/2} \omega_{pe}}$$

- Any external electric field E applied with freq. $\omega < \omega_{pe}$ is unable to penetrate the plasma
- **Debye length**: $\boxed{\lambda_D = v_{th}/\omega_{pe}}$

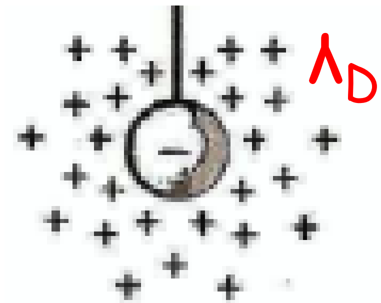
→ E does not penetrate plasma if:

$$\omega < \omega_{pe} \text{ ou } \lambda > \lambda_D$$

PLASMA: Debye length (λ_D)

- λ_D → scale within which separation between charges can be "felt":

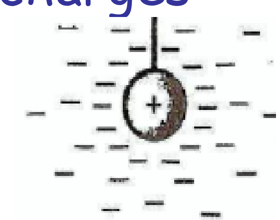
$$\boxed{\lambda_D \simeq \frac{v_{th}}{\omega_{pe}} \simeq \left(\frac{KT_e}{4\pi n_e e^2}\right)^{1/2} = 7\ cm\ (T_e/n_e)^{1/2}}$$



- Within sphere of radius (λ_D) → charge neutrality is not valid: electrostatic external oscillations with $\lambda \leq \lambda_D$ penetrate the sphere and feel the collective effects of the charges → strongly damped (Landau damping)

- Electric Potential Field of a charge within plasma: has its action screened (or partially blocked) by cloud of charges

$$\boxed{\phi(r) = \frac{-e n_1}{r} e^{-r/\lambda_D}}$$



PLASMA: collective behavior

Conditions:

(a) $\tau \ll \tau_n \equiv \dfrac{1}{n_n \sigma v_{\rm th}}$

tokamak: $\tau \ll 2.4 \times 10^6\,\mathrm{s}$

corona: $\tau \ll 2 \times 10^{20}\,\mathrm{s}\,;$

(b) $\lambda \gg \lambda_D \equiv \sqrt{\dfrac{\epsilon_0 kT}{e^2 n}}$

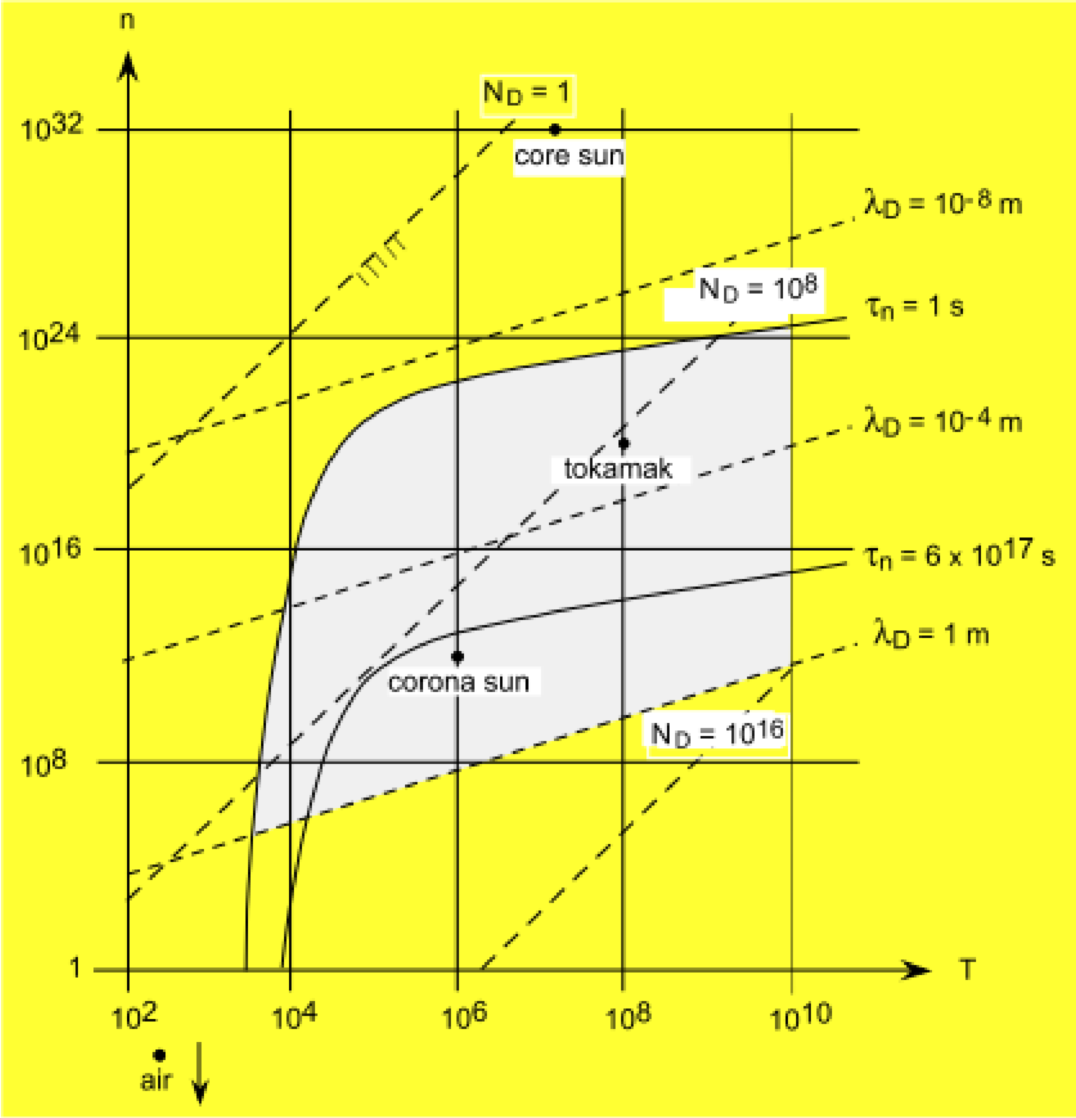
tokamak: $\lambda_D = 7 \times 10^{-5}\,\mathrm{m}$

corona: $\lambda_D = 0.07\,\mathrm{m}\,;$

(c) $N_D \equiv \frac{4}{3}\pi\lambda_D^3\, n \gg 1$

tokamak: $N_D = 1.4 \times 10^8$

corona: $N_D = 1.4 \times 10^9\,.$



PLASMA: Quasi Neutrality

- Tpypical dimension of astrophysical plasmas: $L \gg \lambda_D$

  → **quase neutrality is valid**

  → Internal E fields: little important

  (neutralized by strong oscillations ω_{pe})

  → External E fields typically do not penetrate

  ($\omega \ll \omega_{pe}$, $\lambda > \lambda_D$)

→ Below: macroscopic description of a plasma as a fluid

Plasma Physics

Three theoretical models:

- **Theory of motion of single charged particles** in given magnetic and electric fields;

- **Kinetic theory of a collection of such particles,** describing plasmas *microscopically* by means of particle distribution functions $f_{e,i}(\mathbf{r}, \mathbf{v}, t)$;

- **Fluid theory (magnetohydrodynamics),** describing plasmas in terms of averaged *macroscopic* functions of $\mathbf{r}$ and t .

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Motion of a single charge

Equation of motion

of charged particle in given electric and magnetic field, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$:

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,.$$

- Apply to constant magnetic field $\mathbf{B} = B\mathbf{e}_z\,,\ \mathbf{E} = 0$:

- Systematic solution of Eq. (1) with $\mathbf{v} = d\mathbf{r}/dt = (\dot{x}, \dot{y}, \dot{z})$ gives two coupled differential equations for motion in the perpendicular plane:

$$\begin{aligned} \ddot{x} - (qB/m)\,\dot{y} &= 0\,, \\ \ddot{y} + (qB/m)\,\dot{x} &= 0\,. \end{aligned} \tag{2}$$

$\Rightarrow$ periodic motion about a fixed point $x = x_c$, $y = y_c$ *(the guiding centre)*.

Motion of a single charge

Cyclotron motion

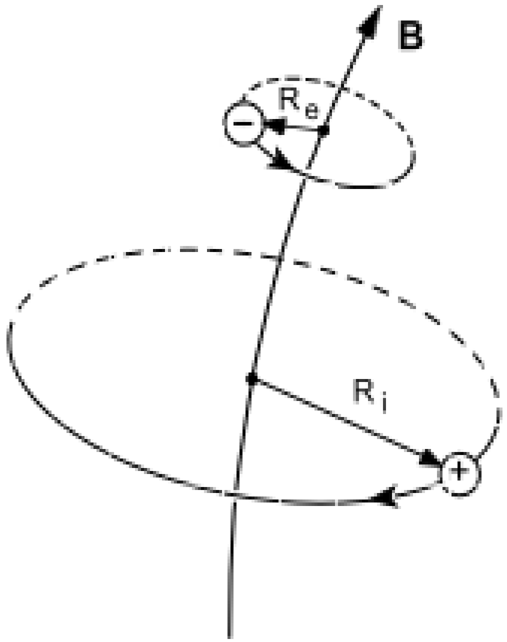
This yields periodic motion in a magnetic field, with *gyro- (cyclotron) frequency*

$$\Omega \equiv \frac{|q|B}{m} \qquad (3)$$

and *cyclotron (gyro-)radius*

$$R \equiv \frac{v_\perp}{\Omega} \approx \frac{\sqrt{2mkT}}{|q|B}\,. \qquad (4)$$

$\Rightarrow$ Effectively, charged particles stick to the field lines.



Opposite motion of electrons and ions about guiding centres with quite different gyro-frequencies and radii, since $m_e \ll m_i$:

$$\Omega_e \equiv \frac{eB}{m_e} \gg \Omega_i \equiv \frac{ZeB}{m_i}\,, \qquad R_e \approx \frac{\sqrt{2m_e kT}}{eB} \ll R_i \approx \frac{\sqrt{2m_i kT}}{ZeB}\,. \qquad (5)$$

Motion of a single charge

Cyclotron motion (cont'd)

Orders of magnitude

- Typical gyro-frequencies, e.g. for tokamak plasma ($B = 3\,\mathrm{T}$):

$$\Omega_e = 5.3 \times 10^{11}\,\mathrm{rad\,s^{-1}} \quad \text{(frequency of } 84\,\mathrm{GHz}\text{)}\,,$$

$$\Omega_i = 2.9 \times 10^{8}\,\mathrm{rad\,s^{-1}} \quad \text{(frequency of } 46\,\mathrm{MHz}\text{)}\,.$$

- Gyro-radii, with $v_\perp = v_{\mathrm{th}} \equiv \sqrt{2kT/m}$ for $T_e = T_i = 1.16 \times 10^8\,\mathrm{K}$:

$$v_{\mathrm{th},e} = 5.9 \times 10^{7}\,\mathrm{m\,s^{-1}} \quad \Rightarrow \quad R_e = 1.1 \times 10^{-4}\,\mathrm{m} \approx 0.1\,\mathrm{mm}\,,$$

$$v_{\mathrm{th},i} = 1.4 \times 10^{6}\,\mathrm{m\,s^{-1}} \quad \Rightarrow \quad R_i = 4.9 \times 10^{-3}\,\mathrm{m} \approx 5\,\mathrm{mm}\,.$$

$\Rightarrow$ Tokamak time scales $(\sim 1\,\mathrm{s})$ and dimensions $(\sim 1\,\mathrm{m})$ *justify averaging.*

Motion of a single charge

- Magnetic fields:
  1. charged particles gyrate around field lines;
  2. fluid and magnetic field move together ("$\mathbf{B}$ frozen into the plasma");

Plasma Physics

Three theoretical models:

- **Theory of motion of single charged particles** in given magnetic and electric fields;
- **Kinetic theory of a collection of such particles,** describing plasmas *microscopically* by means of particle distribution functions $f_{e,i}(\mathbf{r}, \mathbf{v}, t)$;
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Plasma Physics

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- **Kinetic theory of a collection of such particles,** describing plasmas *microscopically* by means of particle distribution functions $f_{e,i}(\mathbf{r},\mathbf{v},t)$;

A plasma consists of a very large number of interacting charged particles $\Rightarrow$ *kinetic plasma theory* derives the equations describing the *collective behavior* of the many charged particles by applying the methods of statistical mechanics. $(\omega \geq \omega_{pe})$

Plasma Physics

Three theoretical models:

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Fluid description: MHD

Macroscopic definition

Macroscopic model: size and time scales are large enough → possible to apply AVERAGES over microscopic quantities: colective plasma oscillations and collective cyclotron motions of ions and electrons

(a) $\tau \gg \Omega_i^{-1} \sim B^{-1}$ (time scale longer than inverse cyclotron frequency);

(b) $\lambda \gg R_i \sim B^{-1}$ (length scale larger than cyclotron radius).

$\Rightarrow$ MHD $\equiv$ magnetohydrodynamics

Fluid description: MHD

Maxwell eqs. + hidrodynamics eqs. = Eqs. MHD

- **Maxwell's equations** describe evolution of electric field $\mathbf{E}(\mathbf{r},t)$ and magnetic field $\mathbf{B}(\mathbf{r},t)$ in response to current density $\mathbf{j}(\mathbf{r},t)$ and space charge $\tau(\mathbf{r},t)$:

MKS

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\,, \qquad \textit{(Faraday)} \qquad (1)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}\,, \qquad c \equiv (\epsilon_0\mu_0)^{-1/2}\,, \qquad \textit{('Ampère')} \qquad (2)$$

$$\nabla \cdot \mathbf{E} = \frac{\tau}{\epsilon_0}\,, \qquad \textit{(Poisson)} \qquad (3)$$

$$\nabla \cdot \mathbf{B} = 0\,. \qquad \textit{(no monopoles)} \qquad (4)$$

- **Gas dynamics equations** describe evolution of density $\rho(\mathbf{r},t)$ and pressure $p(\mathbf{r},t)$:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla\cdot\mathbf{v} \equiv \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0\,, \qquad \textit{(mass conservation)} \qquad (5)$$

$$\frac{\mathrm{D}p}{\mathrm{D}t} + \gamma p\nabla\cdot\mathbf{v} \equiv \frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma p\nabla\cdot\mathbf{v} = 0\,, \qquad \textit{(entropy conservation)} \qquad (6)$$

where

$$\boxed{\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla}$$

is the *Lagrangian time-derivative* (moving with the fluid).

+

Fluid description: MHD

One-fluid approximation:

(combining eqs. of motion of els. and ions)

- Define one-fluid variables that are linear combinations of the two-fluid variables:

$\rho \equiv n_e m_e + n_i m_i \,,$	*(total mass density)*
$\tau \equiv -e\left(n_e - Z n_i\right) ,$	*(charge density)*
$\mathbf{v} \equiv \left(n_e m_e \mathbf{u}_e + n_i m_i \mathbf{u}_i\right)/\rho \,,$	*(center of mass velocity)*
$\mathbf{j} \equiv -e\left(n_e \mathbf{u}_e - Z n_i \mathbf{u}_i\right) ,$	*(current density)*
$p \equiv p_e + p_i \,.$	*(pressure)*

MHD Equations

CGS

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$ *(continuity)*

$$\rho\,(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}) + \nabla p - \boxed{\frac{1}{c}\vec{J}\times\vec{B}} = 0\,,$$ *(momentum)*

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma p \nabla\cdot\mathbf{v} = (\gamma - 1)\eta|\mathbf{j}|^2\,,$$ *(internal energy)*

$$\frac{\partial \mathbf{B}}{\partial t} + c\vec{\nabla}\times\vec{E} = 0\,,$$ *(Faraday)*

where

$$\vec{J} = \frac{c}{4\pi}\vec{\nabla}\times\vec{B}$$ *(Ampère)*

$$\vec{E} = -\frac{\vec{v}_e}{c}\times\vec{B} + \frac{m_e}{e}\vec{g} - \frac{1}{n_e e}\vec{\nabla}p_e + \boxed{\eta\vec{J}}$$ *(Ohm)*

and

$$\nabla\cdot\mathbf{B} = 0$$ *(no magnetic monopoles)*

is initial condition on Faraday's law.

Electric Resistivity: $$\eta = \frac{m_e v_{th}\sigma_{ei}}{Ze^2}$$

Magnetic Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = -c\vec{\nabla} \times \vec{E}$$

$$\vec{E} = -\frac{\vec{v}_e}{c} \times \vec{B} + \frac{m_e}{e}\vec{g} - \frac{1}{n_e e}\vec{\nabla} p_e + \eta \vec{J}$$

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \frac{\eta c^2}{4\pi}\vec{\nabla}^2 \vec{B} - \frac{c}{n_e^2 e}\vec{\nabla} n_e \times \vec{\nabla} p_e}$$

Advection: gas and magnetic field coupling (freezing)

Viscosity: dissipation of the magnetic field

Biermann Battery: only important for generation of B (dynamos) -> non null differential rotation (in general neglected)

MHD Equations: ± usual

CGS

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$ *(continuity)*

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) + \nabla p - \boxed{\frac{1}{c}\vec{J}\times\vec{B}} = 0\,,$$ *(momentum)*

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p + \gamma p \nabla\cdot\mathbf{v} = (\gamma-1)\eta|\mathbf{j}|^2\,,$$ *(internal energy)*

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}_e\times\vec{B}) + \boxed{\nu_M}\vec{\nabla}^2\vec{B}$$ ***(magnetic induction)***

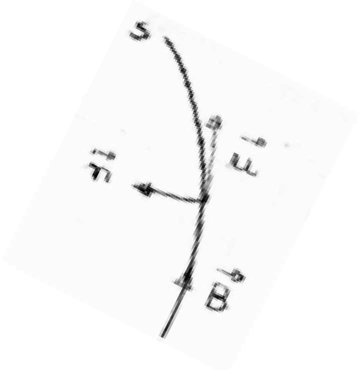
$$\vec{J} = \frac{c}{4\pi}\vec{\nabla}\times\vec{B} \qquad \nabla\cdot\mathbf{B} = 0\,.$$ *(Ampere, divergencia)*

Where magnetic resistivity: $\boxed{\nu_M = \frac{\eta c^2}{4\pi}}$ **(cm^2/s)**

Eq. of state to close the system: $p = (\gamma - 1)\rho e$

Magnetic Force

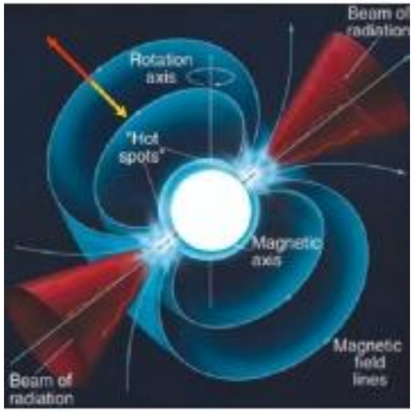
$$\frac{1}{c}\vec{J}\times\vec{B} = \frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B} = \frac{1}{4\pi}(\vec{B}.\vec{\nabla})\vec{B} - \vec{\nabla}\left(\frac{B^2}{8\pi}\right)$$



$$\boxed{\frac{1}{c}(\vec{J}\times\vec{B}) = \frac{B^2}{4\pi}\frac{\vec{n}}{\lambda} - \vec{\nabla}_\perp\left(\frac{B^2}{8\pi}\right)}$$

Tension Force

Magnetic Pressure Force



Dipole magnetic field of a star magnetosphere (ex. pulsar):

tension = pressure magnetic

$$\frac{1}{c}(\vec{J}\times\vec{B}) = 0$$

IDEAL MHD

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \nu_M \vec{\nabla}^2 \vec{B}}$$

Ratio between these two terms:

$$\boxed{\frac{|\vec{\nabla} \times (\vec{v}_e \times \vec{B})|}{\nu_M |\vec{\nabla}^2 \vec{B}|} \sim \frac{L v_e}{\nu_M} = R_{eM}}$$

→ **Magnetic Reynolds number**

In **astrophysical plasmas** in general: $R_{eM} >> 1$ → **ideal MHD**:

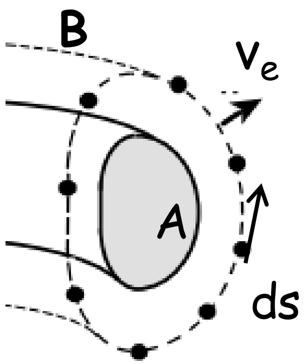
→ $$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \cancel{\nu_M \vec{\nabla}^2} \vec{B}}$$

Exceptions: $R_{eM} \approx 1$: Ex. Magnetic Reconnection → resistive MHD

Ideal MHD → B Flux Freezing

With η = 0: $\nu_M = \frac{\eta c^2}{4\pi}$ =0 *and* $\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B})}$

Integrating over an open surface A surrounded by a closed contour ∂S and using Stokes' theorem:



$$\frac{\partial}{\partial t} \int_A \vec{B}.d\vec{A} + \oint_{\partial S} (\vec{v}_e \times d\vec{s}).\vec{B} = 0$$

→ The magnetic flux through A with closed contour that moves with the electron gas is CONSTANT (if perfectly conductive fluid)

→ Concept of flux freezing

$$\boxed{\frac{d}{dt}\phi = 0}$$

$$\boxed{\phi = \int \vec{B}.d\vec{A}}$$

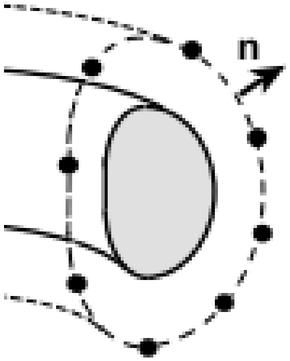
Where d/dt is comoving derivative: $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_e.\vec{\nabla}$

B Flux Freezing: ideal MHD

$$\boxed{\frac{d}{dt}\phi = 0}$$

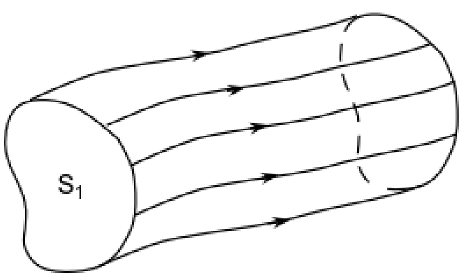
→ **Magnetic flux freezing:**

- It means we can see the lines of force of B as "frozen" in the electron gas and moving along with the gas



- Any motion transverse to the lines of force of the magnetic field, carries them along with the fluid

- A fluid element that moves along a flux tube remains moving with it.



Is B flux freezing always valid?

→ In astrophysical plasmas: **flux freezing valid in general because**

$L, v >> 1 \rightarrow \frac{L v_e}{\nu_M} = R_{eM} >> 1$

• **BUT there are exceptions:**

Ex. 1) magnetic reconnection sites

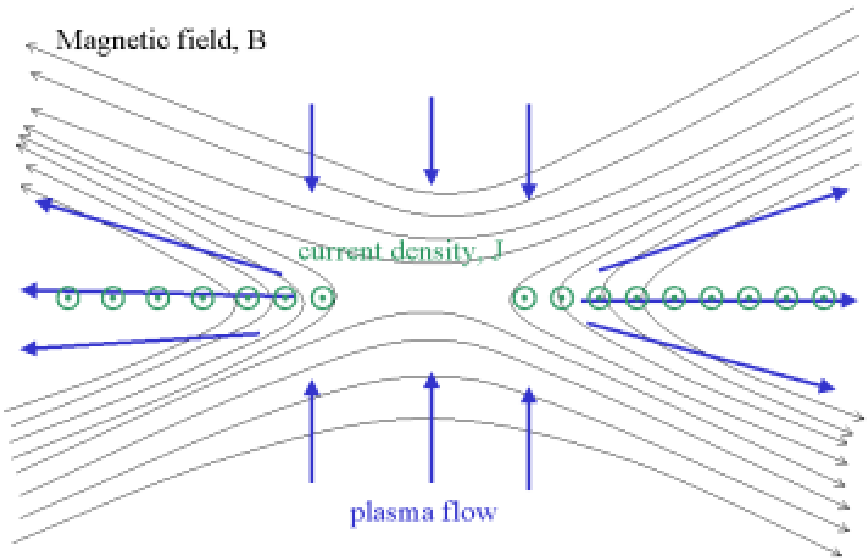
Ex. 2) star formation

Ex. 3) dynamos: magnetic field generation

Is B flux freezing always valid?

→ NO

Ex. 1) magnetic reconnection sites: B flux does not conserve because



Example: Coronal loops (cont'd)

[from recent observations with TRACE spacecraft]

$$\frac{Lv_e}{\nu_M} = R_{eM} \quad \sim 1$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \nu_M \vec{\nabla}^2 \vec{B}$$

Is B flux freezing always valid?

→ NO

Ex. 2) collapse of an interstellar cloud to form a star:

Cloud:
$\rho \sim 10^{-20}\,gcm^{-3}$
$B \sim 10^{-4}\,G$

Star:
$\rho_* \sim 1 gcm^{-3}$
$B_* = ??$

If we use ideal MHD → B flux conservation :
+ mass conservation eq., we obtain:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B})$$

$B_* \sim 10^9\,G$!

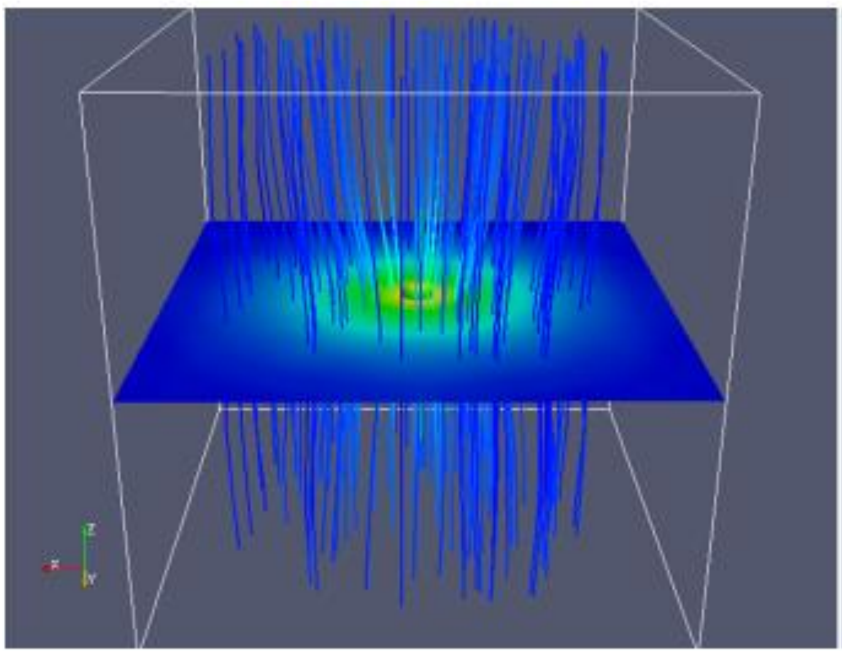
BUT, observations: $B_* \sim 10^3\,G$

Therefore: There was no flux conservation! There were flux removal. **What resistive process did that?**

Self-Gravitating collapsing clouds

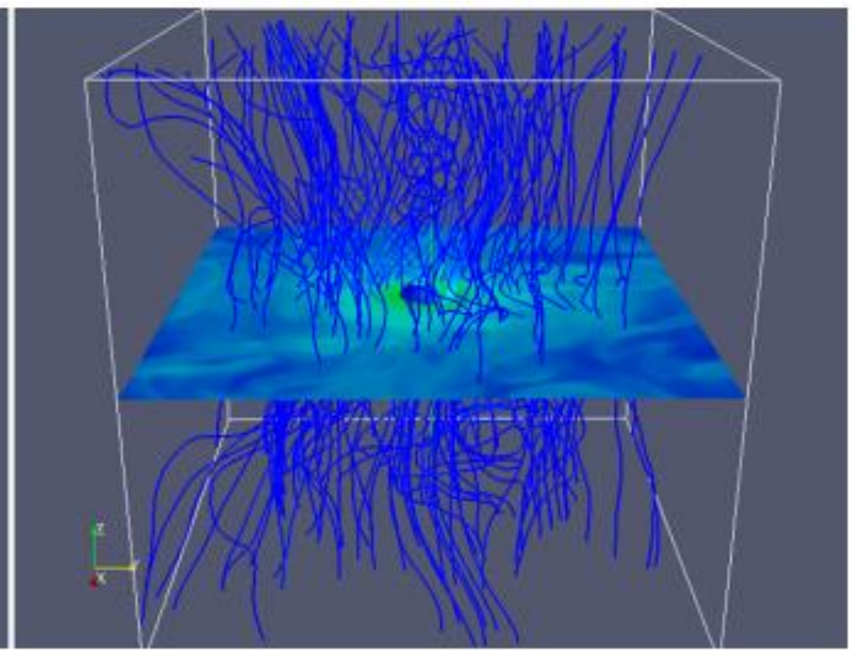
Self-gravitating gas + central spherical potential (~$1/r^2$)

Non-turbulent



Subcritical core

Turbulent

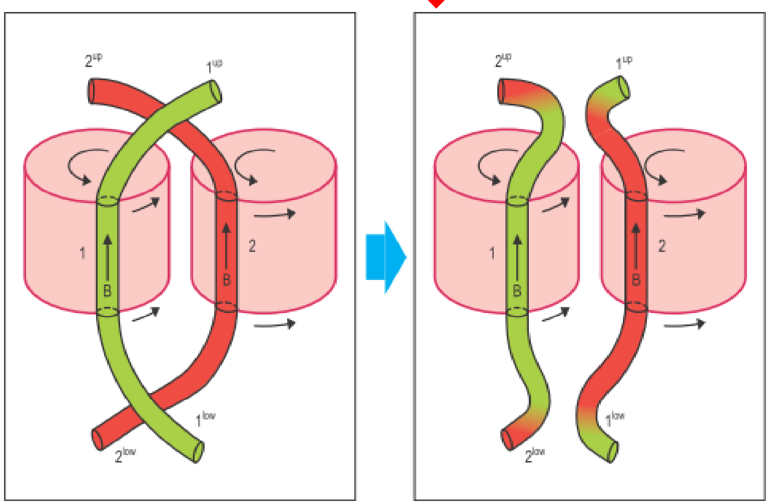


Supercritical core

β=3, n=100 cm^{-3} t~ 40Myr **Leão et al. 2013**

MHD turbulent diffusion: new scenario

In presence of turbulence: field lines reconnect fast (Lazarian & Vishniac 1999) and magnetic flux transport becomes efficient



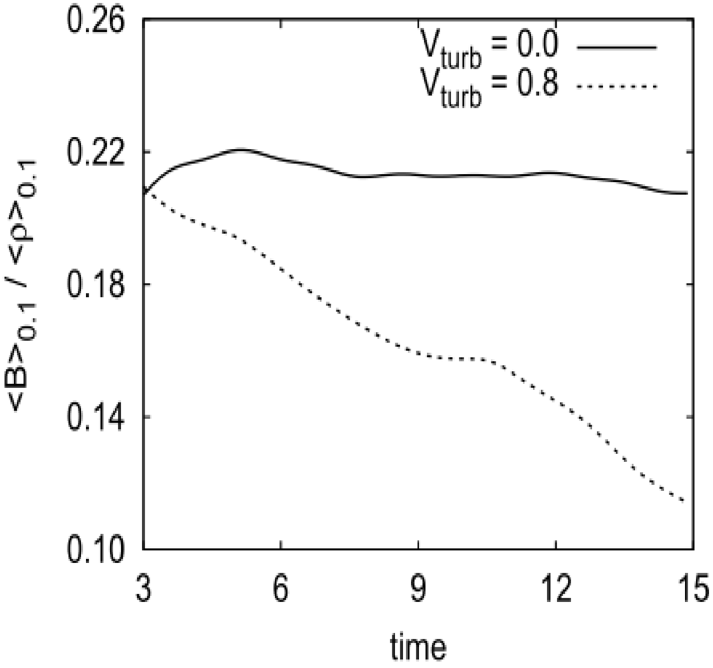
$$\eta_t \sim l_{\rm inj} v_{\rm turb} \qquad \text{if } v_{\rm turb} \geq v_A$$

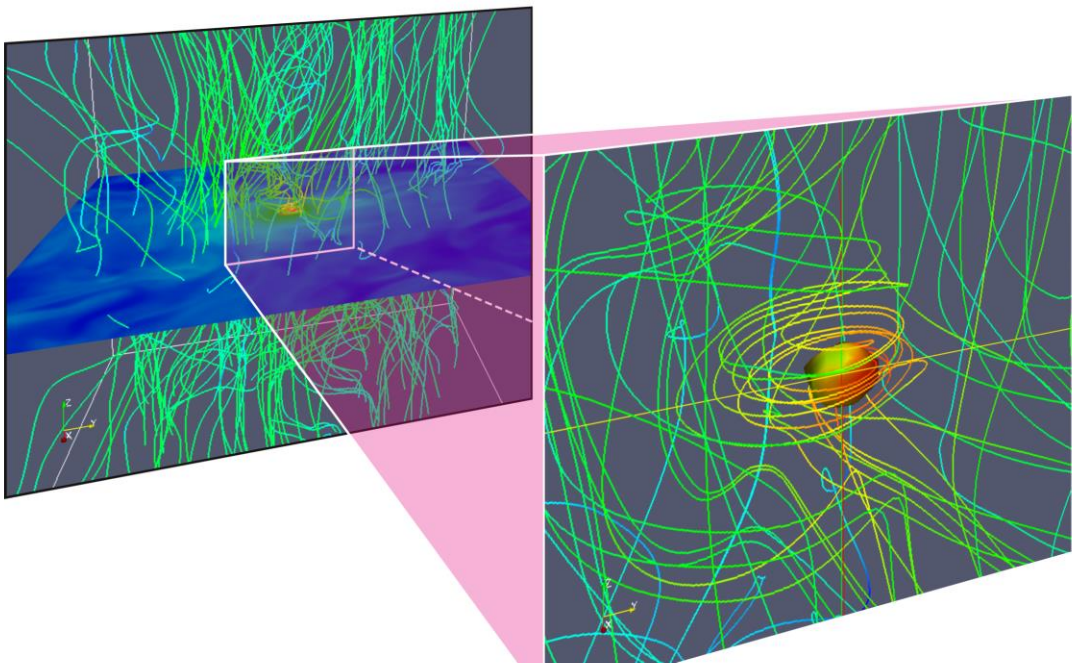
$$\eta_t \sim l_{\rm inj} v_{\rm turb} \left(\frac{v_{\rm turb}}{v_A} \right)^3 \qquad \text{if } v_{\rm turb} < v_A$$

Lazarian 2005, 2012
Santos-Lima et al. 2010, 2012, 2013
de Gouveia Dal Pino et al. 2012

Self-Gravitating collapsing clouds

Turbulent Reconnection Diffusion (Lazarian 06; Santos-Lima, de Gouveia Dal Pino, Lazarian 2010, 2012, 2013)





Leão, dGDP, Santos-Lima, Lazarian 2013

Is B flux freezing always valid?

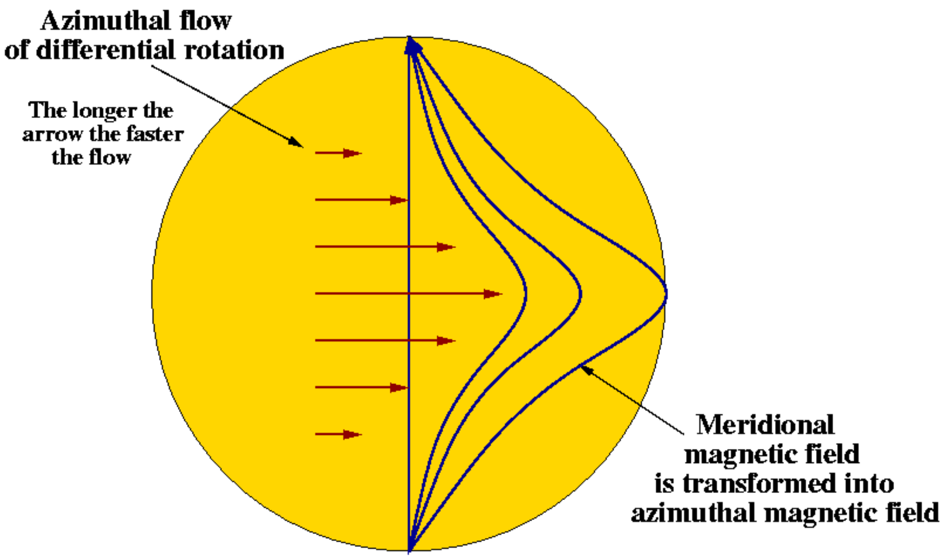
→ NO

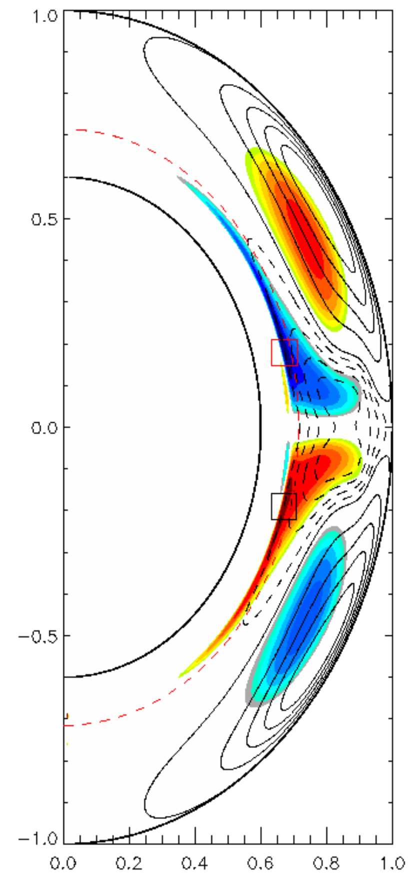
Ex. 3) **dynamo**: generates magnetic fields: obviously does not conserve magnetic flux → NON IDEAL MHD

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \frac{\eta c^2}{4\pi}\vec{\nabla}^2\vec{B} - \frac{c}{n_e^2 e}\vec{\nabla} n_e \times \vec{\nabla} p_e}$$

....+ new terms

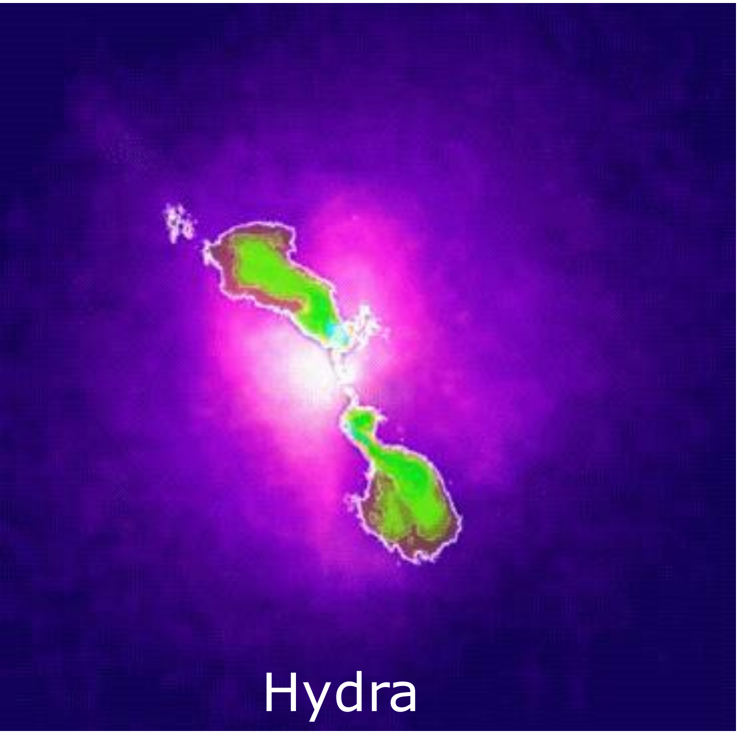
Solar Dynamo Example

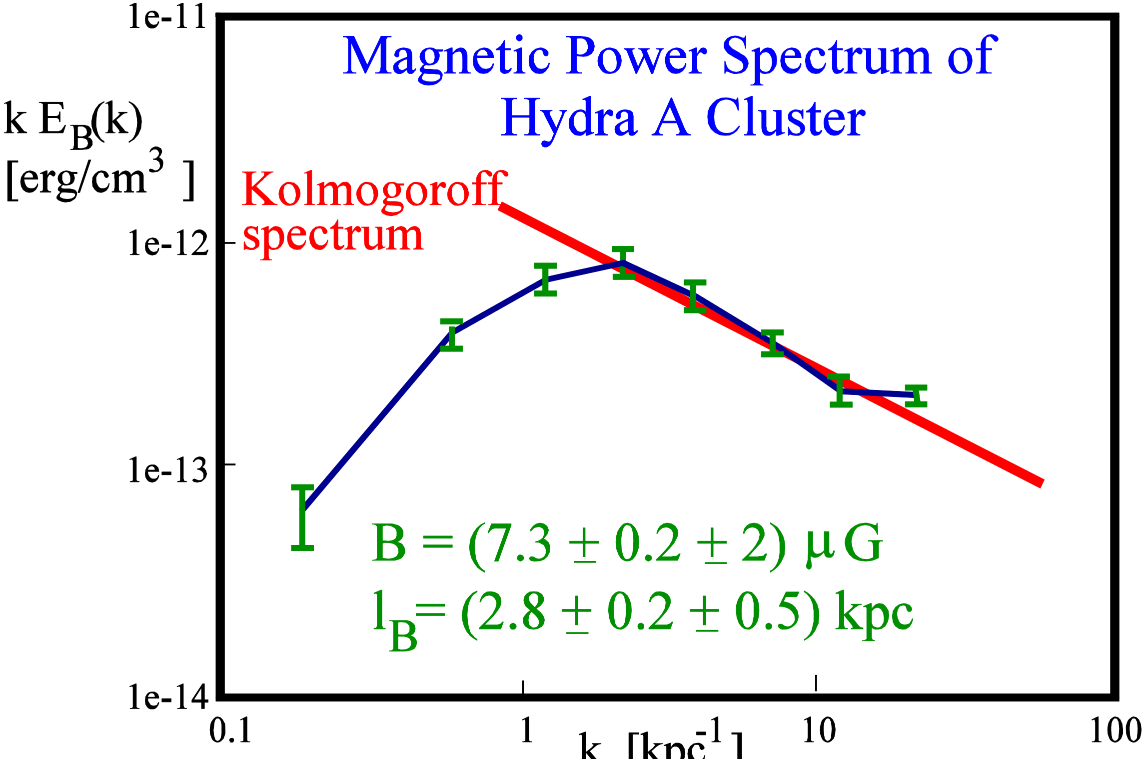




Guerrero & de Gouveia Dal Pino 2008 →

Turbulent Dynamo in ICM





- **Magnetic field is turbulent** (Ensslin & Vogt 2005)
- **Turbulent dynamo operates** (Brandenburg & Subramanian 05)
- **Dynamo amplifies seed fields (AGNs, galactic winds, mergers)**

ICM - COLLISIONLESS

Low density of IGM & ICM:

ion Larmor radius $\mathbf{R_i}$ << mean free path $\boldsymbol{\lambda_{mfp}}$ **~ L**

$$R_i \sim 10^5 \text{ km} \ll \lambda_{mfp} \sim 10^{15} \text{ km} \sim L$$

In absence of collisions: $p_{//} \neq p_{\perp}$

~~MHD~~: INADEQUATE

Kinetic-MHD **description**: $\boxed{P_{ij} = p_\perp \delta_{ij} + (p_\parallel - p_\perp) b_i b_j},$

Kinetic-MHD Turbulence in the ICM

Solve: MHD equations with p_{ij}

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{u}$$

$$\rho\frac{d\mathbf{u}}{dt} = -\nabla_{\mathbf{t}}p_{\perp} - \nabla_{\mathbf{b}}p_{\parallel} + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{p_{\perp}-p_{\parallel}}{B}\nabla_{\mathbf{b}}B$$

$$\frac{d\mathbf{B}}{dt} = -\mathbf{B}(\nabla\cdot\mathbf{u}) + (\mathbf{B}\cdot\nabla)\mathbf{u}$$

Conservation of magnetic momentum + adiabatic law = CGL condition (Chew, Goldberger & Low 1956):

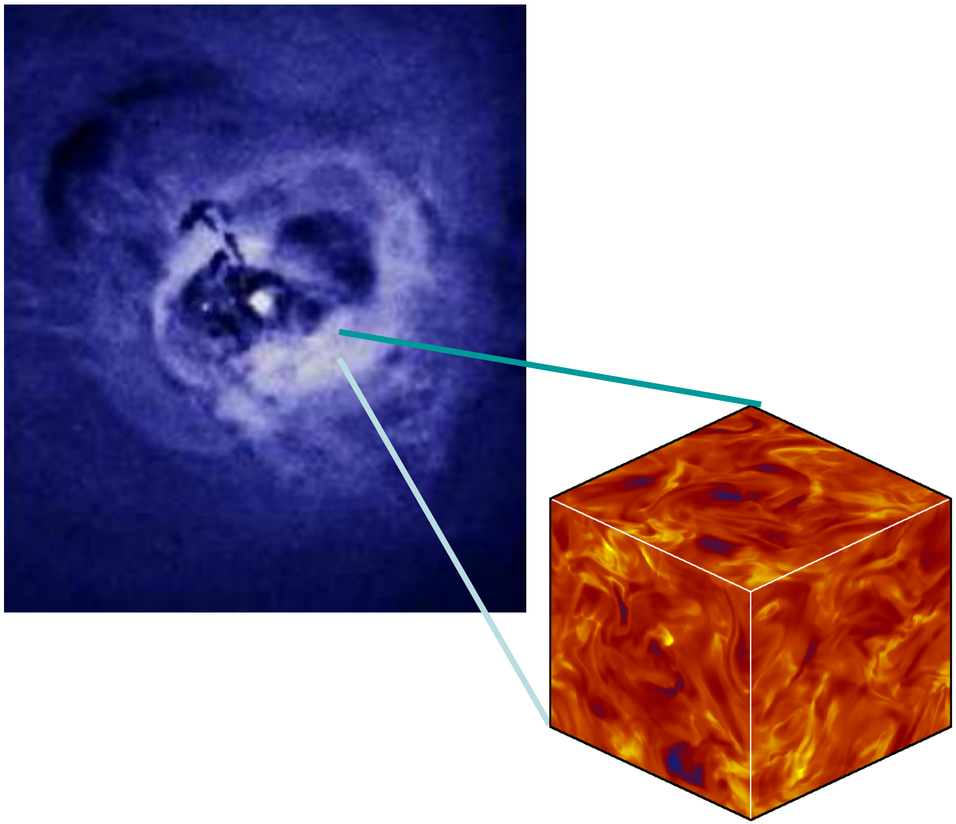
$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0$$

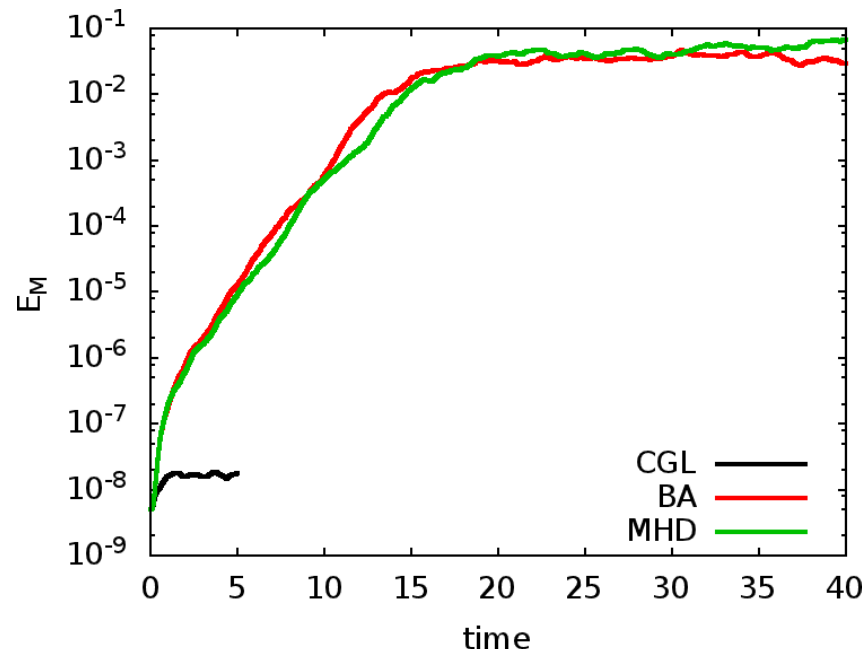
$$\frac{d}{dt}\left(\frac{p_{\parallel}B^2}{\rho^3}\right) = 0$$

$$\Rightarrow \boxed{A = \frac{p_{\perp}}{p_{\parallel}}} = \left(\frac{B}{B_0}\right)^3\left(\frac{\rho}{\rho_0}\right)^{-2}$$

Turbulent Dynamo in the ICM:

Amplification of Cosmic Magnetic Fields





Santos-Lima et al. 2013; Kowal et al. 2011

Scales of actual plasmas

(few more examples)

	l_0 (m)	B_0 (T)	t_0 (s)
tokamak	20	3	3×10^{-6}
magnetosphere Earth	4×10^{7}	3×10^{-5}	6
solar coronal loop	10^{8}	3×10^{-2}	15
magnetosphere neutron star	10^{6}	10^{8} *	10^{-2}
accretion disc YSO	1.5×10^{9}	10^{-4}	7×10^{5}
accretion disc AGN	4×10^{18}	10^{-4}	2×10^{12}
galactic plasma	10^{21} ($= 10^{5}$ ly)	10^{-8}	10^{15} ($= 3 \times 10^{7}$ y)

pulsars, called magnetars, have record magnetic fields of 10^{11} T : the plasma Universe is ever expanding!

+Aplications: (Ideal) MHD: Waves

Normal compressible fluids:

– **Acoustic waves** (release of free energy associated with non-uniform density or velocities)

In plasmas:

- Besides these: **new modes appear**

– **Waves:** modes with real **frequencies**

Perturbing MHD equations

Hypotesis:

- B_o, ρ_o, v_o, p_o : **constant and uniform at equilibrium state**
- Consider perturbations in plasma: **λ << L** (dimension of the system)

Perturbing the system

$$f(\vec{x},\psi) = f_o + f_1(\vec{x},t) + \cancel{f_2(\vec{x},t)} + \ldots$$

Physical quantity at equilibrium (order 0)

$\boxed{f_1(\vec{x},t)}$: 1st order perturbation: $f_1(\vec{x},t) \ll f_o$

Neglect: $f_1^2, f_2, g_2^2, g_2, f_1 g_1,$ etc.

$v = v_o + v_1$, take $v_o = 0$, denote ***$v_1 = v$***

1^{st} order equations

- Coeficients of eqs. are constants:

→ solution:

$$f_1(\vec{x},t) = f_1 exp(nt + i\vec{k}.\vec{x})$$

If n imaginary and k real → WAVE

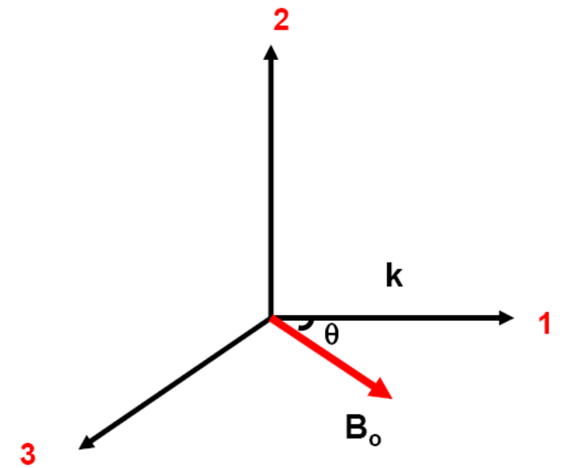
If n real and n>0 → growth rate of INSTABILITY: ***f ~ exp (nt)***

If n complex → wave with growing (or non-growing) amplitude with time

If k real → wave with constant amplitude in space

If k imaginary → wave with growing or damping amplitude in space

Dispersion relation *n(k)*



v_1

$$nv_1 + \frac{\gamma\alpha v_s^2 k^2}{n}v_1 + \frac{k^2 v_A^2}{n+\nu_M k^2} sen\theta(-cos\theta v_3 + sen\theta v_1)$$

$$\frac{4}{3}\nu k^2 v_1 = 0$$

v_2

$$nv_2 + \frac{k^2 v_A^2}{n+\nu_M k^2} cos^2\theta v_2 + \nu k^2 v_2 = 0 \qquad (3.31)$$

v_3

$$nv_3 + \frac{k^2 v_A^2}{n+\nu_M k^2} cos\theta(cos\theta v_3 - sen\theta v_1) + \nu k^2 v_3 = 0 \qquad (3.32)$$

Since these eqs. are homogeneous: solutions only if determinant of coefficients = 0 -> result dispersion relation n(k)

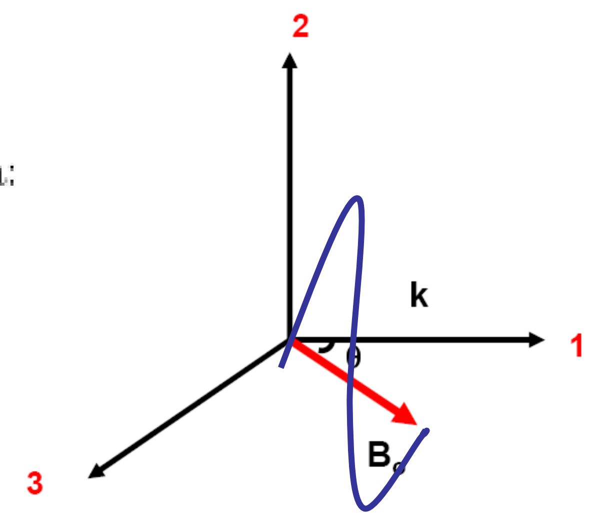
Taking eq. for v_2

$$\boxed{n^2 + n(\nu + \nu_M)k^2 + k^2 v_A^2 cos^2\theta + \nu\nu_M k^4 = 0}$$

No limite que $k\nu << v_A$ e $k\nu_M << v_A$, (3.34) reduz-se a:

$$n = \pm ik v_A cos\theta$$

velocidades de fase ao longo de $\vec{x}_1$ são ($v_\phi = \pm in/k$):



$$\boxed{v_\phi = \mp v_A cos\theta = \mp \frac{B_o}{\sqrt{4\pi\rho_o}} cos\theta}$$

$$\boxed{v_A = \frac{B_o}{(4\pi\rho_o)^{1/2}}}$$

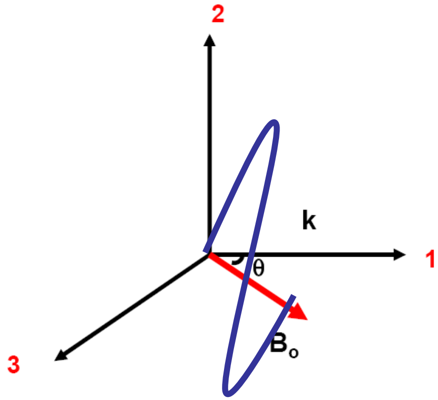
This mode: **Alfven wave**

Alfven wave

First described bt Alfven (1953): **Alfven wave**.

Velocity perturbation transverse to plan *k* and *Bo*

Propagates // Bo



Key role in transmiting forces, like acoustic waves in non-magnetized gas

Since motion transverse to *B*: lines of force are bent by the motion of the wave and a restauring force due to **magnetic tension** is exerted by lines (similar tension in a rope):

Magneto-acoustic Waves

If we consider motions in direction 1 (// k and ⊥ B):

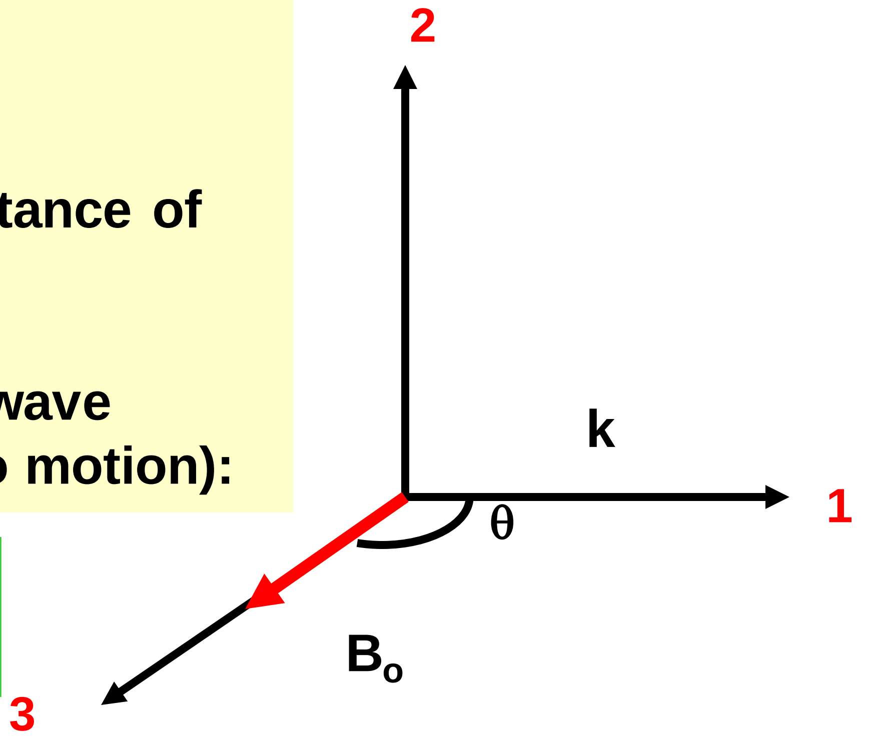
$$v_\phi^4 - (v_A^2 + \gamma v_S^2)v_\phi^2 + \gamma v_S^2 v_A^2 cos^2\theta = 0$$

Velocity // *k implies compression of the gas* (resistance of pressure)

Velocity normal to *B* means compression of B lines (resistance of magnetic pressure)

→ magnetic-acoustic wave (longitudinal to motion):

$$\boxed{v_\phi = \pm(v_A^2 + \gamma v_S^2)^{1/2}}$$



Other aplications

- Shock waves
- Particle Acceleration (next lectures)
- Instabilities
- Magnetic Reconnection (next lectures)
- Winds
- Dynamos
- Turbulence
- Etc….