# MHD turbulence in the solar corona and solar wind

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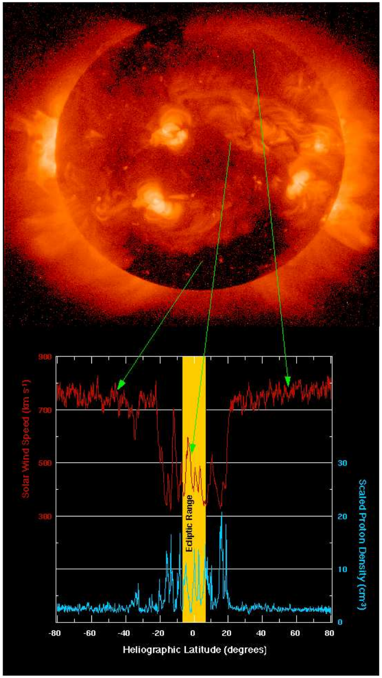
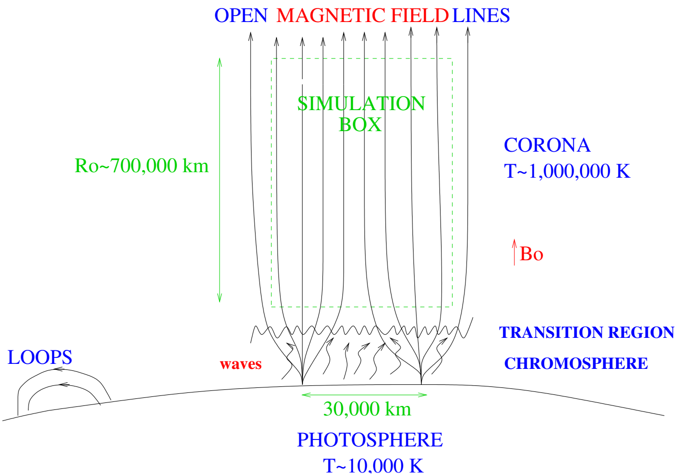
# Heating in open regions: coronal holes




Observational evidence (UVCS/SOHO, Spartan) that the high speed solar wind results from plasma heating ($T \sim 10^6$ K) very close to the Sun.

Fluid models for the acceleration of the solar wind require a **heating dissipation** per unit volume which is a rapidly decreasing function of heliocentric distance. The heating per unit mass has to be extended throughout the model corona.



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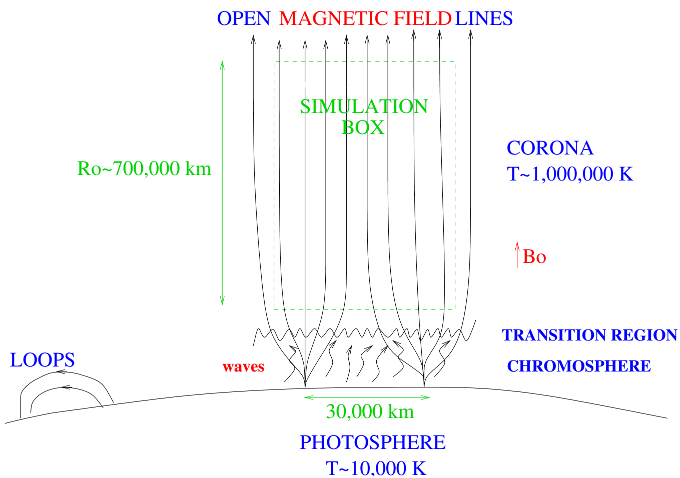


MHD turbulence as in coronal loops $\rightarrow$ RMHD $(B_0 >> b_\perp, u_\perp, \quad \nabla_\parallel << \nabla_\perp)$.

But there is a new problem: how to maintain turbulence in an open region, where Alfven waves can transport the energy out of the region.

# RMHD turbulence driven by waves

RMHD equations using Elsasser variables (can be viewed as propagating fluctuations)
$\mathbf{z}_+ = \mathbf{u} + \mathbf{b}, \quad \mathbf{z}_- = \mathbf{u} - \mathbf{b}, \quad V_A = B_0/\sqrt{4\pi\rho}$

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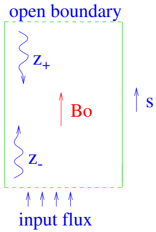
linear solutions (if only one-way propagating fluctuations exist): $\mathbf{z}_\pm \sim \exp[ik_\parallel(s \pm V_A t)]$

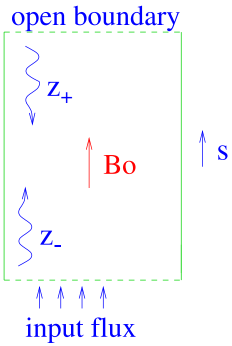


For non-linear terms to be non-zero we need simultaneous presence of both type of fluctuations:

$$\mathbf{z}_- \cdot \nabla_\perp \mathbf{z}_+$$

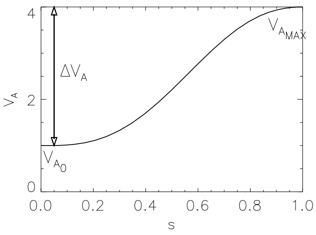
$$\mathbf{z}_+ \cdot \nabla_\perp \mathbf{z}_-$$

so that we can have a turbulent cascade, transfer of energy to small scales, enhance dissipation, produce heating.



# RMHD in an inhomogeneous medium

$$\rho = \rho(s) \rightarrow V_A = V_A(s)$$

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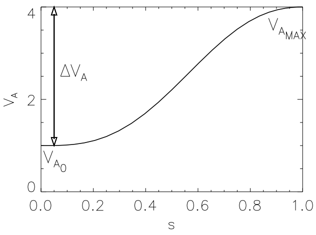


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## Modified RMHD equations in an inhomogeneous medium

$$\frac{\partial \mathbf{z}_+}{\partial t} - V_A \frac{\partial \mathbf{z}_+}{\partial s} = -\frac{1}{2}\frac{dV_A}{ds}\mathbf{z}_+ + \frac{1}{2}\frac{dV_A}{ds}\mathbf{z}_- - \nabla_\perp p' - \mathbf{z}_- \cdot \nabla_\perp \mathbf{z}_+ + \eta \nabla_\perp^2 \mathbf{z}_+$$

$$\frac{\partial \mathbf{z}_-}{\partial t} + V_A \frac{\partial \mathbf{z}_-}{\partial s} = \frac{1}{2}\frac{dV_A}{ds}\mathbf{z}_- - \frac{1}{2}\frac{dV_A}{ds}\mathbf{z}_+ - \nabla_\perp p' - \mathbf{z}_+ \cdot \nabla_\perp \mathbf{z}_- + \eta \nabla_\perp^2 \mathbf{z}_-$$

## Boundary conditions

$$\frac{\partial \mathbf{z}_-}{\partial s}^{\text{bot}}(\mathbf{k}_\perp) = A(k_\perp)\cos(2\pi f\ t) \quad , \text{if } 2 \le k_\perp \le 5$$

$$\frac{\partial \mathbf{z}_+}{\partial s}^{\text{top}}(\mathbf{k}_\perp) = 0 \qquad , \forall\ \mathbf{k}_\perp$$

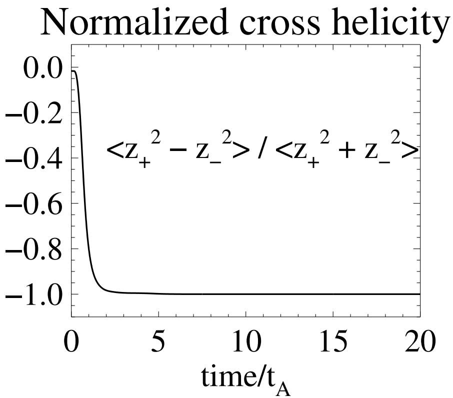
$\mathbf{f}$ = (low) frequency forcing
fluctuations amplitude (at the base) = $\delta\mathbf{u_0} \sim 30 - 50$ km/s
(for a coronal hole)
perpendicular structures size (at the base) = $\mathbf{l_0} \sim 10 - 30$ Mm
(supergranules or inter-network length)
unit of parallel length = $L \sim R_s \sim 700$ Mm (solar radius).
unit of time $\mathbf{t_0} = \mathbf{l_0}/\delta\mathbf{u_0}$ (typical timescale of the forcing structures)

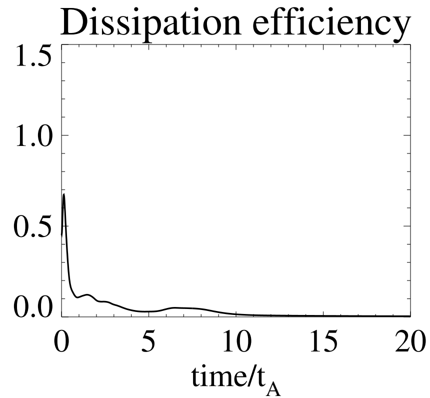
## Numerical simulations

Pseudospectral code: **Fourier** $(x, y)$ + **Chebyshev** $(s)$.

Resolution $512 \times 512 \times 33$.

# Simulations without reflections



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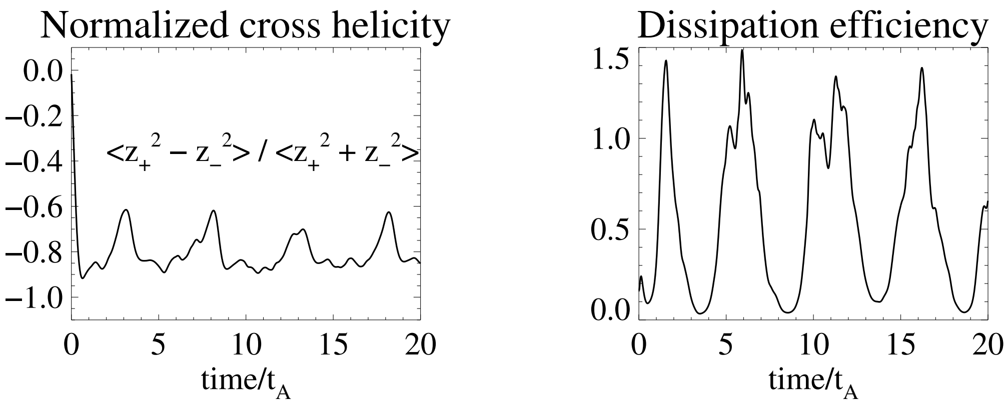
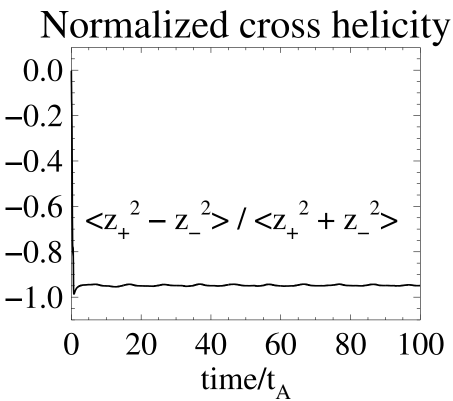
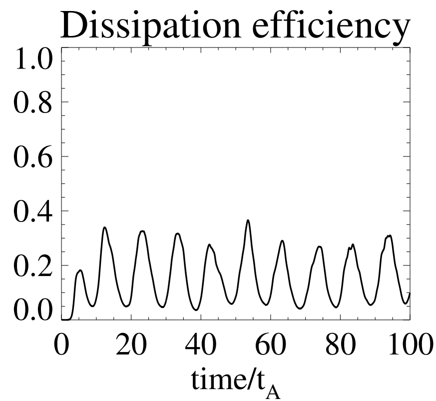


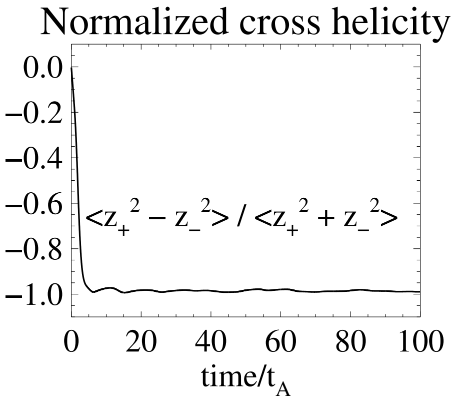
# Simulations with weak reflections

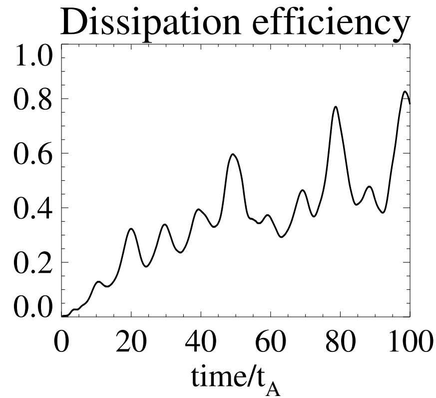


# Simulations with weak reflections and small correlation length

$t_0 = l_0/\delta u_0 << t_A = R_s/V_A$



## Conditions for sustaining turbulence (efficiently)

- Presence of reflections (not neccesarily strong)
- Presence of non-propagating fluctuations
- Timescale ordering: $t_{NL} < t_0 < t_A \sim t_R < t_f < t_\eta$

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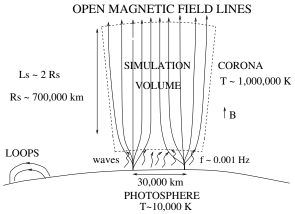
$t_{NL} = l_k/\delta u_k$=nonlinear time; $t_0 = l_0/\delta u_0$=forcing; $t_A = R_s/V_A$=Alfven crossing time; $t_R = R_s/\Delta V_A$=reflection; $t_f = 1/f$=forcing period; $t_\eta = l_0^2/\eta$=dissipative time.

# Heating distribution profile

## Consider RMHD equations in a radially expanding box.




$$\frac{\partial \mathbf{z}_+}{\partial t} - V_A \frac{\partial \mathbf{z}_+}{\partial r} = R_1 \mathbf{z}_- - R_2 \mathbf{z}_+ - \nabla_\perp p' - \mathbf{z}_- \cdot \nabla_\perp \mathbf{z}_+ + \eta \nabla_\perp^2 \mathbf{z}_+$$

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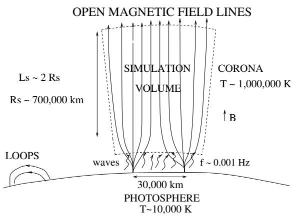


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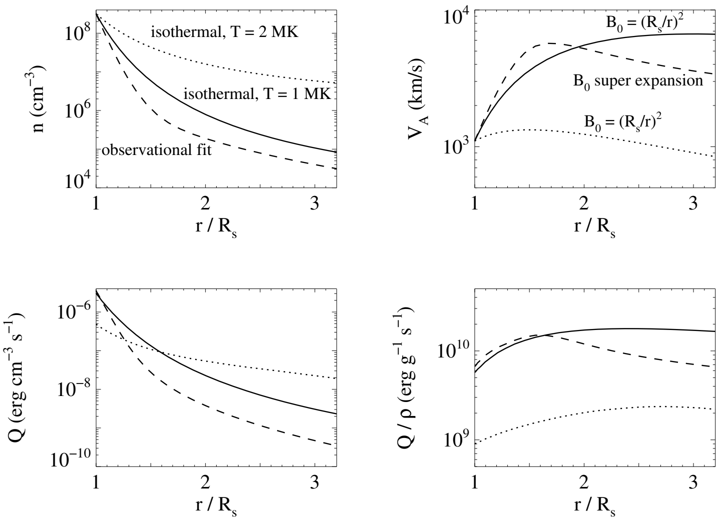
$$R_1(r) = \frac{1}{2} \frac{dV_A}{dr}, \qquad R_2(r) = \frac{1}{2} \left( \frac{dV_A}{dr} + \frac{V_A}{A} \frac{dA}{dr} \right) \text{ (reflection rates)}$$

$$A(r) = A_0 \frac{B_0}{B(r)} \text{ (cross section area)}$$

# Compute heating profile from turbulent dissipation rate

$$Q(r) = \rho(r)\ \epsilon_{\text{turb}}(r)$$




Dmitruk et al, ApJ 2002

Top left: number density profiles; Top right: corresponding Alfven speed profiles.
Bottom left: heating per unit volume; Bottom right: heating per unit mass.

The previous results show explicitly (from simulations) the connection between the assumed Alfven speed profile (density) and the heating profile. Further understanding of this connection: phenomenological model

Replace non-linear terms $\mathbf{z}_{\mp} \cdot \nabla_{\perp} \mathbf{z}_{\pm}$ with modeling $Z_{\mp} \mid Z_{\pm} \mid /2\lambda_{\perp}(r)$, with $\lambda_{\perp}(r)$=correlation length scale (for instance, linearly increasing with $r$), and obtain one dimensional equations for $Z_{\mp}(r)$.

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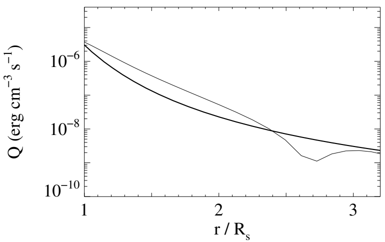


Comparison of heating profile from direct numerical simulation and from numerical solution of the phenomenological model.

Asymptotic limit as $\lambda_\perp \longrightarrow 0$ ($\lambda_\perp = 1 \longrightarrow 30000$ km, $\lambda_\perp = 0.1 \longrightarrow 3000$ km) of the phenomenological model solution.

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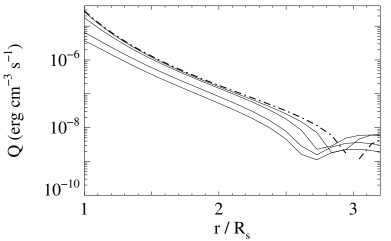


In this limit, $| Z_+ | << | Z_- |$ and $Q = \rho\epsilon \approx \rho Z_-^2 \mid Z_+ \mid / \lambda_\perp \approx \rho Z_-^2 \mid dV_A/dr \mid$. From flux $F = \rho A V_A Z_-^2$ balance equation $dF/dr = -A(r)Q$ we can obtain

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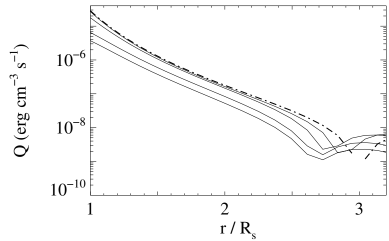


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$$Q(r) \approx F_{A_0}\ (A_0/A) \mid dV_A/dr \mid (V_{A_0}/V_A^2) \quad r < r_m$$

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with $F_{A_0}$ flux at the base, $r_m$ radial distance where $V_A = V_{A_m}$=maximum.

This express again (in this case for an asymptotic limit in the phenomenological model) the connection between the heating profile and the Alfven speed profile.