$\mathcal{N}=8$ gauged supergravity and Extended field theory

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Abstract

We present a unified description of the bosonic sector of $\mathcal{N}=8$ gauged supergravity (GSG) in four dimensions. The theory is based on an extended manifold of $1+3+56$ dimensions. The bosonic degrees of freedom are unified in a generalized metric. The diffeomorphism and gauge symmetries are promoted to generalized diffeomorphism $Diff(4)\times E_{7(7)}\times\mathbb{R}^+$.

Motivation

Different attempts to promote T-duality, a distinctive symmetry of string theory, to a symmetry of a field theory, can be grouped into the name of Double Field theory (DFT) [1]. Interestingly enough, the Scherk-Schwarz (SS) reduction of (bosonic) DFT leads to an action that can be identified with the bosonic sector of $\mathcal{N}=4$ gauged supergravities [2]. The aim of the present work is to extend the above ideas to incorporate the full stringy U-duality symmetry group in four dimensions. We name it Extended Field Theory (EFT).

Generalized diffeomorphisms on $E_{7(7)}\times\mathbb{R}^+$

We start by defining the generalized diffeomorphism in the $E_{7(7)}\times\mathbb{R}^+$ group compatible with supersymmetry.

Generalized Lie derivative over an $E_{7(7)}$ frame

$$(L_{E_{\bar A}}E_{\bar B})^M = E^P_{\bar A}\partial_P E^M_{\bar B} - 12P^M{}_P{}^Q{}_R\partial_Q E^R_{\bar A}E^P_{\bar B} + \frac{\omega}{2}\partial_P E^P_{\bar A}E^M_{\bar B}$$

$$(L_{E_{\bar A}}E_{\bar B})^M = F_{\bar A\bar B}{}^{\bar C}E^M_{\bar C} \qquad ; \qquad F_{\bar A\bar B}{}^{\bar C} = X_{\bar A\bar B}{}^{\bar C} + T_{\bar A\bar B}{}^{\bar C}$$

$P : 56\times 56 \to 133 = Adj(E_{7(7)})$; ω conformal weight of $\mathbb{R}^+$

Linear constraints

$$T_{\bar A\bar B}{}^{\bar C} = -\vartheta_{\bar A}\delta^{\bar C}_{\bar B} + 8\vartheta_{\bar K}P^{\bar K}{}_{\bar A}{}^{\bar C}{}_{\bar B}\ ;\ P^{\bar C}{}_{\bar B}{}^{\bar M}{}_{\bar N}X_{\bar A\bar M}{}^{\bar N} = X_{\bar A\bar B}{}^{\bar C}$$

$$X_{\bar A\bar B}{}^{\bar B} = X_{\bar A\bar B}{}^{\bar A} = 0\ ;\ X_{\bar A[\bar B\bar C]} = 0$$

$$X_{(\bar A\bar B\bar C)} = 0 \quad ; \quad X_{\bar A\bar B}{}^{\bar C}\in 912 \quad ; \quad \vartheta_{\bar A}\in 56$$

Closure condition (quadratic constraints)

Generalized bracket

$$[[E_{\bar A}, E_{\bar B}]] = \frac{1}{2}(L_{E_{\bar A}}E_{\bar B} - L_{E_{\bar B}}E_{\bar A})$$

We demand

$$[L_{E_{\bar A}}, L_{E_{\bar B}}]E_{\bar C} - L_{[[E_{\bar A},E_{\bar B}]]}E_{\bar C} = \triangle_{[\bar A\bar B]} = 0$$

On the other hand gaugings expressed in $SU(8)$ indices should transform as scalars

$$\delta_\xi F_{\bar M}{}^{\bar N\bar R} = \xi^{\bar P}D_{\bar P}F_{\bar M}{}^{\bar N\bar R} - \xi^{\bar Q}\triangle_{(\bar Q\bar M)}{}^{\bar N\bar P} \quad\Rightarrow\quad \triangle_{(\bar Q\bar M)}{}^{\bar N\bar P} = 0$$

$$\delta_\xi\vartheta_{\bar M} = \xi^{\bar P}D_{\bar P}\vartheta_{\bar M} - \xi^{\bar Q}\hat\triangle_{\bar Q\bar M} \quad\Rightarrow\quad \hat\triangle_{\bar Q\bar M} = 0$$

Remarkably, in the particular case when the gaugins are constant we find exactly the conditions found in [3]

$$[F_{\bar M}, F_{\bar N}] = -F_{\bar M\bar N}{}^{\bar R}F_{\bar R} \quad ; \quad F_{(\bar M\bar N)}{}^{\bar P}F_{\bar P\bar R}{}^{\bar Q} = F_{(\bar M\bar N)}{}^{\bar P}\vartheta_{\bar P} = 0$$

Space-time extension

We now want to couple the gaugings with a four-dimensional metric $g_{\mu\nu}$ and vectors A^M_μ and perform the SS reduction.

Generalized diffeomorphism

$$E_{\bar{\mathcal{A}}}{}^{\mathcal{M}}(x,Y) = \mathcal{E}_{\bar{\mathcal{A}}}{}^{\mathcal{B}}(x)U_{\mathcal{B}}{}^{\mathcal{M}}(Y)$$

$$\mathcal{L}_\xi V^{\mathcal{M}} = \xi^{\mathcal{P}}\partial_{\mathcal{P}}V^{\mathcal{M}} - A^{\mathcal{M}}{}_{\mathcal{N}}{}^{\mathcal{P}}{}_{\mathcal{Q}}\partial_{\mathcal{P}}\xi^{\mathcal{Q}}V^{\mathcal{N}}$$

The mixed components of $A^{\mathcal{M}}{}_{\mathcal{N}}{}^{\mathcal{P}}{}_{\mathcal{Q}}$ can only take the form

$$A^\mu{}_\nu{}^M{}_N = a_1\delta^\nu_\mu\delta^N_M\ ;\ A^M{}_N{}^\mu{}_\nu = a_2\delta^\nu_\mu\delta^N_M$$
$$A^M{}_\nu{}^\mu{}_N = a_3\delta^\nu_\mu\delta^N_M\ ;\ A^\mu{}_N{}^M{}_\nu = a_4\delta^\nu_\mu\delta^N_M$$

Master flux

$$(\mathcal{L}_{E_{\bar{\mathcal{A}}}}E_{\bar{\mathcal{B}}})^{\mathcal{M}} = F_{\bar{\mathcal{A}}\bar{\mathcal{B}}}{}^{\bar{\mathcal{C}}}E^{\mathcal{M}}_{\bar{\mathcal{C}}}$$

Bein

$$\mathcal{E}_{\bar{\mathcal{A}}}{}^{\mathcal{M}}(x) = \begin{pmatrix} e^\mu_{\bar\alpha} & e^\nu_{\bar\alpha}A^M_\nu \\ 0 & \Phi^M_{\bar A}\end{pmatrix}$$

$$U_{\mathcal{B}}{}^{\mathcal{M}}(Y) = \begin{pmatrix} \delta^\mu_\beta & 0 \\ 0 & U^M_B\end{pmatrix}$$

Gauge symmetry and four dimensional diffeomorphisms

$$\xi^{\mathcal{M}} = (\lambda^\mu, \Lambda^M)$$

$$\hat\delta e^\mu_{\bar\alpha} = \mathcal{L}_\lambda e^\mu_{\bar\alpha} + \frac{1}{2}\vartheta_N\Lambda^N e^\mu_{\bar\alpha}$$
$$\hat\delta A^M_\mu = \mathcal{L}_\lambda A^M_\mu - F_{NP}{}^M\Lambda^N A^P_\mu - \partial_\mu\Lambda^M - \frac{1}{2}\vartheta_N\Lambda^N A^M_\mu$$
$$\hat\delta\Phi^M_{\bar A} = \mathcal{L}_\lambda\Phi^M_{\bar A} - F_{NP}{}^M\Lambda^N\Phi^P_{\bar A}$$

$$F_{MN}{}^P = F_{MN}{}^P \qquad \text{(gaugings)}$$
$$F_{\mu\nu}{}^M = 2\partial_{[\mu}A_{\nu]}{}^M + F_{NP}{}^M A_\mu{}^N A_\nu{}^P$$
$$F_{[\mu M]}{}^P = (\Phi^{-1})_M{}^{\bar N}\partial_\mu\Phi_{\bar N}{}^P + F_{[RM]}{}^P A_\mu{}^R$$
$$F_{(\mu M)}{}^P = F_{(RM)}{}^P A_\mu{}^R$$
$$F_{\mu\nu}{}^\gamma = 2\hat\Gamma_{[\mu\nu]}{}^\gamma = -2(e^{-1})_{[\nu}{}^{\bar\beta}\partial_{\mu]}e_{\bar\beta}{}^\gamma$$

Conclusions

We have shown that the idea of DFT can be extended to a more general context to include the whole U-duality group. We treated the diffeomorphisms and gauge transformations in a unified way. We provide a higher dimensional origin for the gaugings of maximal supergravity in terms of an extended geometry over a generalized manifold. The generalized Lie derivative provides gaugings that are compatible with supersymmetry, therefore satisfying automatically the linear constraint. We also compute the closure conditions of the algebra and extract the quadratic constraints, which generalize those found in maximal supergravity to the case of non-constant fluxes. Based on these results, we give a first step towards an action written in terms of the generalized fluxes which we hope can be interpreted as generalized Ricci scalar.

References

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[3] Arnaud Le Diffon, Henning Samtleben, Mario Trigiante, "$\mathcal{N}=8$ Supergravity with Local Scaling Symmetry", JHEP 04 (2011) 079 [arXiv:1103.2785 [hep-th]].

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