Effective Field Theories

Lecture 5

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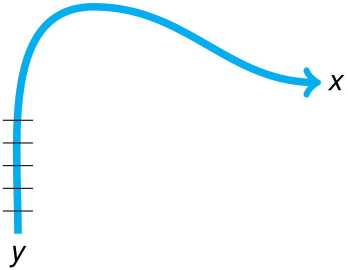
Wilson Lines



$$W(x,y) = P\exp\left\{-ig\int_y^x A_\mu(z)\,\mathrm{d}z^\mu\right\}$$
$$= \prod_i e^{-igA_\mu(z_i)\Delta z_i^\mu}$$

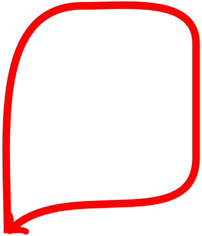
$$(t\cdot D)\,W(x,y) = \delta(x-y)$$

$$W(x,y) \to U(x)W(x,y)U(y)^\dagger \qquad \text{show this}$$

$$W(x,y)\psi(y) \to U(x)\;W(x,y)\psi(y)$$

Transports charge from y to x

Wilson Loop



$$W_C(x) = P \exp\left\{ -ig \oint_C A_\mu(z)\, \mathrm{d}z^\mu \right\}$$

$$W_C(x) \to U(x)\ W_C(x)\ U(x)^\dagger$$

$$\mathrm{Tr}\, W_C(x)$$

is gauge invariant. Depends on the curve C, but not on the starting point x. show this

Collinear Wilson Lines

n-collinear Wilson line:

$$W_n(x) = P \exp \left\{ -ig \int_{-\infty}^{0} \bar{n} \cdot A_n(x + s\bar{n}) \, \mathrm{d}s \right\}$$

An integral along the $\bar{n}$ direction from $-\infty$ to x.

$W_n(x)$ is the Green's function for $i\bar{n} \cdot D_n$.

$$W_n(x)^\dagger \bar{n} \cdot \mathcal{P} W_n(x) = \bar{n} \cdot D_n$$

convert an ordinary derivative into a covariant derivative.

Cannot do this in a normal gauge theory because there is no preferred path to x. Around closed loops

$$P \exp ig \oint A_\mu(z) \mathrm{d}z^\mu \neq 0$$

Here we have direction labels $n, \bar{n}$.

$$\mathcal{L} = \overline{\xi}_{n,p}(x) \left\{ [in \cdot (D_{us} + D_n)] + \left(i\not{D}_{n,\perp}\right) W_n(x) \frac{1}{i\bar{n} \cdot \mathcal{P}} W_n^\dagger(x) \left(i\not{D}_{\perp,n}\right) \right\} \frac{\not{\bar{n}}}{2}\, \xi_{n,p}(x)$$

You can also show that the Wilson lines are needed by collinear gauge invariance. Under gauge transformations,

$$\begin{aligned} \psi &\to U\psi \\ g\,A^\mu &\to U\,g\,A^\mu U^\dagger + i\partial^\mu U\,U^\dagger \end{aligned}$$

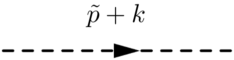
Under n-collinear gauge transformations, the x dependence on $U_n(x)$ has momentum $(1, \lambda^2, \lambda)Q$.

$$\begin{aligned} \xi_{n,p}(x) &\to U_n(x)\;\xi_{n,p}(x) \\ \xi_{\bar{n},p} &\to \xi_{\bar{n},p} \\ A_{us} &\to A_{us} \\ W_n(x) &\to U_n(x)\;W_n(x) \end{aligned}$$

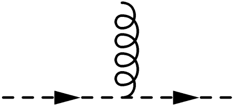
so

$$W_n(x)^\dagger \xi_{n,p}(x)$$

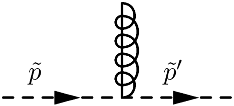
is n-collinear gauge invariant.



$$= i\frac{\not n}{2}\frac{\bar n\cdot\tilde p}{n\cdot k\,\bar n\cdot\tilde p+\tilde p_\perp^2+i\epsilon}$$



$$= igT^A n_\mu \frac{\not{\bar n}}{2}$$



$$= igT^A\left(n_\mu + \frac{\gamma_\mu^\perp \tilde{\not p}_\perp}{\bar n\cdot\tilde p}+\frac{\tilde{\not p}'_\perp\gamma^\perp_\mu}{\bar n\cdot\tilde p'}-\frac{\tilde{\not p}'_\perp\tilde{\not p}_\perp}{\bar n\cdot\tilde p\;\bar n\cdot\tilde p'}\bar n_\mu\right)\frac{\not{\bar n}}{2}$$

Notation for fields

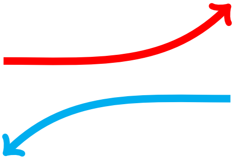
What makes EFT complicated?

HQET:

$$p = m_Q v + k$$

v is constant, i.e. the label is conserved.

NRQCD/NRQED and SCET:



Can have label-changing interactions, so the label is a dynamical variable, and not a constant.

quarks can change direction, n-collinear interactions can change p.

SCET Matching for the Current

Suppose you have a current $\overline{\psi}\gamma^\mu\psi$ in QCD, as in $e^+e^- \to q\overline{q}$.

It matches to

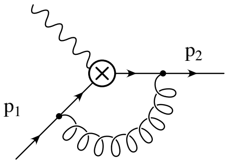
$$\overline{\psi}\gamma^\mu\psi \to C\left[\overline{\xi}_{\bar{n},p}W_{\bar{n}}\right]\gamma^\mu\left[W_n^\dagger\xi_{n,p}(x)\right]$$

by collinear gauge invariance.

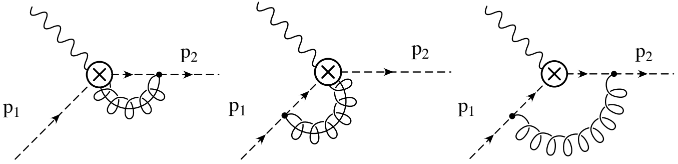
$$C = 1 + \mathcal{O}\left(\alpha_s\right)$$

High scale matching: $\mu \sim Q$

- full theory:



- EFT:



SCET Matching: $J^\mu \to q\bar{q}$

$$J^\mu = \overline{\psi}\,\gamma^\mu \psi$$

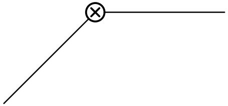
In the EFT, this matches to

$$J^\mu = C\left[\overline{\xi}_{n,p} W\right]\,\gamma^\mu \left[W^\dagger \xi_{n,p}\right]$$

Compute the on-shell matrix element at one loop.

Start with scattering kinematics $q^2 = -Q^2 < 0$.

$\sqrt{\mathcal{R}_\psi}$



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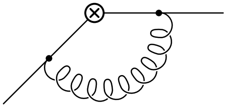
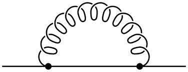


$$\frac{i\mathcal{R}_\psi}{\not{p}} \qquad \text{as} \qquad p \to 0$$

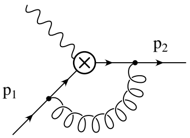


$$I_1 = i\not{p}$$

Use Feynman gauge

$$\begin{aligned} I_2 &= -g^2 C_F \int \frac{\mathrm{d}^d k}{(2\pi)^d} \gamma^\alpha \frac{\not{p} + \not{k}}{(p+k)^2} \gamma_\alpha \frac{1}{k^2} \\ &= C_F \frac{\alpha_s}{4\pi} i\not{p} \left[\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right] \end{aligned}$$

$$\mathcal{R}_\psi^{-1} = 1 + C_F \frac{\alpha_s}{4\pi} i\not{p} \left[-\frac{1}{\epsilon_{IR}} \right]$$



$$I_V = -g^2 C_F \int \frac{\mathrm{d}^d k}{(2\pi)^d} \gamma^\alpha \frac{\not{p}_2 + \not{k}}{(p_2+k)^2} \gamma^\mu \frac{\not{p}_1 + \not{k}}{(p_1+k)^2} \gamma_\alpha \frac{1}{k^2}$$

We want the graph on-shell, so $\ldots \not{p}_1 = 0, \quad \not{p}_2 \ldots = 0$.

Combine $(p_1+k)^2$ and $(p_2+k)^2$ first using x, and then k^2 using z.

The entire integral is

$$
C_F \frac{\alpha_s}{4\pi} \gamma^\mu (4\pi\tilde{\mu}^2)^\epsilon \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z\, z \left\{ \frac{(2-d)^2}{2} \Gamma(\epsilon) \left[Q^2 x(1-x) z^2 \right]^{-\epsilon} \right.
$$

$$
+ Q^2 \Gamma(1+\epsilon) \left[Q^2 x(1-x) z^2 \right]^{-1-\epsilon} \times
$$

$$
\left. \left[2(1-xz)((1-x)z-1) - (d-4) x(1-x) z^2 \right] \right\}
$$

$$
= C_F \frac{\alpha_s}{4\pi} \gamma^\mu \left[-\frac{2}{\epsilon_{IR}^2} - \frac{2}{\epsilon_{IR}} \ln \frac{\mu^2}{Q^2} + \frac{1}{\epsilon_{UV}} - \frac{4}{\epsilon_{IR}} - \ln^2 \frac{\mu^2}{Q^2} \right.
$$

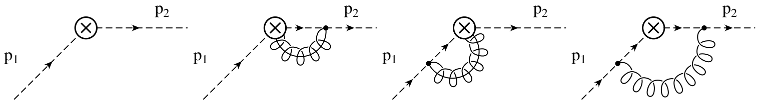
$$
\left. - 3 \ln \frac{\mu^2}{Q^2} + \frac{\pi^2}{6} - 8 \right]
$$

The on-shell matrix element is

$$1 + C_F \frac{\alpha_s}{4\pi} \gamma^\mu \left[-\frac{2}{\epsilon_{IR}^2} - \frac{2}{\epsilon_{IR}} \ln \frac{\mu^2}{Q^2} - \frac{3}{\epsilon_{IR}} - \ln^2 \frac{\mu^2}{Q^2} - 3 \ln \frac{\mu^2}{Q^2} + \frac{\pi^2}{6} - 8 \right]$$

No UV divergence since the current has no anomalous dimension.

The EFT graphs are



The n-collinear graph gives

$$I_n = -ig^2\mu^{2\epsilon}C_F \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\bar{\not{n}}\, n^\alpha}{2} \frac{\not{n}\, \bar{n}\cdot(p_2-k)}{2(p_2-k)^2} \gamma^\mu \frac{1}{-\bar{n}\cdot k} \bar{n}_\alpha \frac{1}{k^2}$$

$$= -2ig^2\mu^{2\epsilon}C_F \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\bar{n}\cdot(p_2-k)}{(p_2-k)^2} \gamma^\mu \frac{1}{-\bar{n}\cdot k} \frac{1}{k^2}$$

The only scale is p_2^-, so the integral vanishes.

The ultrasoft graph is

$$I_s = -ig^2 C_F \int \frac{\mathrm{d}^d k}{(2\pi)^d} n^\alpha \frac{1}{[n\cdot(p_2-k)]} \gamma^\mu \frac{1}{[\bar{n}\cdot(p_1-k)]} \bar{n}_\alpha \frac{1}{k^2}$$

and also vanishes.

QCD:

$$1 + C_F \frac{\alpha_s}{4\pi} \gamma^\mu \left[-\frac{2}{\epsilon_{IR}^2} - \frac{2}{\epsilon_{IR}} \ln \frac{\mu^2}{Q^2} - \frac{3}{\epsilon_{IR}} - \ln^2 \frac{\mu^2}{Q^2} - 3 \ln \frac{\mu^2}{Q^2} + \frac{\pi^2}{6} - 8 \right]$$

SCET:

$$1 + C_F \frac{\alpha_s}{4\pi} \gamma^\mu \left[\frac{2}{\epsilon_{UV}^2} - \frac{2}{\epsilon_{IR}^2} + \frac{2}{\epsilon_{UV}} \ln \frac{\mu^2}{Q^2} - \frac{2}{\epsilon_{IR}} \ln \frac{\mu^2}{Q^2} + \frac{3}{\epsilon_{UV}} - \frac{3}{\epsilon_{IR}} \right]$$

Matching:

$$C(\mu) = 1 + C_F \frac{\alpha_s}{4\pi} \left[-\ln^2 \frac{\mu^2}{Q^2} - 3 \ln \frac{\mu^2}{Q^2} + \frac{\pi^2}{6} - 8 \right]$$

Anomalous dimension:

$$\mu \frac{\mathrm{d}C(\mu)}{\mathrm{d}\mu} = C_F \frac{\alpha_s}{4\pi} \left[-4 \ln \frac{\mu^2}{Q^2} - 6 \right] C(\mu)$$

$\ln Q^2$ in the anomalous dimension: Sums the Sudakov double logs.

For the time-like case,

$$\ln Q^2 = \ln\left(Q^2 - i0^+\right) \to \ln\left(-q^2 - i0^+\right) = \ln q^2 - i\pi$$

Matching:

$$C(\mu) = 1 + C_F \frac{\alpha_s}{4\pi}\left[-\ln^2 \frac{\mu^2}{q^2} - 3\ln\frac{\mu^2}{q^2} - 2i\pi \ln\frac{\mu^2}{q^2} + \frac{7\pi^2}{6} - 3i\pi - 8\right]$$

$$\mu \frac{\mathrm{d}C(\mu)}{\mathrm{d}\mu} = \gamma(\mu) C(\mu)$$

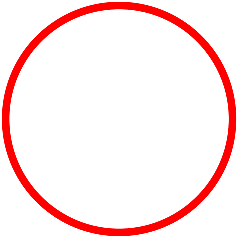
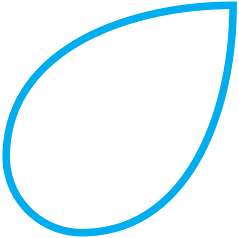
Then

$$\ln \frac{C(\mu_2)}{C(\mu_1)} = \int_{\mu_1}^{\mu_2} \frac{\mathrm{d}\mu}{\mu}\, \gamma(\mu)$$

$$\gamma(\mu) = A(\alpha_s) \ln \frac{\mu^2}{Q^2} + B(\alpha_s)$$

- Can prove only a single log to all orders in perturbation theory
- Gives the Sudakov resummation: $\ln F = \mathsf{L} f_0(\alpha_s \mathsf{L}) + f_1(\alpha_s \mathsf{L}) + \ldots$.
- $A(\alpha_s)$ is the cusp anomalous dimension $\Gamma(\alpha_s)$

Cusp Anomalous Dimension

A smooth Wilson line has renormalization of g and proportional to its length (like mass renormalization).

If there are kinks, additional anomalous dimension Γ that depends on the angle between tangent vectors at the point.

SCET developed to study processes such as $B \to \pi\pi$ with energetic hadrons.

Applied to study jets.

Can also be used to study electroweak corrections at the LHC.

At high energies, W and Z are effectively massless. Can be used to compute electroweak radiative corrections and sum the Sudakov double logs.

$$\alpha_W \; \log^2 \frac{s}{M_{W,Z}^2}$$

Might think that these are tiny, but they are acutally relevant.

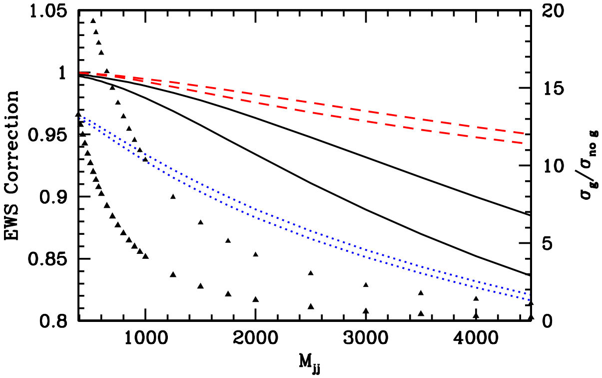
Advantages are:

- factored the scale s from M_W, M_Z, M_h.
- Can do the calculation to NLL for all LHC scattering processes.
- Much simpler than existing fixed order results
- Can include the complete mass dependence on m_t/M_Z, M_W/M_Z and M_h/M_Z
- Can include top-Yukawa corrections

$t\bar{t}$ Invariant Mass distribution

AM, M. Trott, PLB711 (2012) 312

Solid black lines which are the corrections for 7 and 8 TeV LHC:



Seen in the data.

Event Shapes

Thrust:

$$\tau = 1 - T = 1 - \max_{\mathbf{n}} \frac{\sum_i |\mathbf{n} \cdot \mathbf{p}_i|}{\sum_i |\mathbf{p}_i|}$$

C parameter:

$$C = \frac{3}{2} \frac{\sum_{i,j} |\mathbf{p}_i| \, |\mathbf{p}_j| \sin^2 \theta_{ij}}{\left(\sum |\mathbf{p}_i|\right)^2}$$

In the two jet region $\tau \to 0$, $C \to 0$.

Gehrmann et al

Becher and Schwartz

Abbate et al.

Compute fixed order to α_s^3 and resummation to N^3LL and $1/Q$ power correction

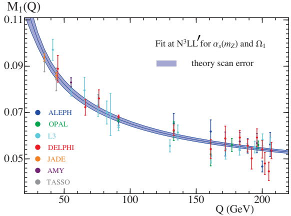
Slides from I. Stewart talk.

Moments:

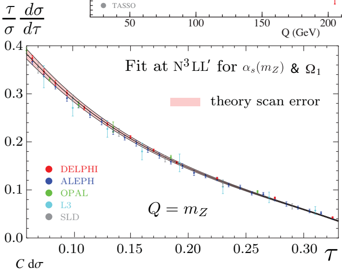
$$M_n = \int_0^1 \tau^n \, p(\tau) \, \mathrm{d}\tau$$

Global Fits

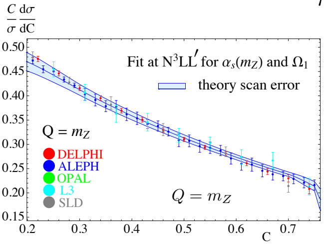
thrust moment

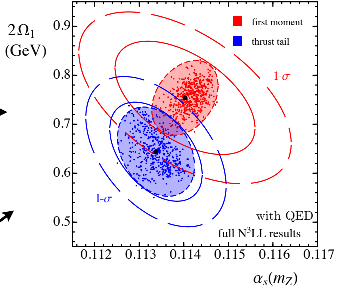


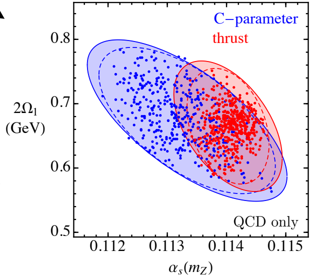
thrust tail



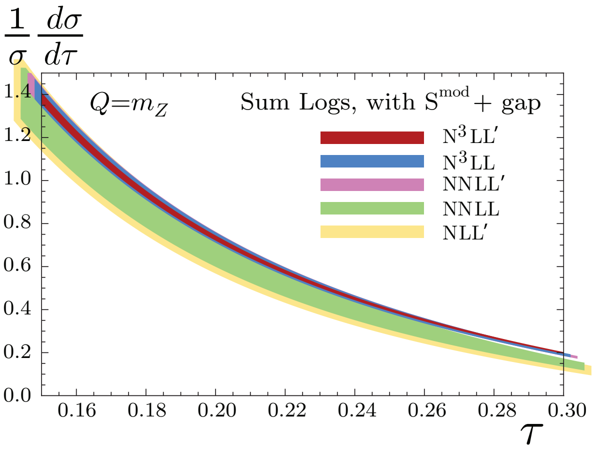
C-param

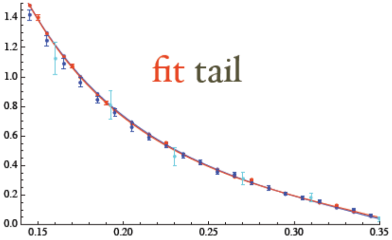


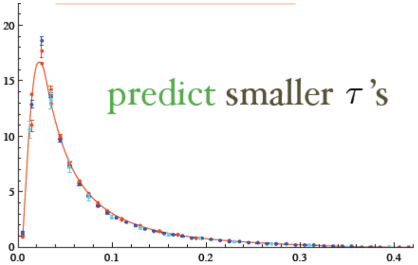




universality? $\Omega_1^C \leftrightarrow \Omega_1^\tau$


$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
$Q=m_Z$
Sum Logs, with $\mathrm{S}^{\mathrm{mod}}$ + gap
$\mathrm{N^3LL'}$
$\mathrm{N^3LL}$
NNLL′
NNLL
NLL′
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.16
0.18
0.20
0.22
0.24
0.26
0.28
0.30
τ


fit tail
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.15
0.20
0.25
0.30
0.35


predict smaller τ's
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4

Jet Vetoes

Look at $W + 0$ jets, $W + 1$ jet, etc.

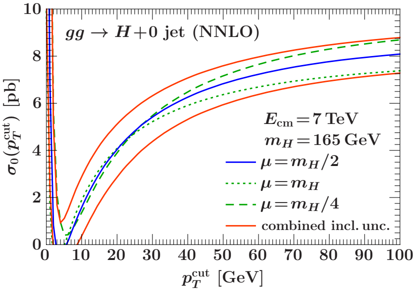
Similarly for the Higgs, there is a jet veto.

$$\sigma_0(p_T^{\rm cut}) = \sigma_{\rm total} - \sigma_{\geq 1}(p_T^{\rm cut})$$
$$= \left[1 + \alpha_s + \alpha_s^2 + \cdots\right] - \left[\alpha_s(L^2 + \cdots) + \alpha_s^2(L^4 + \cdots) + \cdots\right]$$

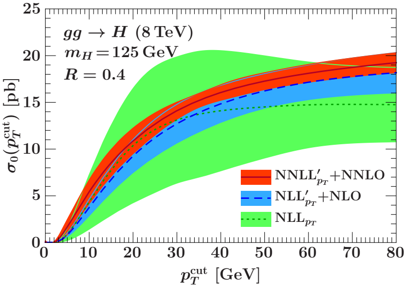
Resummation of Veto Logs

NNLO Fixed Order: FeHIP, HNNLO

Uncertainty procedure: IS, Tackmann



IS, Tackmann, Walsh, Zuberi (in prep)



Resummation of Veto Logs

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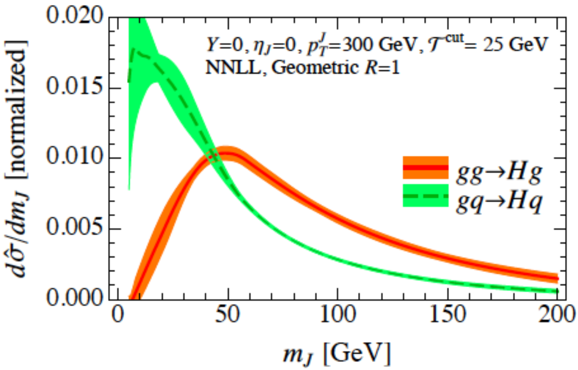
Jet Mass Distribution

Try and distinguish between quark and gluon jets.



Jet Mass Distribution