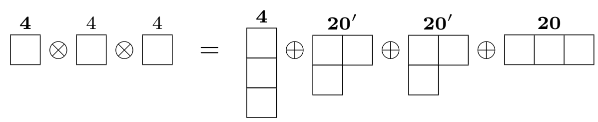
Exercises: QCD at large Nc

(29/5/13 - C.Schat)

1. Derive the Fierz identity for $SU(N)$ for the fundamental representation

$$(t^A)^a_b (t^A)^c_d = \frac{1}{2}\delta^a_d \delta^c_b - \frac{1}{2N}\delta^a_b \delta^c_d \qquad (1)$$

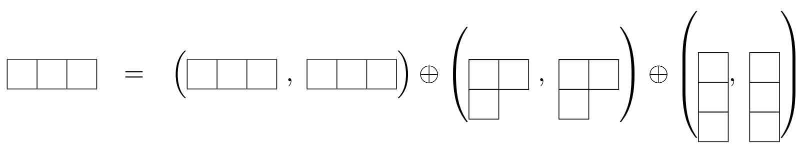
2. (a) For $SU(4)$ show that $\mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} = \mathbf{4} \oplus \mathbf{20'} \oplus \mathbf{20'} \oplus \mathbf{20}$ using Young tableaux:



(2)

   (b) Repeat for $SU(6)$. Identify the irrep of the ground state baryons.

3. Decomposition of $SU(XY)$ irreps in $SU(X) \times SU(Y)$. Ref: C.Itzykson, M.Nauenberg, Rev.Mod.Phys.38(1966)95. In the general case



(3)

   (a) Compute for $SU(6) \supset SU(2) \otimes SU(3)$ and identify the octet and decuplet states. Are there ground state baryons in a spin 1/2 decuplet ? Why ? How could you have baryons in an octet with spin 3/2 ?

4. (a) Derive the N_c scaling of the m-meson vertex.

   (b) Derive the N_c scaling of the m-mesons and r-glueball vertex.

5. (a) Derive the 't Hooft equations

$$\mu^2 \phi(x) = \left(\frac{M^2}{x} + \frac{M^2}{1-x}\right)\phi(x) - \frac{g^2}{\pi}\int_0^1 \mathrm{dy}\,\frac{\mathrm{P}}{(\mathrm{x}-\mathrm{y})^2}\phi(\mathrm{y}) \qquad (4)$$

   where $M^2 = m^2 - g^2/\pi$.

   (b) Discuss the solutions.

   Refs: G.'t Hooft Nucl.Phys.B75(1974)461 (original calculation); S.Coleman, 1/N lectures in Aspects of Symmetry (pedagogical presentation); R.L.Jaffe, P.F.Mende, Nucl.Phys.B369(1992)189 (numerical solution and plots).

6. (a) Write the most general mass operator for ground state baryons, including the explicit ϵ and $1/N_c$ factors.

$$M = c_0 + c_1 S^2 + c_2 T^8 + c_3 S^i G^{i8} + c_4 S^2 T^8 + c_5 T^8 T^8 + c_6 T^8 S^i G^{i8} + c_7 T^8 T^8 T^8 \qquad (5)$$

(b) Compute the matrix elements $\langle M \rangle$ between the baryons in the octet and the decuplet (ground state baryons $N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega$). Show first that

$$\begin{aligned} \langle T^8 \rangle &= \frac{1}{\sqrt{12}}(N_c - 3N_s) \qquad (6) \\ G^{i8} &= \frac{1}{\sqrt{12}}(S^i - 3S_s^i) \qquad (7) \end{aligned}$$

$$S^2 - 3S \cdot S_s = \frac{1}{2}\left(3I^2 - S^2 - 3S_s^2\right) \qquad (8)$$

where N_s is the number of strange quarks and S_s^i the spin of the strange quarks.

(c) Derive the mass relation

$$\Delta - 3\Sigma^* + 3\Xi^* - \Omega = \mathcal{O}(\epsilon^3/N_c^2) \qquad (9)$$

Refs: E.E.Jekins, R.F.Lebed, Phys.Rev. D52 (1995) 282; A.Manohar, hep-ph/9802419 (Large N QCD).

7. For the subspace of ground state baryons, proof the operator identity

$$4I^a G^{ia} = (N_c + 2)S^i \qquad (10)$$

Hint: Use the Fierz identity for Pauli matrices and the spin-flavor symmetry of the states.
Ref: A.Manohar, hep-ph/9802419 (Large N QCD).