

Broad-area lasers, laser solitons and patterns in optics

Part II: Laser solitons



Thorsten Ackemann



Solitons started in Scotland

"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; ..., assuming the form of a **large solitary elevation**, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently **without change of form or diminution of speed**. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour [14 km/h], preserving its original figure some thirty feet [9 m] long and a foot to a foot and a half [300–450 mm] in height. Its height gradually diminished, and after a chase of one or two miles [2–3 km] I lost it in the windings of the channel. ... Singular and beautiful

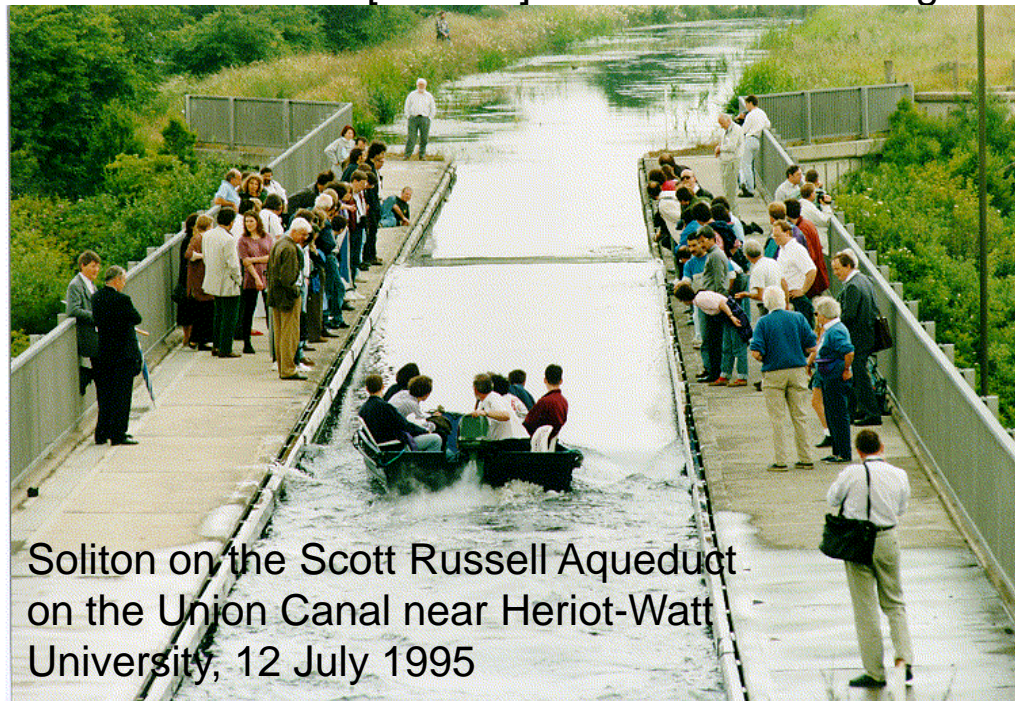
phenomenon which I have called the **Wave of translation**.

- Published: 1845
Russell, **Report on Waves**.

Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844.

- Impact: 1960s
computers, plasmas, ...

- Fibre optics: 1970s
<http://www.ma.hw.ac.uk/solitons>



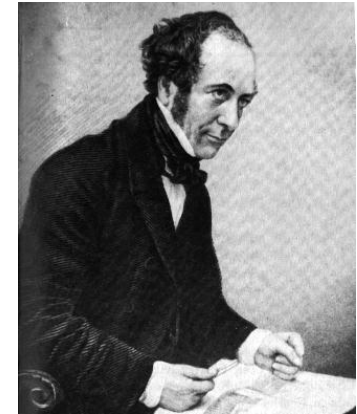
Soliton on the Scott Russell Aqueduct
on the Union Canal near Heriot-Watt
University, 12 July 1995

Solitons started in Scotland



Coast (BBC2 24/04/13)

John Scott Russel



<http://www.ma.hw.ac.uk/solitons>

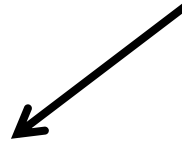
1834: “... large solitary elevation, without change of form or diminution of speed.”

Russell, **Report on Waves.**

Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844.

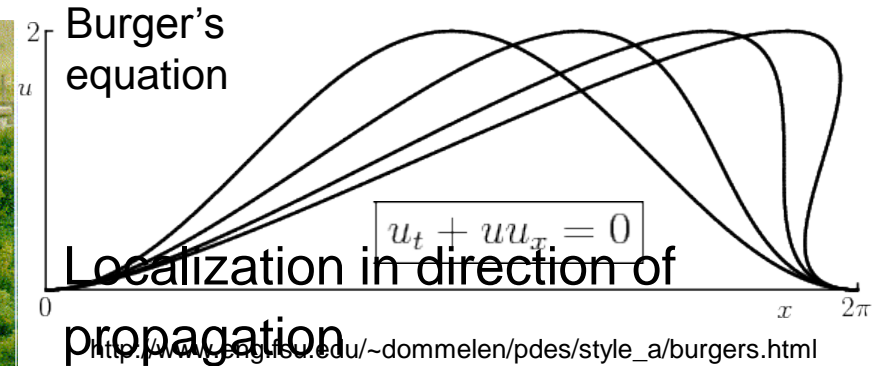
Solitons: Origin

Wave propagation includes **dispersion** and **nonlinearities**



balancing of **Pulse broadening** and **Steepening, shocks, new frequencies**

Soliton = **wave packet propagating with constant shape**



→ **Temporal soliton**

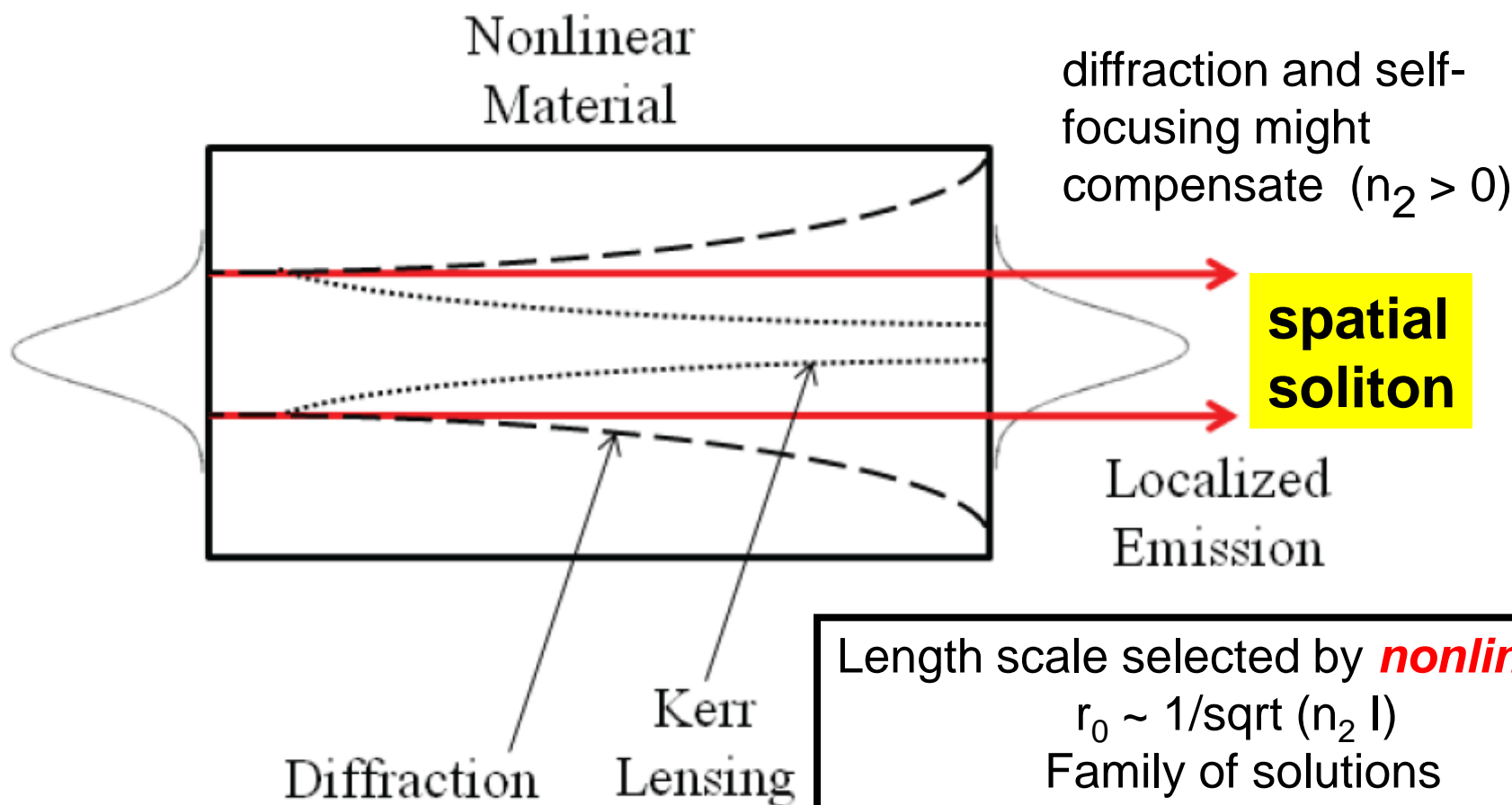
longitudinal soliton
meaning self-localized in
direction of propagation

Optical transverse soliton (propagation)

Nonlinear Schroedinger equation (NLS) for propagation in z

$$\frac{\partial}{\partial z} \mathcal{E} = -i \left[\frac{1}{2k_l} \nabla_{\perp}^2 \mathcal{E} + k_l n_2 |\mathcal{E}|^2 \mathcal{E} \right]$$

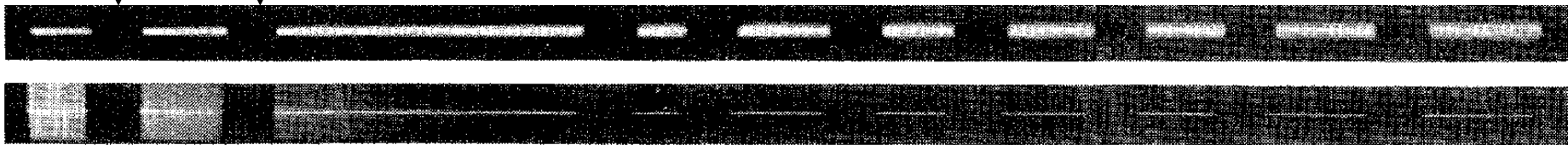
conservative



Experiment

side view of a cell containing sodium vapor
mounting straps

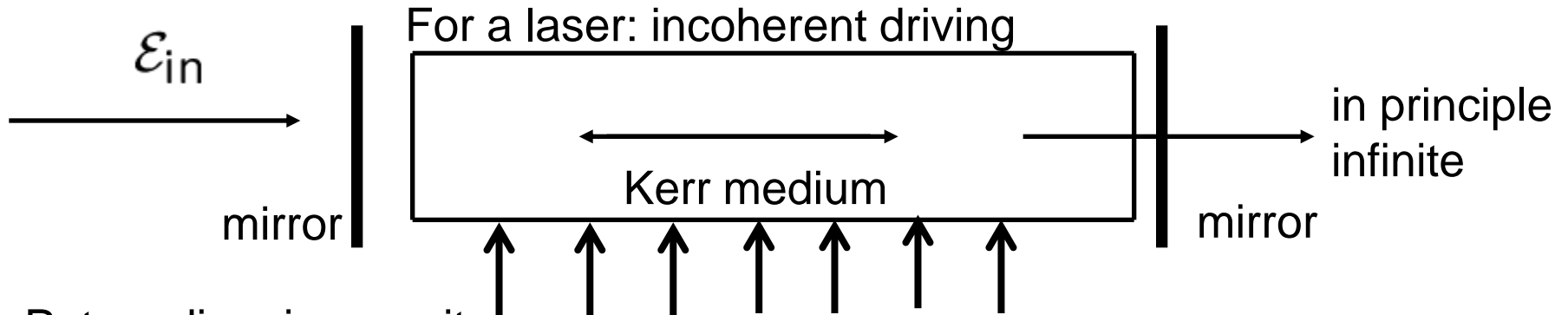
propagation direction →



Na (Bjorkholm and Ashkin 1974)

- linear propagation (top): diffractive spreading
- nonlinear propagation (bottom): self-guiding, spatial soliton
- pure $\chi(3)$ Kerr nonlinearity → collapse in 2D
(need to stabilize by high-order terms,
e.g. in Na, two-level atom saturable nonlinearity, “saturable Kerr”)
- in a propagation experiment you will never know whether your soliton is long-term stable because you run out of material

Cavity solitons

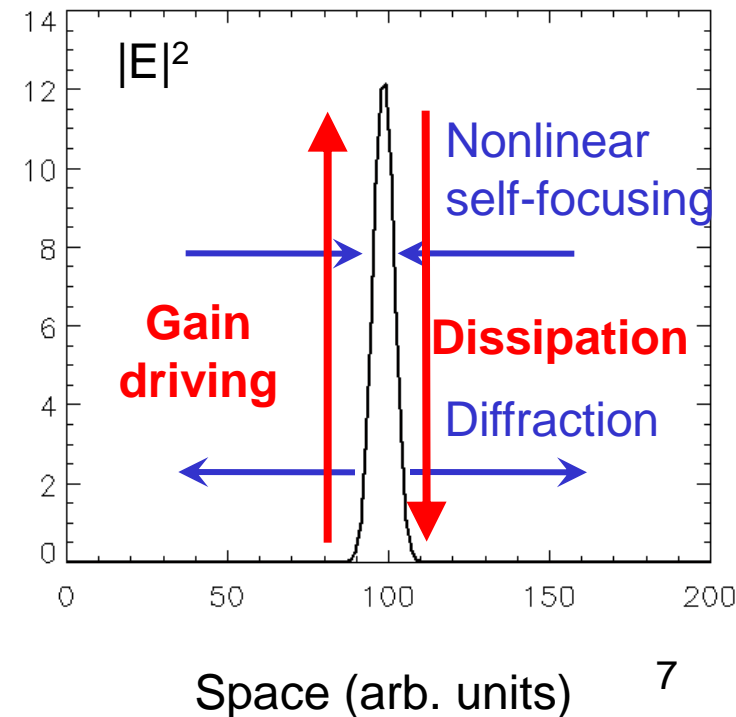


- Put medium in a cavity
 - propagation replaced by time evolution
- **Dissipation**: light leaks out of the mirrors
 - compensate by **driving**
- second balance condition:
 - family collapses to single solution

"attractor"

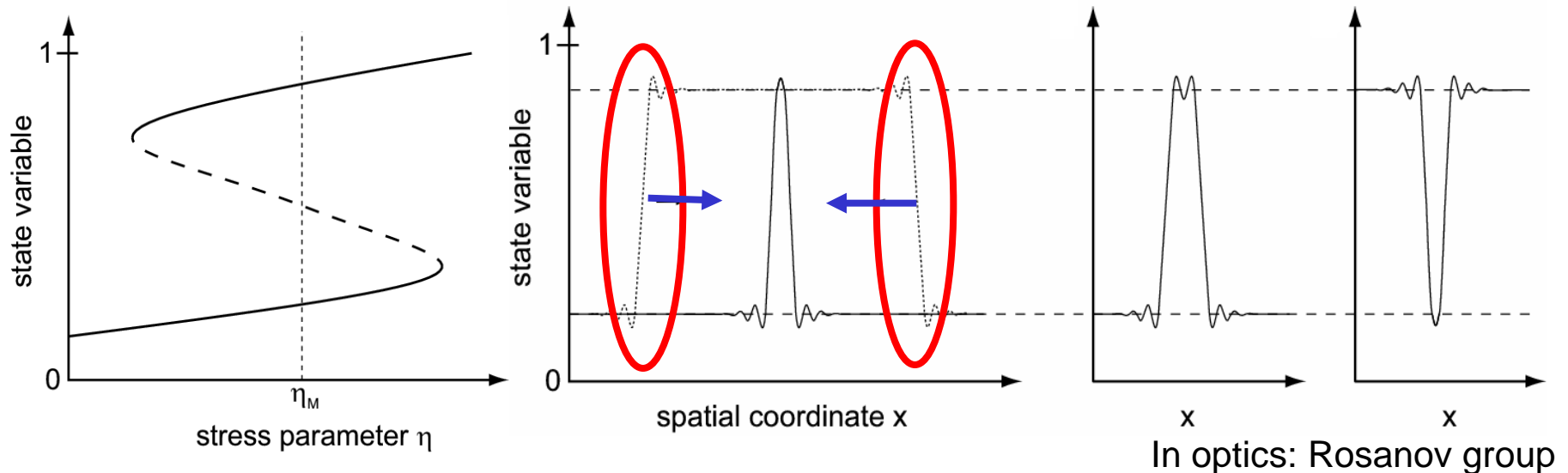
First approach to cavity solitons:

„soliton in a box“



What is a dissipative soliton?

- imagine **bistability** between two states: e.g. low and high amplitude



- in a spatially extended system different spatial regions might be in different states → in between there will be a **front!**
- this front can **move** → one state invades the other and the system becomes homogenous again
- fronts can **lock** and leave an island of one state in the other
→ **localized state, localized structure, or dissipative soliton**

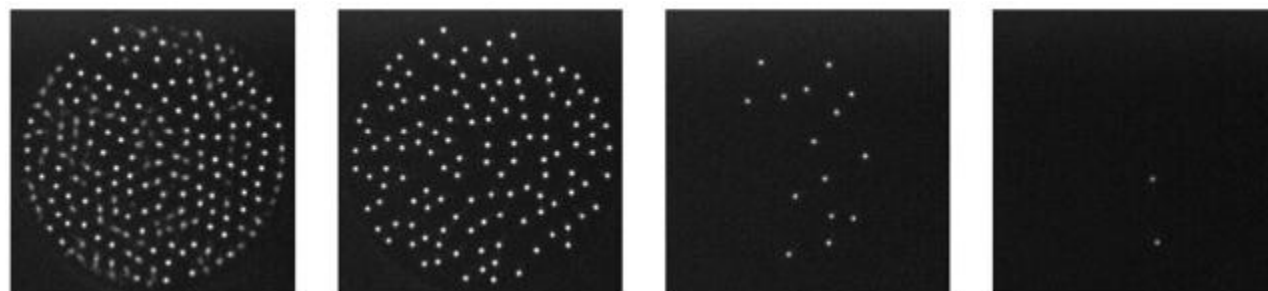
Ubiquitous in Nonlinear Science ...

Many aspects of VCSEL patterns and solitons are **universal** for self-organizing spatially extended systems driven out of **thermodynamic equilibrium**

- nonlinear optics
- hydrodynamics
- gas discharges
- ...

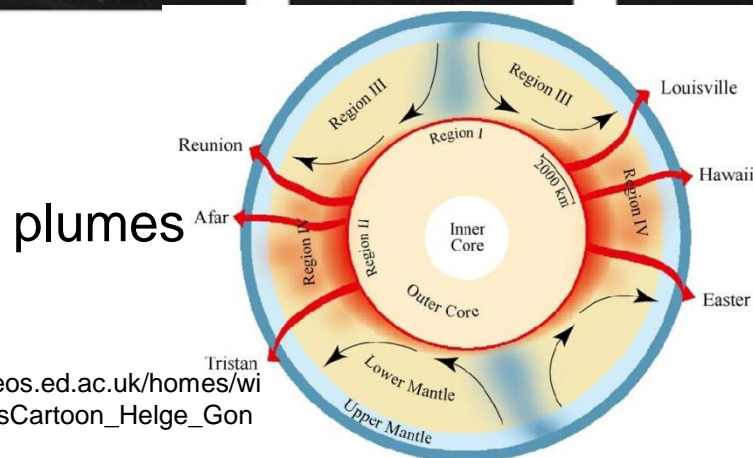
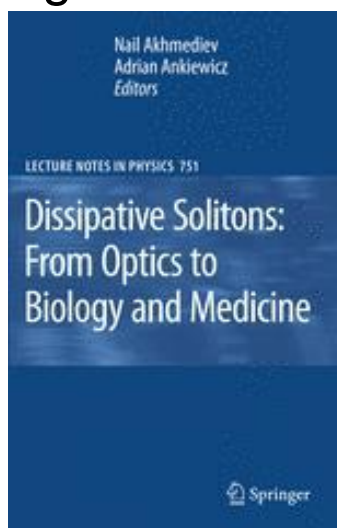
AC gas discharge

Purwins et al., Adv. Phys. 59, 485 (2010)

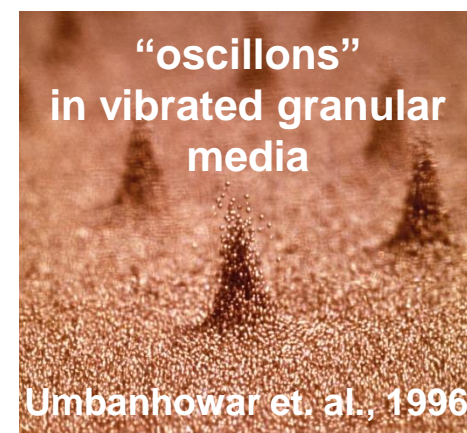


sodium vapour +
optical feedback

Ackemann+Lange
(U Münster)



plumes



Umbanhowar et. al., 1996

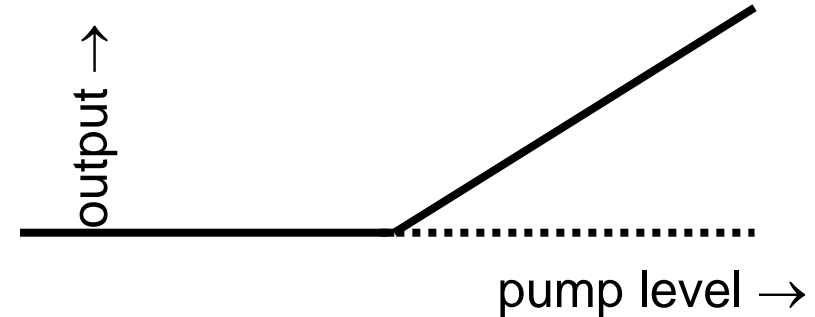
- chemistry
- biology
- nature

➤ Not only structured,
but **self-localized**

http://www.geos.ed.ac.uk/homes/williams/PlumesCartoon_Helge_Gonermann.jpg

Cavity soliton laser needs bistability

But **normal** laser has a continuous turn-on from threshold: **no** cavity solitons.



bistable laser schemes

laser with
injected signal

laser with
frequency-selective feedback

laser with
saturable absorber

gain

gain

filter

gain

SA

Hachair et al.,
IEEE Sel. Top. QE **12**,
339 (2006)

Truly free running laser with phase invariance

Go for this!

First semiconductor based CSL:
Tanguy et al., PRL **100**, 013907 (2008)

Genevet et al., PRL **101**, 123905 (2008)

Still slaving by external phase reference

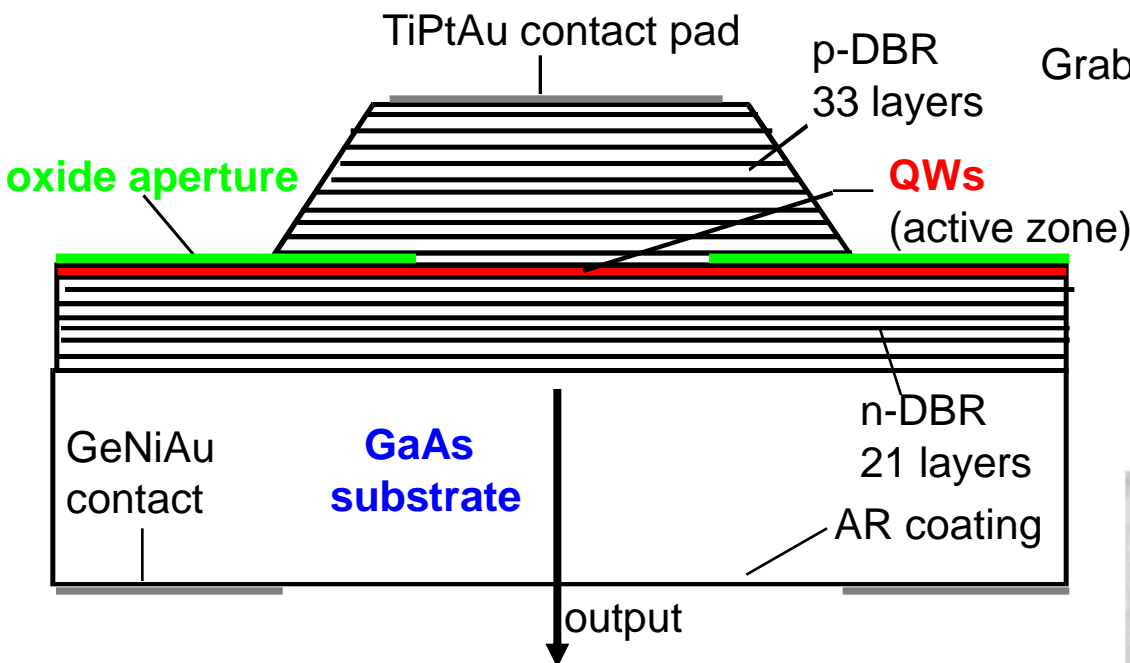
first CSL using photorefractives, dyes: Bazhenov et al. (1992); Saffman et al. (1994); Taranenko et al. (1997)

Thanks

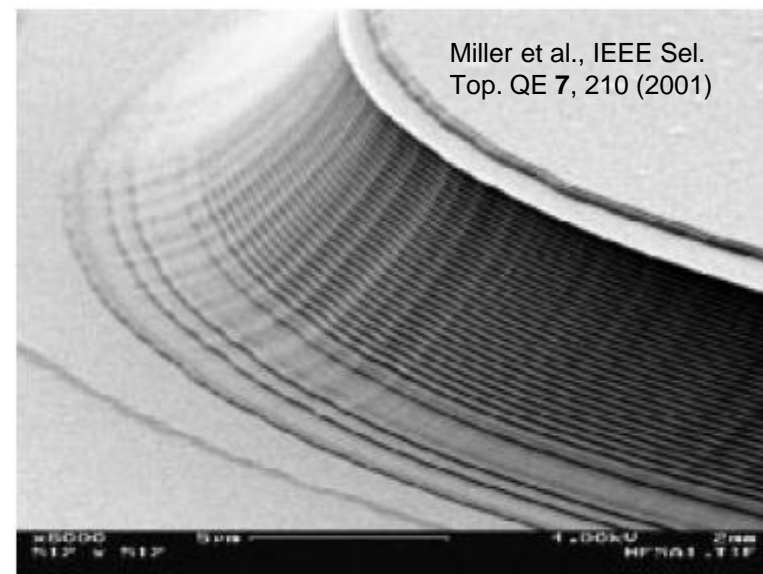
Cavity soliton laser:

- Experiment: Y. Noblet*, J. Jimenez**, N. Radwell*, Y. Tanguy
- Devices: R. Jaeger (Ulm Photonics)
- Theory: C. McIntyre*, W. J. Firth, G.-L. Oppo (Strathclyde),
P. V. Paulau (Minsk, Strathclyde, Palma, now University of Oldenburg),
D. Gomila, P. Colet (IFISC, Palma de Mallorca),
N. A. Loiko (Minsk), N. N. Rosanov (St. Petersburg)
- Funding: *EPSRC DTA, ** Conayt, EU FP6 FunFACS, British Council,
Royal Society, DAAD

Our devices: High power VCSELs



Grabherr et al., IEEE STQE **5**, 495 (1999)



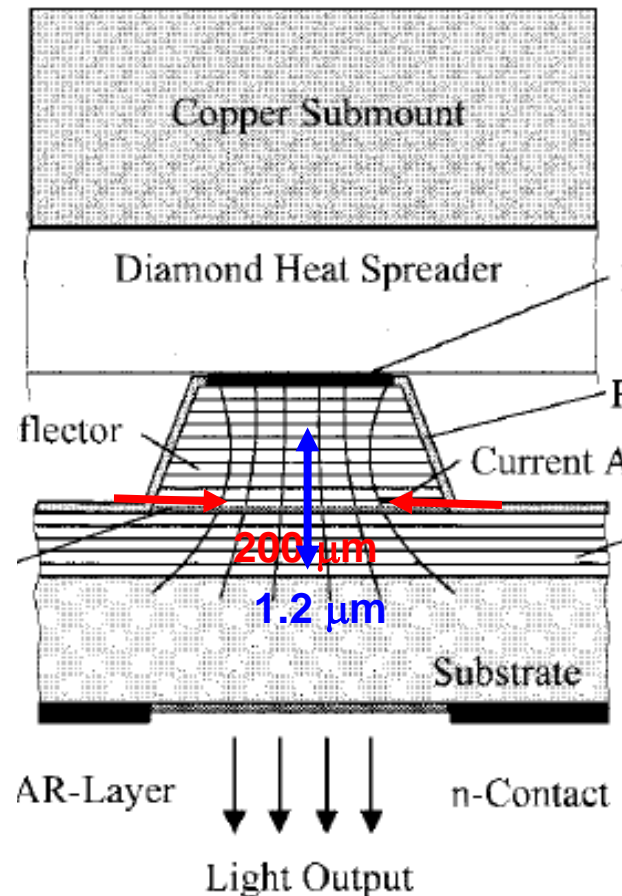
Miller et al., IEEE Sel. Top. QE **7**, 210 (2001)

Fig. 2. Wet chemically etched mesa with full p-contact, p-type DBR, and active region.

- **three InGaAs/GaAs quantum wells**
(gain maximum ≈ 980 nm)
- **oxide layer**
→ current and optical confinement
- **Emission through substrate**

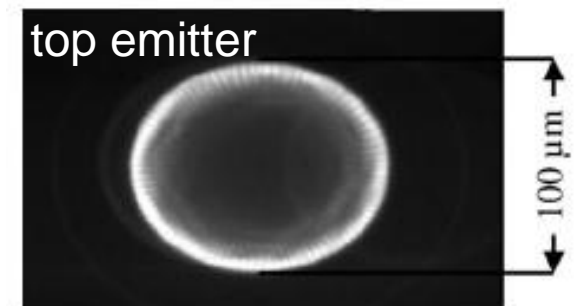
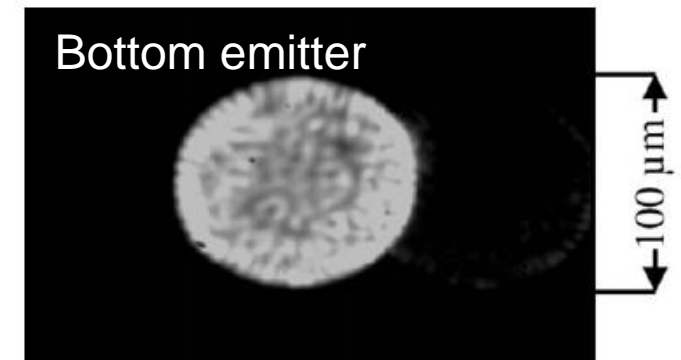
Technology and mounting

Grabherr et al., IEEE STQE **5**, 495 (1999)



A **disk**, not a tube!

- for wavelength > 880 nm:
emission through
transparent substrate
→ **bottom emitter**
- heat sinking from top
- much more homogeneous



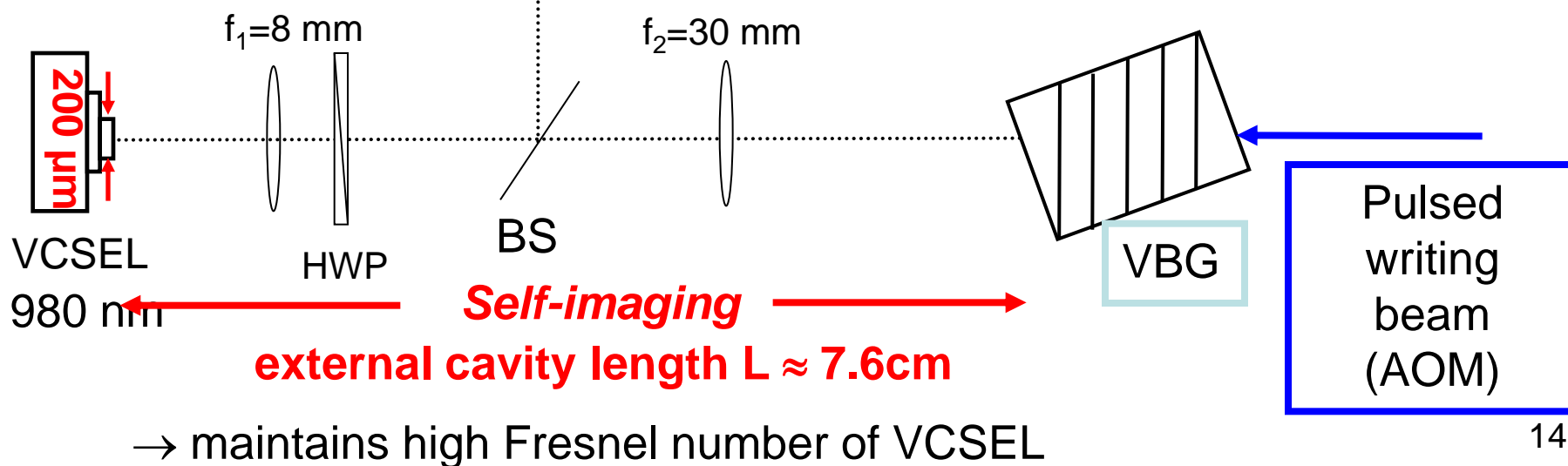
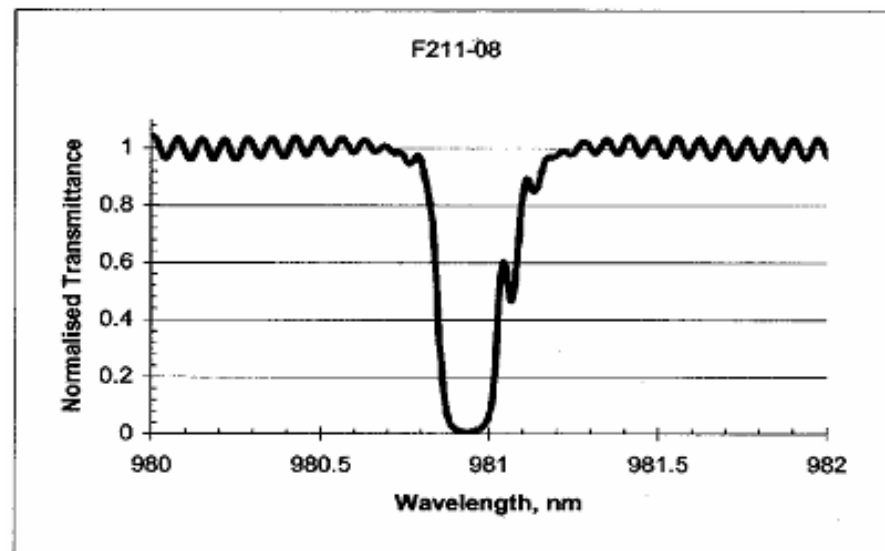
Setup with volume Bragg grating

Volume Bragg grating VBG:

Compact frequency-selective element

Radwell+Ackeman, IEEE QE **45**, 1388 (2009)

Detection Branch:

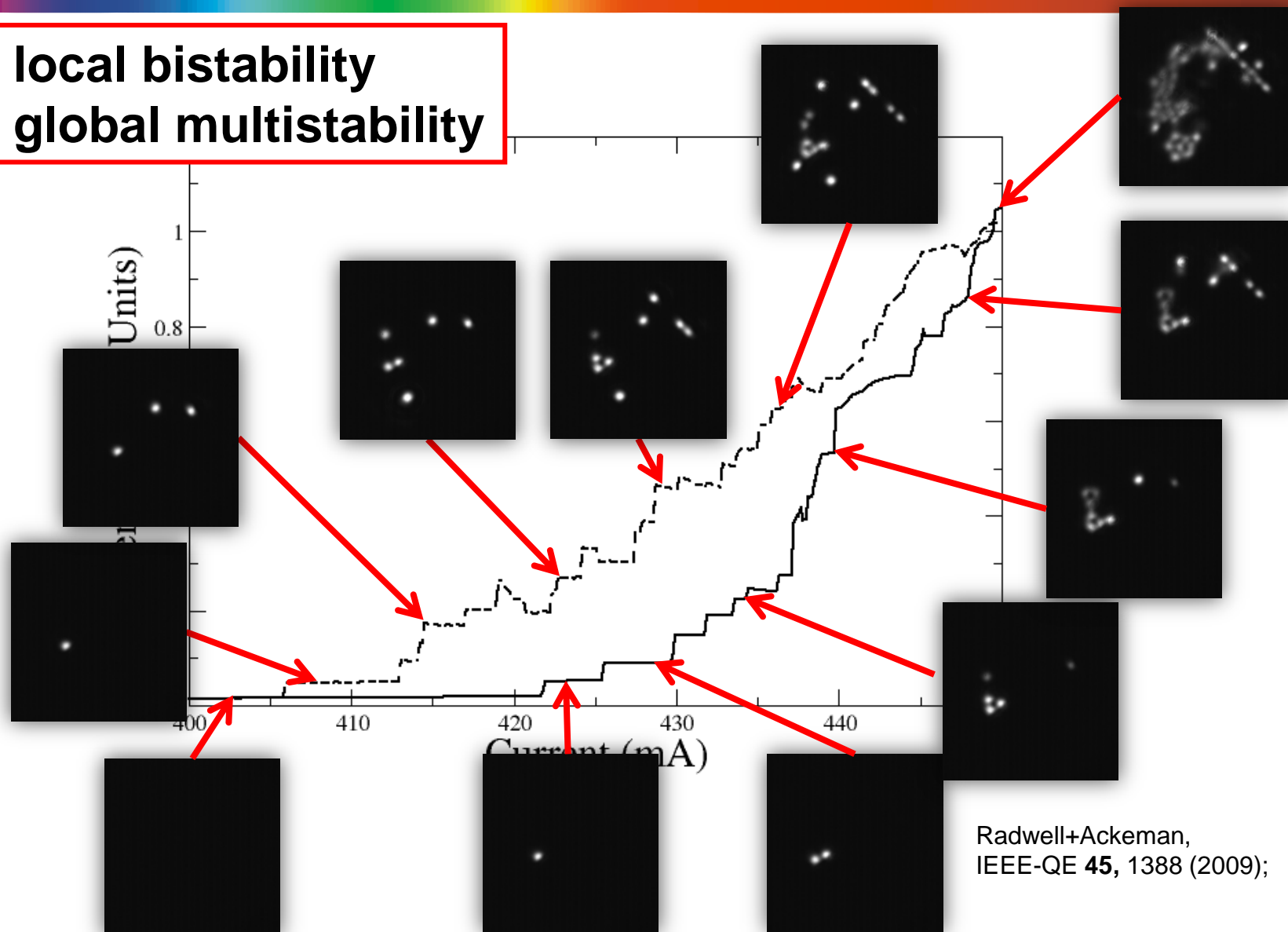




Experiment: Current ramp

LI-curve of whole device

- ✓ local bistability
- ✓ global multistability



Radwell+Ackeman,
IEEE-QE **45**, 1388 (2009);

Current sweeps detuning, not gain

Switching threshold of first soliton

Switch-on

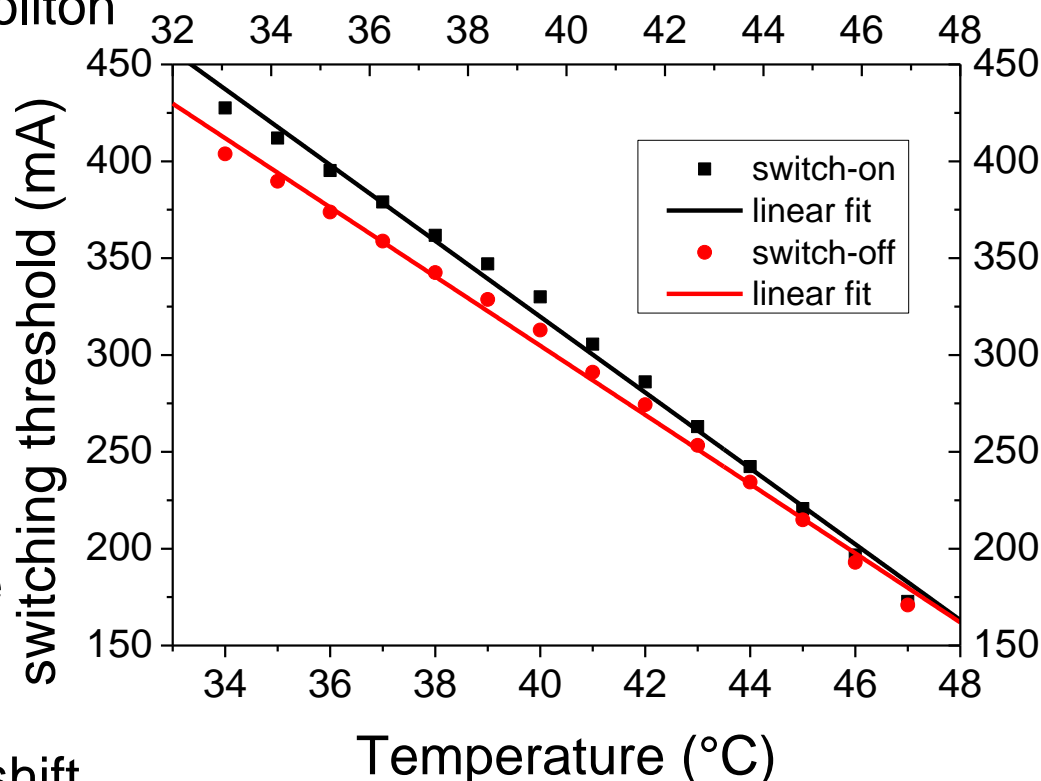
$$dI / dT \approx -19.6 \pm 0.4 \text{ mA/K}$$

Switch-off

$$dI / dT \approx -17.9 \pm 0.4 \text{ mA/K}$$

From other measurements:

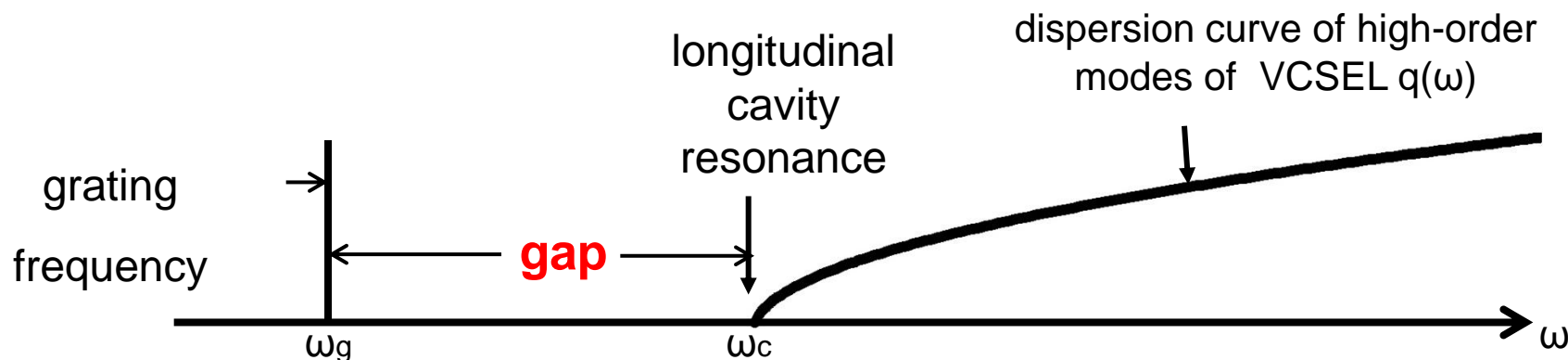
- Shift of cavity resonance with temperature
 $d\lambda / dT \approx 0.066 \text{ nm/K}$
- Ohmic heating induced shift
 $d\lambda / dI \approx 0.0035 \text{ nm/mA}$
- Combined
 $1 / (dT / dI) \approx 19 \text{ mA/K}$



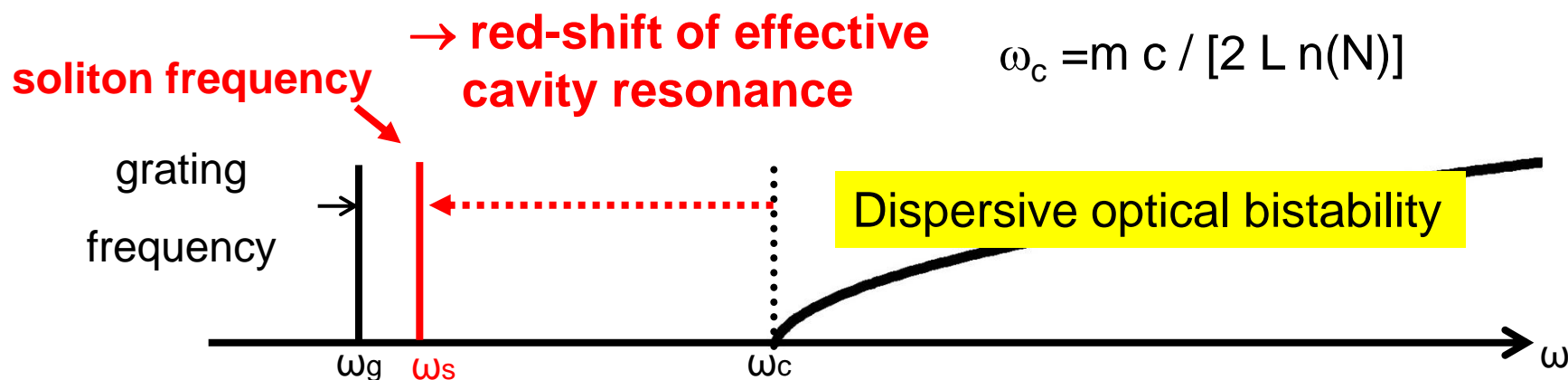
Main effect of current is **red-shift of resonance** until **detuning** between cavity and VBG so small that switching occurs via carrier nonlinearity

Bistability: Qualitative interpretation

Low-amplitude state: laser off, carrier density high, refractive index low



High-amplitude state: laser on, carrier density low \rightarrow refractive index high

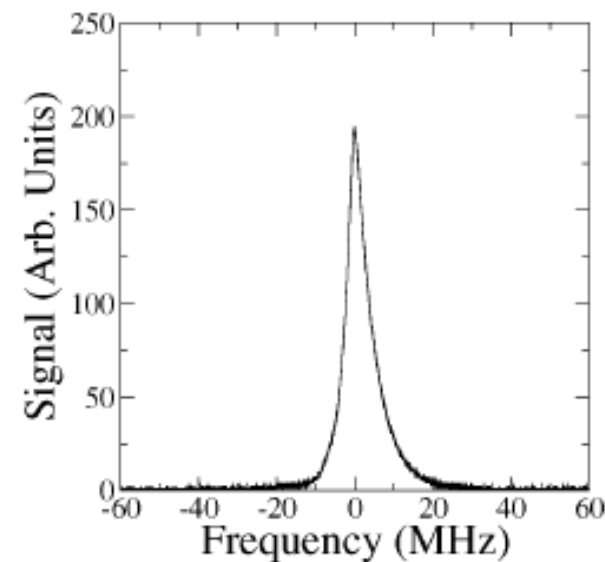
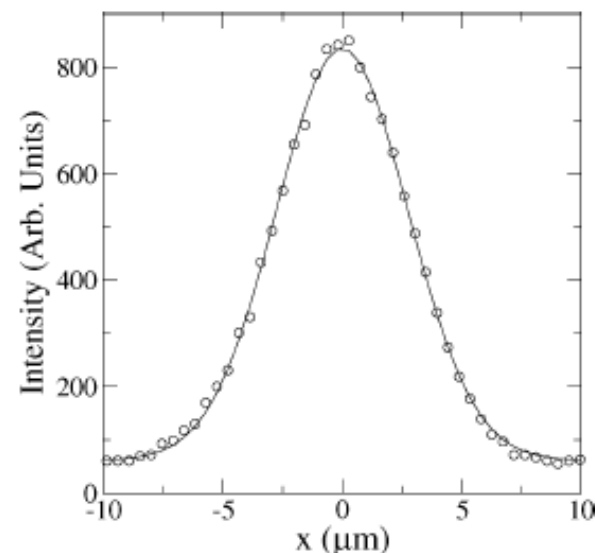


Localized high-amplitude state stabilized by self-focusing

Coherent emitters

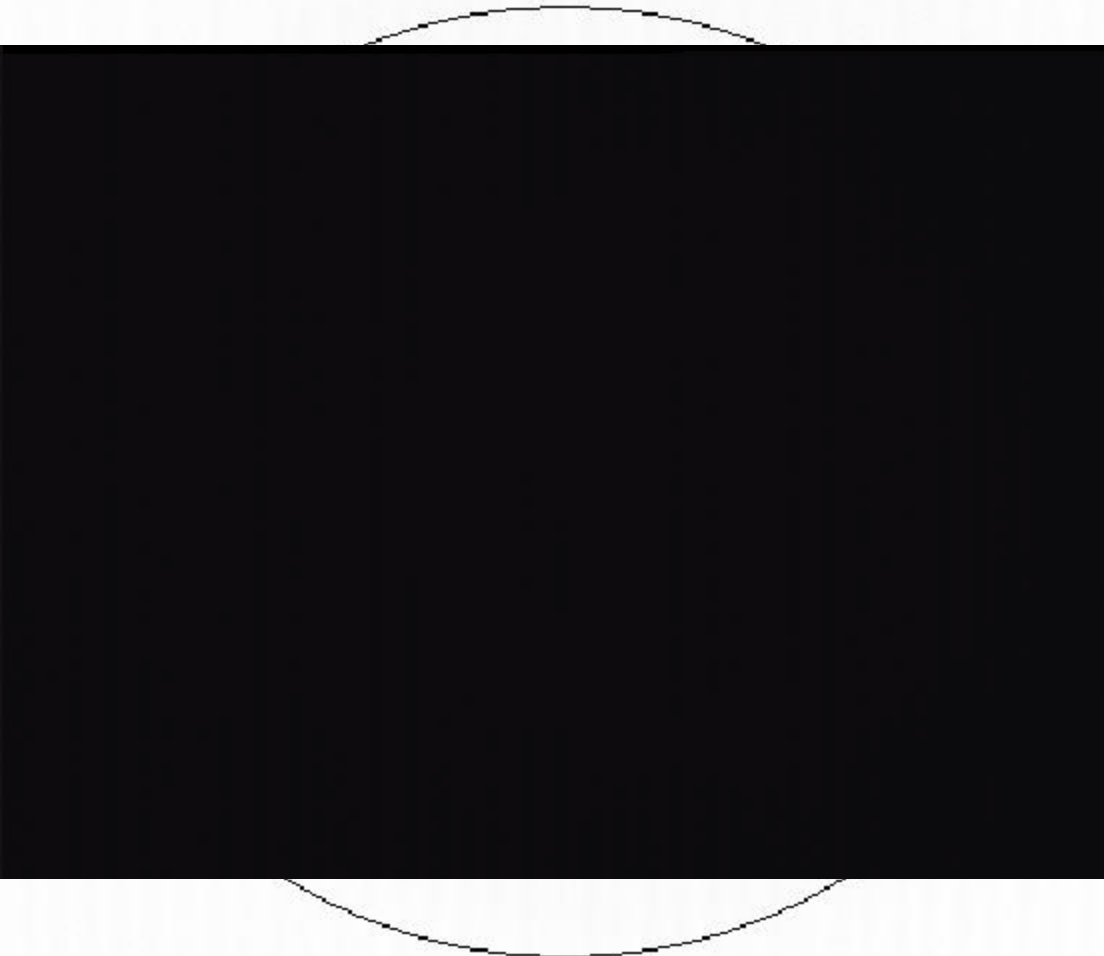
- Well defined, **circularly symmetric spots** with size of 4.8-5.8 μm ($1/e^2$ -radius of intensity)
- Angular width in far field: 57-69 mrad (centre on-axis within ≤ 18 mrad)
- Close to being diffraction-limited
→ **high spatial coherence**
- Linewidth in single-mode: ≈ 6 MHz
→ **high temporal coherence**
- these are coherent emitters → ***microlasers***

Mutual coherence? -- Later



Spatial solitons: Switch-on/-off

Transverse plane (\perp cavity axis)
of broad-area VCSEL



- bright spots much smaller than pumped aperture
- stabilized by nonlinearities
- bistable

demonstration of
independent writing and
erasure:
all 8 possible
configurations of 3 bits
→ **solitonic character**

→ ***Laser (cavity) solitons***

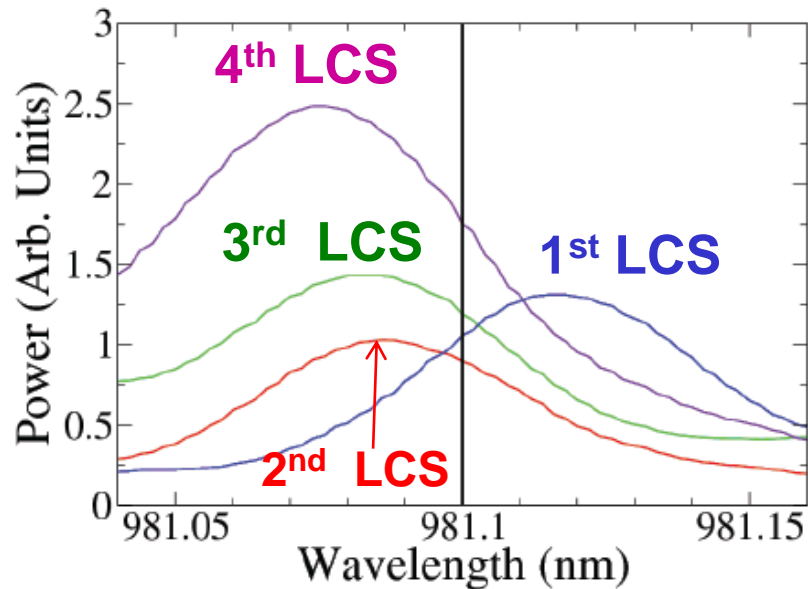
Independence and mobility



- **“Stir”** LCS with cw writing beam
- WB attracts LCS and pulls them out of center of trap
- moves them from one trap to another
- pulls them where they can't exist
- mechanism for **erasure**

- VBG isotropic → no preferred direction for switch-on/switch-off
- but symmetry broken by imperfections of device

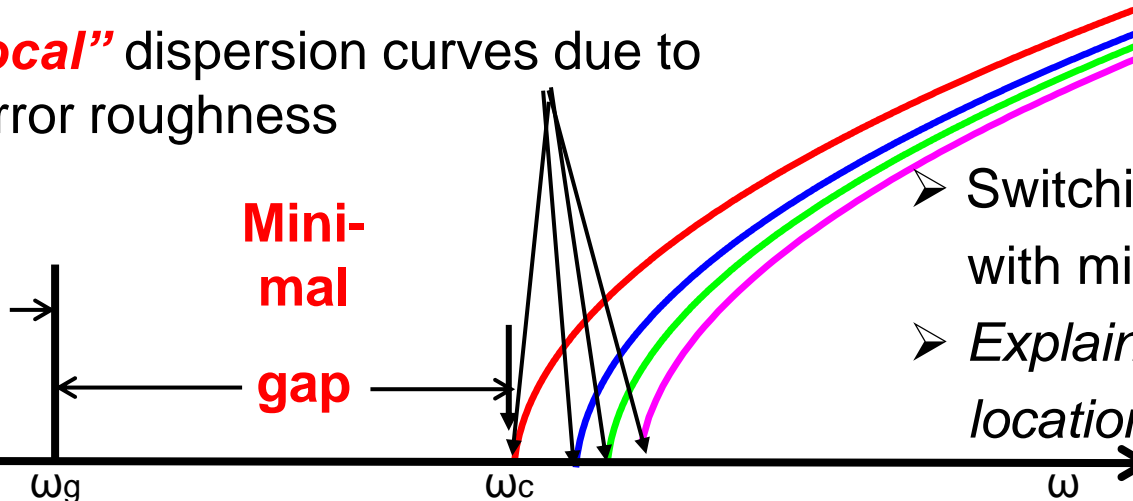
Frequency of solitons and disorder



- Different solitons have different threshold frequencies
- The most **reddish** solitons come first!
- Threshold increases with increasing detuning

“local” dispersion curves due to mirror roughness

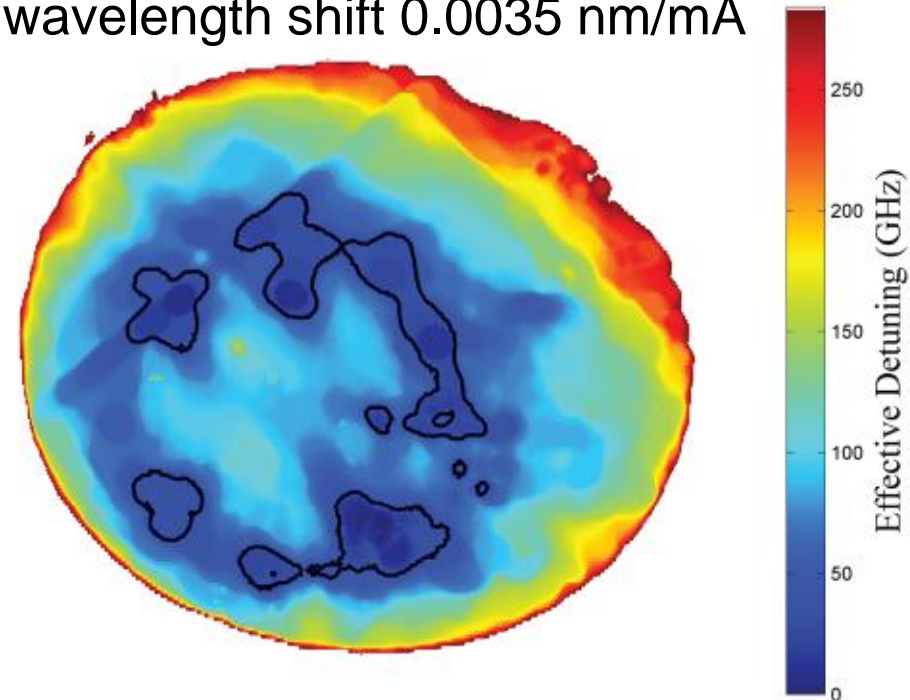
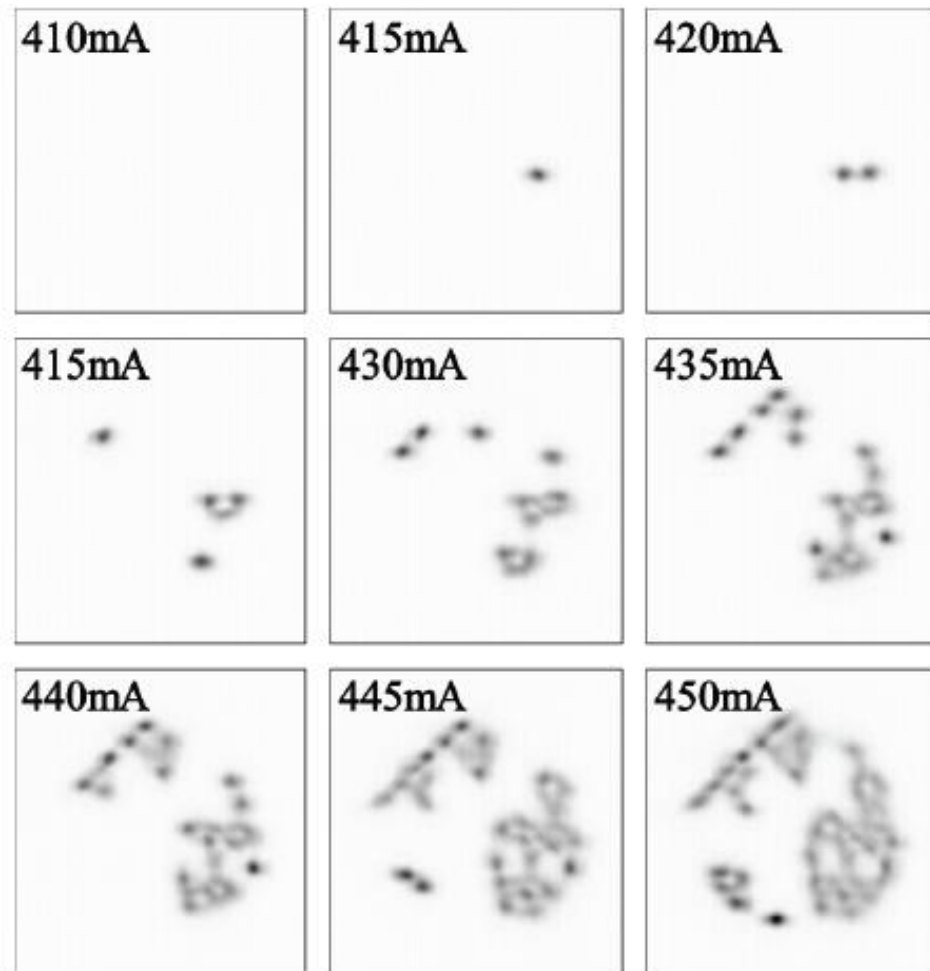
**Mini-
mal
gap**



- Switching will occur at locations with minimal gap
- Explains why most **reddish** locations switch first!

Application in disorder mapping

Convert threshold value to detuning by wavelength shift 0.0035 nm/mA

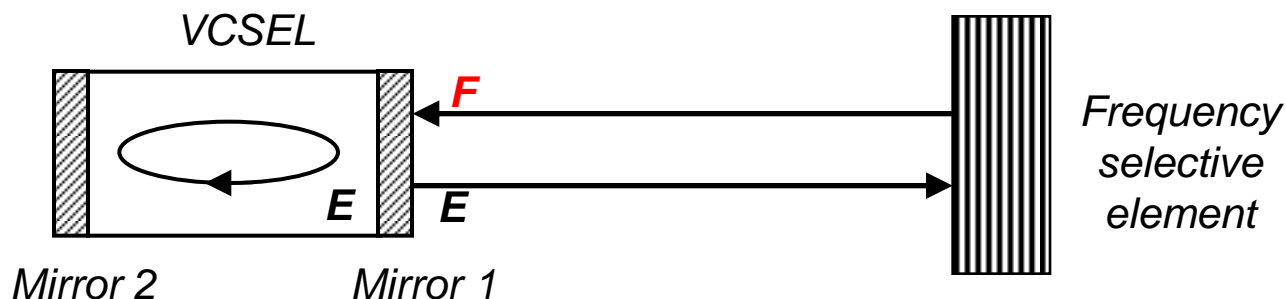


- potentially useful to characterize disorder in VCSEL on relevant scales with fairly high resolution, Opt. Lett. **37**,1079 (2012)
- ***monolayer fluctuations***

Theoretical model (class B)

Assume:

- perfect self-imaging
- feedback only dependent on frequency, not on wavevector



feedback

VCSEL field $\longrightarrow \partial_t E = -(1+i\theta)E + i\nabla^2 E - i\sigma(\alpha+i)(N-1)E + \frac{2\sqrt{T_1}}{T_1+T_2} F$

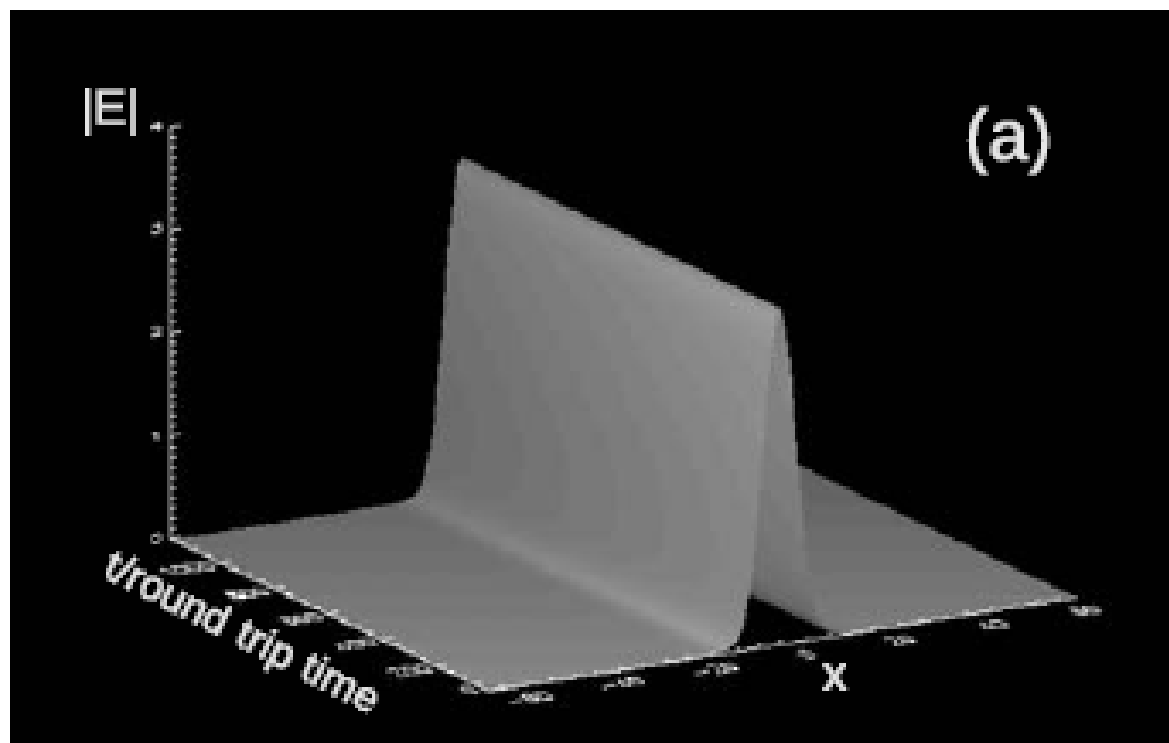
carriers $\longrightarrow \partial_t N = -\gamma [N - J + |E|^2 (N-1) + D\nabla^2 N] \quad \gamma \approx 0.01$

external cavity field $\longrightarrow F(t) = e^{-i\delta\tau_f} \hat{G}(t - \tau_f/2) [-r_1 F(t - \tau_f) + t_1 E(t - \tau_f)]$

frequency filter $\longrightarrow \hat{G}(t)[\cdot] = \frac{r_g}{2\beta} \int_{t-2\beta}^t e^{i\Delta_g(t'-t)} [\cdot] dt'$

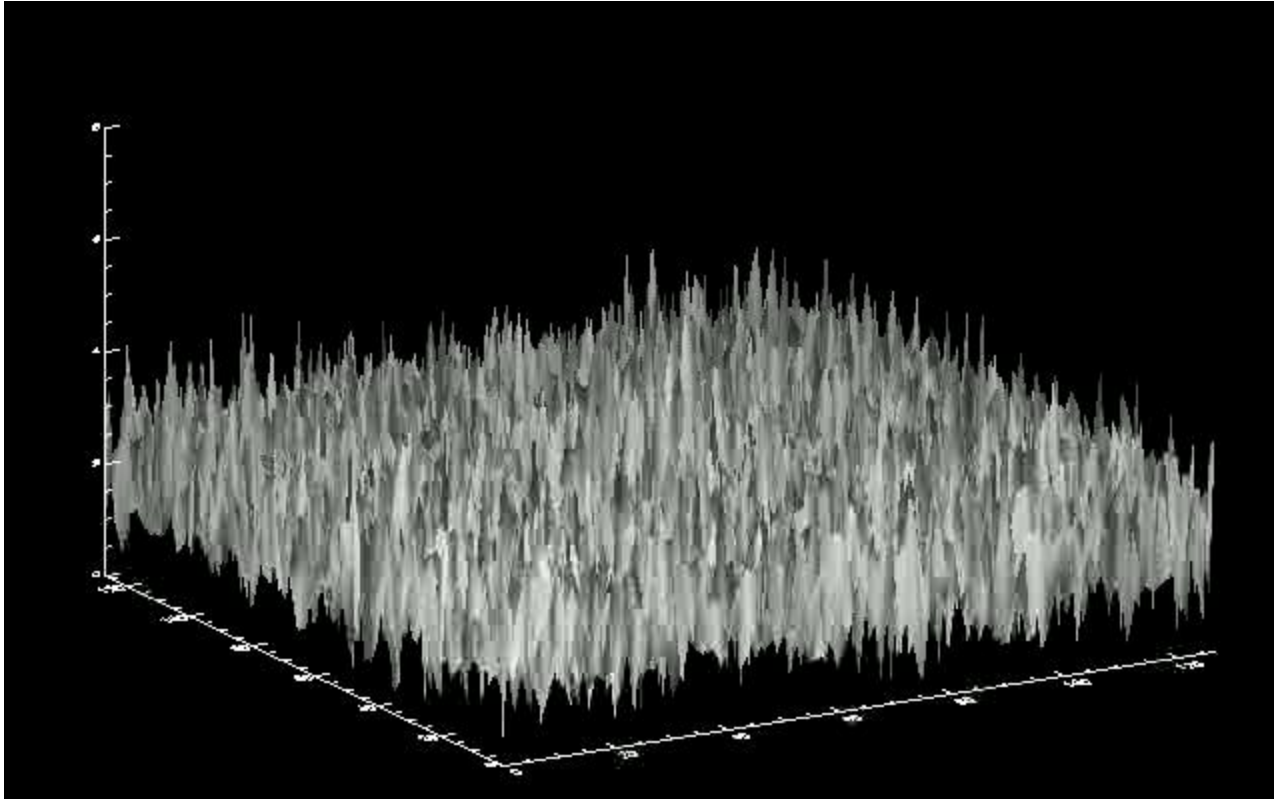
NOT Lang-Kobayashi:
All round-trips

Numerics (1D)



- Stationary LCS, single frequency, one external cavity mode
- Width $\approx 8 \mu\text{m}$

Works also in 2D



- Good reproduction of experimental results
- Applies also to transient dynamics (Radwell et al., Eur. Phys. J. D **59**, 121 (2010))
- Bifurcation structure and instabilities analyzed in simpler Ginzburg-Landau model plus linear filter (Paulau, Firth, ...)

A simpler class A model

- Adiabatic elimination of carriers
 - same stationary states but stability properties might change
 - but note: carriers slower than field, loss of relaxation oscillations
- Take into account delay or not

$$\frac{\partial E}{\partial t} = -\kappa E + \frac{\kappa\mu E}{1 + |E|^2} + \frac{i\alpha\kappa\mu E}{1 + |E|^2} - i\Delta_{\perp}E + F + i\omega_s E - i\alpha\kappa E$$

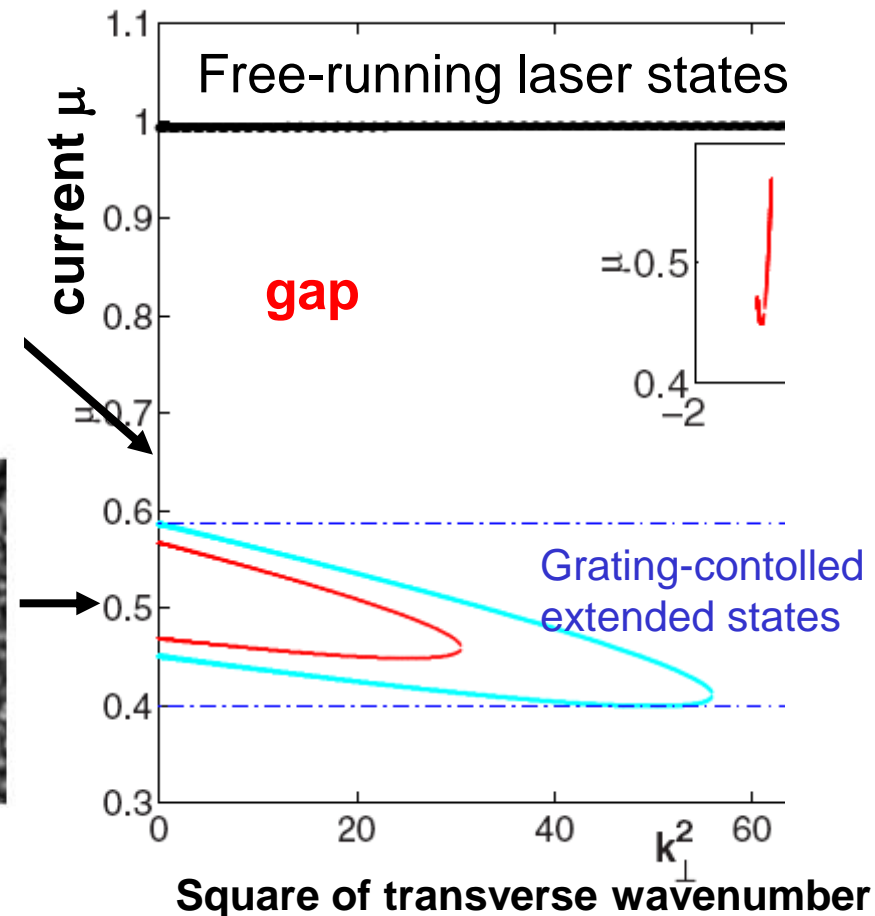
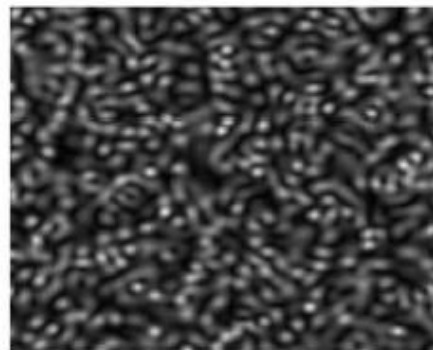
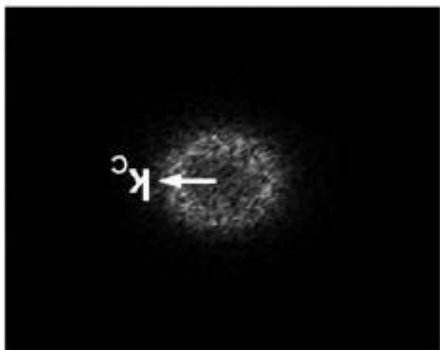
$$\frac{dF}{dt} = -\lambda F + \sigma\lambda E(t - \tau)$$

- Lorentzian filter
- Lang-Kobayashi for simplicity

Paulau et al., PRE **78**, 016212 (2008)

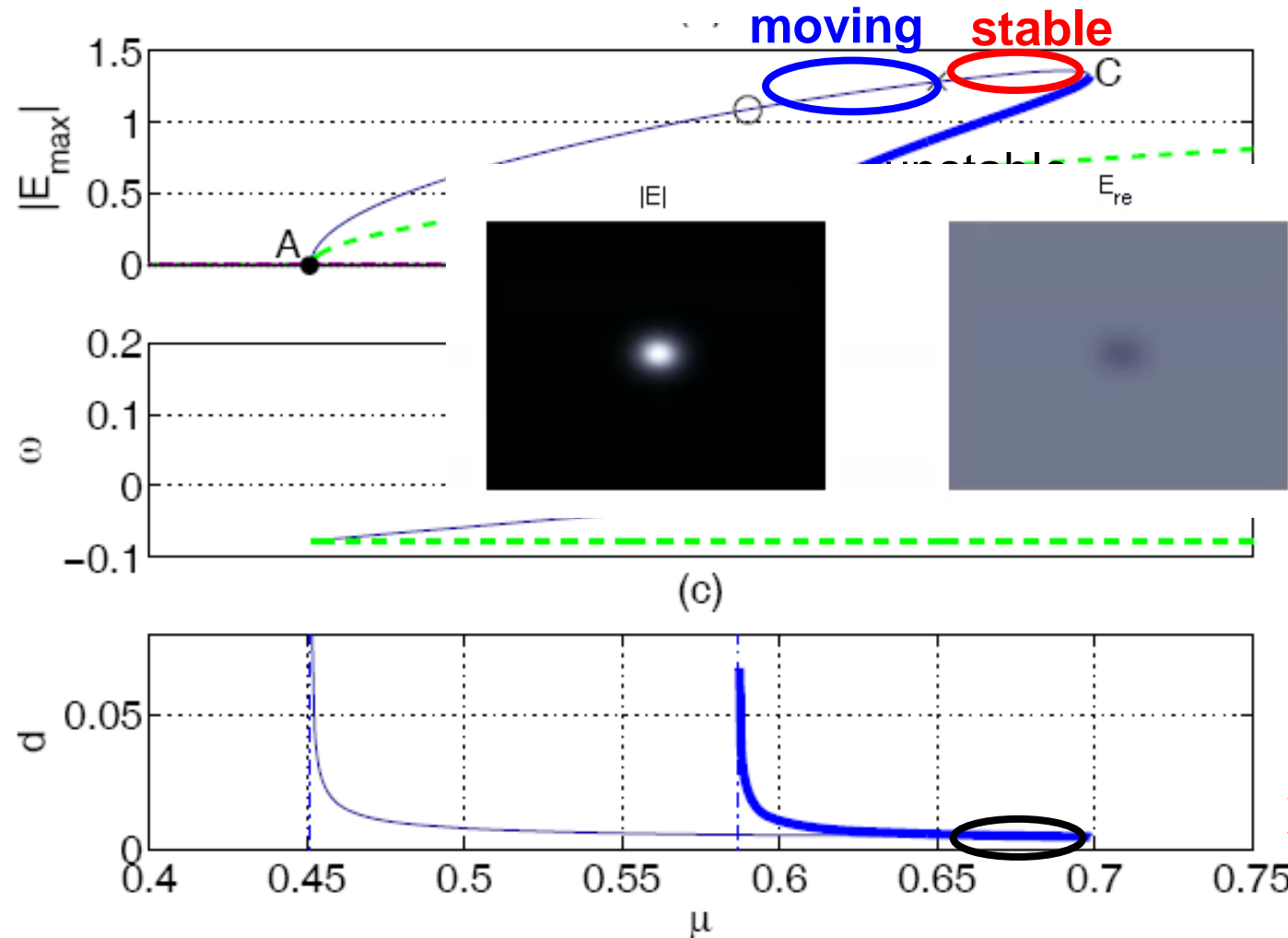
Filamentation vs. solitons

In **gap**: **solitons** with center on axis



Extended states with
irregular spatio-temporal dynamics
Interpretation: filamentation, modulational instability

Bifurcation diagram



- LCS branches start from homogeneous solution with infinite width, A, B merge in saddle-node bifurcation, C **stable** section:

- ✓ blue detuning to grating
- ✓ width 10-13 μm

- spontaneous motion: **drift instability**

Even simpler: Ginzburg-Landau model

Ginzburg-Landau model + linear filter, Firth + Paulau, Eur. Phys. J. D **59**, 13 (2010)

Linear loss (gain) and
frequency detuning

Nonlinear gain (loss) and self focusing (defocusing)

Diffraction

Filtered Feedback

$$\begin{cases} \frac{\partial E}{\partial t} = g_0 E + g_2 |E|^2 E + i\Delta_{\perp} E + F \\ \frac{dF}{dt} = -\lambda F + \tilde{\sigma} E, \end{cases}$$

Width of filter

Feedback strength

- in GLE: Chirped-sech soliton solutions known but unstable
- Stabilized by **coupling to resonant filter** suppressing background
- simplest model for laser with FSF

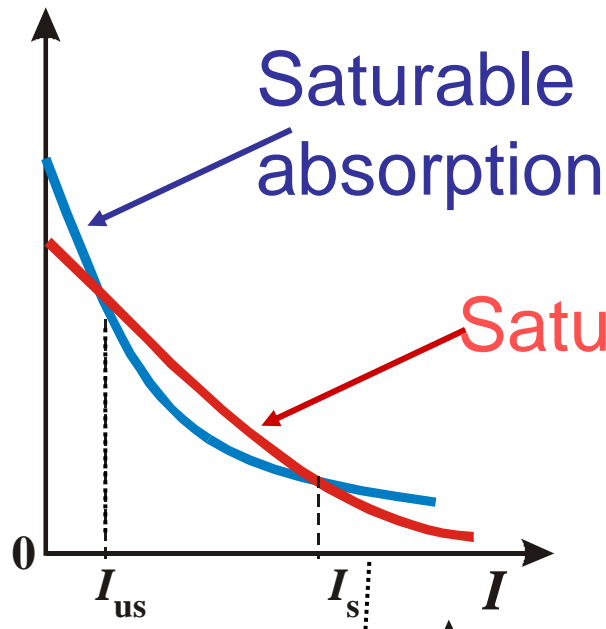
L.M. Hocking and K. Stewartson, Proc. R. Soc. Lond., A **326**, 289 (1972); N.R. Pereira and L. Stenflo, Phys. Fluids **20**, 1733 (1977).

P.A. Bélanger, L. Gagnon, and C. Paré, Opt. Lett. **14**, 943 (1989).

C. Paré, L. Gagnon, and P.A. Bélanger, Opt. Commun. **74**, 228 (1989).

$$\begin{aligned} g_0 &= \kappa(1 + i\alpha)(\mu - 1) - i\omega_m, \\ g_2 &= -\kappa(1 + i\alpha)\mu, \\ \tilde{\sigma} &= \sigma\lambda, \end{aligned}$$

Laser with saturable absorber



Saturable absorber:

Linear absorption coefficient at low power
Bleaching of absorption at high power

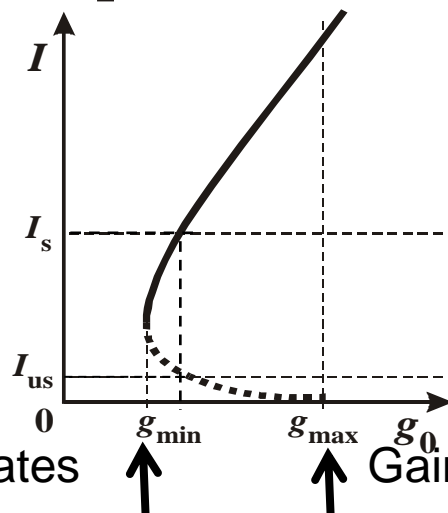
Saturable gain

Pioneer: **Rosanov** (St. Petersburg)
Here: Material from Cargese
summer school 2006

Bistability for

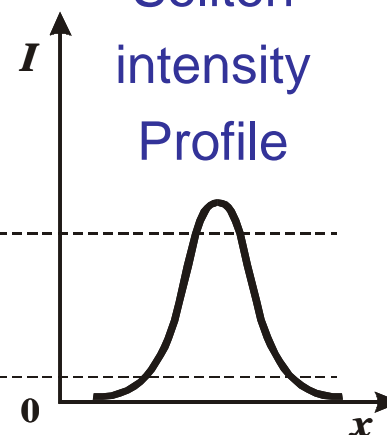
$$g_{\min} < g_0 < g_{\max}$$

Gain compensates
nonsaturable losses



Gain compensates saturable and
nonsaturable losses

Soliton intensity Profile



“Homoclinic Connection”

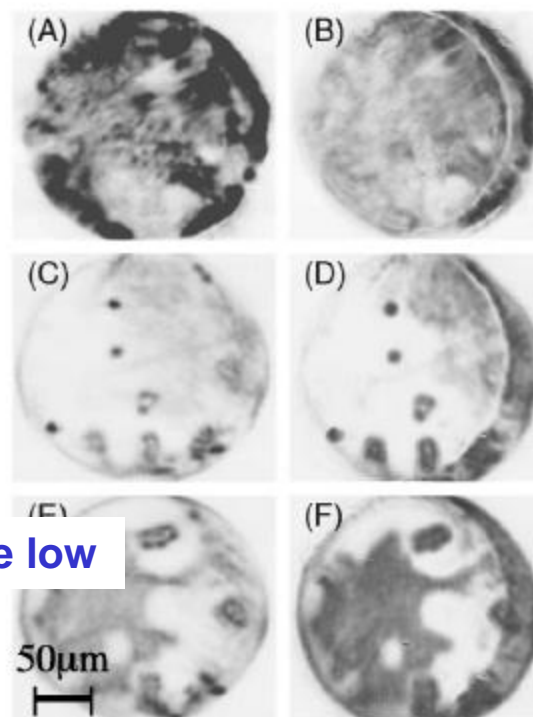
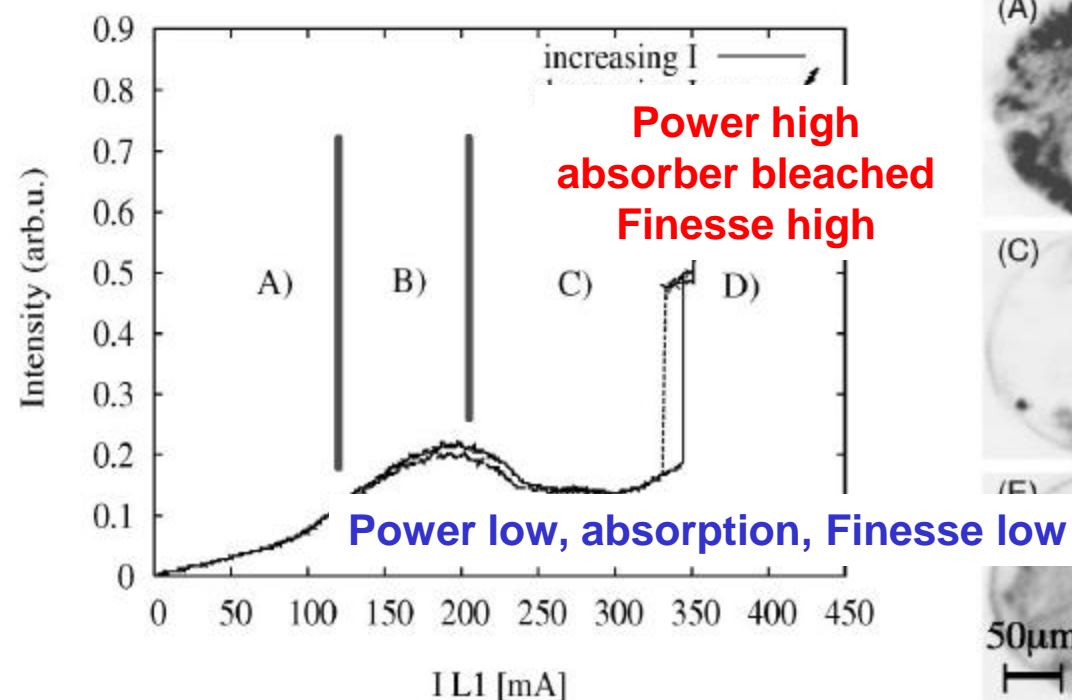
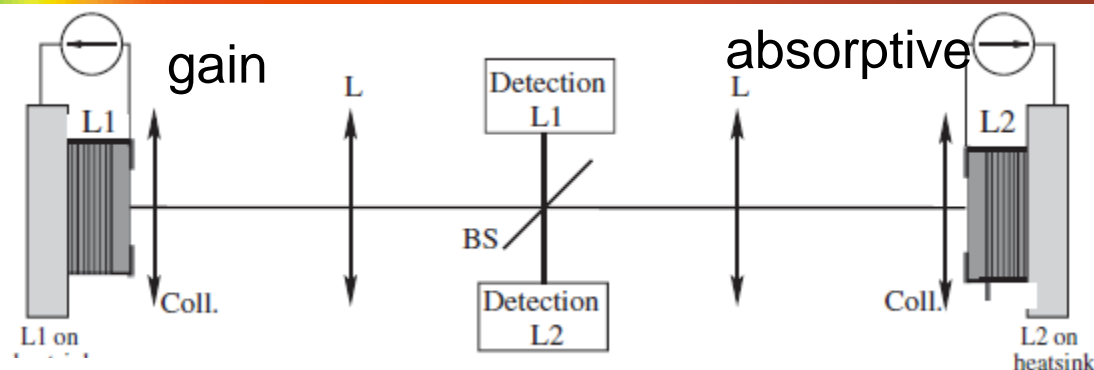
Saturable
absorption
stabilizes
background:

**Absorptive
bistability**

Laser solitons due to saturable absorption

INLN group: Genevet et al.,
PRL 101, 123905 (2008)

Face-to-face coupled VCSELs

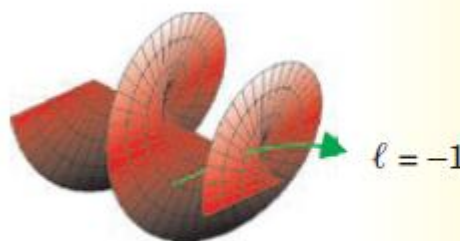


➤ absorptive
optical
bistability

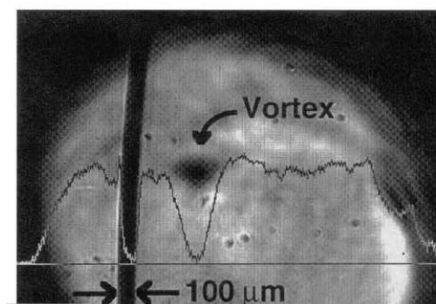
➤ solitons
➤ independ-
ently
controllable

For the orbital angular momentum lover

- A doughnut beam or **optical vortex** has a spiral phase structure, a singularity at the centre and carries orbital angular momentum
- Stable soliton solution for **self-defocusing** wave equation
- But unstable in **self-focusing** medium, decays into bright solitons



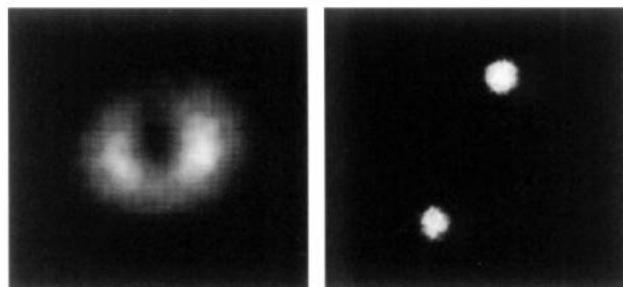
Padgett et al., Phys. Today **57**(5), 35 (2004)



Stable

Swartzlander, OPN
10, 10 (1993)

(defect in bright
homogeneous state)



Unstable

Different
for cavity
solitons?

Tikhonenko, JOSAB **12**, 2046 (1995)

self-focusing media ?

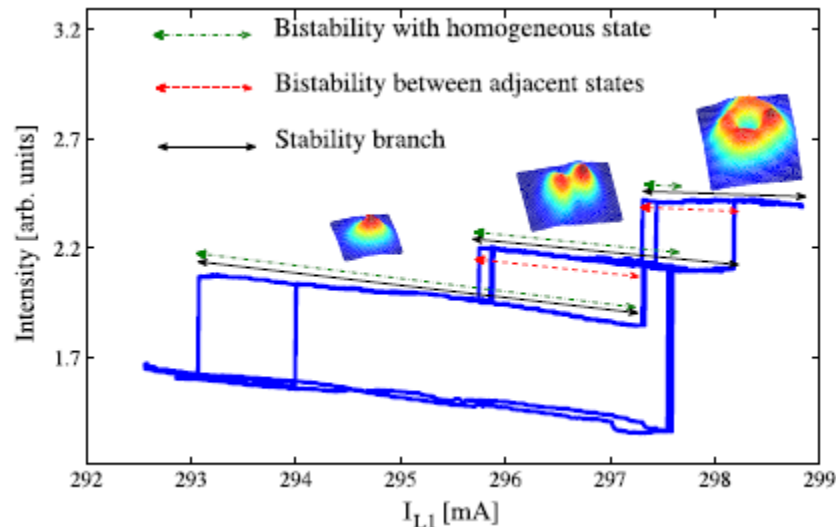
Cavity vortex solitons

➤ *absorptive:*

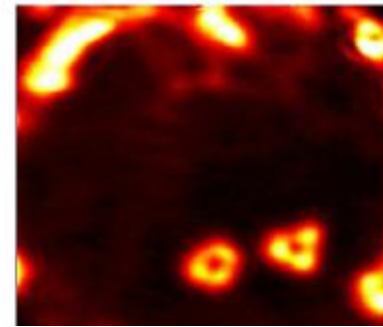
- theory: Rosanov group, e.g. Federov et al., IEEE QE **39**, 197 (2003)
- first experiment: INLN, Genevet et al., PRL **104**, 223902 (2010)

Nested hysteresis curves

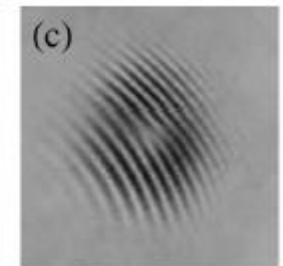
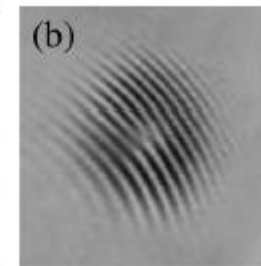
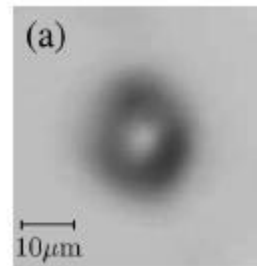
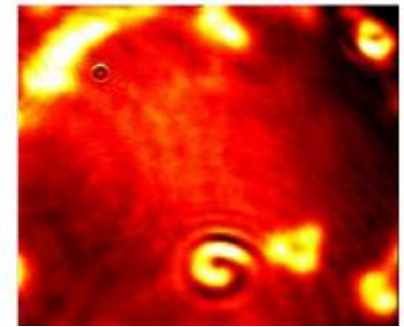
Clusters of solitons and “rings”



intensity



Interferogram with
magnified part of ring



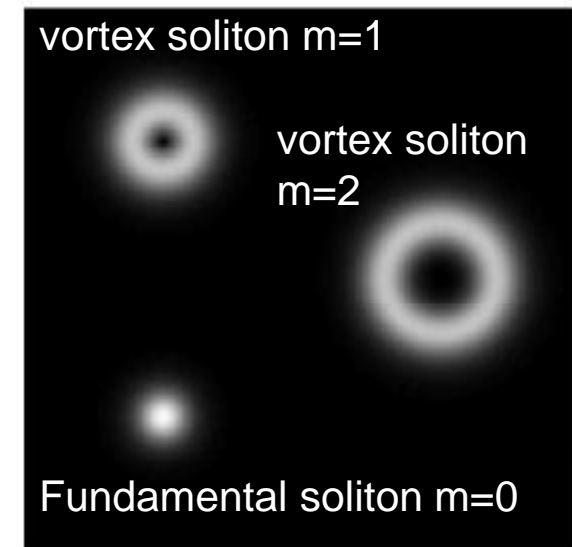
Tilted reference →
forks indicate singularity

Cavity vortex solitons II

➤ *self-focusing:*

- theory: e.g. Crasovan et al., Phys. Rev. E **63**, 016605 (2000)
- experiment: none to our knowledge
- specific prediction in a cubic complex Ginzburg-Landau equation with filter
→ simplest model for a laser with frequency-selective feedback

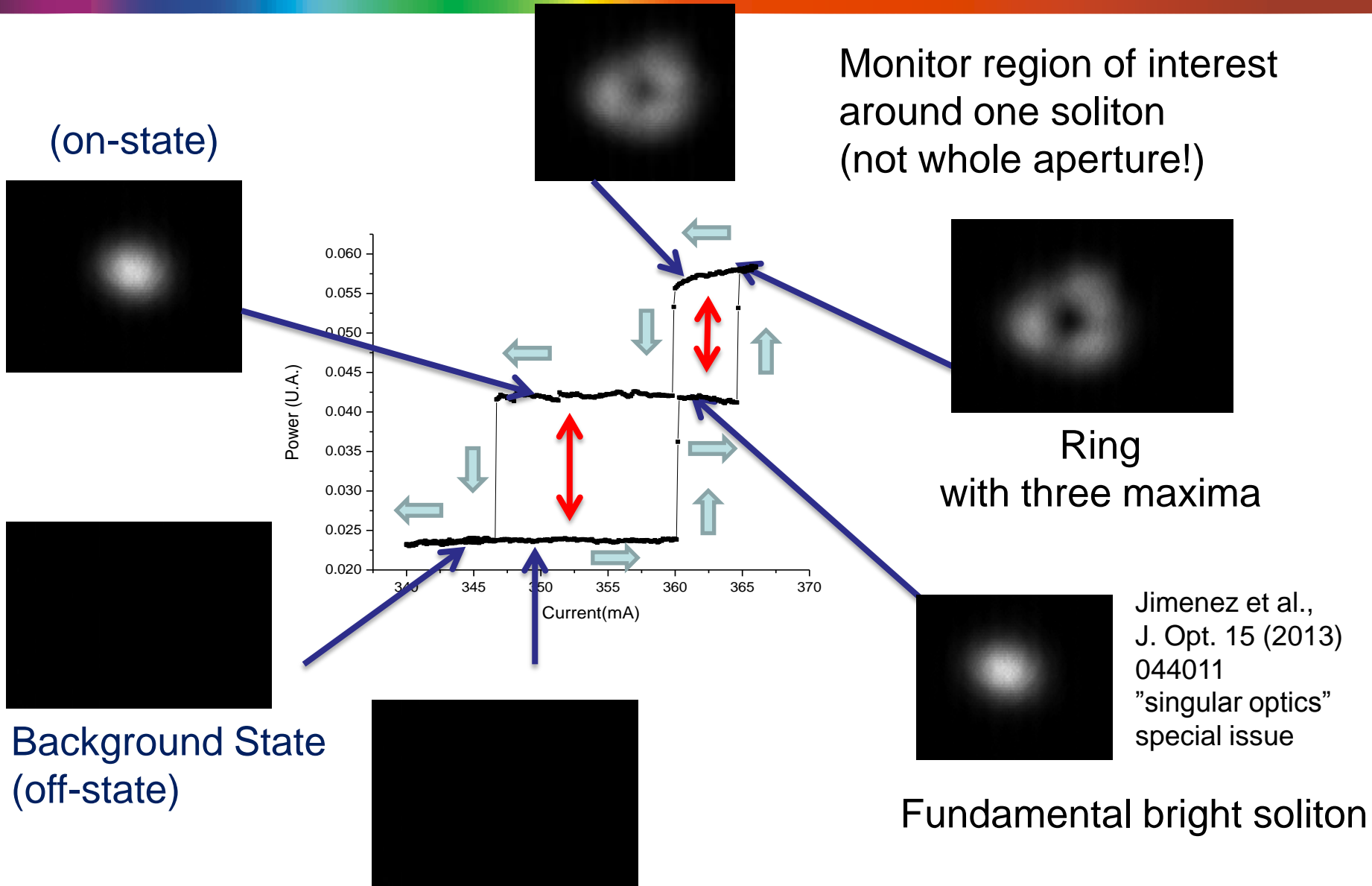
Paulau et al., Opt. Exp. **18**, 8859 (2010);
PRE **84**, 036213 (2011)



Note:

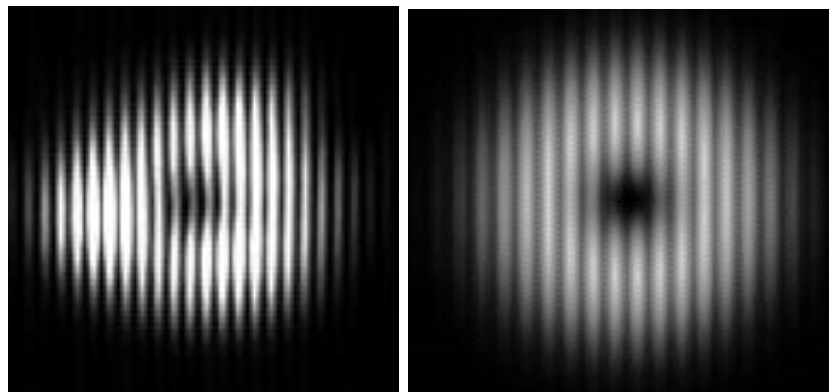
- vortex solitons with integer m form **discrete family of 2D high order solitons**
- this possibility exists only in systems in which the **phase is free**
i.e. in lasers and other oscillators without coherent injection

FSF: LI-curve and bistability

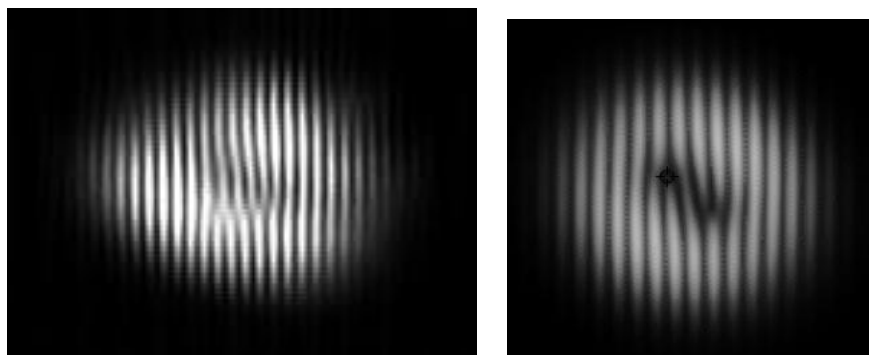


Interference Patterns (self-interference)

Self-interference (overlapping)

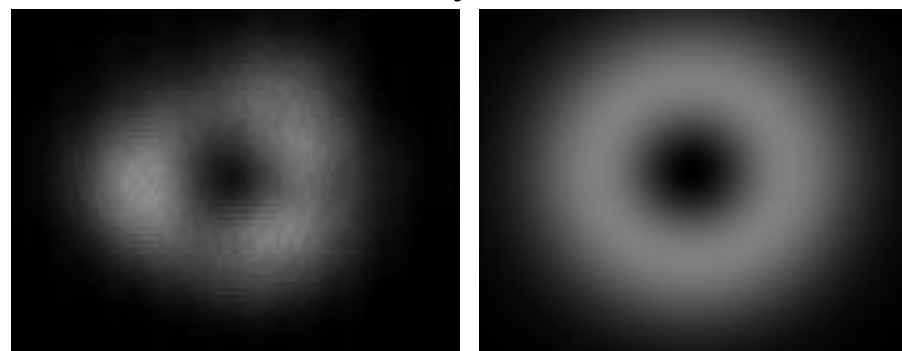


On top of each other



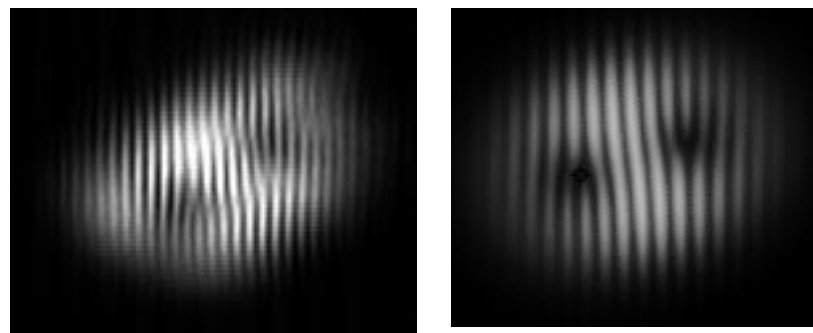
Shifted sideways, slightly
vertically

intensity



Experimental

Simulation



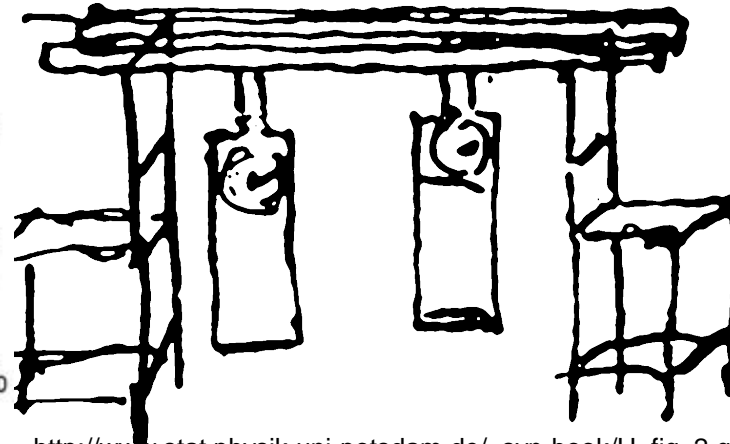
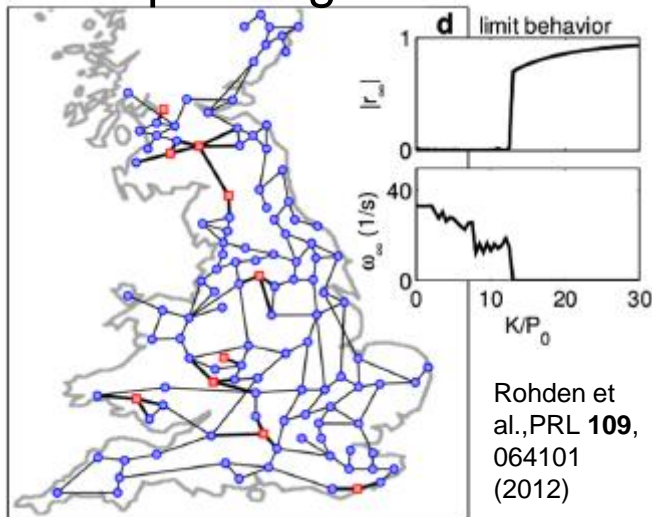
Stronger shifts

Evidence for one phase singularity with $m=1 \rightarrow$ vortex

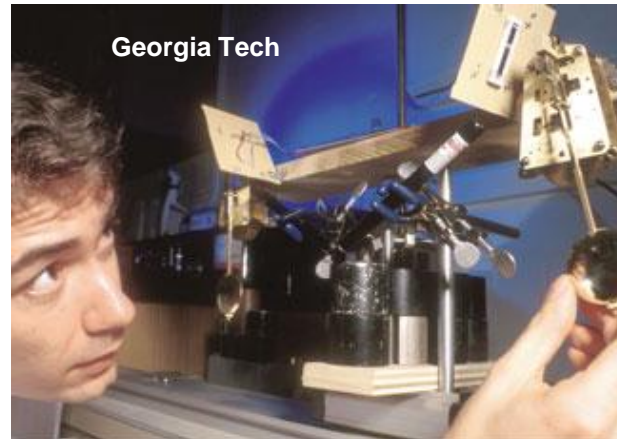
Synchronization

- We saw that solitons are in generally mutually incoherent due to disorder, i.e. have different frequencies and phases.
- What happens if two are together and interact?

UK power grid



http://www.stat.physik.uni-potsdam.de/~syn-book/H_fig_2.gif



Huygens 1665

Coupled clocks
synchronize:
Frequency- and
phase-locking

$$\frac{d^2 \phi_j}{dt^2} = P_j - \alpha \frac{d\phi_j}{dt} - \sum_i K_{ij} \sin(\phi_i - \phi_j),$$

What about the solitons?

Adler scenario

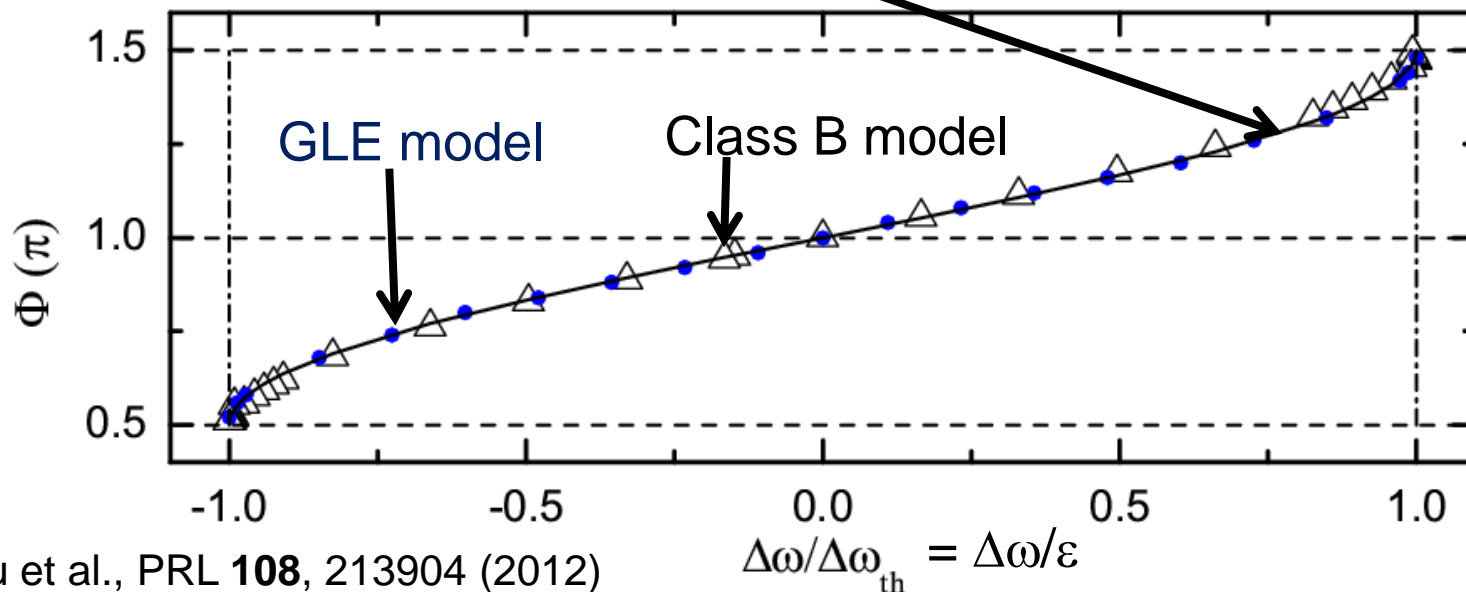
➤ **Adler equation:**

Adler, Proc. IRE (1946)

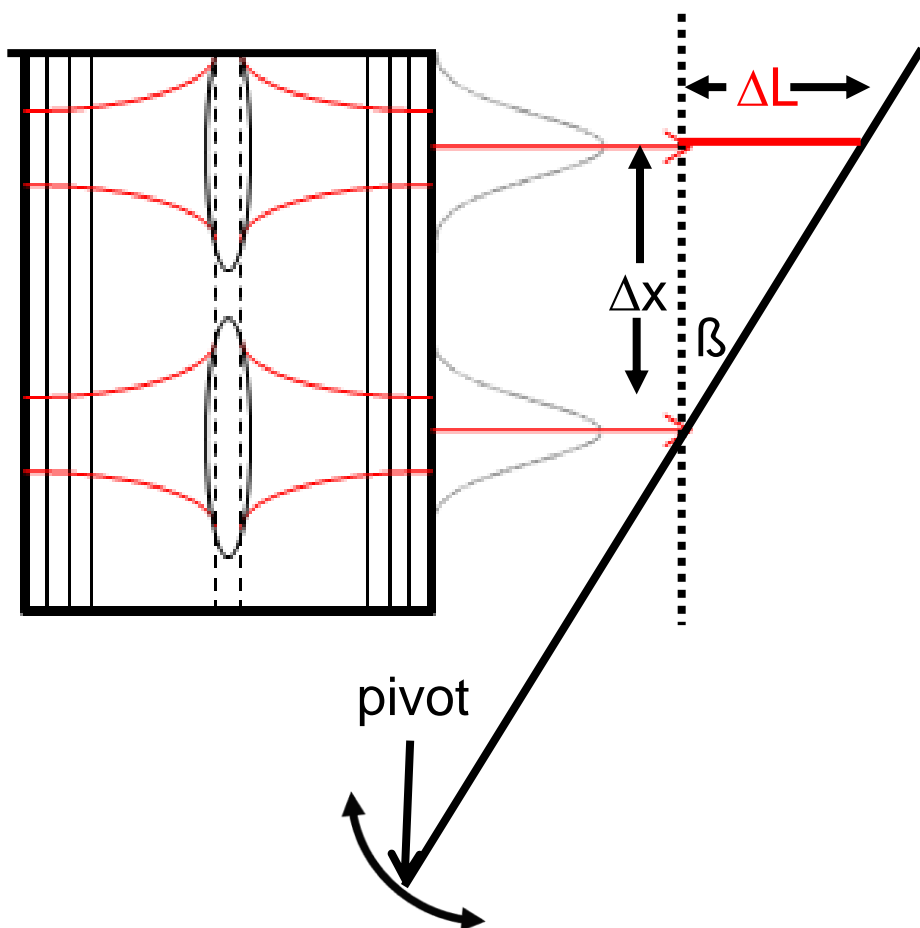
Archetypical equation for frequency- and phase-locking of nonlinear oscillators in presence of detuning

$$\frac{d\Phi}{dt} = \Delta\omega - \varepsilon \sin(\Phi)$$

- Stable locking at 0 or π for $\Delta\omega=0$
- with detuning still locking for $\Delta\omega < \varepsilon$



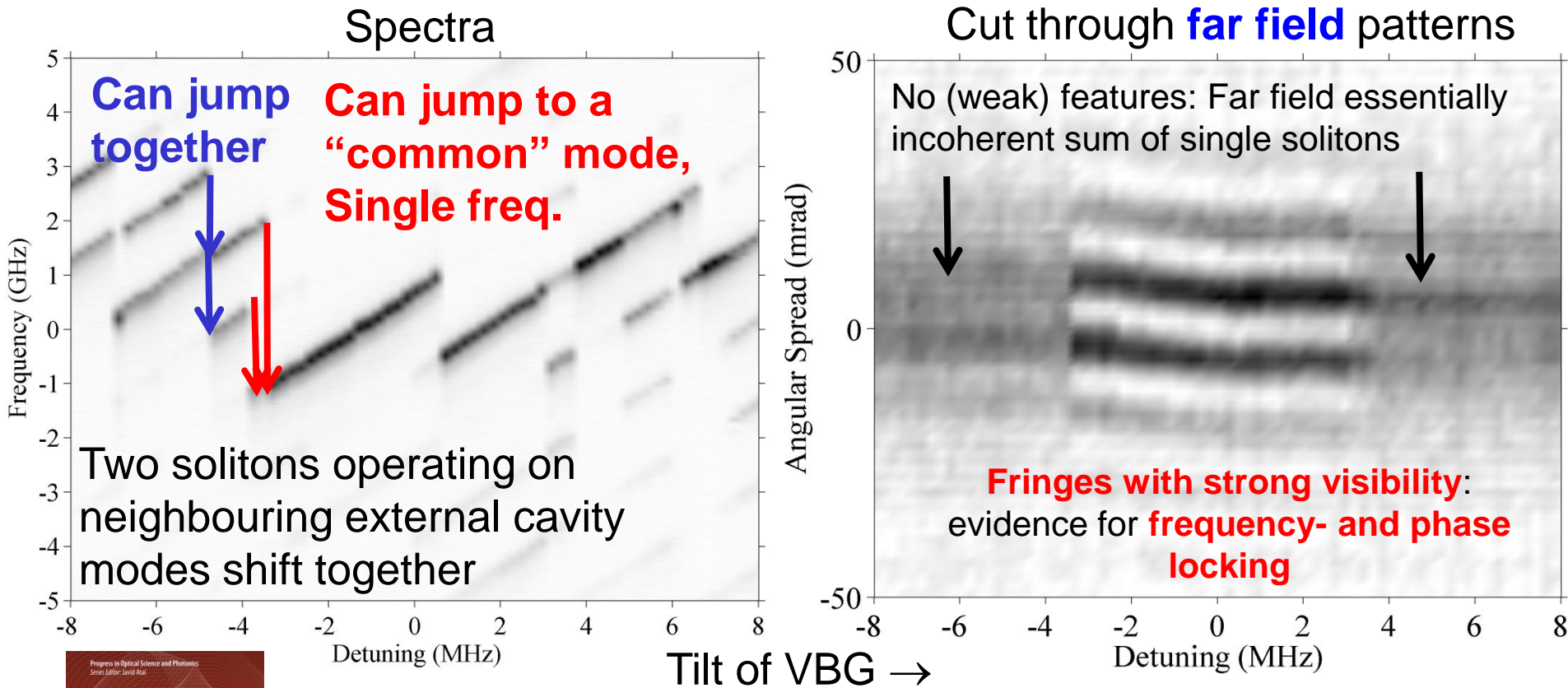
Experiment: How to control detuning?



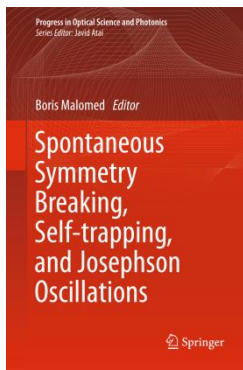
- **Tilt β of VBG** → controls detuning in external cavity (feedback phase)
- **different arm length ΔL** → mutual detuning or offset of combs
→ align to be “zero” or multiples of free spectral range
- Shift ΔL by a few μm by PZT
→ change detunings by a few tens of MHz
- Near and far field profiles of solitons unaffected but positions are

Similar to control of detuning between coupled microchip lasers (R. Roy et al.)

Experiment: Phase locking



Shift of fringe pattern → **change of locking phase**



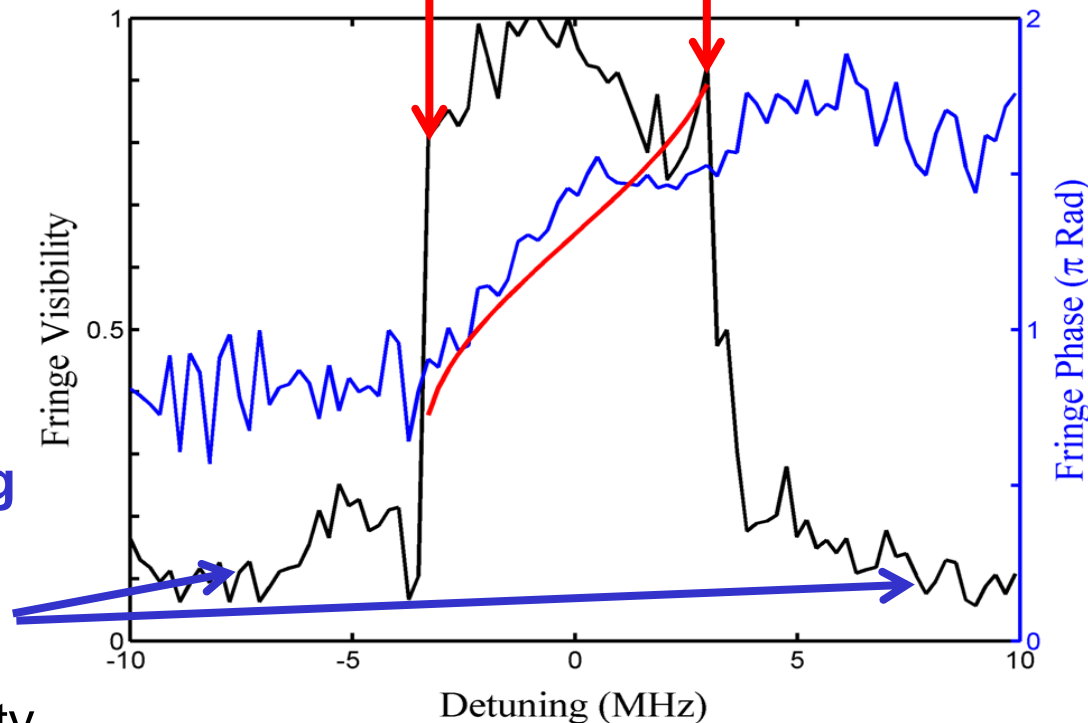
Ackemann et al., Book chapter (2013)

Experiment: Adler

(Nearly) **complete locking:**

Adler range

(analytical curve (red) scaled in x and shifted in y)



partial locking

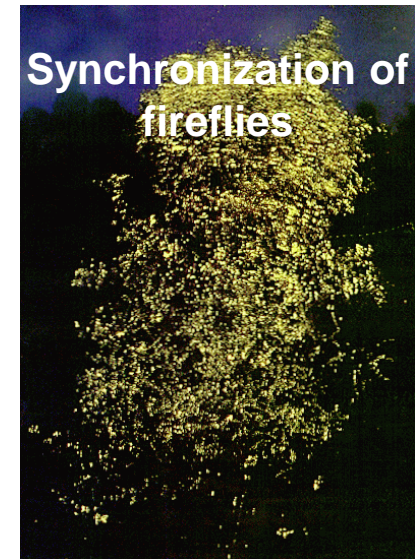
via some
sharing of
external cavity
modes

➤ Nice qualitative agreement

42

Disorder and locking

- Temporal and spatial systems react very different to disorder
- **Temporal (longitudinal):** Bound states with $\pi/2$ phase between constituents predicted for complex cubic-quintic Ginzburg Landau equation are actually observed in mode-locked fiber lasers (Grelu) averaging along cavity axis → each LCS sees **all** disorder
- **Spatial:** Each LCS sees only **local** disorder
 - Translational modes strongly damped
 - **synchronization dynamics, Adler scenario**
frequency **and** phase locking
- Nevertheless new features
 - **self-localized**
 - **bistable**
 - potential of reasonably **large disordered networks**
- **Ideas to control disorder locally**



Mode-locking and temporal solitons

- Pulses in ultrafast, mode-locked lasers can be understood in many (not all!) configurations as temporal dissipative solitons
 - Balance dispersion and self-phase modulation
 - Cavity losses and driving
 - Bistable (self-starting problem)
- Simplified treatment by cubic-quintic Ginzburg-Landau equation
$$i\psi_z + D\psi_{tt}/2 + |\psi|^2\psi + \nu|\psi|^4\psi = i\delta\psi + i\varepsilon|\psi|^2\psi + i\beta\psi_{tt} + i\mu|\psi|^4\psi$$
- Major contributors
 - Fundamental theory: Akhmediev
 - Experiment on molecules, dynamics, ...: Grelu, Cundiff
 - For high-power lasers: Keller, Wise
- Recent review: Grelu, Akhmediev, Nat. Phot. 6, 84 (2012)

Summary: Laser solitons

➤ **Cavity soliton laser**

- Optically controllable **microlasers** based on **spatial dissipative solitons!**
- **Disorder** important in realization (FSF as tool to probe)
- **Synchronization: Frequency and phase-locking (Adler scenario)**
- **Vortex** solitons as high order states

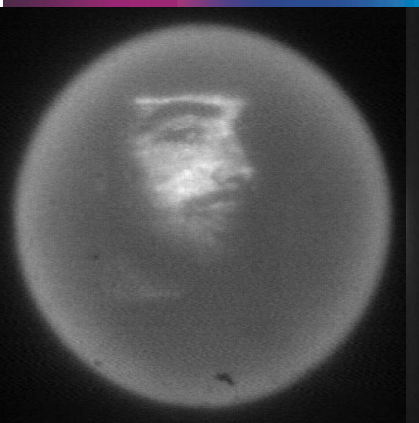
Review on CS :
Ackemann, Firth, Oppo,
Adv.At. Mol. Opt. Phys.
57, 323 (2009)

➤ Different mechanisms, but common features

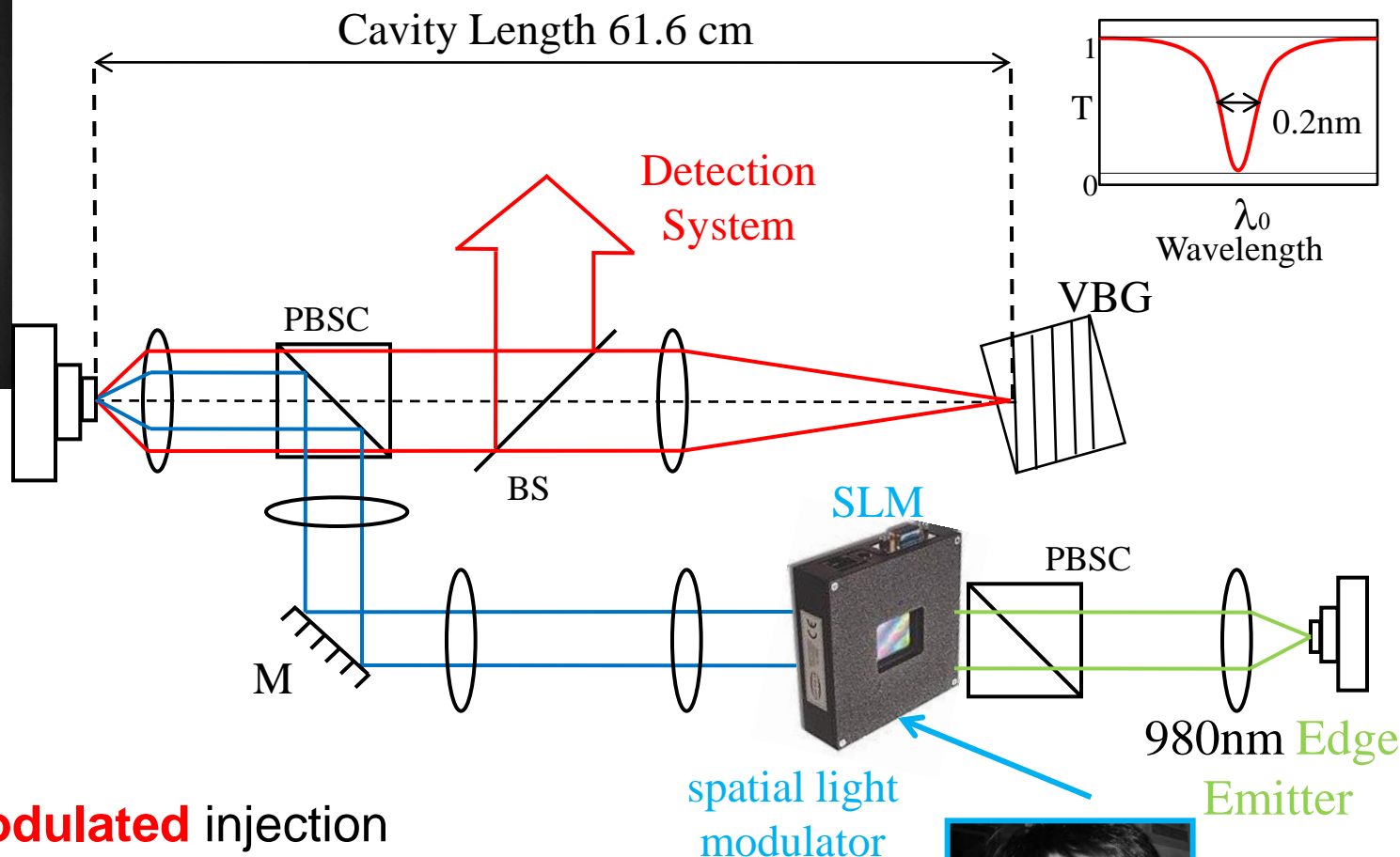
- **Dispersive** vs. **absorptive** optical bistability
- Decisive is that there is a mechanism **suppressing lasing in the background** (absorptive or off-resonant to filter)

➤ **Outlook:** 3D localization, mode-locking of spatial solitons (**Friday!**);
networks of phase-locked LCS, local control of inhomogeneities;
miniaturization, monolithic integration; cluster of solitons and understanding
of connection to high-order solitons and inhomogeneities

Setup for compensation



980 nm **VCSEL**
200 μm Aperture



- **spatially modulated** injection
- spatially modulated carrier distribution
- spatially modulated refractive index
- **compensation of variations in cavity resonance**



Demonstration of control

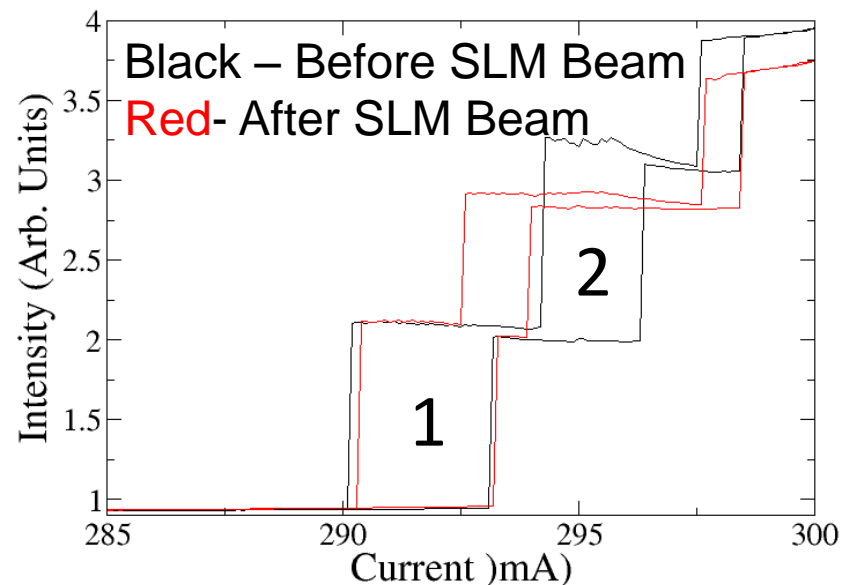
1. Find 2 solitons with similar thresholds



2. Apply SLM beam locally to soliton with higher threshold



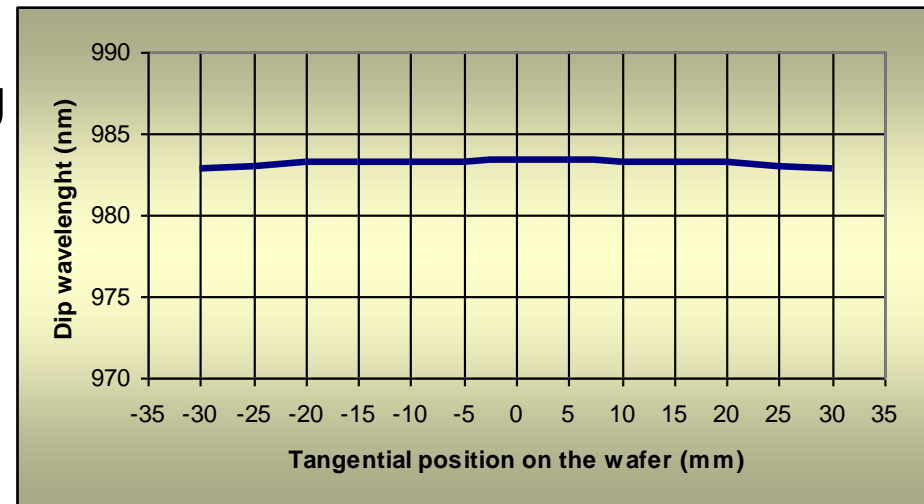
3. Soliton thresholds overlap



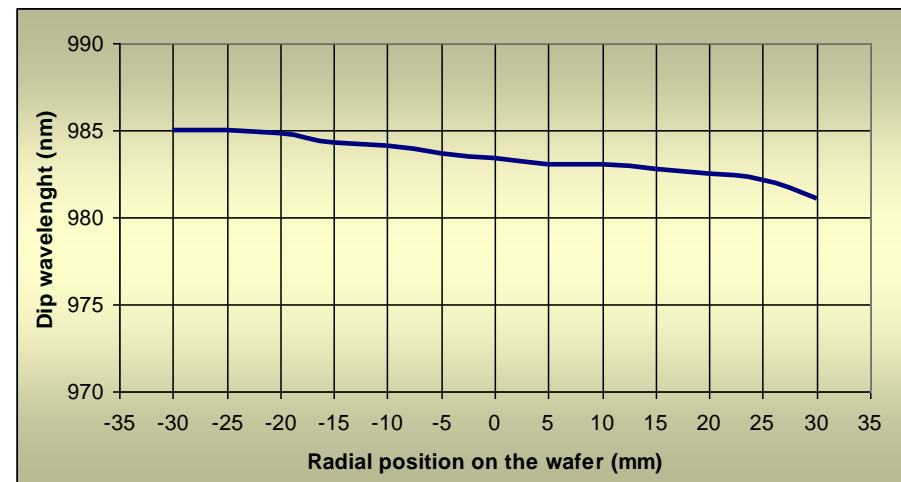
- SLM for VIS, efficiency in NIR low
 - low power
 - needs to be resonant to microcavity
- **first step towards homogenization**

Large-scale homogeneity

tangential



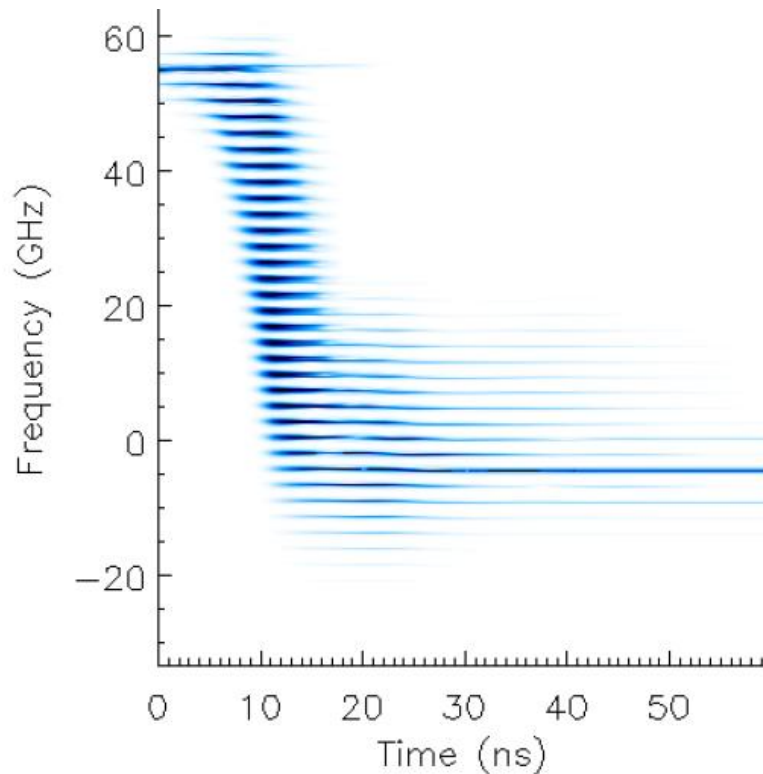
radial



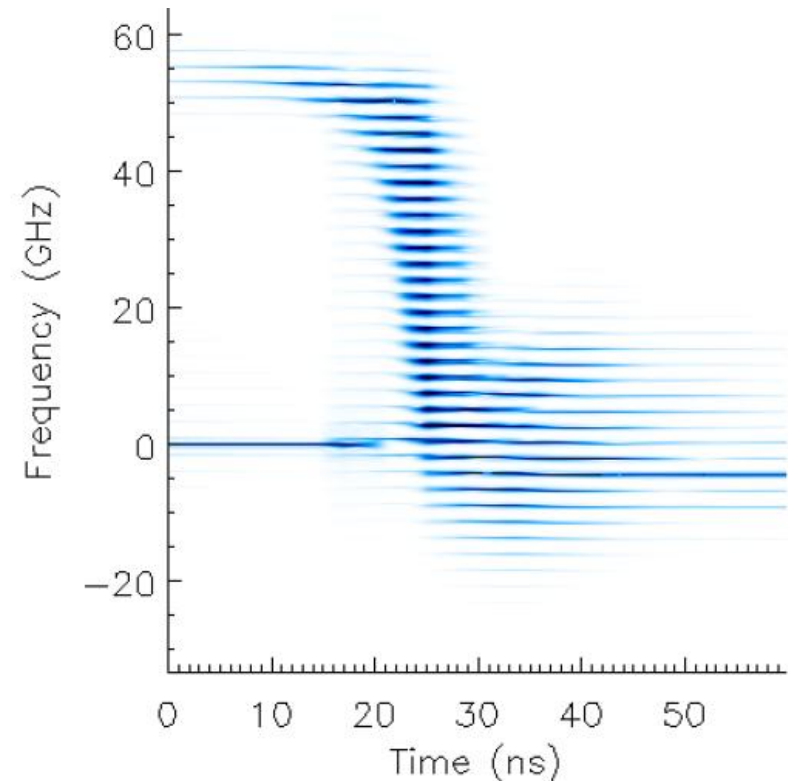
- improve beam shape
 - optimize temperature difference between bottom and top heating filament of the effusions cells
- improve homogeneity of substrate temperature
 - reduce temperature level of growth
 - enhance uniformity of substrate holder rings
- result:
 - **< 0.012 GHz / μm**
 - **< 2.5 GHz / 200 μm**

Time-resolved optical spectrum

Blue detuned excitation (20 ns pulse)



Excitation at grating frequency



Shift of carrier frequency →
evolution via **unstable LCS** →
spectral simplification

Still evolution via **unstable LCS**

Even simpler: Ginzburg-Landau model

Ginzburg-Landau model + linear filter, Firth + Paulau, Eur. Phys. J. D **59**, 13 (2010)

Linear loss (gain) and frequency detuning

Nonlinear gain (loss) and self focusing (defocusing)

Diffraction

Filtered Feedback

$$\begin{cases} \frac{\partial E}{\partial t} = g_0 E + g_2 |E|^2 E + i\Delta_{\perp} E + F + in(x)E, \\ \frac{dF}{dt} = -\lambda F + \tilde{\sigma} E, \end{cases}$$

Width of filter

Feedback strength

Pinning potential

L.M. Hocking and K. Stewartson, Proc. R. Soc. Lond., A **326**, 289 (1972); N.R. Pereira and L. Stenflo, Phys. Fluids **20**, 1733 (1977).

P.A. Bélanger, L. Gagnon, and C. Paré, Opt. Lett. **14**, 943 (1989).

C. Paré, L. Gagnon, and P.A. Bélanger, Opt. Commun. **74**, 228 (1989).

$$n(x) = \frac{-n_j}{2} \left[\cos \left(\frac{\pi(x-x_j)}{W} \right) + 1 \right]$$

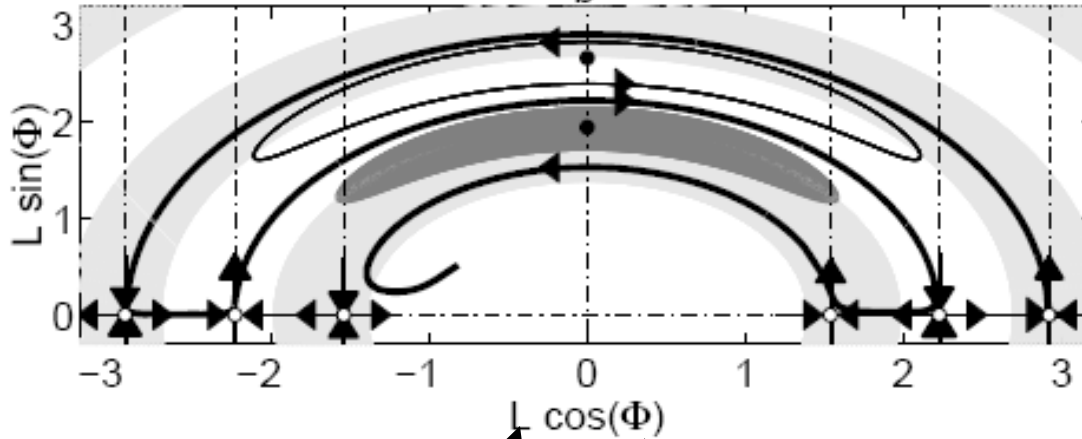
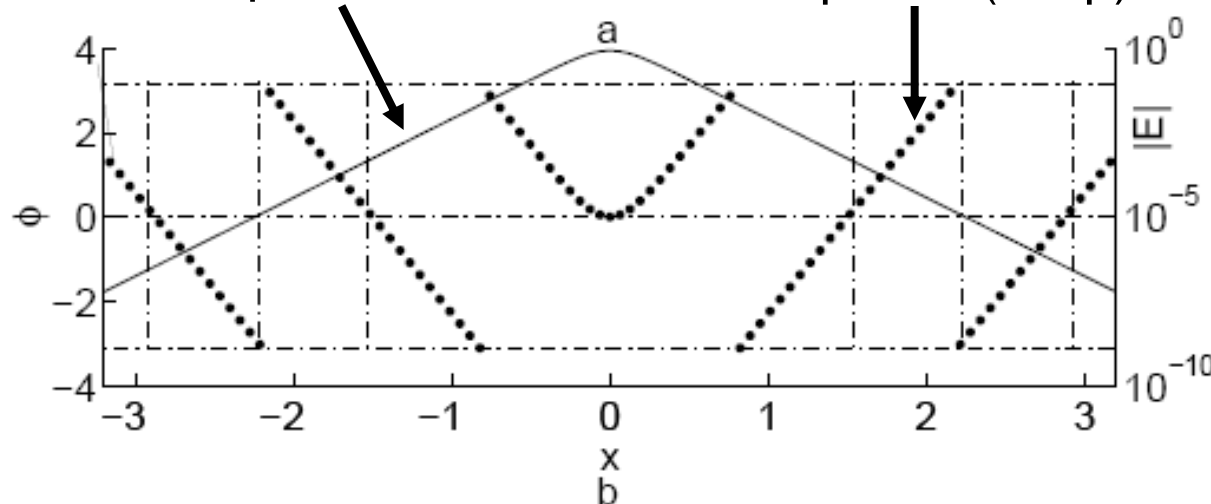
$$\begin{aligned} g_0 &= \kappa(1 + i\alpha)(\mu - 1) - i\omega_m, \\ g_2 &= -\kappa(1 + i\alpha)\mu, \\ \tilde{\sigma} &= \sigma\lambda, \end{aligned}$$

Homogeneous system

Work by P. Paulau, note $\alpha = 0.5 \ll$ than in experimental system $\alpha \approx 5$

Soliton amplitude

Soliton phase (chirp)



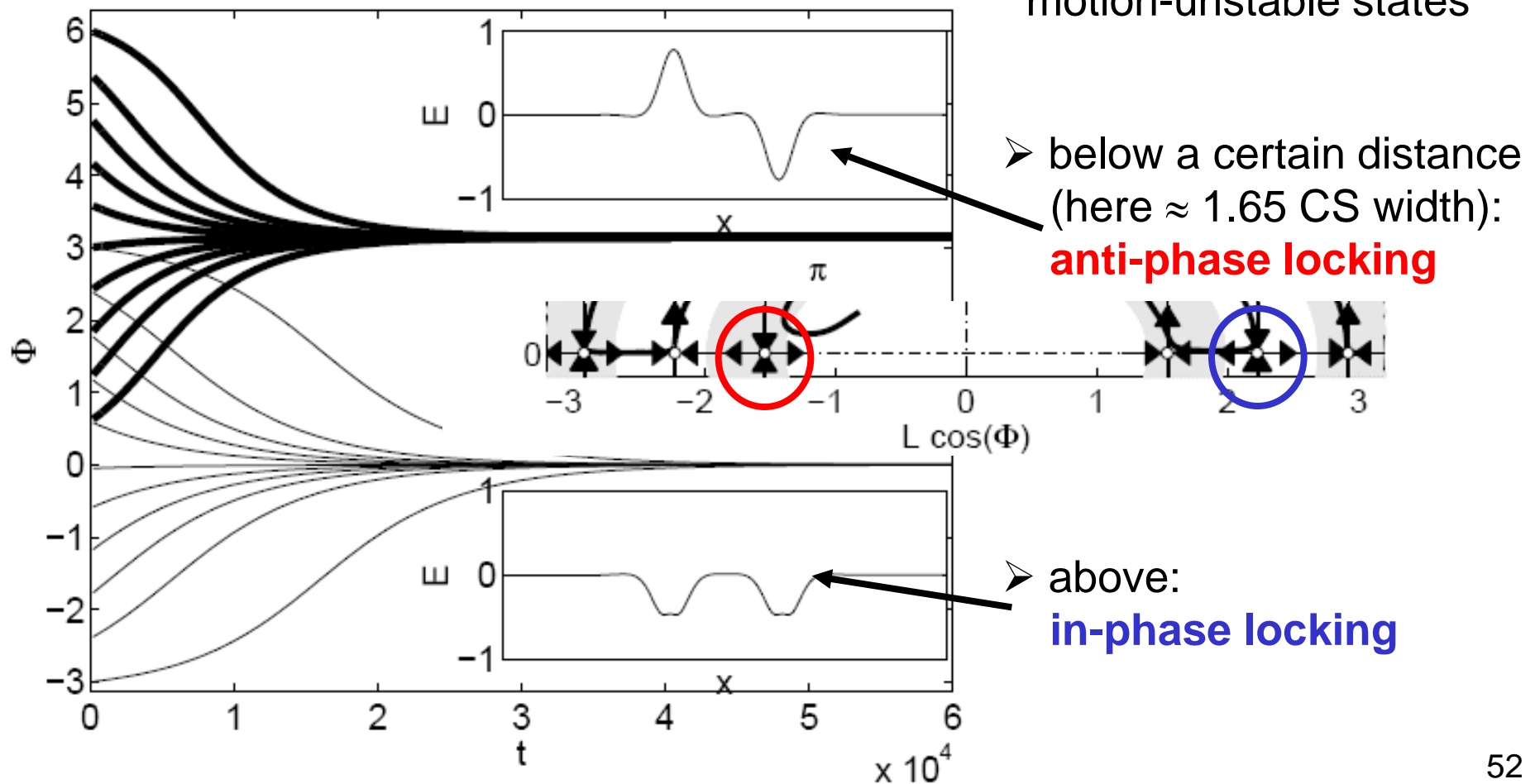
Interaction plane: distance - phase

- **Foci** at phase $\pi/2$
stable
- **Saddles** at phases
0 and π
always unstable
- qualitatively very
similar to perturbatively
obtained diagram for
cubic-quintic GLE;
temporal solitons in fiber
lasers; Akhmediev et al, PRL
79, 4047 (1997); Tuarev et al,
PRE 75, 045601(R)(2007) ⁵¹

Results with traps of equal depths

Trap depth about 0.48 GHz

➤ Traps destroy translational symmetry and stabilize motion-unstable states



Results with unequal traps

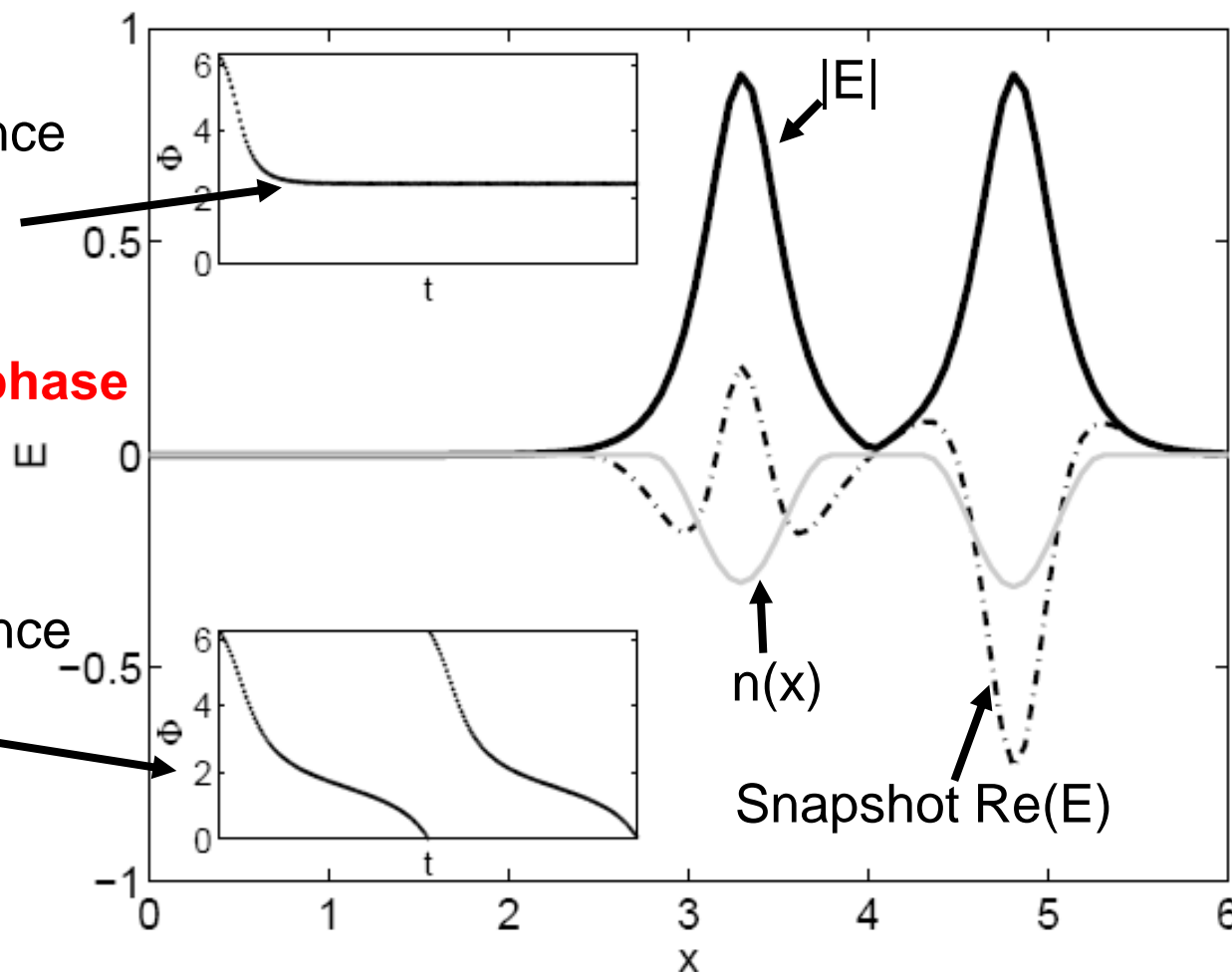
Trap depth different by 3%

➤ below a certain distance
(here 2.06 CS width)

- **synchronization**
- **“non-standard” phase**
- here 2.415

➤ above a certain distance

- unlocked



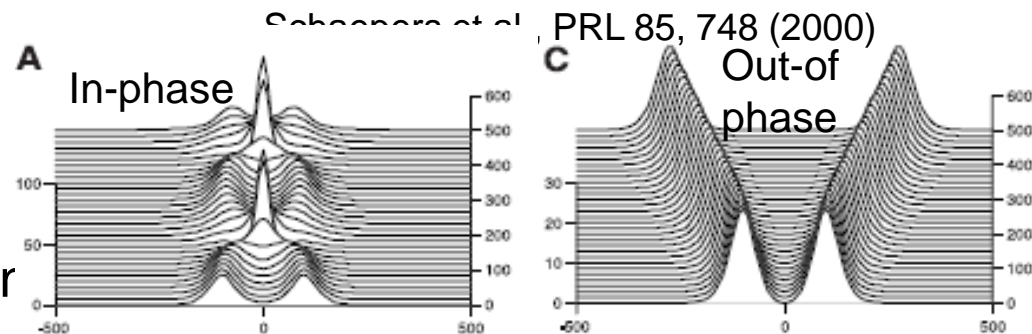
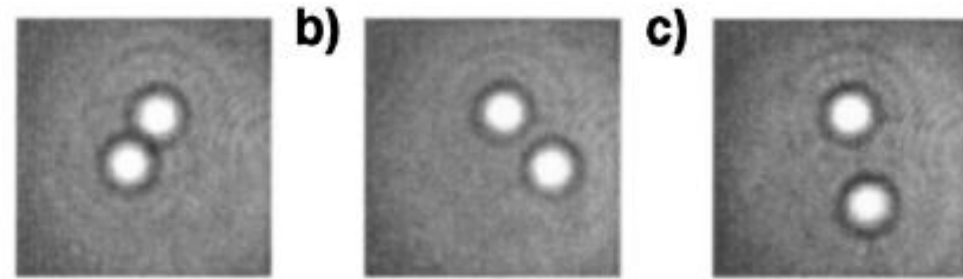
Outlook: Complex networks of LCS

➤ **Coherently driven systems:**

complex interactions via intensity (oscillating tails) but **no** phase dependence

➤ **Propagational solitons:**

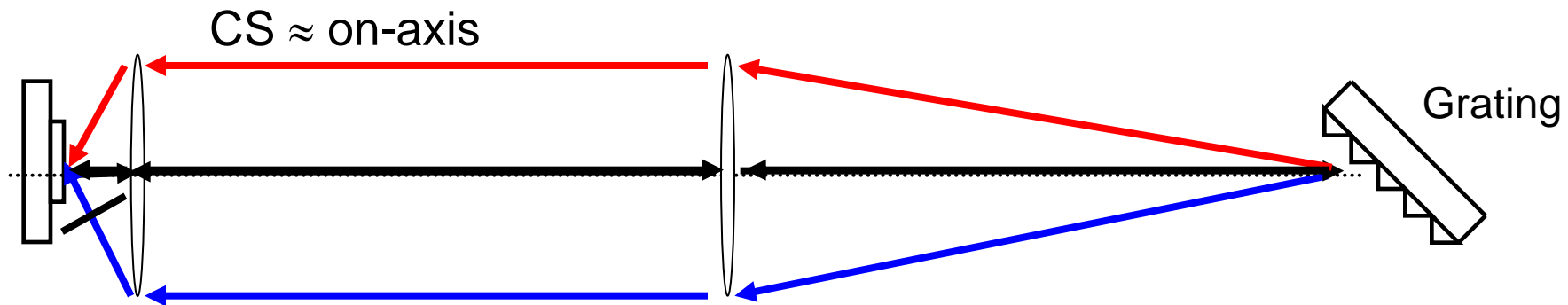
phase sensitive interactions, depending on launching condition



Stegeman + Segev, Science 286, 1518 (1999)

- ## ➤ **Laser cavity solitons** combine features of these two cases and adds new
- (optical) phase, spatial phase and polarization (phase between x, y comp.) are Goldstone modes
 - **Phase, location, and polarization are free to change during dynamics**
 - Complex network with many degrees of freedom
 - pioneering work: Akhmediev, Vladimirov, Rosanov, + coworkers

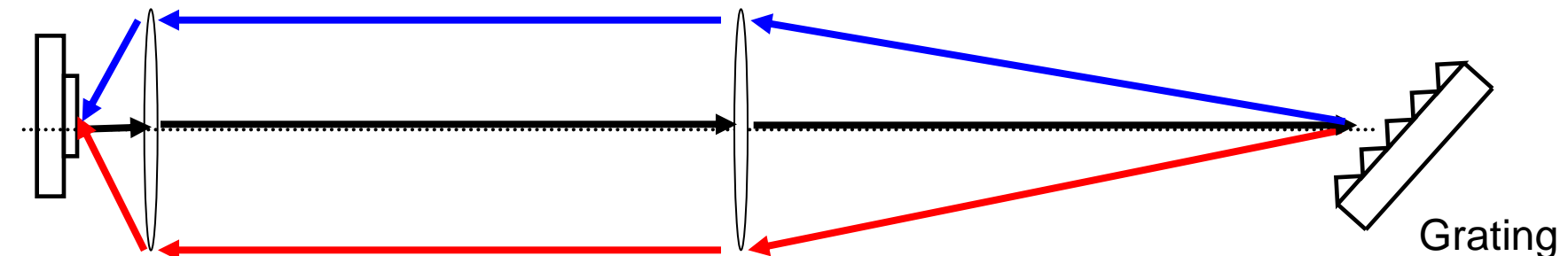
Asymmetry from the grating



- CS exactly at Littrow freq. \rightarrow exact retro-reflection
- CS slightly blue detuned \rightarrow **angle** \rightarrow wavefront tilted to the **right**

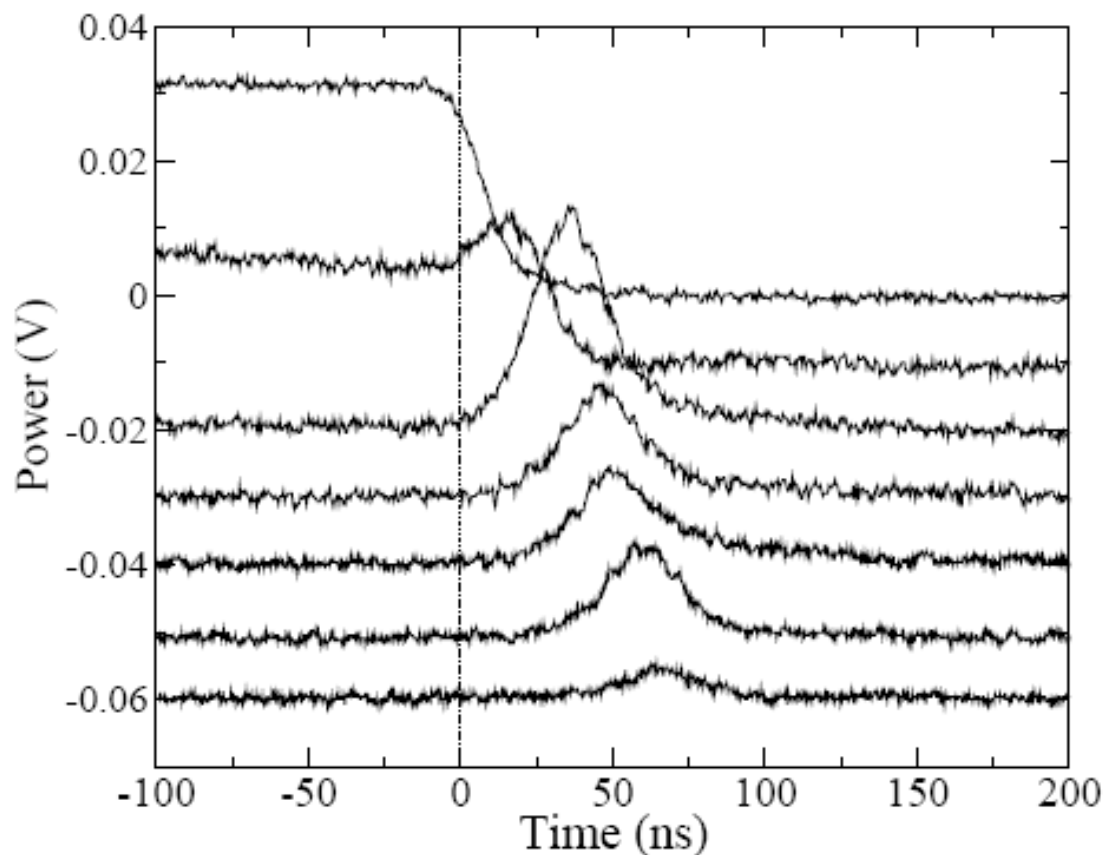
**Tilted wavefront
induces **drift**!**

- CS slightly red detuned \rightarrow **angle** \rightarrow wavefront tilted to the **left**



Drifting excitations

Ignite CS in a situation, in which it is only transient, monitor by APD array



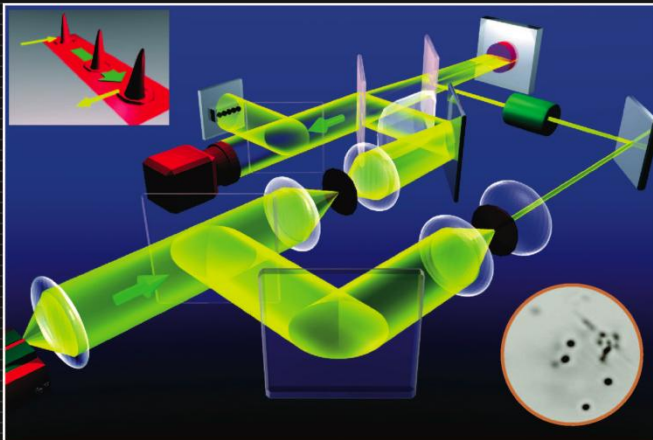
- velocity: $1.4 \mu\text{m} / \text{ns}$
- drift distance: $> 50 \mu\text{m}$
- Delay: $\sim 40 \text{ ns}$

Caution:
Measurement in far field
indicate that angle increase
by 2.5°
soliton character likely but
needs to be confirmed

CS Application – All-optical delay line

Articles published week of 7 JANUARY 2008
Volume **92** Number **1**

APPLIED PHYSICS LETTERS



In CSL

- velocity: $1.4 \mu\text{m} / \text{ns}$
- drift distance: $> 50 \mu\text{m}$
- Delay: $\sim 40 \text{ ns}$

In amplifier exp. (Nice)

- velocity: $4.7 \mu\text{m} / \text{ns}$
- drift distance: $36 \mu\text{m}$
- Delay: 7.5 ns

Motivation:

**All-optical delay line as
buffers in photonic networks**

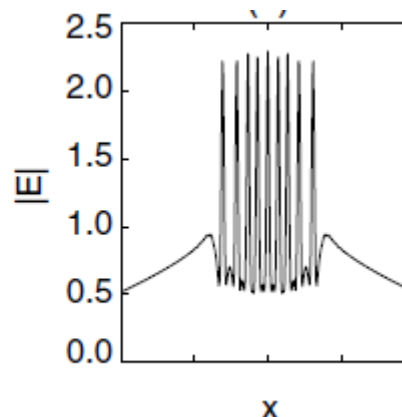
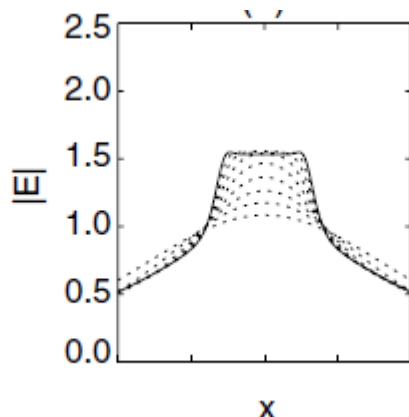
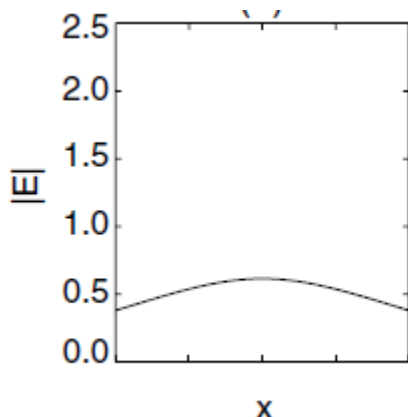
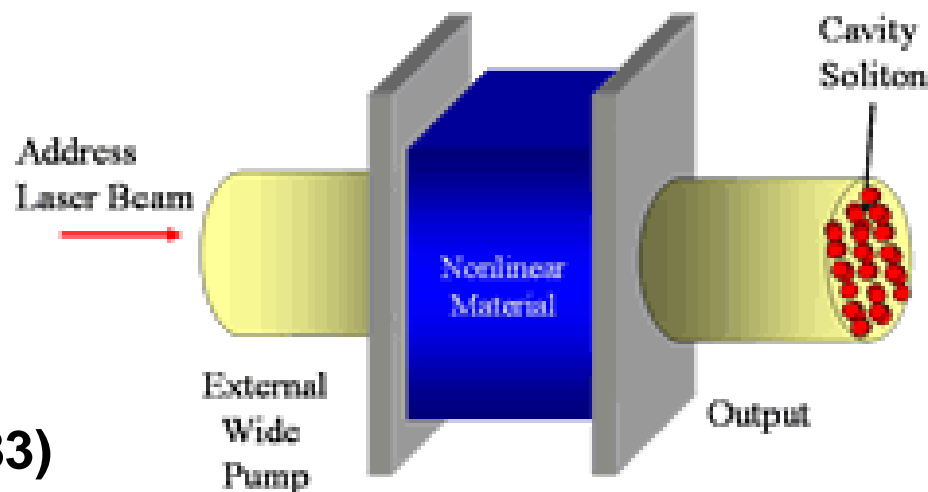
History: Cavity solitons

cavity soliton = (spatially) self-localized, bistable solitary wave in a cavity

- stabilized by counteraction of
 - nonlinearity vs. diffraction
 - driving vs. dissipation

driven cavity (optical bistability)

Probably first simulations:
McLaughlin, et al. PRL **51**, 758 (1983)

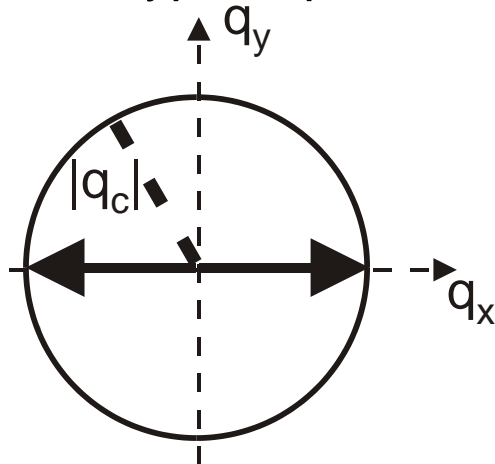


Experiments later

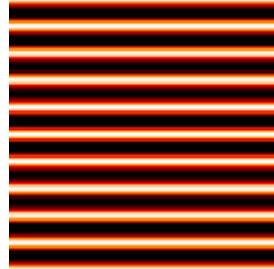
First analysis: Rosanov et al. Opt. Spectrosc. **65**, 449 (1988);
J. Opt. Soc. Am. B **7**, 1057 (1990)

Bifurcations of hexagons

tilted wave argument selects length scale
not type of pattern

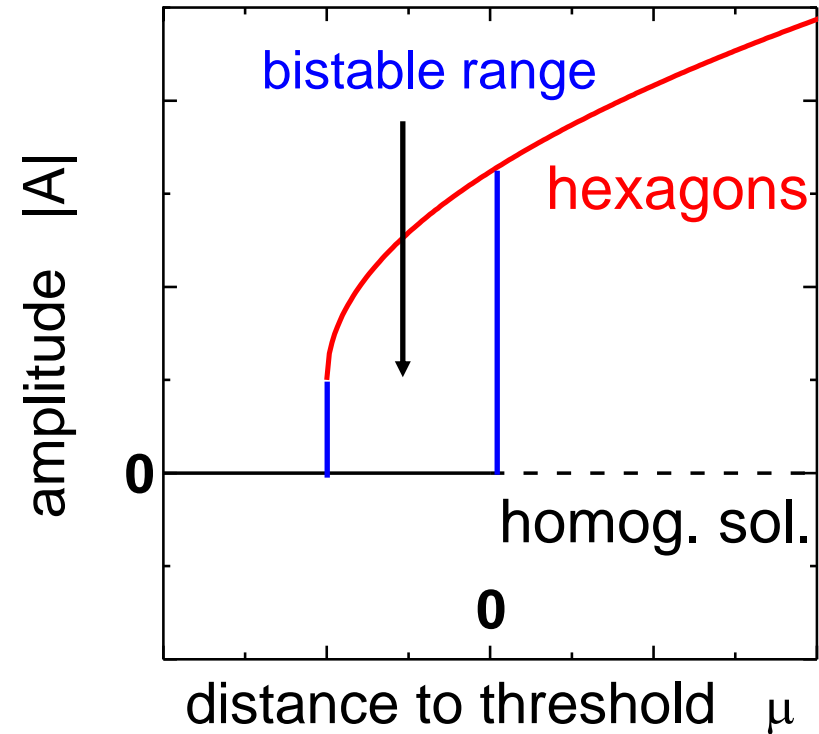
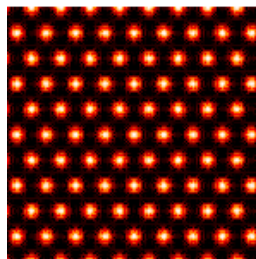


stripes



generic in 2D

hexagons

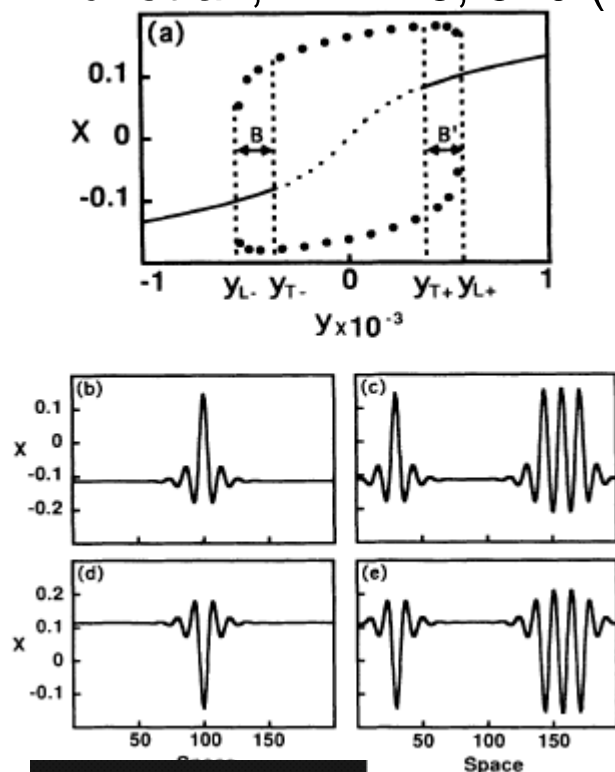


bifurcation to hexagons **subcritical**

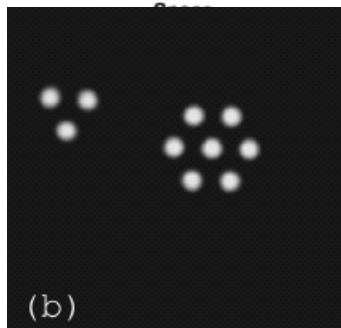
region of **coexistence** of
homogeneous solution and hexagons

In bistable range: Cavity solitons and patterns

Tlidi et al., PRL 73, 640 (1994)

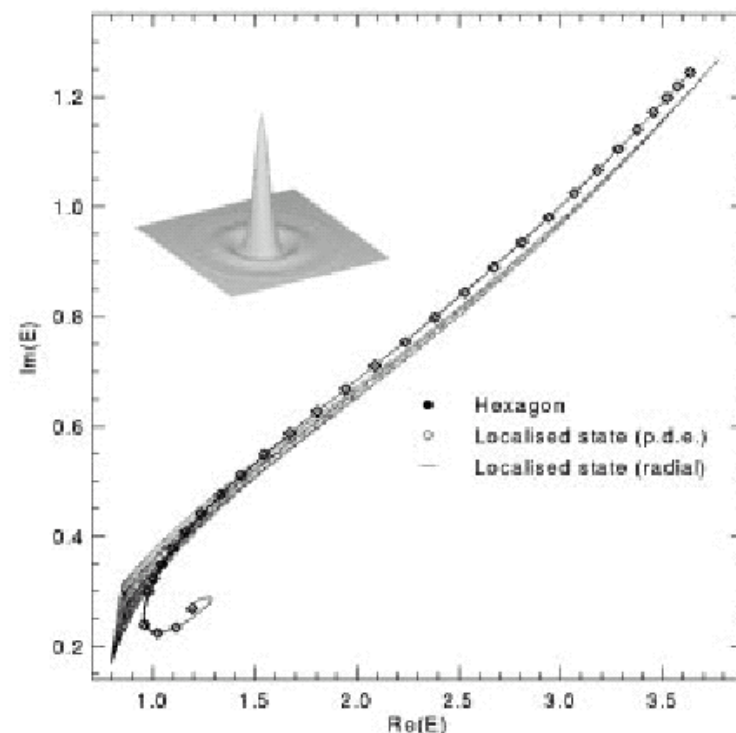


CS very
similar to
constituent
of a
hexagon



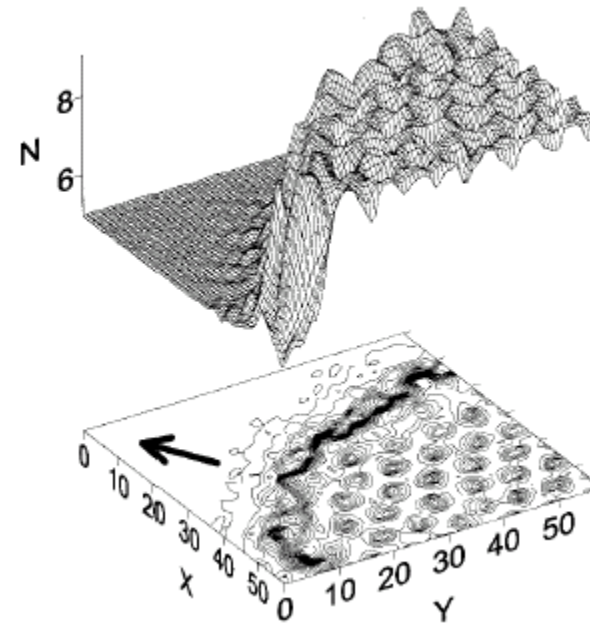
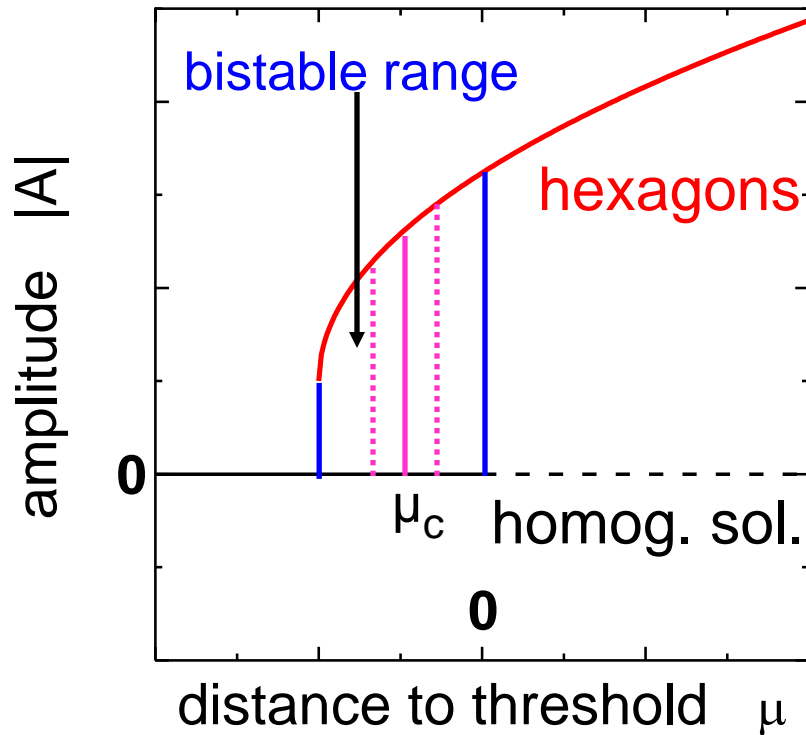
„localized
pattern“

Firth+Scroggie, PRL 76, 1623(1996)



CS (or LS) = part of a pattern

Fronts and cavity solitons



general prediction (1D):

LS / CS occur in vicinity of Pomeau front

Coullet et al., PRL **84**,
3069 (2000)

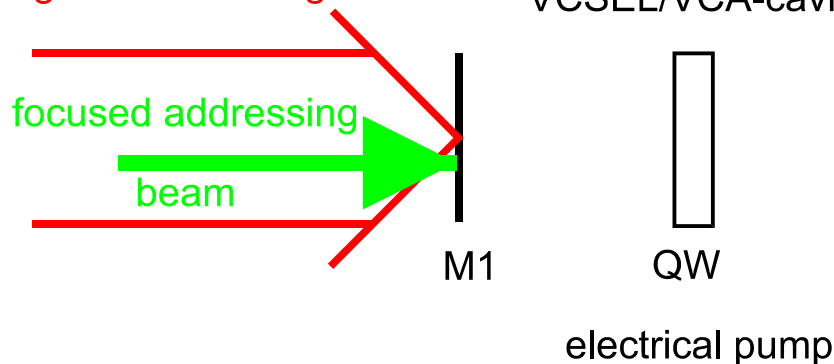
a) **front** between half-plane with hexagons and half-plane with homogeneous solution stationary at **one** value of stress parameter

b) **Pomeau-front** stationary for **finite** range of stress parameter due to locking at modulated interface

Vertical-cavity (regenerative) amplifier

homogeneous holding beam

VCSEL/VCA-cavity

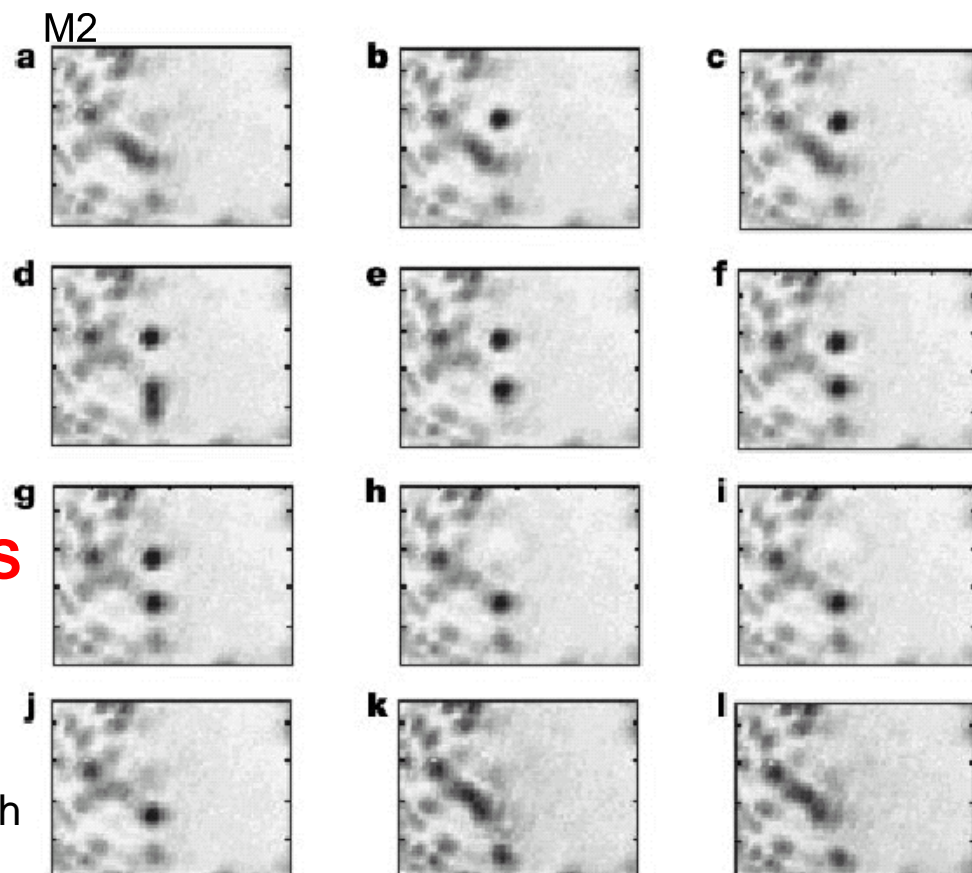


hexagons,
stripes,
honeycombs
localized structures

Barland et al.,
Nature **41**, 699
(2002)

electrically pumped above
transparency, but below threshold

**VCA = vertical-cavity
(regenerative) amplifier**
+ **homogeneous holding beam**

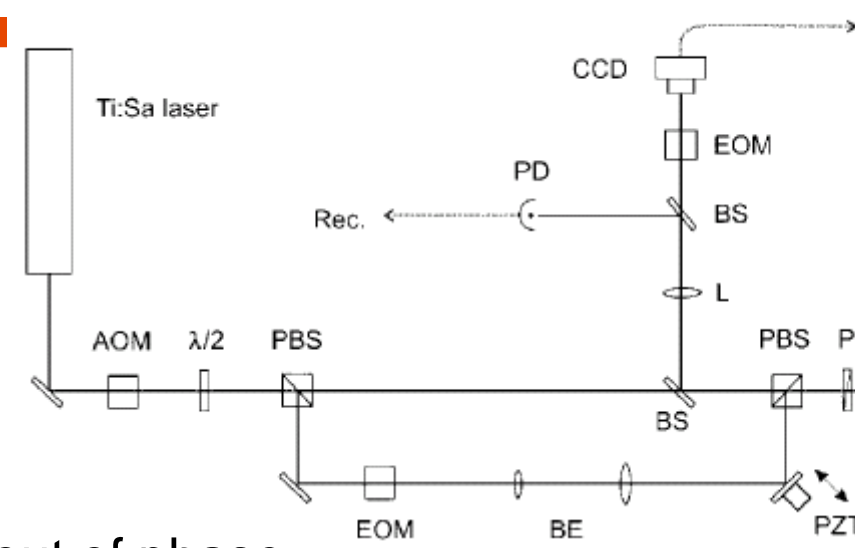
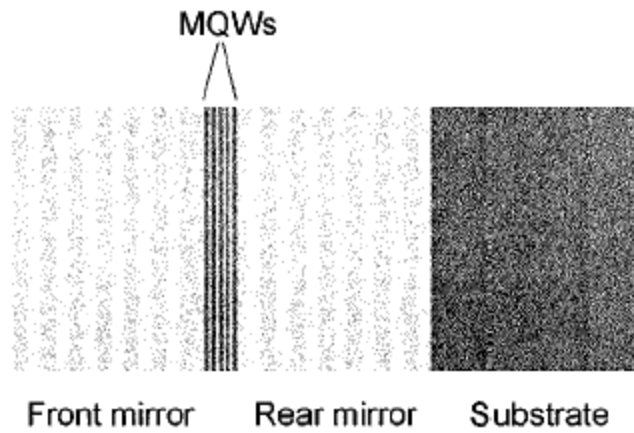


Independent manipulation of two CS

“Cavity solitons as
pixels in semiconductor
microcavities”

$60 \times 60 \mu\text{m}^2$

out of a device with
diameter $150 \mu\text{m}$



pulse, in phase

pulse, out of phase

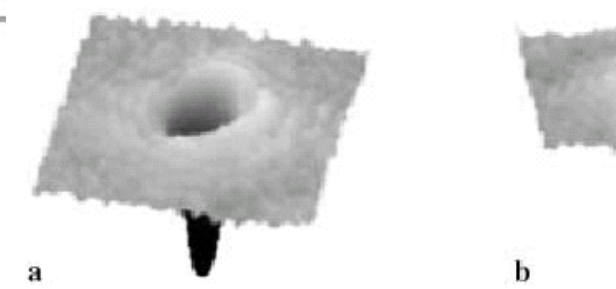
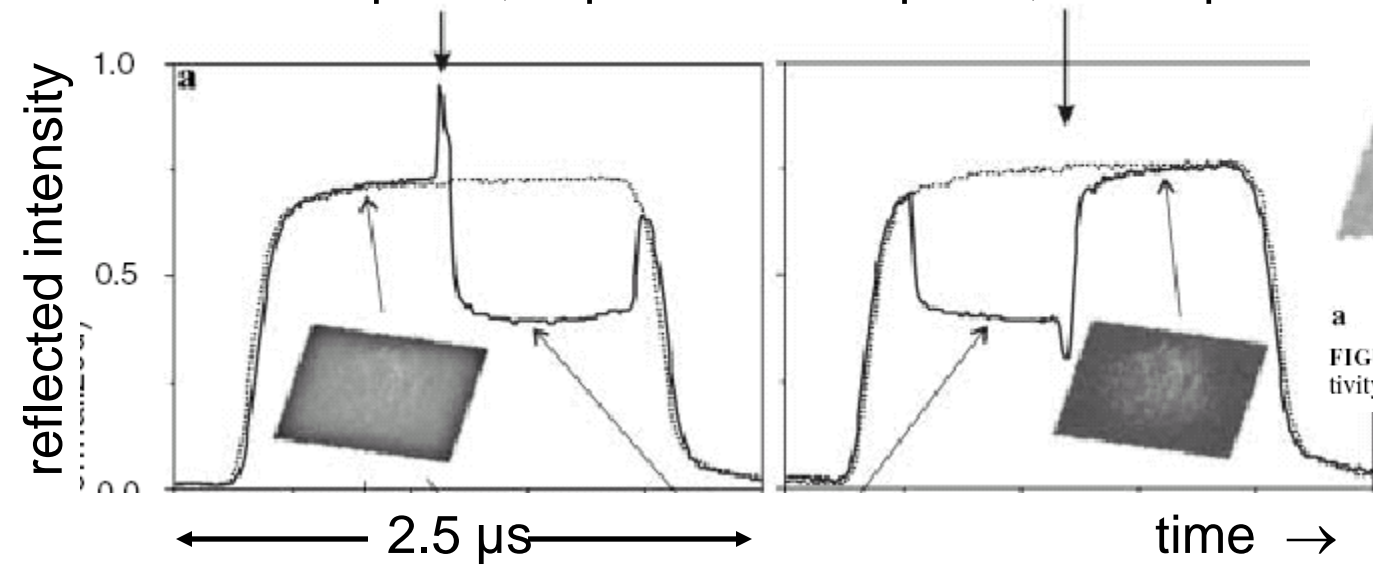


FIGURE 2 Bright soliton (dark in reflection). 3D representation: **a** view from above; and **b** view from below

switching of a single

Experiments: CS in cavities

- Kreuzer, Neubecker (Darmstadt): liquid crystals
Mol. Cryst. Liq. Cryst. 207, 219 (1991); JMO 41, 885 (1994)
 - Bistable single spots; no external control
 - Identification as “self-induced modes”
 - An experiment before its time (later Louvergneaux (Louvain))
- Taranenko, Weiss (PTB): absorbing and self-defocusing driven VCSELs
PRA 61, 063818 (2000); APB 75, 75 (2002) ...
 - Probably first observation of CS in semiconductor microcavity
 - But complete independent manipulation of two not demonstrated

