Lecture 3

Part 1: Fano resonances
Part 2: Airy beams
Part 3: Parity-time symmetric systems

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Part 1:

Fano resonances
Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano
National Bureau of Standards, Washington, D. C.
(Received July 14, 1961)

of phase normalizations. These curves are represented
by
\[
\frac{|(\psi_E | T | \bar{\epsilon})|^2}{|\psi_E | T | i)|^2} = \frac{(q+\epsilon)^2}{1+\epsilon^2} \frac{q^2 - 1 + 2q\epsilon}{1 + \epsilon^2}.
\]

(21)

This function is plotted in Fig. 1 for a number of values
of \(q\), which is regarded as constant in the range of
interest. Notice that

\[
\frac{(q + \epsilon)^2}{1 + \epsilon^2}
\]

Fano and his formula

Ugo Fano (1912-2001)

narrow band + flat background → Fano resonance

Fig. 1. Natural line shapes for different values of \(q\). (Reverse the scale of abscissas for negative \(\epsilon\).)
Fano resonances in nanoscale structures

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(Published 11 August 2010)

The simplest model
Fano resonances in Mie scattering

Gustav Mie (1868-1951)

Fano Resonances: A Discovery that Was Not Made 100 Years Ago

Andrey E. Miroshnichenko, Sergej Flach, Andrey V. Gorbach, Boris S. Luk’yanchuk, Yuri S. Kivshar and Michael I. Tribelsky
Fano resonances in photonic crystals
Waveguide + defect coupling

- Interference between different photon pathways
- Bandwidth modulation with small refractive index variation ($\frac{\delta n}{n} < 10^{-4}$)

How can we use optical resonators to control light?

1 μm
Fano resonance and nonlinear switch

\[ e^{i kn} \]

\[ R \, e^{-i kn} \]

\[ T \, e^{i kn} \]

\[ A_{-2} \rightarrow A_{-1} \rightarrow A_0 \rightarrow A_1 \rightarrow A_2 \]

\[ A_\alpha \]

\[ |T(I_{in})|^2 \]

\[ I_{out}(I_{in}) \]

\[ I_\alpha(I_{in}) \]

Frequency, \( \omega = a/\lambda \)
Concepts of nonlinear devices

Canberra, 2002

MIT, 2002

Stanford, 2003

Sydney, 2003
Fano resonance with nanoantennas

M. Rybin et al, PRB (2013)
RF experimental Fano antennas

M. Rybin et al, PRB (2013)
Fano interference governs wave transport in disordered systems

Alexander N. Poddubny, Mikhail V. Rybin, Mikhail F. Limonov & Yuri S. Kivshar

DOI: 10.1038/ncomms1924

Part 2:

Airy beams
Airy Function

George Biddell Airy

\[
W(m) = \int_{0}^{\infty} \cos \left( \frac{\pi}{2} (\omega^3 - m\omega) \right) d\omega,
\]

\[
\int_{-\infty}^{\infty} |Ai(x)|^2 dx = \infty
\]


Widely used:

\[
Ai(x) = \frac{1}{\pi} \int_{0}^{\infty} \cos \left( \frac{z^3}{3} + xz \right) dz
\]

\[
Ai(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i(z^3/3 + xz)} dz
\]
Airy wave packets

Free particle Schrödinger equation:

\[-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t},\]

Initial Airy distribution:

\[\psi(x,0) = Ai(Bx/\hbar^{2/3}),\]

Airy solution:

\[\psi(x,t) = Ai\left[\frac{B}{\hbar^{2/3}}\left(x - \frac{B^3t^2}{4m^2}\right)\right] e^{(iB^3t/2mh)[x-(B^3t^2/6m^2)]}.\]

Features:

- Asymmetric field Profile
- Non-spreading
- Self-deflection

\[\int_{-\infty}^{\infty} |Ai(x)|^2 \, dx = \infty\]

Finite energy Airy beams

Free particle Schrödinger equation:
\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - i\hbar \frac{\partial \psi}{\partial t} = 0 \]

Paraxial wave equation in free space:
\[ -\frac{1}{2} \frac{\partial^2 \phi}{\partial s^2} - i \frac{\partial \phi}{\partial \xi} = 0 \quad (s = x/x_0, \quad \xi = z/kx_0^2) \]

(1) Initial distribution (extended):
\[ \phi(0, s) = \text{Ai}(s) \]
\[ \phi(\xi, s) = \text{Ai}(s - (\xi/2)^2) \exp(i(s\xi/2) - i(\xi^3/12)) \]

(2) Initial distribution (truncated):
\[ \phi(0, s) = \text{Ai}(s) \exp(as) \]
\[ \phi(\xi, s) = \text{Ai}(s - (\xi/2)^2 + ia\xi) \exp(as - (a\xi^2/2) - i(\xi^3/12) + i(a^2\xi/2) + i(s\xi/2)) \]

Initial Airy distribution

\[ \phi(0, s) = \text{Ai}(s) \exp(as) \]

After Fourier transform:

\[ \Phi_0(k) \propto \exp(-ak^2) \exp(ik^3/3) \]

Airy beam generation:

Airy beam manipulation

Tilting incident beam

Phase mask arrangement

Nonlinear process


Self-healing properties of Airy Beams

Self-healing: restore the initial beam profiles after perturbations

Airy Caustic

Babinet’s Principle


M. Born, and E. Wolf, Principles of optics
Energy flow during self-healing

Paraxial wave equation in spatiotemporal domain

\[
    i \frac{\partial \psi}{\partial Z} + \frac{1}{2} \left( \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} + \frac{\partial^2 \psi}{\partial T^2} \right) = 0, \quad \psi = \phi(Z,T)U(Z,X,Y)
\]

Spatial Bessel (U)+Temporal Airy (\(\phi\))

Spatial Airy (U)+Temporal Airy (\(\phi\)) (Airy3)


Plasmonic Airy Beam


\[ E_y(x,y,z) = A(x,z) \exp(ik_0z) \exp(-\alpha_d y), \]

\[ k_0 = k_0 \sqrt{\varepsilon_d \varepsilon_m / (\varepsilon_d + \varepsilon_m)}. \]

\[ \frac{\partial^2 A}{\partial x^2} + 2ik_z \frac{\partial A}{\partial z} = 0. \]
Airy plasmons: Experimental generation

FIB FEI Helios 600
150nm thick gold film
11 periods of 200nm thick slits (in z-direction) and varying width in x-direction from 2μm to 200nm

NSOM imaging of the Airy plasmon
Near field imaging

Experiment

Numerics
Plasmonic Airy beams with dynamically controlled trajectories

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Plasmonic Airy Beam Generated by In-Plane Diffraction

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Physicists curve light on metal 'chip'

By Liz Tay on Aug 17, 2011 3:22 PM (4 days ago)
Filed under Hardware

Applications hinge on development of plasmonic circuits.

Researchers have generated self-healing, curved beams of light on a flat metal surface in a move that could yield improved optical computing components.

A team of physicists from the Australian National University (ANU) in Canberra generated the so-called Airy beams on a 150-nanometre-thick gold film.
Scattering of Airy plasmons

- Strong sensitivity to the beam tilt
- Control of the focusing spot
- Good agreement with the theory

A. Klein et al. Opt Lett 2012 (one of 10 most downloaded papers of August)
Plasmonic potentials

Plasmonic Airy beam in linear potentials

linear potential: \( n(x) = n_0 - a \theta x / h_0^2, \quad |\theta x| \ll h_0, \quad n_0 = n_{\text{eff}}(h_0). \)

\[ E_y(x, y, z) = A(x, y)\psi(x, z) \exp(in_0kz) \]

Eigen field
Envelope function

\[ i \frac{\partial \psi}{\partial \xi} + f s \psi + \frac{1}{2} \frac{\partial^2 \psi}{\partial s^2} = 0, \quad s = x / x_0, \quad \xi = z / (n_0 k x_0^2), \]

\[ f = -a \theta k^2 n_0 x_0^3 / h_0^2 \]
Plasmonic Airy beam in linear potentials

\[ i \frac{\partial \psi}{\partial \xi} + f s \psi + \frac{1}{2} \frac{\partial^2 \psi}{\partial s^2} = 0, \]

Solution (Fresnel transform):

\[
\psi(s, \xi) = \sqrt{\frac{1}{2\pi i \xi}} \exp(-i \frac{f^2 \xi^3}{6}) \int_{-\infty}^{+\infty} \psi(\chi, 0) \exp\left(\frac{i}{2\xi}[s - f \frac{\xi^2}{2} - \chi]^2\right) d\chi,
\]

For incident truncated Airy beam:

\[\psi(s, 0) = A i(s) \exp(as)(a > 0)\]

Corresponding solution:

\[
\psi(s, \xi) = A i\left[s - \frac{1}{4}(1 + 2f)\xi^2 + ia\xi\right] \exp(as - \frac{af\xi^2}{2} - \frac{a^2\xi^2}{2}) \exp\left[i\left(-\frac{f^2\xi^3}{6} + fs\xi - \frac{f^2\xi^3}{4} - \frac{\xi^3}{12} + \frac{a^2\xi^2}{2} + \frac{s\xi}{2}\right)\right].
\]

when

\[f = -1/2, \quad \theta = \theta_c = h_0^2/(2ak^2n_0x_0^3)\]

\[\theta = 0\]

\[\theta = \theta_c\]
Airy plasmons in linear potentials

Paraxial wave equation

\[ i \frac{\partial \psi}{\partial \xi} + f s \psi + \frac{1}{2} \frac{\partial^2 \psi}{\partial s^2} = 0 \]

Stationary solution

\[ n(x) = n_0 - a \theta x / h_0^2 \]

\[ \theta = \theta_c = h_0^2 / (2 \alpha k^2 n_0 x_0^3) \]

Deflection manipulation through changing tilting angles

\[ \theta = -0.35^\circ \]

\[ h = 60 \text{ nm} \]

\[ \lambda = 632.8 \text{ nm} \]
Self-healing properties

\[ i \frac{\partial \psi}{\partial \xi} + f_s \psi + \frac{1}{2} \frac{\partial^2 \psi}{\partial s^2} = 0, \]

Solution (Fresnel transform):

\[ \psi(s, \xi) = \sqrt{\frac{1}{2\pi i \xi}} \exp(-i \frac{f^2 \xi^3}{6}) \int_{-\infty}^{\infty} \psi(\chi, 0) \exp\left(\frac{i}{2\xi} [(s - \frac{f \xi^2}{2}) - \chi]^2\right) d\chi, \]

\( \lambda = 632.8 \text{ nm}, \ h_0 = 60 \text{ nm}, \ a = 0.1, \ x_0 = 500 \text{ nm} \) and \( \theta_c = 0.175^\circ \).
Different wavelength for Airy beams in linear potentials

\[ \theta = \theta_c = \frac{h_0^2}{(2a k^2 n_0 x_0^3)} \]

Wavelength dependent

\[ \theta_c \text{ (deg)} \]

\[ \lambda \text{ (µm)} \]

\[ z \text{ (µm)} \]

\[ x \text{ (µm)} \]
Part 3:

Parity-time symmetric systems and their applications in optics

Complex quantum potentials

Quantum particle on the line:

\[ i\hbar \psi_t = -\frac{\hbar^2}{2m} \psi_{xx} + U(x)\psi, \]

\[ \psi(x, t) = \exp \left( -i \frac{E}{\hbar} t \right) \psi(x), \quad -\frac{\hbar^2}{2m} \psi_{xx} + U(x)\psi = E\psi \]

What if \( U(x) = V(x) + iW(x) \)? Commonly, \( E = E_r + i\gamma \); hence

\[ \psi(x, t) = \exp \left( -i \frac{E_r}{\hbar} t \right) \exp \left( \frac{\gamma}{\hbar} t \right) \psi(x), \quad \text{no good.} \]
Parity-Time (PT) symmetric quantum potentials

Is it possible that, despite $W \neq 0$, all eigenvalues are real?
Yes! Example:

$$U(x) = -(ix)^N, \quad N \text{ real.}$$

- $N \geq 2$: infinite sequence of real, positive, eigenvalues
- $1 < N < 2$: finite number of real positive + infinite sequence of complex conjugate pairs
- $N \leq 1$: no real eigenvalues

(C M Bender & S Boettchner, PRL 80 5243 (1998))

The necessary condition for the entirely real spectrum is $\mathcal{PT}$ symmetry:

$$U^*(-x) = U(x), \quad \Rightarrow \quad V(-x) = V(x), \quad W(-x) = -W(x)$$
Parity-Time (PT) Symmetry

Parity operator: $P$

\[
\hat{p} \rightarrow -\hat{p}, \quad \hat{x} \rightarrow -\hat{x}
\]

Time operator: $T$

\[
\hat{p} \rightarrow -\hat{p}, \quad \hat{x} \rightarrow \hat{x}, \quad i \rightarrow -i,
\]

Hamiltonian: $H$

\[
\hat{H} = \frac{\hat{p}^2}{2} + V(x)
\]

Requirement:

\[
V(x) = V^*(-x)
\]

- Quantum field theory
- Complex Lie algebra
- Complex crystals
- Condensed matter system
- Population biology
- Optics
Observation of PT-symmetry breaking in complex optical potentials

Guo et al. PRL (2009)

- PT symmetry: supermodes do not experience gain or loss; zero gain/loss on average for arbitrary inputs

- Broken PT symmetry (unbalanced gain and loss): mode confinement and/or amplification in the waveguide with gain

Observation of parity-time symmetry in optics

Ruter et al., Nature Physics (2010)
Nonlinear PT-symmetric coupler—a dimer

Optical coupler

Model

\[
i \frac{da_1}{dz} + i \rho a_1 + Ca_2 + \gamma |a_1|^2 a_1 = 0
\]
\[
i \frac{da_2}{dz} - i \rho a_2 + Ca_1 + \gamma |a_2|^2 a_2 = 0
\]

PHYSICAL REVIEW A 82, 043818 (2010)

Nonlinear suppression of time reversals in $\mathcal{PT}$-symmetric optical couplers

Andrey A. Sukhorukov, Zhiyong Xu, and Yuri S. Kivshar
Properties of nonlinear modes

Stationary states:

\[ a_1 = \sqrt{I} \cos[\theta(z)] \exp[+i\varphi(z)/2 + i\beta(z)] \]
\[ a_2 = \sqrt{I} \sin[\theta(z)] \exp[-i\varphi(z)/2 + i\beta(z)] \]

\[ \beta_{\pm} = \gamma I_0 / 2 + C \cos(\varphi_{\pm}), \]
\[ I = I_0, \theta = \pi / 4, \beta = \beta_{\pm}z, \]
\[ \sin(\varphi_{\pm}) = \rho / C, \]
\[ \cos(\varphi_{\pm}) = \mp \sqrt{1 - (\rho / C)^2} \]

Propagation dynamics of nonlinear modes

Periodic evolution of nonlinear modes

Nonlinearity-induced symmetry breaking

PT-symmetric dimer in a linear chain

\[
\begin{align*}
\dot{a}_j - & C_1 a_{j-1} + C_1 a_{j+1} = 0, \quad j \neq 0, 1 \\
\dot{a}_0 - & i\rho a_0 + C_1 a_{-1} + C_2 a_1 = 0 \\
\dot{a}_1 - & i\rho a_1 + C_2 a_0 + C_1 a_2 = 0
\end{align*}
\]

- \(a_j\) – mode amplitudes at waveguides
- \(C\) – coupling coefficient between the waveguide modes
- \(\rho\) – coefficient of gain/loss in waveguides 0,1

PT symmetry breaking for planar lattice

- Boundary conditions \( a_{N+2} \equiv 0, \quad a_{-N-1} \equiv 0 \)

- Consider eigenmodes:
  \[
  a_n = A_n \exp(i \phi_n + i \beta z)
  \]

- PT symmetry:
  \[
  \text{Im}(\beta) = 0, \quad \frac{d|a_n|}{dz} = 0
  \]

- For \( n \neq 0, 1 \):
  \[
  C_1 |a_n| [\sin(\phi_{n+1} - \phi_n)|a_{n+1}| + \sin(\phi_{n-1} - \phi_n)|a_{n-1}|] = 0
  \]

- For \( |n| \geq 1 \):
  \[
  |a_n a_{n+1}| \sin(\phi_{n+1} - \phi_n) = 0
  \]

- Consider \( n = 0, 1 \):
  \[
  |a_0| = |a_1| \text{ and } C_2 \sin(\phi_1 - \phi_0) + \rho = 0
  \]

- Solvability of last relation defines PT symmetry
PT symmetry breaking for a straight array

- Stability condition

\[ |C_2| > |\rho| \]

- Same stability condition as for isolated PT coupler!
- Does not depend on lattice coupling outside the active region

PT symmetry breaking for a circular array

- Consider ratio \( J = B_+ \exp(-ik)/F_- \)
  
  \[
  |J|^2 = \frac{c_1^2 - c_2^2 + \rho^2 - 2c_1 [\rho - 2c_2 \text{Im}(J)] \sin(k)}{c_1^2 - c_2^2 + \rho^2 + 2c_1 \rho \sin(k)}
  \]

- PT symmetry breaking occurs at a given \( k \) when solutions disappear
- Threshold corresponds to real \( k \)
- Stability condition:
  
  \[
  ||C_1|| - ||C_2|| \geq |\rho|
  \]

- Threshold depends on all lattice parameters

Nonlocal effects

- PT-defect – non-Hermitian
- Quantum-mechanical context: interaction of a non-Hermitian system with the Hermitian world

- Dynamics can be sensitive to a potential at distant locations
- Continuing debate on the meaning of nonlocality and relevance to real physical systems

H. F. Jones, Phys. Rev. D 76, 125003 (2007); M. Znojil, Phys. Rev. D 80, 045009 (2009); …
PT-symmetric dimer in a nonlinear chain

- Distant boundaries (infinite lattice limit)

- Kerr-type nonlinearity

\[ i \frac{d}{dz} a_j + C_1 a_{j-1} + C_1 a_{j+1} + \gamma |a_j|^2 a_j = 0, \quad j \neq 0, 1, \]

\[ i \frac{d}{dz} a_0 + i \rho a_0 + C_1 a_{-1} + C_2 a_1 + \gamma |a_0|^2 a_0 = 0, \]

\[ i \frac{d}{dz} a_1 - i \rho a_1 + C_2 a_0 + C_1 a_2 + \gamma |a_1|^2 a_1 = 0, \]

- Conservative solitons exist on either sides of PT coupler

\[ a_j = \text{Asech}[\delta (j - j_0 - 2 C_1 v z)] e^{i [v (j - j_0) + (\delta^2 - v^2) C_1 z + \alpha]} \]
Soliton scattering

• both reflected and transmitted waves are amplified

Scattering nonreciprocity

• Transmitted waves do not change but reflection depends of the position of gain and loss waveguides

Localized modes

• PT symmetric defect supports a localized mode

Soliton scattering by PT coupler

- Lattice parameters
  \[ C_1 = 2, \quad C_2 = 4, \quad \rho = 1.5, \quad \gamma = 1 \]
- Soliton velocity \[ \nu = 0.5 \]
- Localized mode at PT coupler is excited when soliton amplitude is increased (right)
Controlling soliton scattering with localized PT modes

**Soliton scattering**
- $\alpha$ - soliton phase
- Labels – localized PT mode amplitude

**PT symmetry breaking**
- Mode amplitude 1.4
- Left: $\alpha = 3.67$
  PT symmetry preserved
- Right: $\alpha = 3.75$
  nonlinear PT symmetry breaking
Unidirectional soliton scattering

Multi-soliton scattering

Unidirectional soliton flows in PT-symmetric potentials

U. Al Khawaja¹, S. M. Al-Marzoug²,³, H. Bahlouli²,³, and Yuri S. Kivshar⁴
PT symmetric waveguide arrays

\[ i \dot{u}_n + C(u_{n+1} - 2u_n + u_{n-1}) + 2|u_n|^2u_n = -v_n + i \gamma u_n, \]
\[ i \dot{v}_n + C(v_{n+1} - 2v_n + v_{n-1}) + 2|v_n|^2v_n = -u_n - i \gamma v_n \]

S Suchkov, B Malomed, S Dmitriev, and Y Kivshar, PRB 84 (2011)

\[ iu_t + u_{xx} + 2|u|^2u = -v + i \gamma u, \]
\[ iv_t + v_{xx} + 2|v|^2v = -u - i \gamma v . \]
Invariant manifolds and solitons

\[ iu_t + u_{xx} + 2|u|^2u = -v + i\gamma u, \]
\[ iv_t + v_{xx} + 2|v|^2v = -u - i\gamma v. \]

Change of variables:

\[ u(x, t) = e^{i(\Omega t - \theta)} U(x, t), \quad v(x, t) = e^{i\Omega t} V(x, t), \]
\[ \sin \theta = \gamma, \]

casts the \( \mathcal{PT} \)-system to

\[ iU_t + U_{xx} - \Omega U + 2|U|^2U = -\cos \theta V + i\gamma(U - V), \]
\[ iV_t + V_{xx} - \Omega V + 2|V|^2V = -\cos \theta U + i\gamma(U - V), \]

whose \( U = V \) reduction is

\[ i\phi_t + \phi_{xx} - a^2\phi + 2|\phi|^2\phi = 0, \quad \Omega = a^2 + \cos \theta \]
\[ \phi(x) = a \text{sech}(ax) \]
Soliton dynamics and instabilities

High-frequency solitons

Low-frequency solitons

A new type of breathers

Related publications from our group

Pseudo-Parity-Time Symmetry in Optical Systems

Xiaobing Luo (罗小兵), Jiahao Huang (黄嘉豪), Honghua Zhong (钟宏华), Xizhou Qin (秦锡洲), Qiongtao Xie (谢琼涛), Yuri S. Kivshar, and Chaohong Lee (李朝红)