Lecture 3

Part 1: Fano resonances
Part 2: Airy beams
Part 3: Parity-time symmetric systems

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Australian National University

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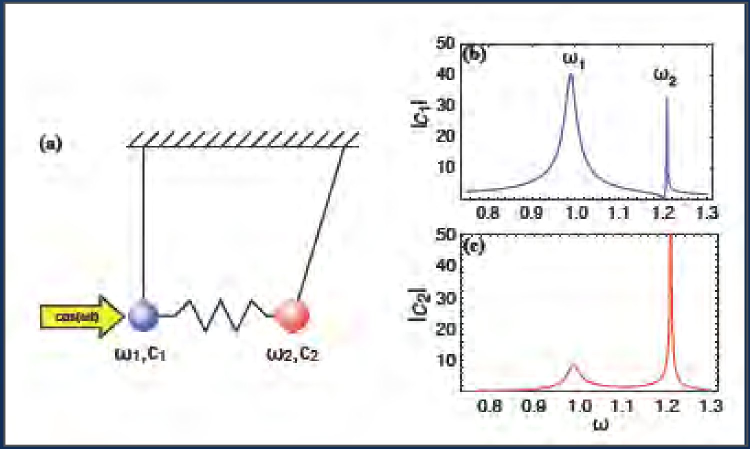
Part 1:



Fano resonances

Fano and his formula

Ugo Fano (1912-2001)

CAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. FANO
National Bureau of Standards, Washington, D. C.
(Received July 14, 1961)

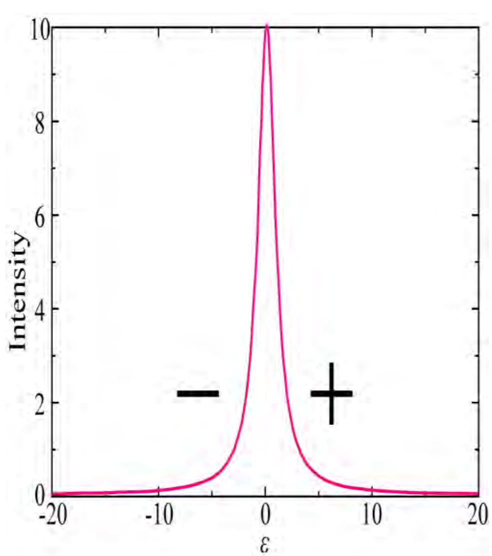
of phase normalizations. These curves are represented by

$$\frac{|(\Psi_E|T|i)|^2}{|(\psi_E|T|i)|^2}=\boxed{\frac{(q+\epsilon)^2}{1+\epsilon^2}}=1+\frac{q^2-1+2q\epsilon}{1+\epsilon^2}. \qquad (21)$$

This function is plotted in Fig. 1 for a number of values of q, which is regarded as constant in the range of interest. Notice that

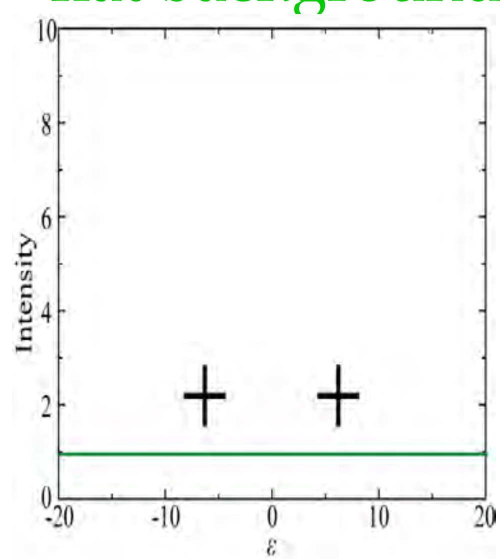
$$\boxed{\frac{(q+\varepsilon)^2}{1+\varepsilon^2}}$$

narrow band



+

flat background



→

Fano resonance

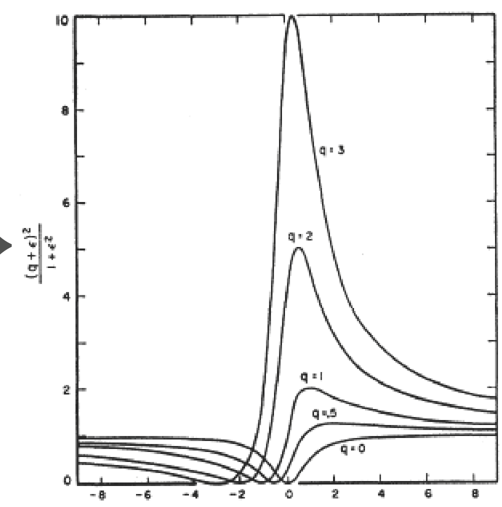


FIG. 1. Natural line shapes for different values of q. (Reverse the scale of abscissas for negative q.)

Our review paper in Rev Mod Phys

REVIEWS OF MODERN PHYSICS, VOLUME 82, JULY–SEPTEMBER 2010

Fano resonances in nanoscale structures

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Nonlinear Physics Centre and Centre for Ultrahigh Bandwidth Devices for Optical Systems (CUDOS), Research School of Physics and Engineering, Australian National University, Canberra, Australian Capital Territory 0200, Australia

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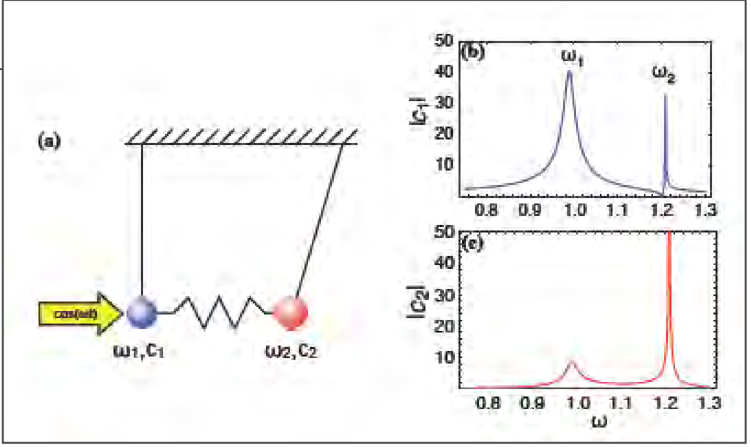
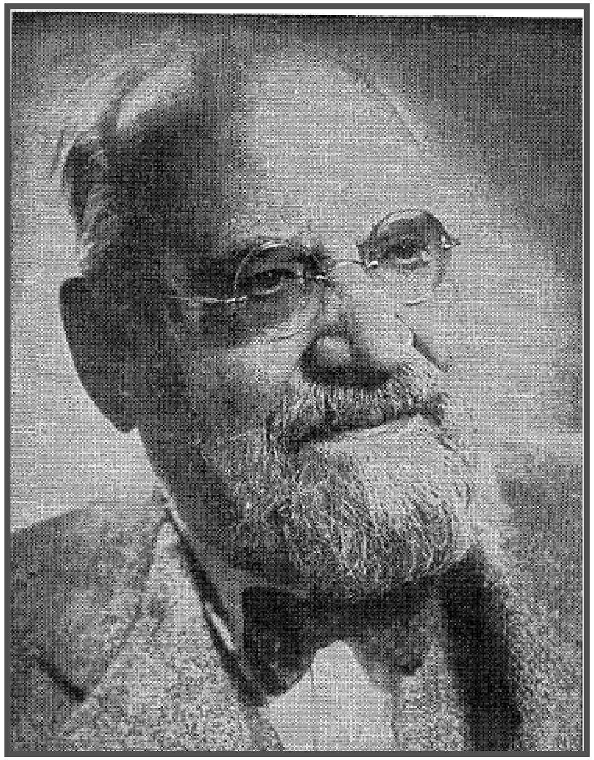
(Published 11 August 2010)

The simplest model



Fano resonances in Mie scattering



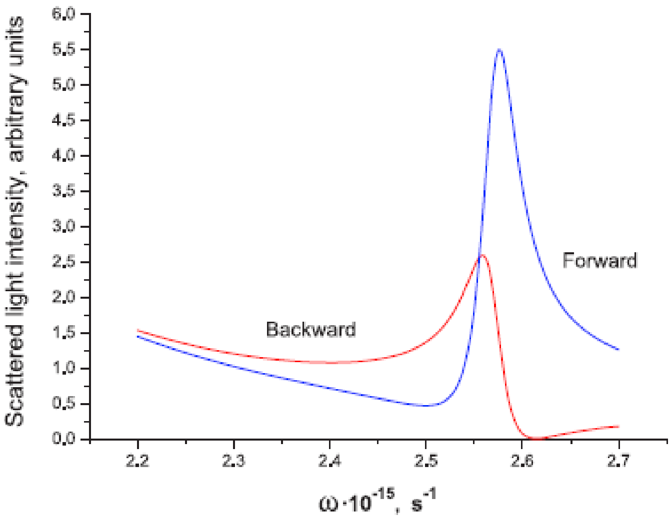
Gustav Mie (1868-1951)

1908. № 3.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 25.

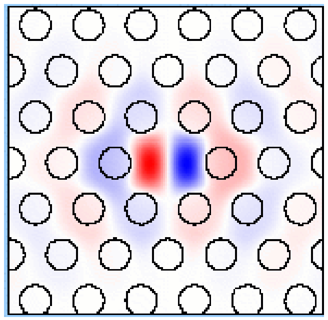
1. *Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen;*
von Gustav Mie.

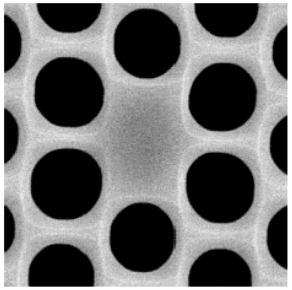


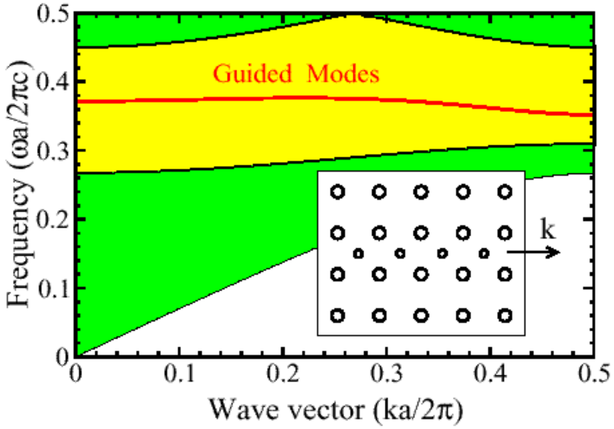
Fano Resonances: A Discovery that Was Not Made 100 Years Ago

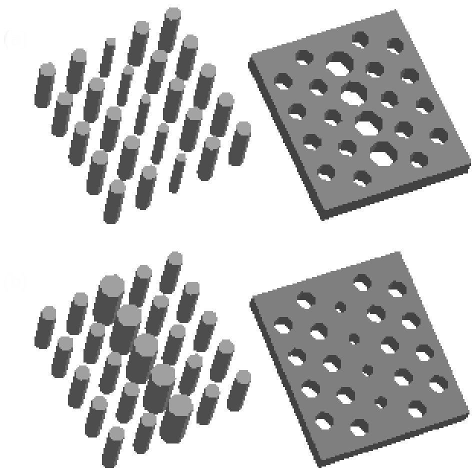
Andrey E. Miroshnichenko, Sergej Flach, Andrey V. Gorbach, Boris S. Luk'yanchuk, Yuri S. Kivshar and Michael I. Tribelsky

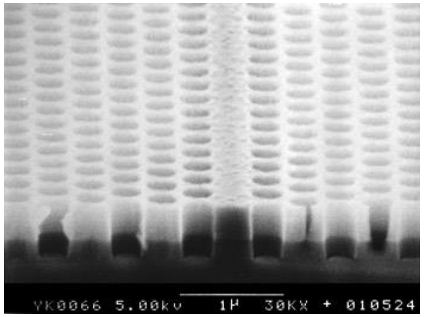
Fano resonances in photonic crystals

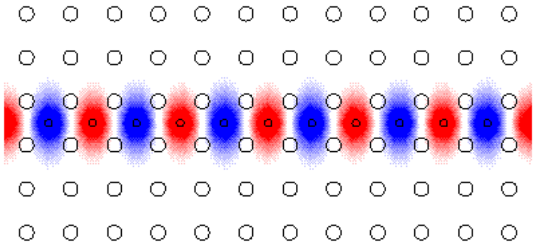




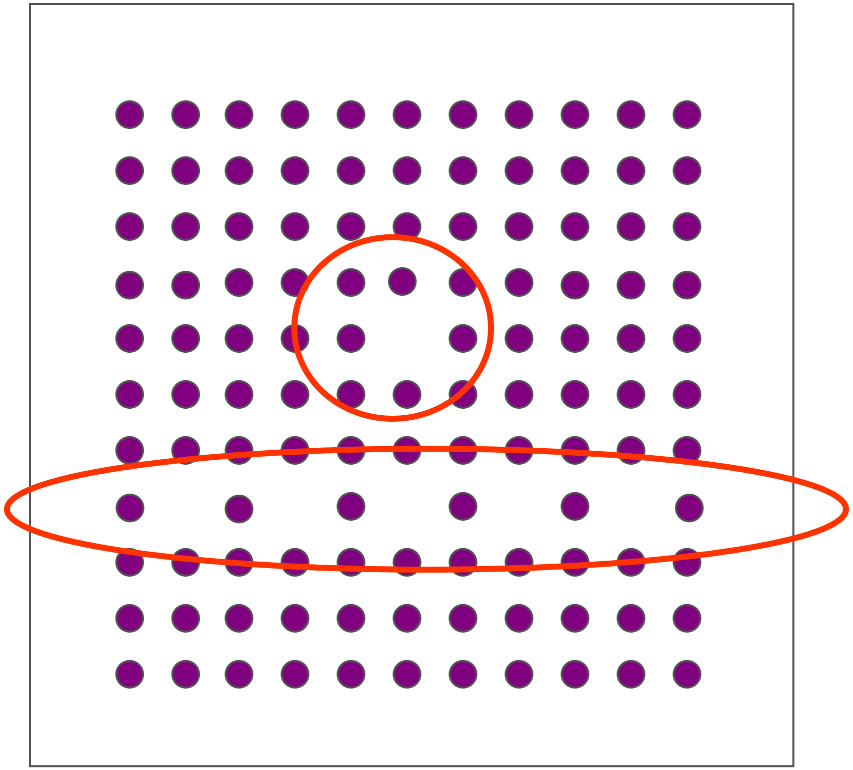


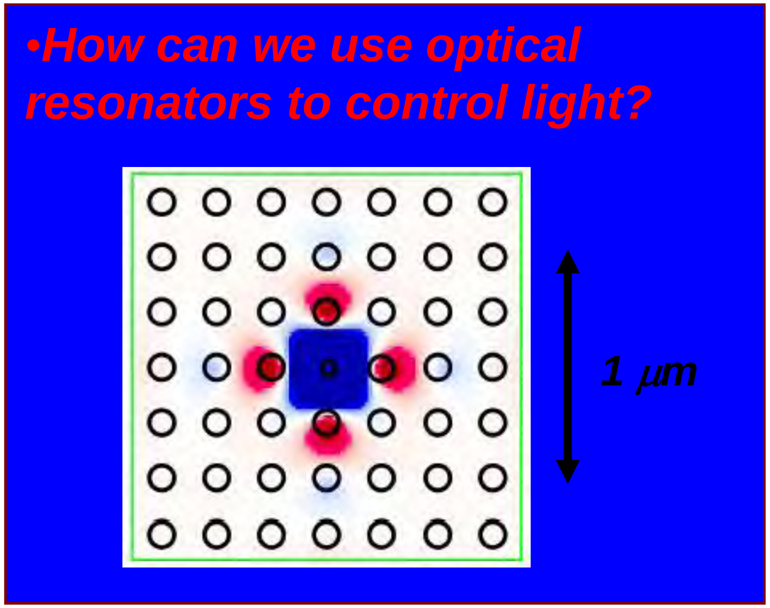






Waveguide + defect coupling

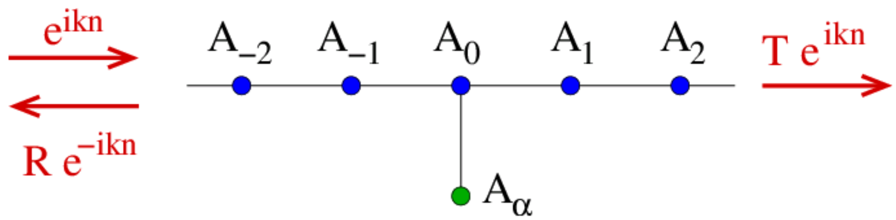


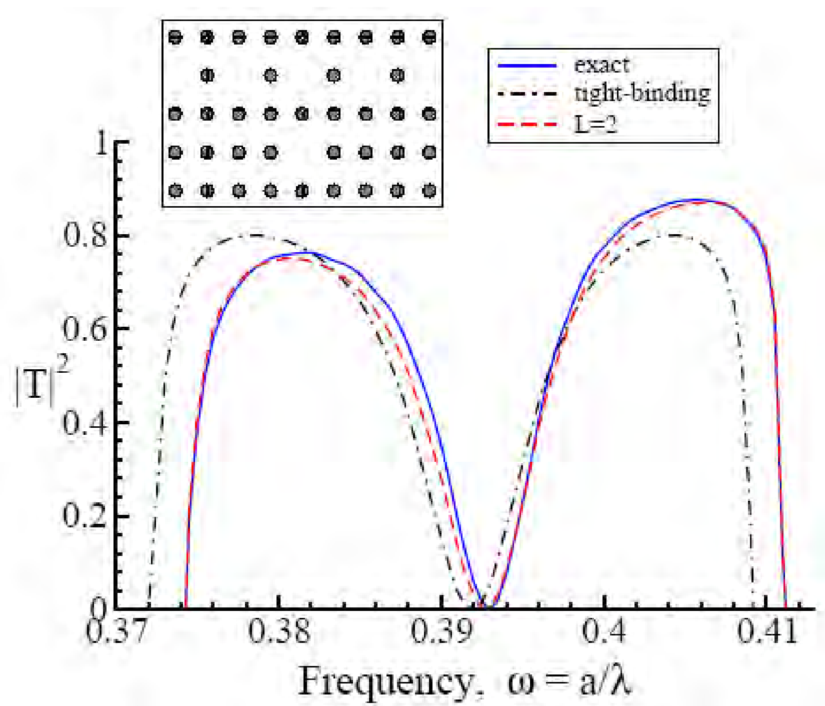


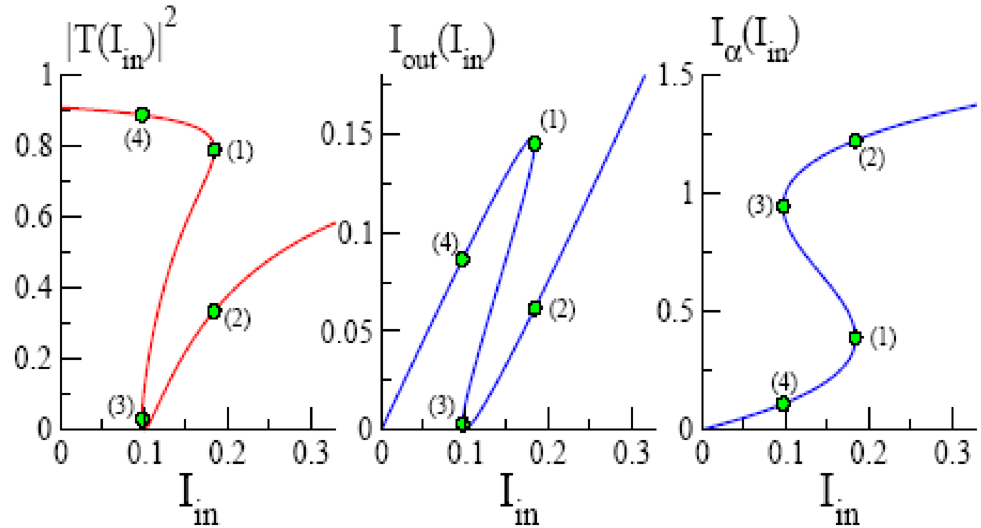
•Interference between different photon pathways

•Bandwidth modulation with small refractive index variation ($\delta n/n<10^{-4}$)

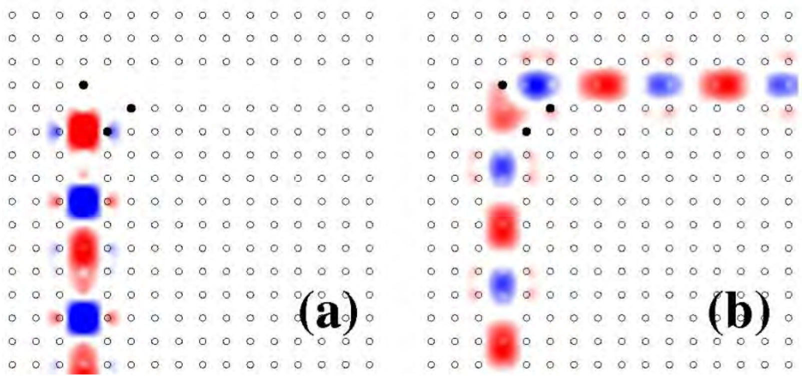
Fano resonance and nonlinear switch





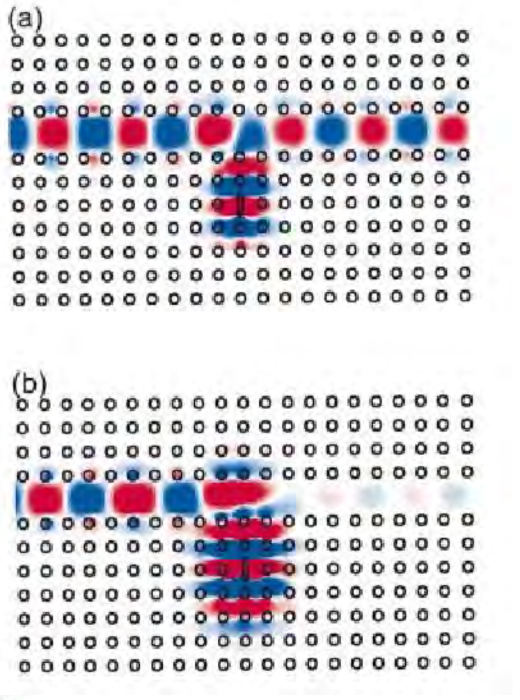


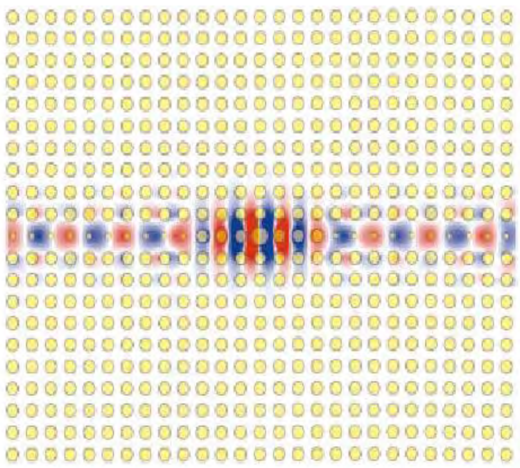
Concepts of nonlinear devices



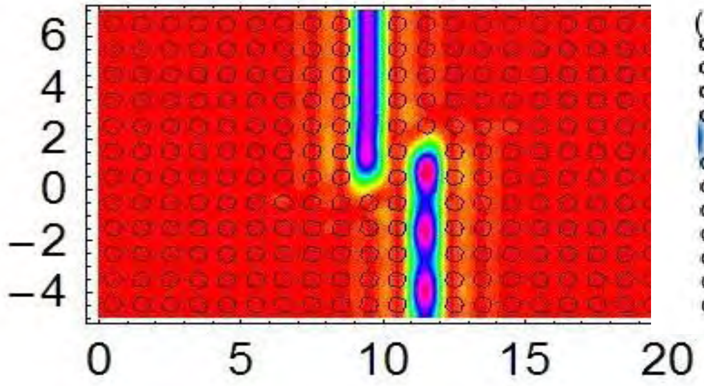
Canberra, 2002

Stanford, 2003



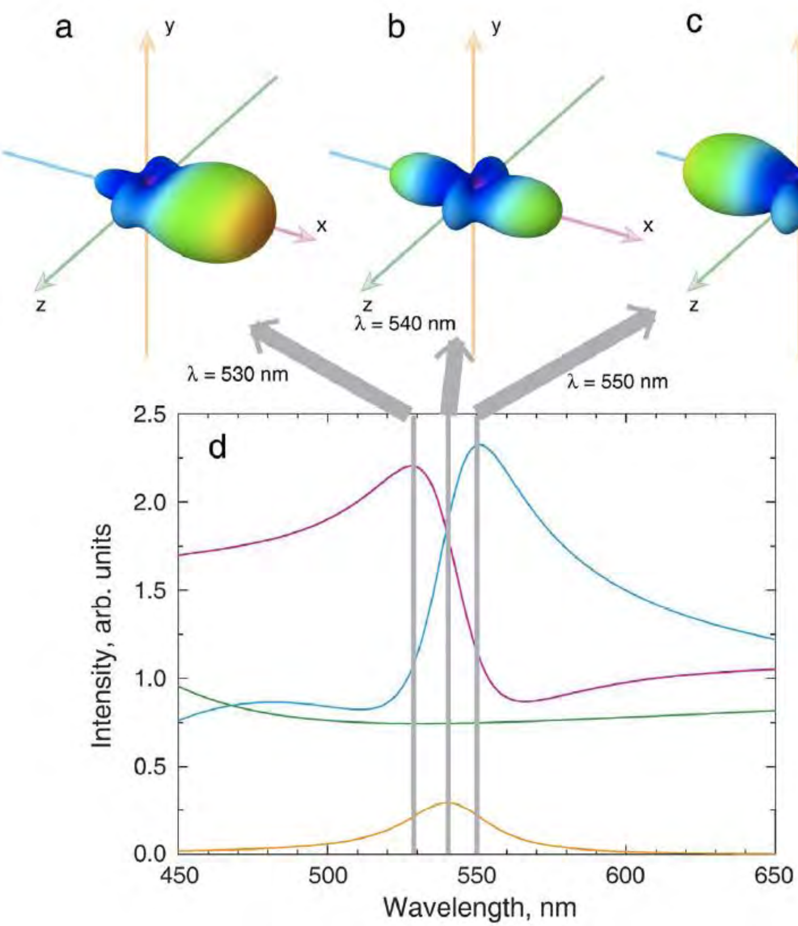


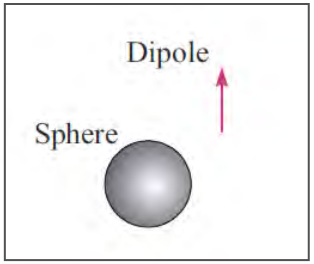
MIT, 2002

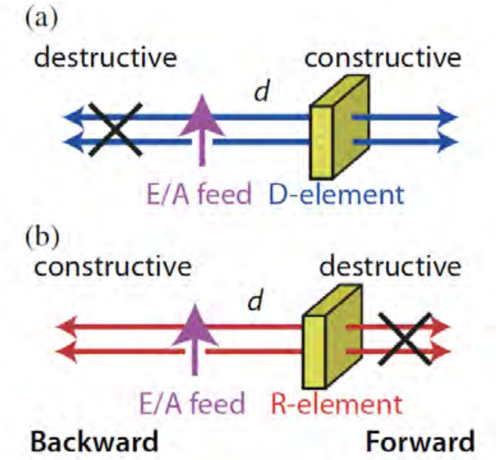


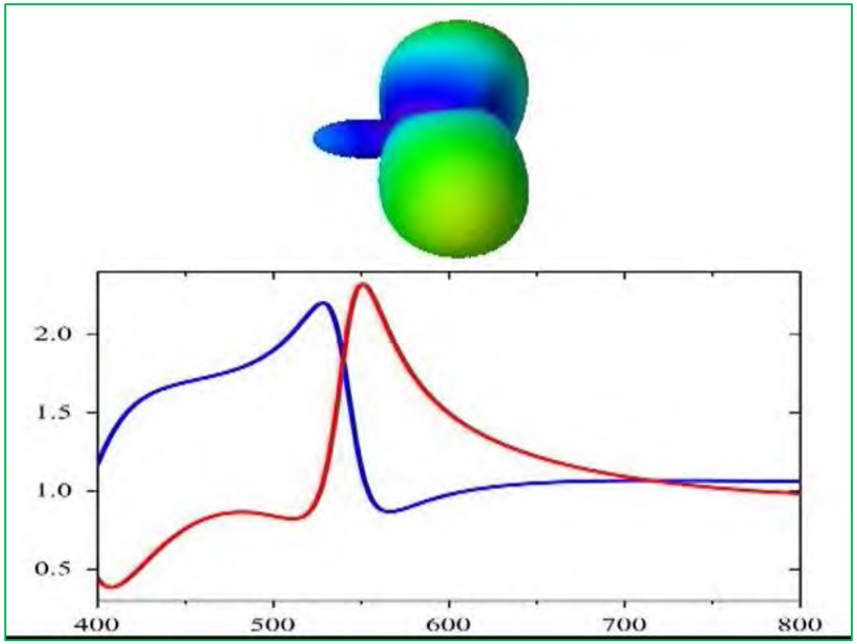
Sydney, 2003

Fano resonance with nanoantennas



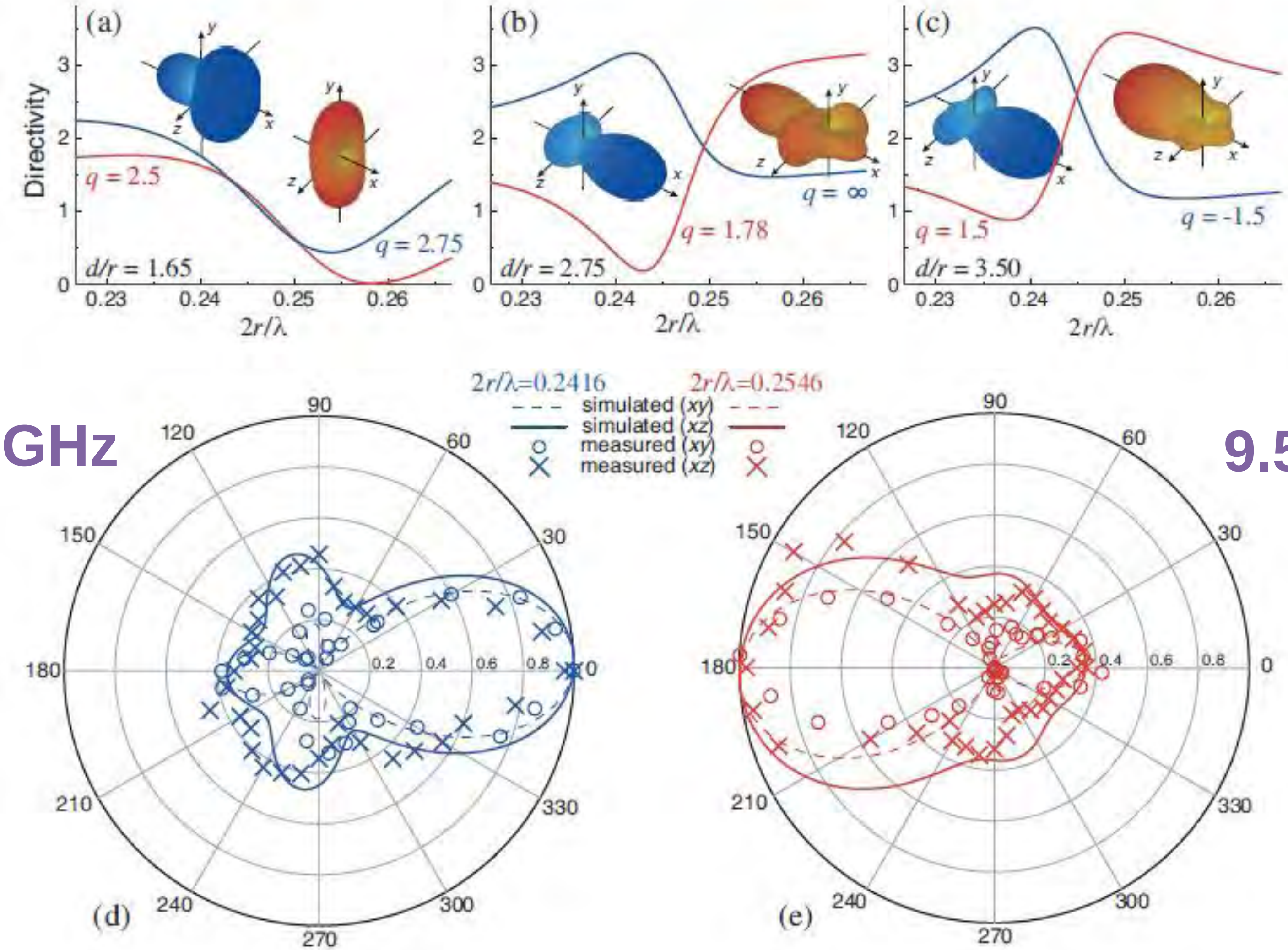






M. Rybin et al, PRB (2013)

RF experimental Fano antennas



9.06 GHz

9.55 GHz

M. Rybin et al, PRB (2013)

Fano effect and disorder

nature COMMUNICATIONS

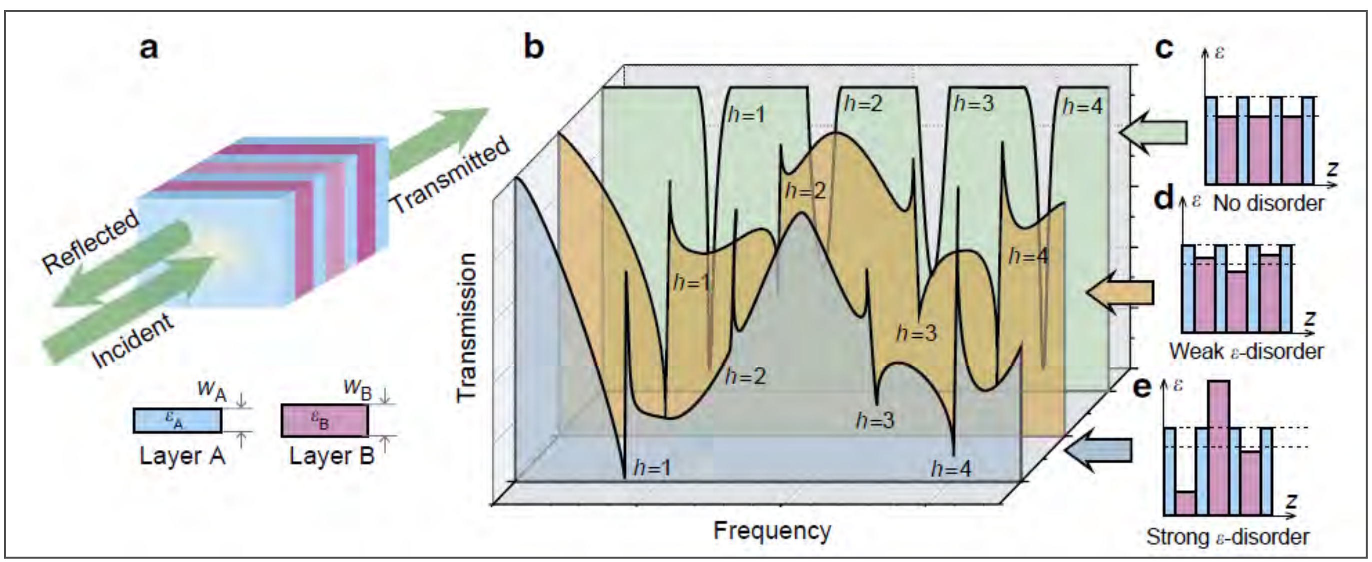
ARTICLE

Received 9 Feb 2012 | Accepted 23 May 2012 | Published 26 Jun 2012

DOI: 10.1038/ncomms1924

Fano interference governs wave transport in disordered systems

Alexander N. Poddubny[1,2], Mikhail V. Rybin[1,2], Mikhail F. Limonov[1,2] & Yuri S. Kivshar[1,3]



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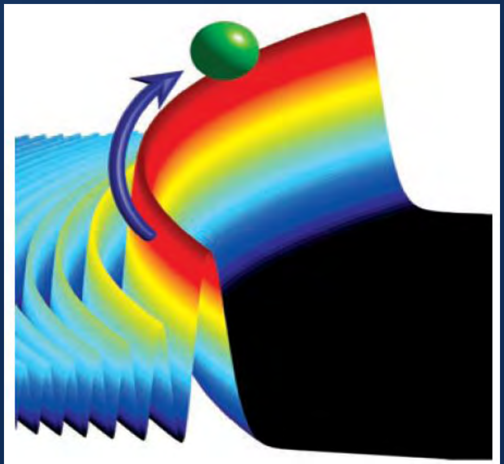


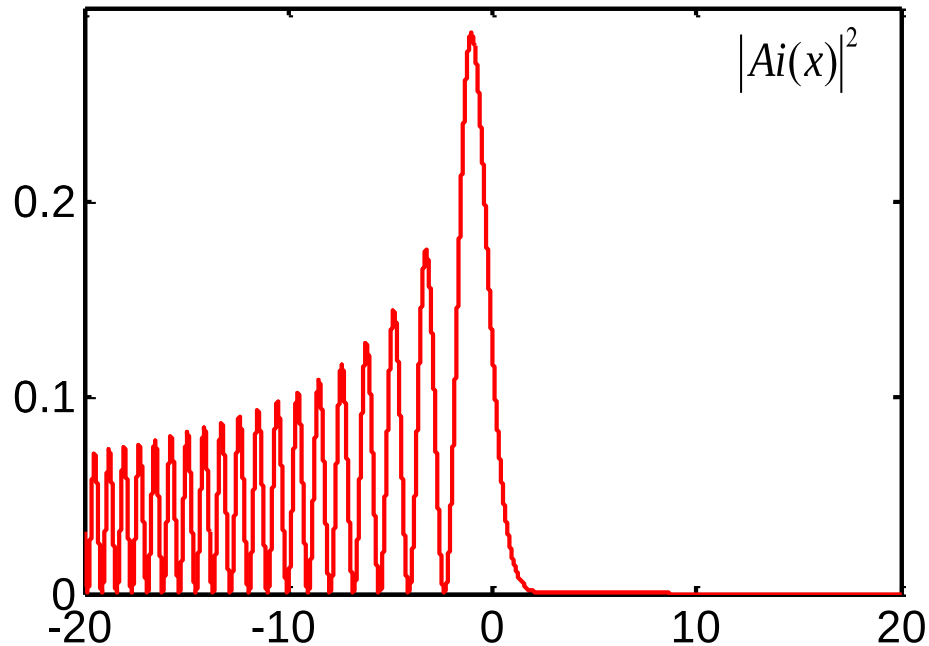
Airy beams

Airy Function

Widely used :

$$Ai(x) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{z^3}{3} + xz\right)\mathrm{d}z \qquad Ai(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{e}^{\mathrm{i}(z^3/3+xz)}\mathrm{d}z$$

George Biddell Airy



$$W(m) = \int_0^\infty \cos\left[\frac{\pi}{2}\left(\omega^3 - m\omega\right)\right]\mathrm{d}\omega,$$

$$\int_{-\infty}^{\infty} |Ai(x)|^2 dx = \infty$$

O. Vallée, and M. Soares, Airy functions and applications to physics (World Scientific, NJ, 2004).

Airy wave packets

Free particle Schrödinger equation:

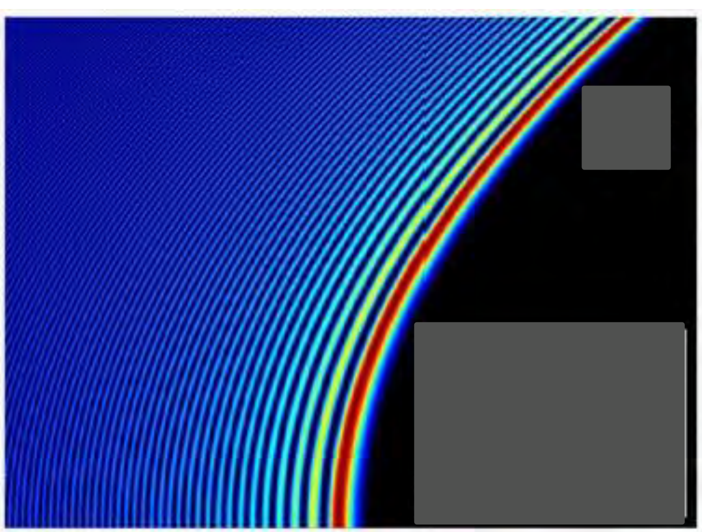
$$-\frac{\hbar^2}{2m}\frac{\partial^2 x}{\partial x^2} = i\hbar\frac{\partial\psi}{\partial t},$$

Initial Airy distribution:

$$\psi(x,0) = \mathrm{Ai}(Bx/\hbar^{2/3}),$$

Airy solution:

$$\psi(x,t) = \mathrm{Ai}\left[\frac{B}{\hbar^{2/3}}\left(x - \frac{B^3t^2}{4m^2}\right)\right] e^{(iB^3t/2m\hbar)[x-(B^3t^2/6m^2)]}.$$

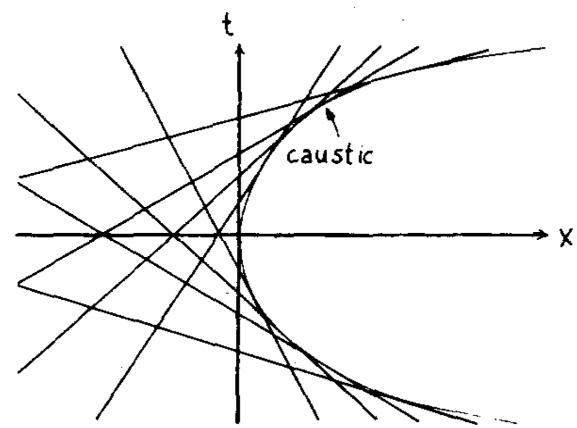


Features:

- Asymmetric field Profile
- Non-spreading

$$\int_{-\infty}^{\infty} |Ai(x)|^2 dx = \infty$$

- Self-deflection



$$t = 0,\ x_0 = -\frac{p_0^2}{B^3},\quad x = x_0 + \frac{p_0 t}{m}\ (p_0 = p)$$

M. V. Berry, and N. L. Balazs, Am J Phys 47, 264-267(1979).

Finite energy Airy beams

Free particle Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}-i\hbar\frac{\partial\psi}{\partial t}=0$$

Paraxial wave equation in free space:

$$-\frac{1}{2}\frac{\partial^2\phi}{\partial s^2}-i\frac{\partial\phi}{\partial\xi}=0\ (s=x/x_0,\ \xi=z/kx_0^2)$$

(1) Initial distribution (extended):

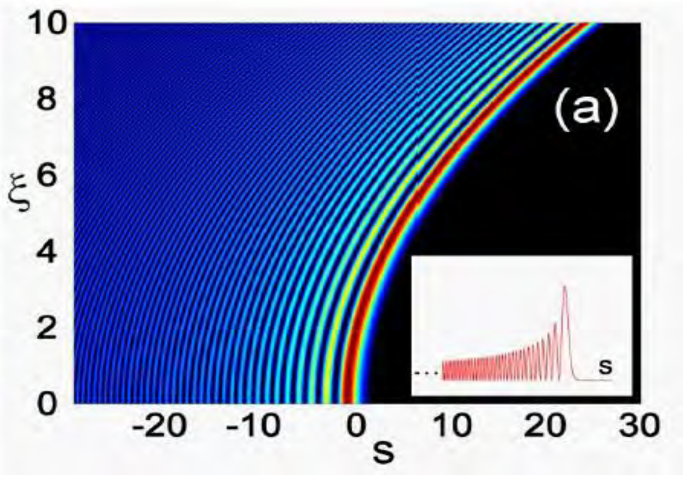
$$\phi(0,s)=\mathrm{Ai}\,(s)$$

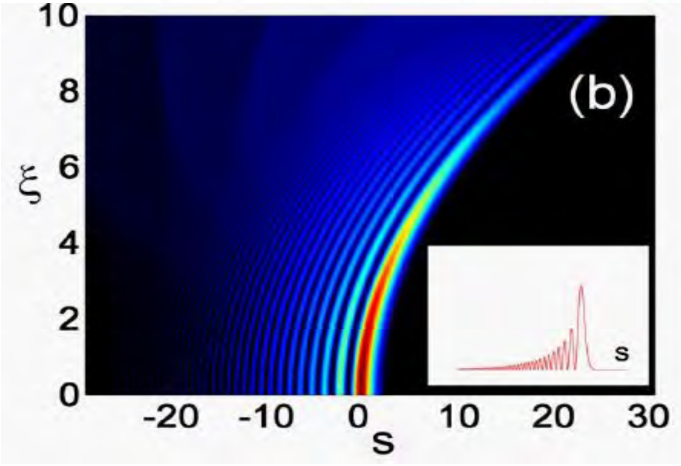
$$\phi(\xi,s)=\mathrm{Ai}(s-(\xi/2)^2)\exp(i(s\xi/2)-i(\xi^3/12)).$$

(2) Initial distribution (truncated):

$$\phi(0,s)=\mathrm{Ai}\,(s)\exp(as)$$

$$\phi(\xi,s)=\mathrm{Ai}\,(s-(\xi/2)^2+ia\xi)\exp(as-(a\xi^2/2)$$
$$-i(\xi^3/12)+i(a^2\xi/2)+i(s\xi/2)).$$





G. A. Siviloglou et. al, Phys. Rev. Lett. 99, 213901 (2007).

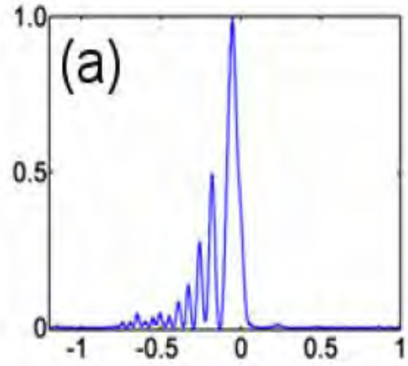
Airy beam generation

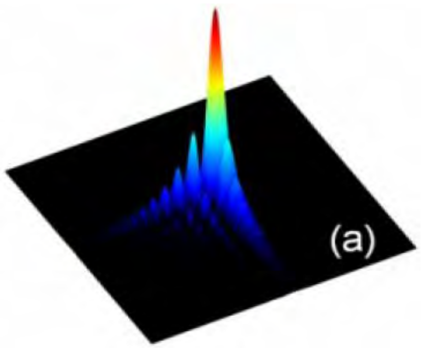
Initial Airy distribution

$$\phi(0, s) = \mathrm{Ai}(s)\exp(as)$$

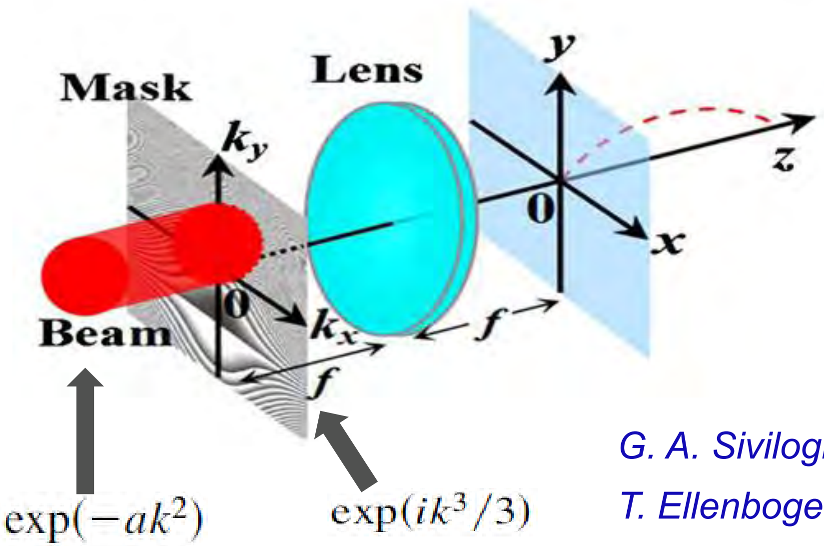
After Fourier transform:

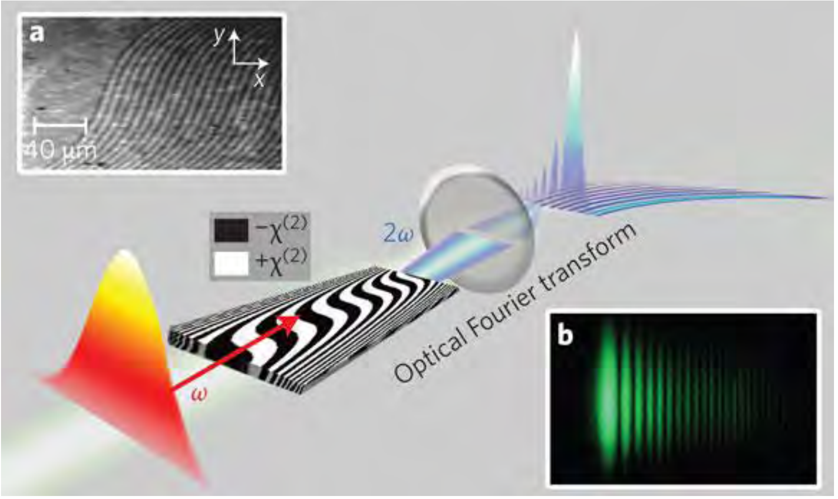
$$\Phi_0(k) \propto \exp(-ak^2)\exp(ik^3/3)$$





Airy beam generation:





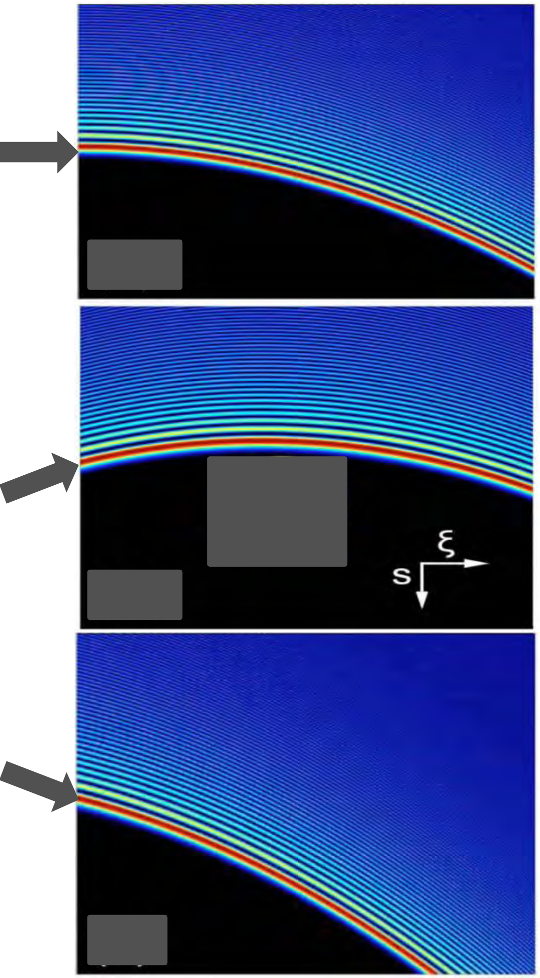
G. A. Siviloglou, and D. N. Christodoulides, Opt Lett 32, 979-981 (2007).

T. Ellenbogen et. al, Nat Photonics 3, 395-398 (2009).

Y. Hu et. al, Opt Lett 35, 2260-2262 (2010).

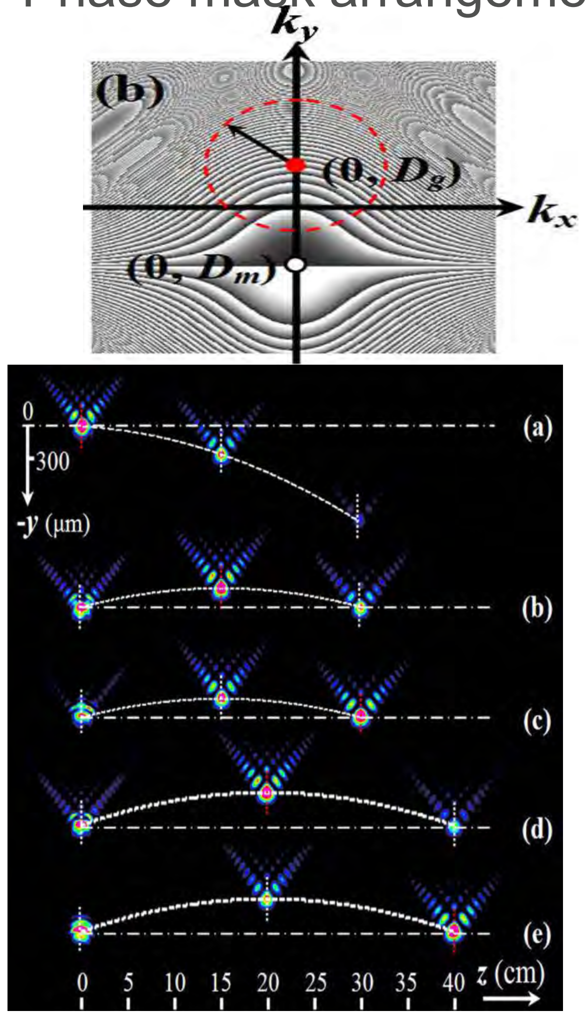
Airy beam manipulation

Tilting incident beam



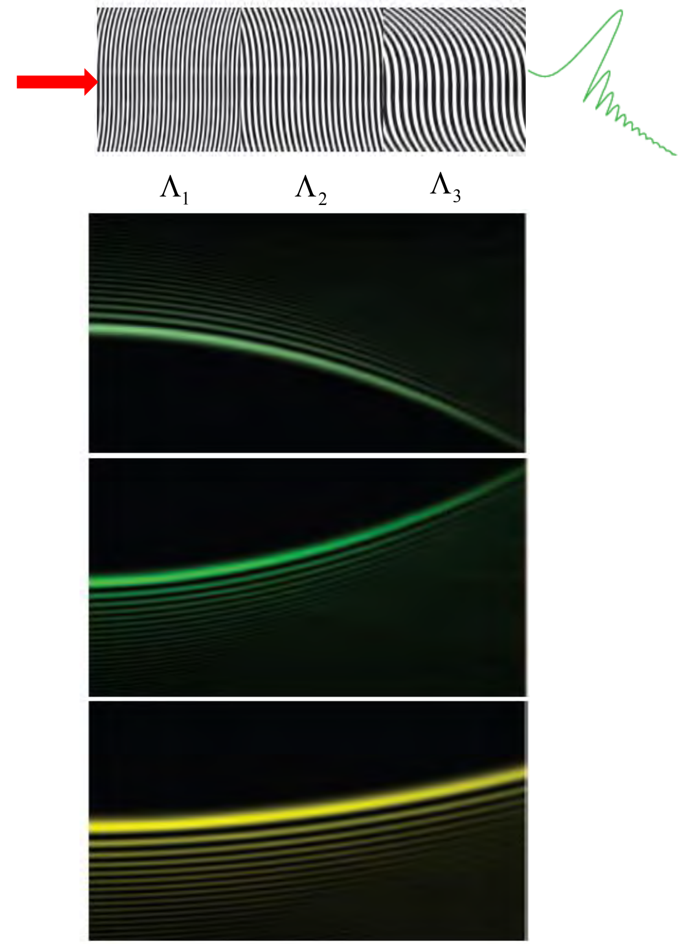
G. A. Siviloglou et. al,
Opt Lett 33, 207-209 (2008).

Phase mask arrangement



Yi Hu et. al,
Opt. Lett. 35, 2260-2262 (2010)

Nonlinear process

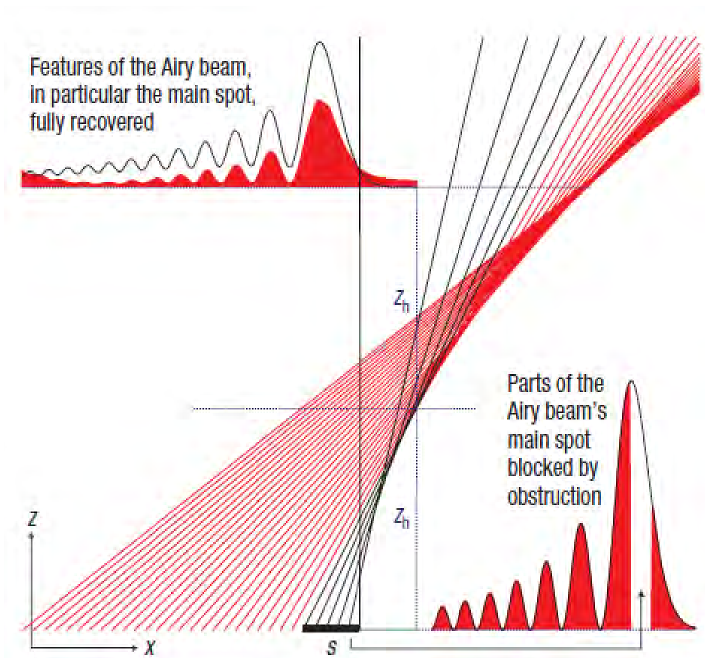


T. Ellenbogen et. al,
Nat Photonics 3, 395-398 (2009).

Self –healing properties of Airy Beams

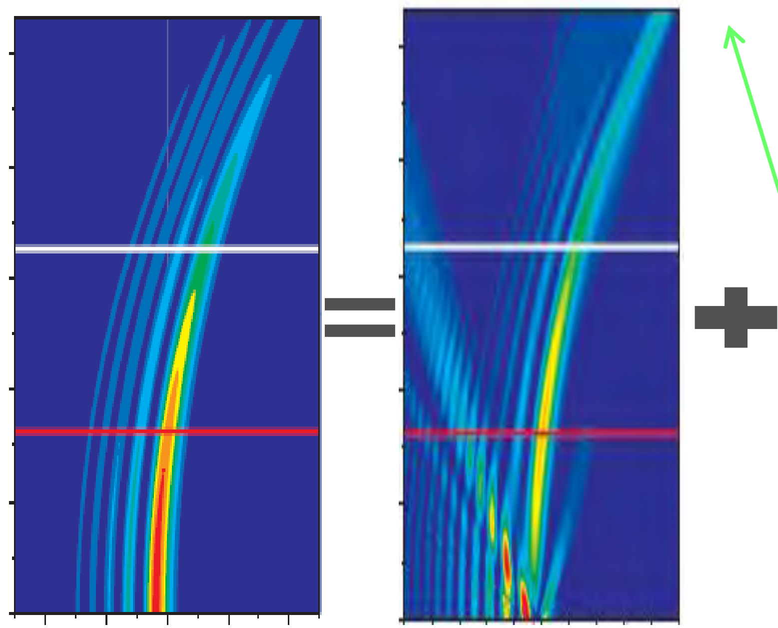
Self-healing: restore the initial beam profiles after perturbations

Airy Caustic

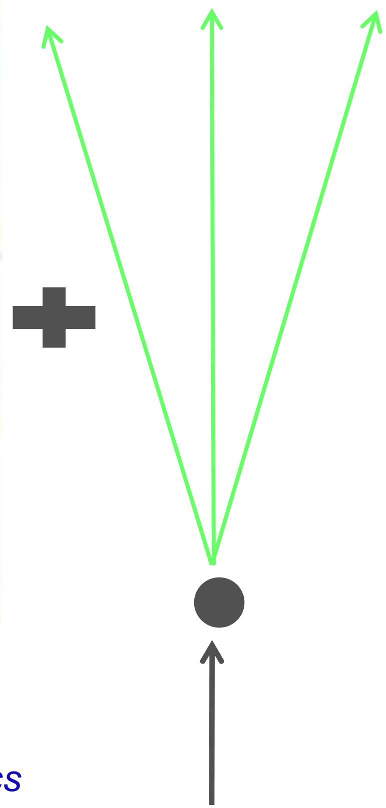


J. Baumgartl et. al,
Nat Photonics 2, 675-678 (2008).

Babinet's Principle



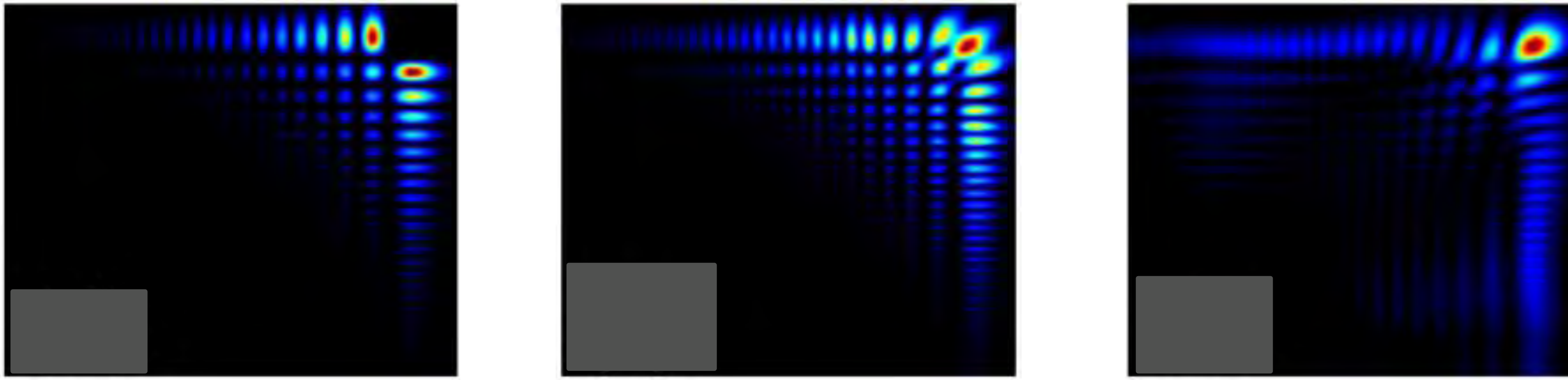
Perturbation
diffraction

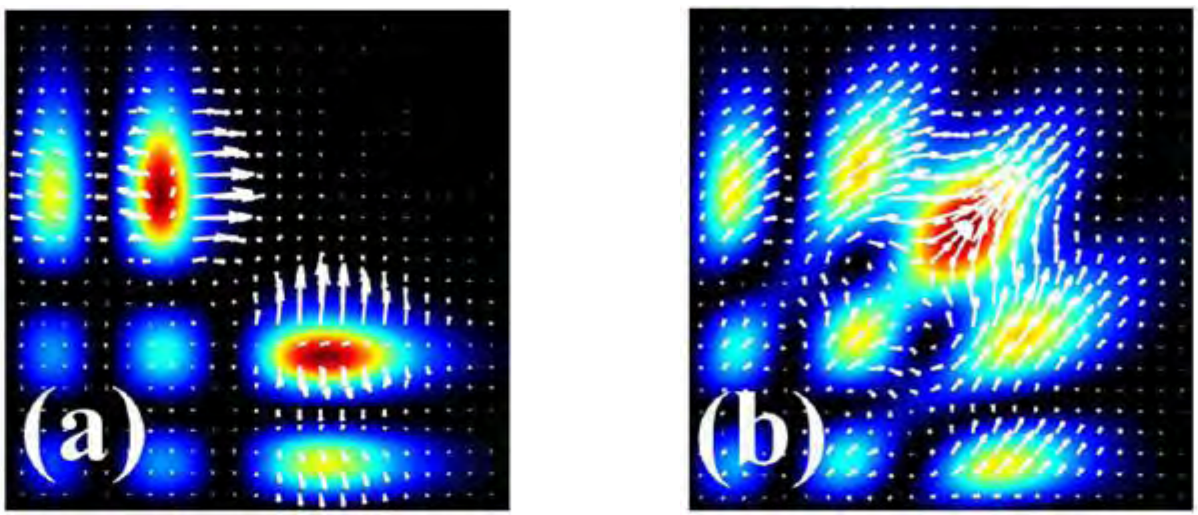


Z. Bouchal et. al,
Opt Commun 151, 207-211 (1998).

M. Born, and E. Wolf, Principles of optics

Energy flow during self-healing





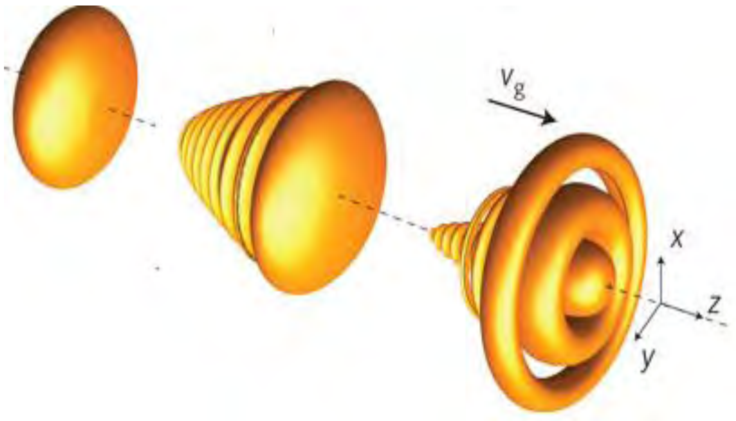
J. Broky et. al, Opt Express 16, 12880-12891 (2008).

Airy bullets

Paraxial wave equation in spatiotemporal domain

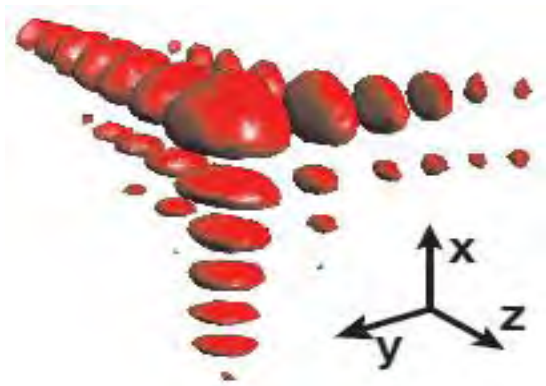
$$i\frac{\partial\psi}{\partial Z}+\frac{1}{2}\left(\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}+\frac{\partial^2\psi}{\partial T^2}\right)=0, \qquad \psi=\phi(Z,T)U(Z,X,Y)$$

Spatial Bessel (U)+Temporal Airy (ϕ)



A. Chong et, al,
Nat Photonics 4, 103-106 (2010).

Spatial Airy (U)+Temporal Airy (ϕ) (Airy3)



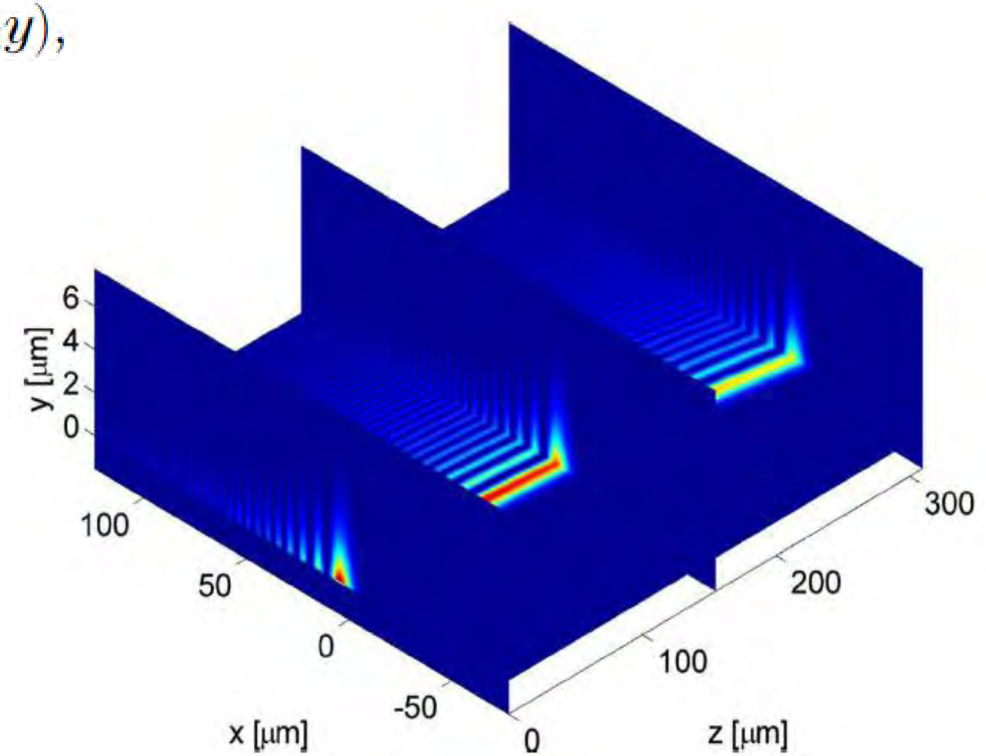
D. Abdollahpour et. al,
Phys. Rev. Lett. 105, 253901 (2010).

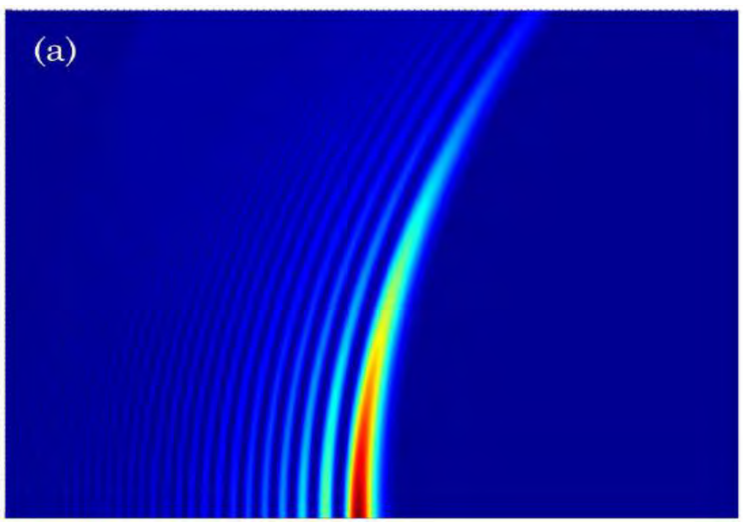
Plasmonic Airy Beam

$$E_y(x,y,z)=A(x,z)\exp(ik_z z)\exp(-\alpha_d y),$$

$$k_z = k_0\sqrt{\varepsilon_d\varepsilon_m/(\varepsilon_d+\varepsilon_m)}.$$

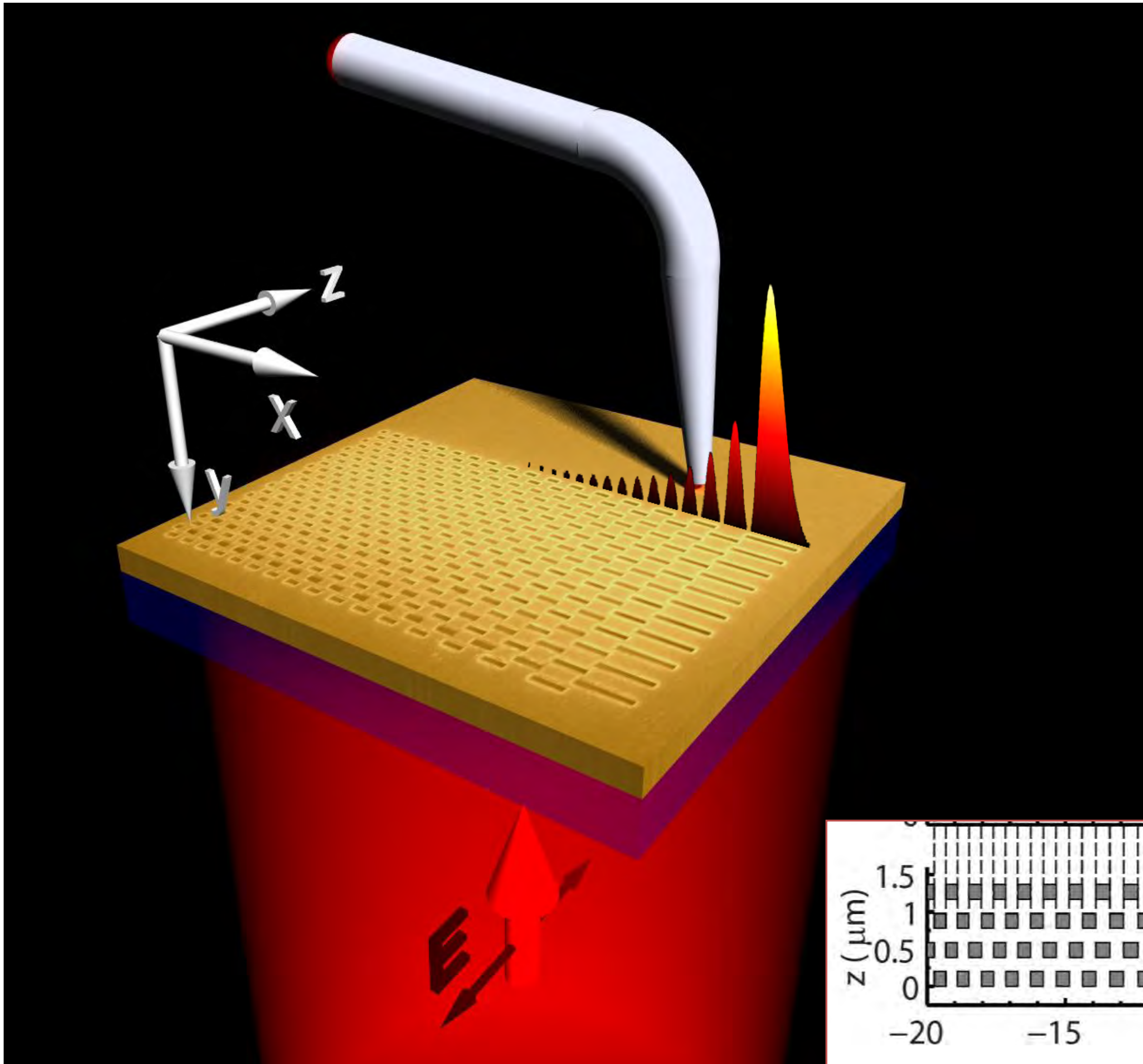
$$\frac{\partial^2 A}{\partial x^2}+2ik_z\frac{\partial A}{\partial z}=0.$$





A. Salandrino, and D. N. Christodoulides, Opt Lett 35, 2082-2084 (2010).

Airy plasmons: Experimental generation

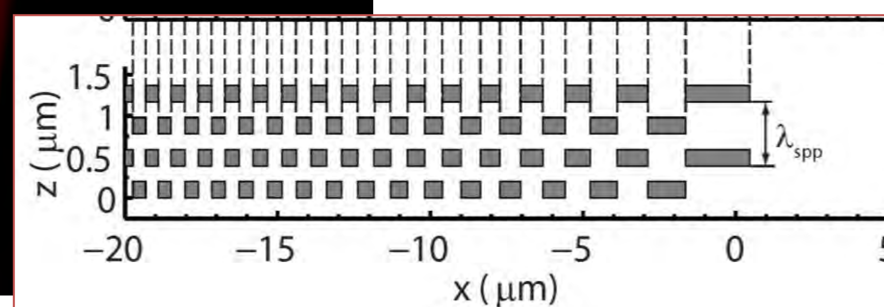


FIB FEI Helios 600

150nm thick gold film

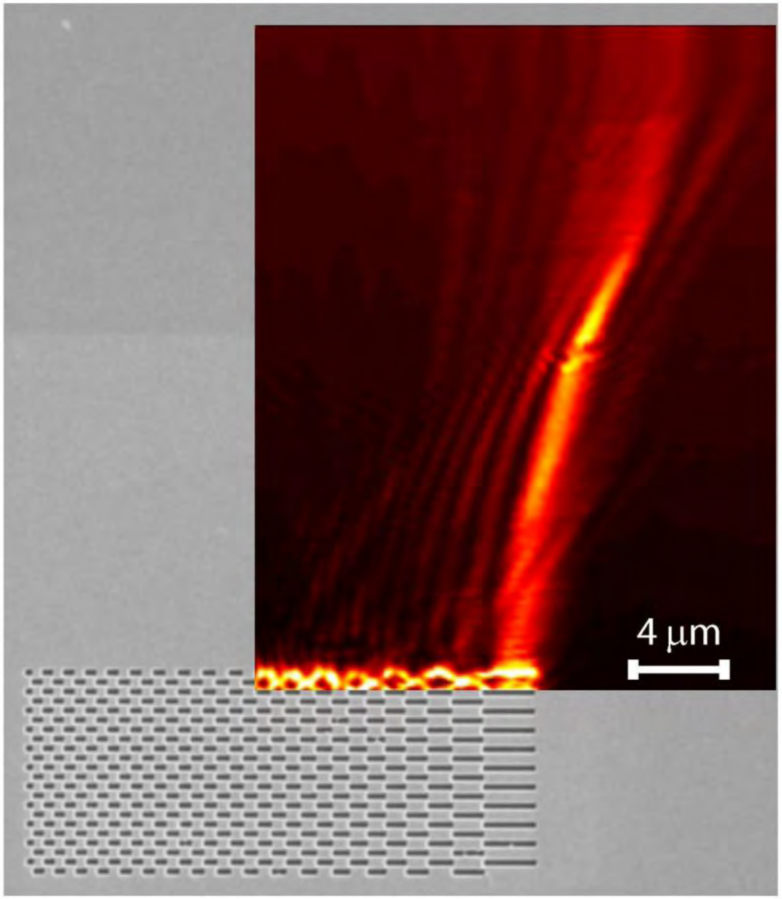
11 periods of 200nm thick slits (in z-direction) and varying width in x-direction from 2μm to 200nm

NSOM imaging of the Airy plasmon

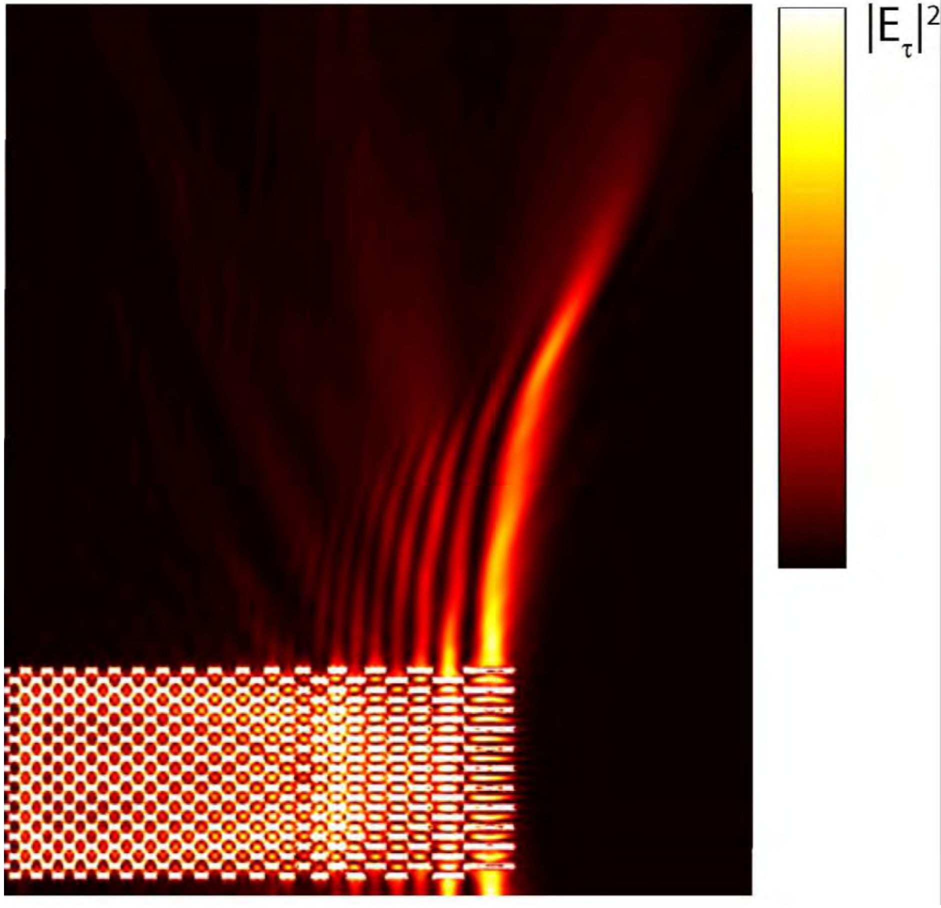


Near field imaging

Experiment



Numerics



Airy plasmonics: publicity

August 15, 2011 / Vol. 36, No. 16 / OPTICS LETTERS 3191

Plasmonic Airy beams with dynamically controlled trajectories

Peng Zhang,[1,2,†] Sheng Wang,[1,†] Yongmin Liu,[1,†] Xiaobo Yin,[1,3] Changgui Lu,[1] Zhigang Chen,[2] and Xiang Zhang[1,3,*]
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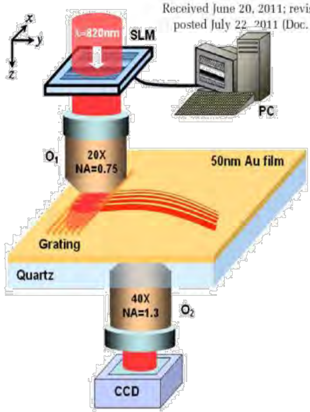
Received June 20, 2011; revised July 21, 2011; accepted July 21, 2011; posted July 22, 2011 (Doc. ID 149581); published August 11, 2011



Accepted Paper in Condensed Matter: Electronic Prope

Next >

Generation and near-field imaging of Airy surface plasmons

Alexander Minovich, Angela E. Klein, Norik Janunts, Thomas Pertsch, Dragomir N. Neshev, and Yuri S. Kivshar

Accepted Tuesday Jul 12, 2011

We demonstrate experimentally the generation and near-field imaging of nondiffracting surface waves-plasmonic Airy beams, propagating on the surface of a gold metal film. The Airy plasmons are excited by an engineered nanoscale phase grating, and

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS

PRL **107**, 126804 (2011) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2011

Plasmonic Airy Beam Generated by In-Plane Diffraction

L. Li, T. Li,* S. M. Wang, C. Zhang, and S. N. Zhu
National Laboratory of Solid State Microstructures, College of Physics, College of Engineering and Applied Sciences, Nanjing University†*, Nanjing 210093, China*

(Received 14 July 2011; revised manuscript received 23 August 2011; published 15 September 2011)

Physicists curve light on metal 'chip'

By Liz Tay on Aug 17, 2011 3:22 PM (4 days ago)
Filed under Hardware

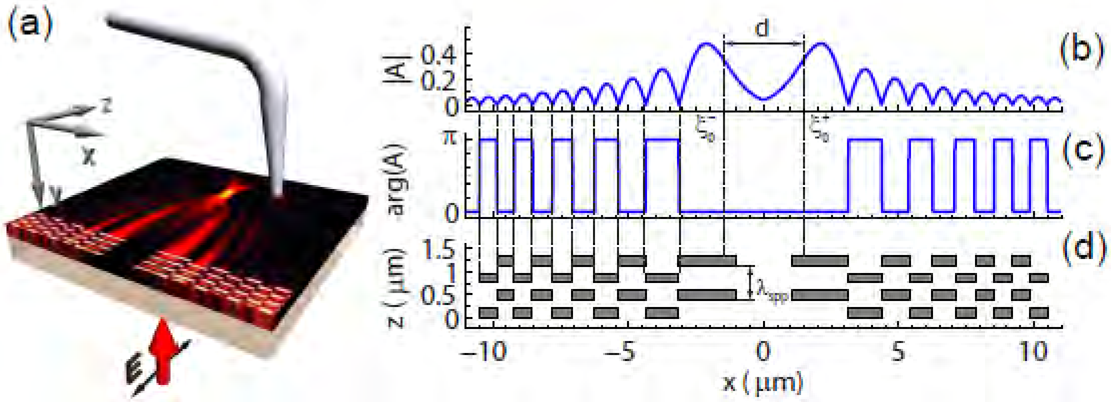
Like Tweet 7 +1 0 Share Comment Now

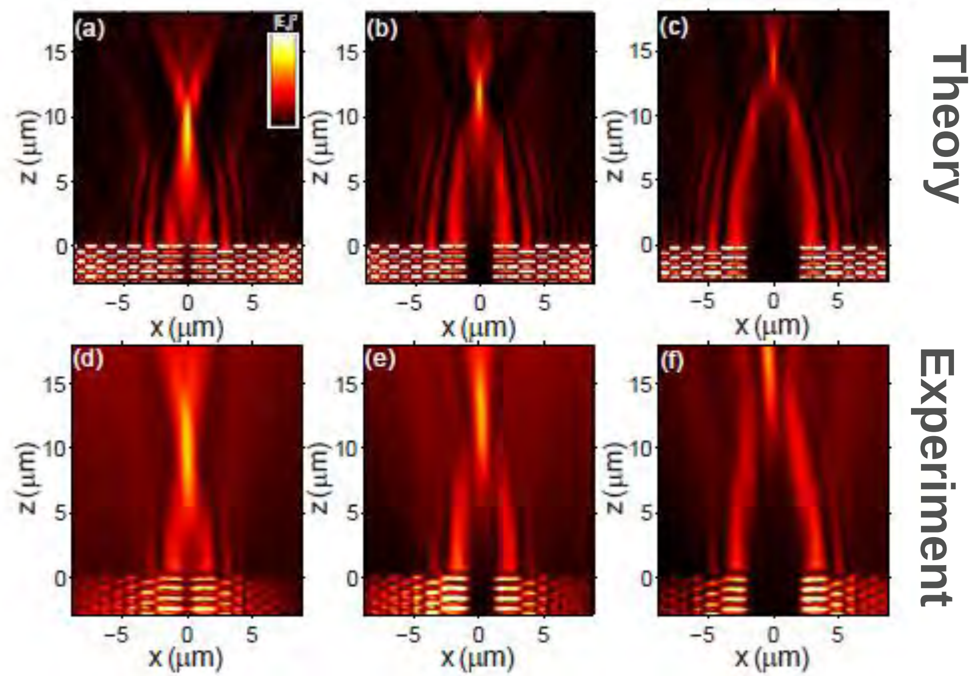
Applications hinge on development of plasmonic circuits.

Researchers have generated self-healing, curved beams of light on a flat metal surface in a move that could yield improved optical computing components.

A team of physicists from the Australian National University (ANU) in Canberra generated the so-called Airy beams on a 150-nanometre-thick gold film.

Scattering of Airy plasmons





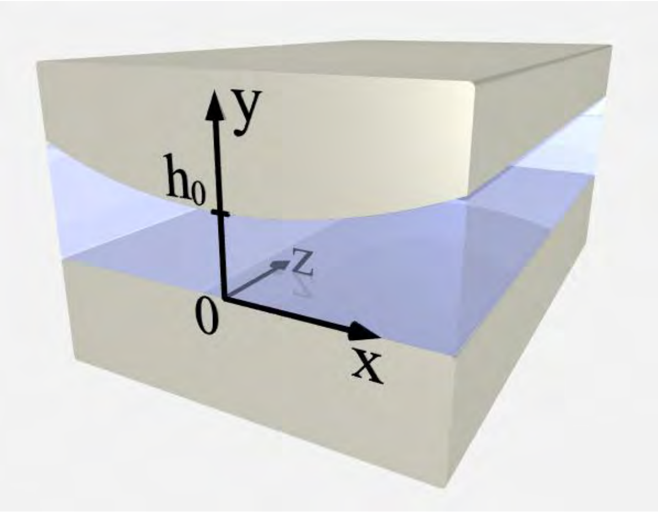
Strong sensitivity to the beam tilt

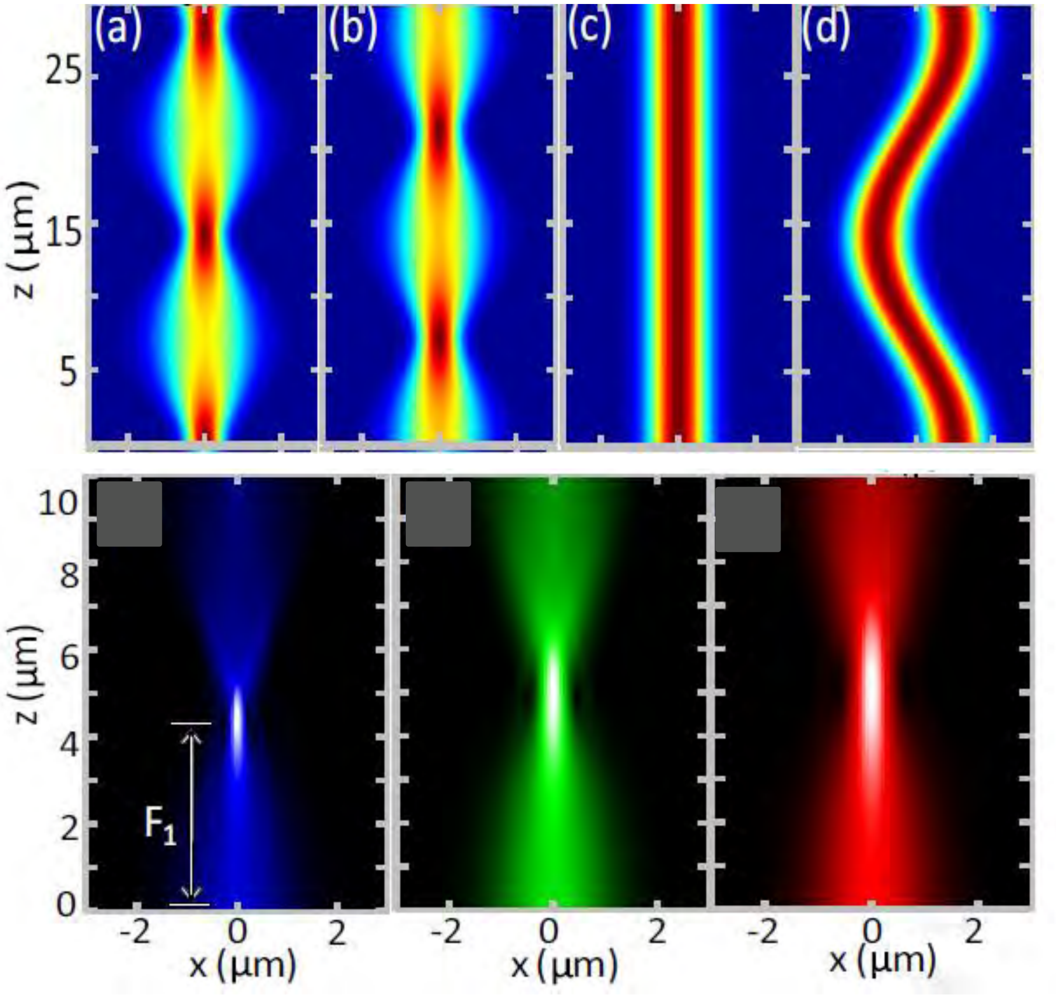
Control of the focusing spot

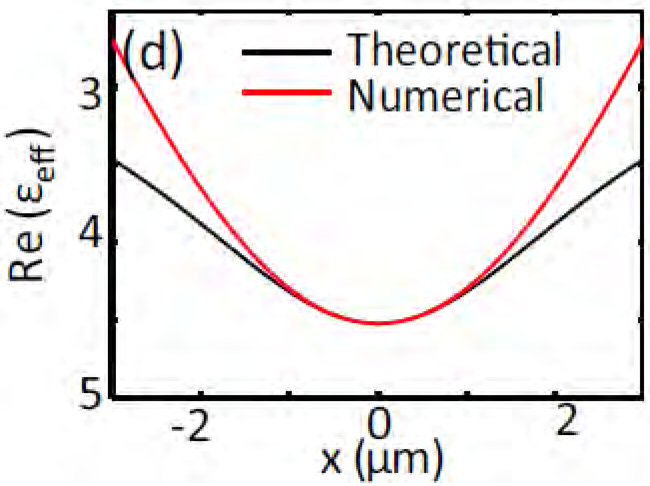
Good agreement with the theory

A. Klein et al. Opt Lett 2012 (one of 10 most downloaded papers of August)

Plasmonic potentials

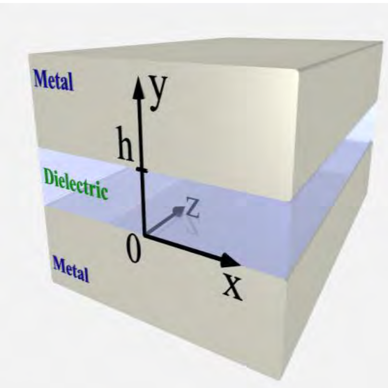


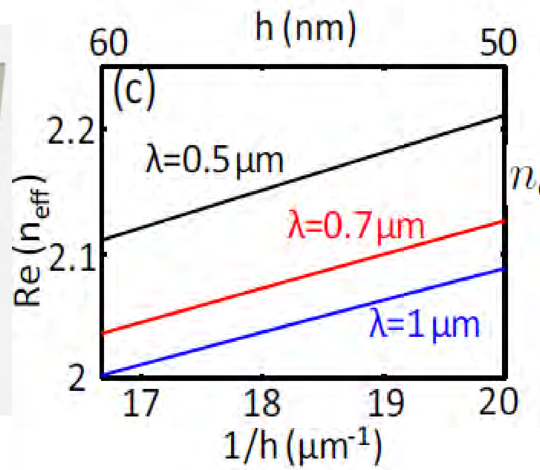




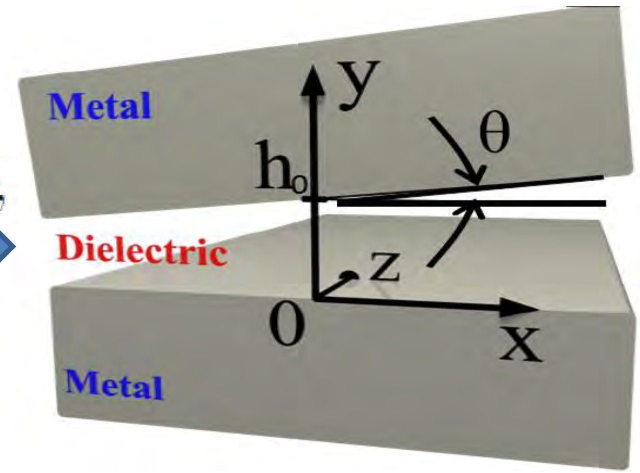
W. Liu et. al, "Polychromatic nanofocusing of surface plasmon polaritons", PRB (2012)

Plasmonic Airy beam in linear potentials





$n_{\rm eff}(h) = a/h + b,$



linear potential: $n(x) = n_0 - a\theta x/h_0^2, \;\; |\theta x| \ll h_0, \;\; n_0 = n_{\rm eff}(h_0).$

$$E_y(x,y,z) \;=\; A(x,y)\psi(x,z)\exp(in_0kz)$$

Eigen field   Envelope function

$$i\frac{\partial\psi}{\partial\xi} + fs\psi + \frac{1}{2}\frac{\partial^2\psi}{\partial s^2} = 0, \qquad s \;=\; x/x_0,\; \xi \;=\; z/(n_0kx_0^2),$$

$$f \;=\; -a\theta k^2 n_0 x_0^3/h_0^2$$

$$i\frac{\partial\psi}{\partial\xi} + fs\psi + \frac{1}{2}\frac{\partial^2\psi}{\partial s^2} = 0,$$

when

$$f = -1/2, \quad \theta = \theta_c = h_0^2/(2ak^2 n_0 x_0^3)$$

Solution (Fresnel transform):

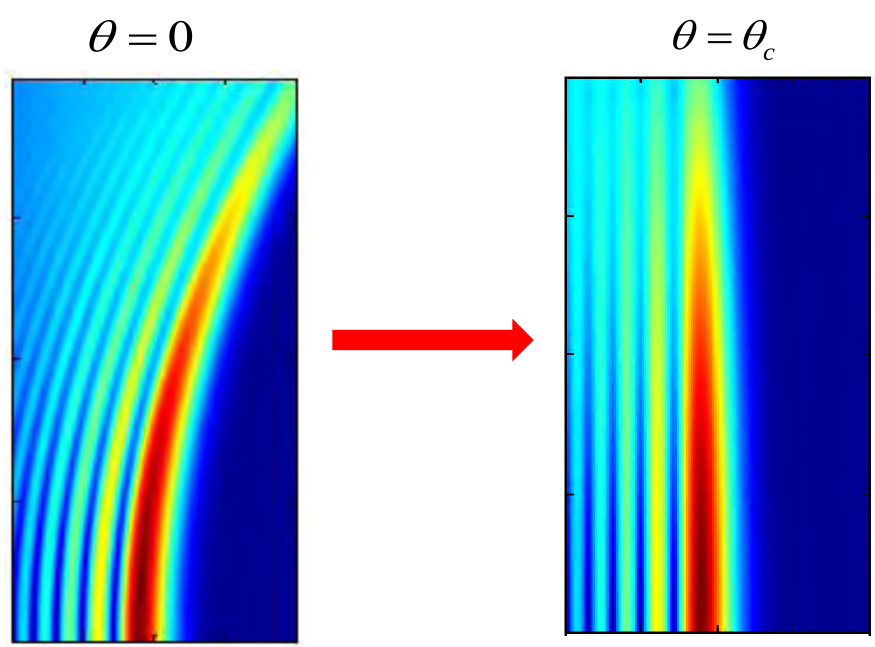
$$\psi(s,\xi) = \sqrt{\frac{1}{2\pi i\xi}}\exp(-i\frac{f^2\xi^3}{6})\int_{-\infty}^{+\infty}\psi(\chi,0)$$
$$\exp(\frac{i}{2\xi}[(s-\frac{f\xi^2}{2})-\chi]^2)d\chi,$$

For incident truncated Airy beam:

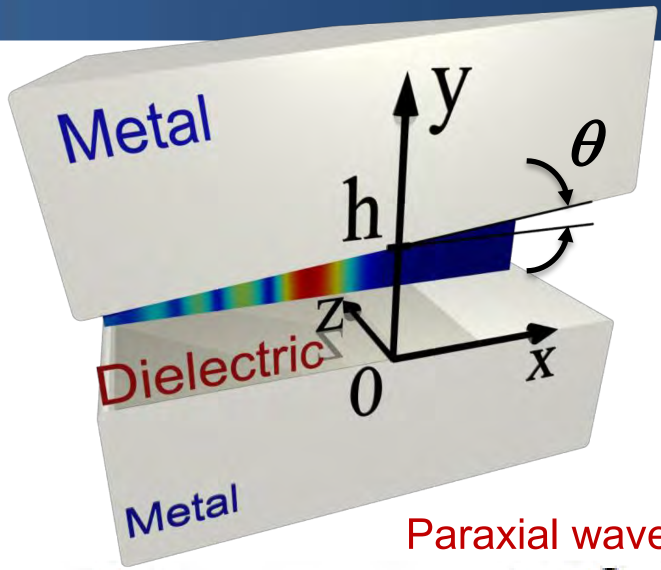
$$\psi(s,0) = Ai(s)\exp(as)(a>0)$$

Corresponding solution:

$$\psi(s,\xi) = Ai[\boxed{s-\frac{1}{4}(1+2f)\xi^2} + ia\xi]\exp(as - \frac{af\xi^2}{2}$$
$$-\frac{a\xi^2}{2})\exp[i(-\frac{f^2\xi^3}{6} + fs\xi - \frac{f\xi^3}{4} - \frac{\xi^3}{12} + \frac{a^2\xi}{2} + \frac{s\xi}{2})].$$



Airy plasmons in linear potentials



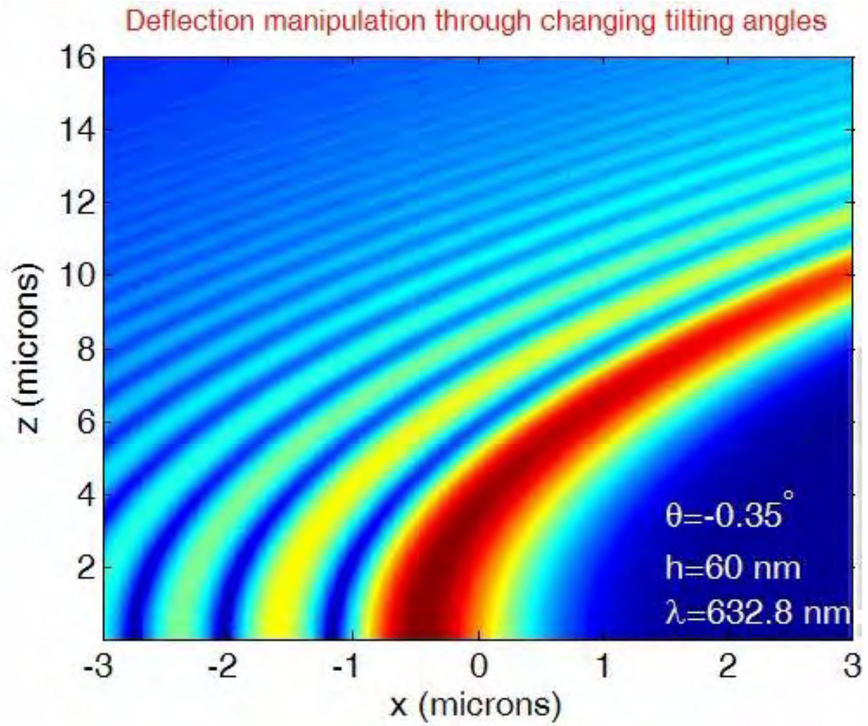
Paraxial wave equation

$$i\frac{\partial\psi}{\partial\xi}+fs\psi+\frac{1}{2}\frac{\partial^2\psi}{\partial s^2}=0$$

$$\boxed{n(x)=n_0-a\theta x/h_0^2}$$

$$\theta=\theta_c=h_0^2/(2ak^2n_0x_0^3)$$

Stationary solution

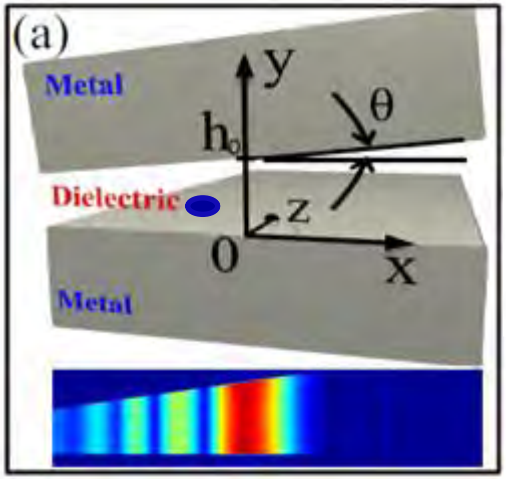


Self-healing properties

$$i\frac{\partial \psi}{\partial \xi} + fs\psi + \frac{1}{2}\frac{\partial^2 \psi}{\partial s^2} = 0,$$

Solution (Fresnel transform):

$$\psi(s,\xi) = \sqrt{\frac{1}{2\pi i\xi}}\exp(-i\frac{f^2\xi^3}{6})\int_{-\infty}^{+\infty}\psi(\chi,0)$$
$$\exp(\frac{i}{2\xi}[(s-\frac{f\xi^2}{2})-\chi]^2)d\chi,$$



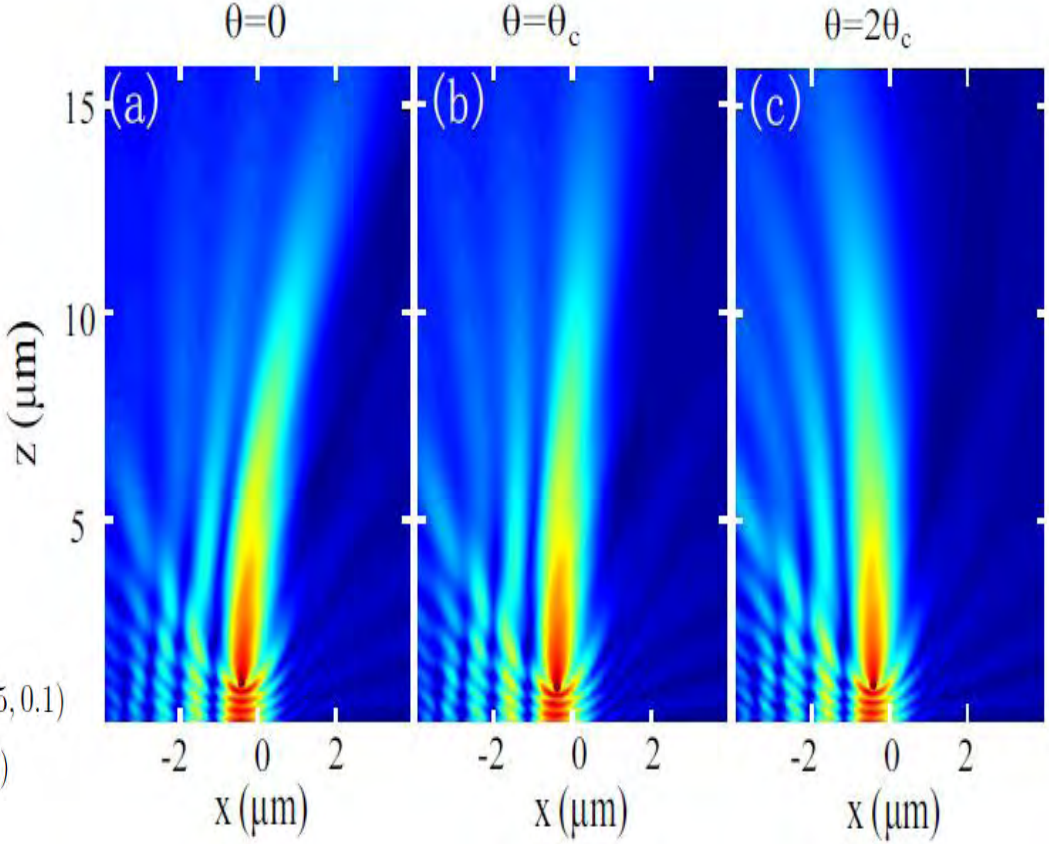
$\epsilon = 2.25$

$(R_x, R_y, R_z) = (0.1, 0.025, 0.1)$

$(x, y, z) = (-0.4, 0.025, 1)$

$\lambda = 632.8\,\mathrm{nm},\ h_0 = 60\,\mathrm{nm},\ a = 0.1,$

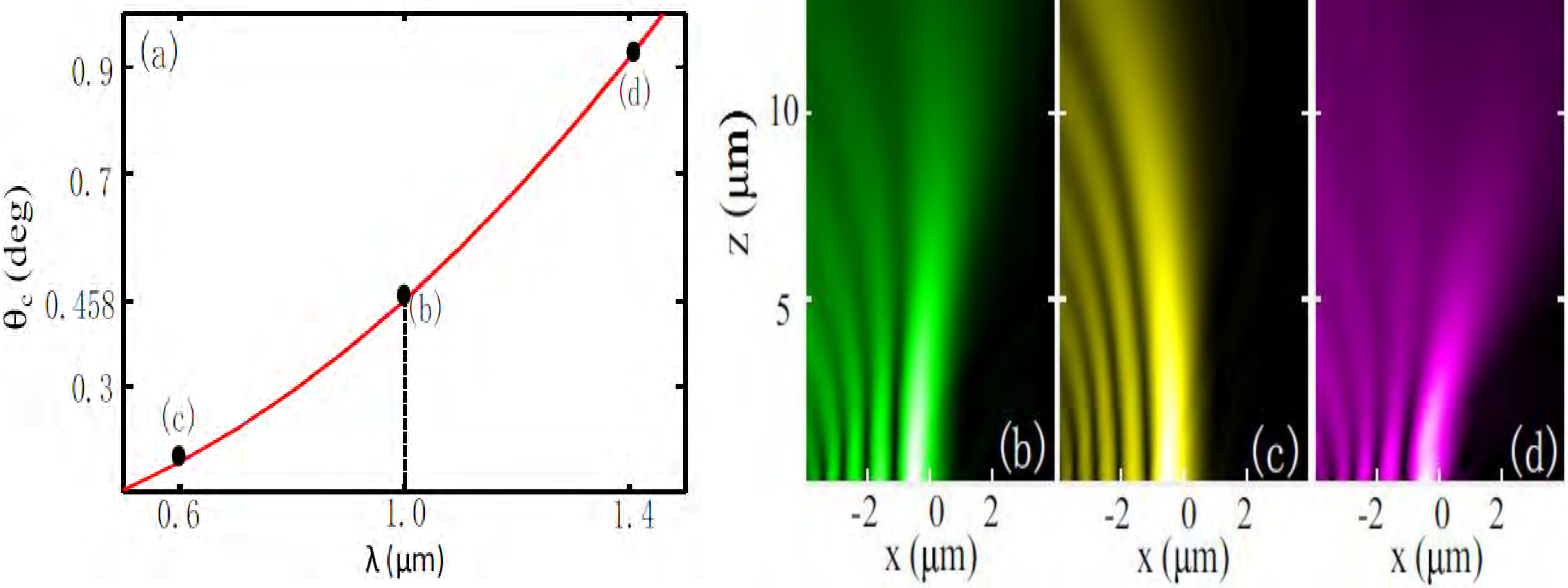
$x_0 = 500\,\mathrm{nm}$ and $\theta_c = 0.175°$.

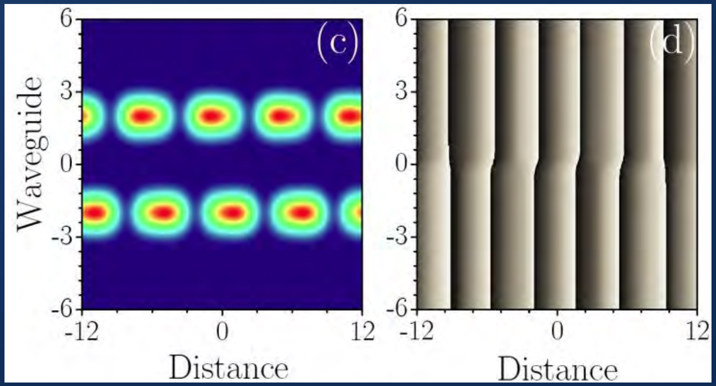


Different wavelength for Airy beams in liner potentials

$$\theta = \theta_c = h_0^2 / (2ak^2 n_0 x_0^3)$$

Wavelength dependent





Part 3:

Parity-time symmetric systems and their applications in optics

Complex quantum potentials

Quantum particle on the line:

$$i\hbar\psi_t = -\frac{\hbar^2}{2m}\psi_{xx} + U(x)\psi,$$

$$\psi(x,t) = \exp\left(-i\frac{E}{\hbar}t\right)\Psi(x), \quad -\frac{\hbar^2}{2m}\Psi_{xx} + U(x)\Psi = E\Psi$$

What if $U(x) = V(x) + iW(X)$? Commonly, $E = E_r + i\gamma$; hence

$$\psi(x,t) = \exp\left(-i\frac{E_r}{\hbar}t\right)\exp\left(\frac{\gamma}{\hbar}t\right)\Psi(x), \quad \text{no good.}$$

Parity-Time (PT) symmetric quantum potentials

Is it possible that, despite $W \neq 0$, all eigenvalues are real?

Yes! Example:

$$U(x) = -(ix)^N, \quad N \text{ real.}$$

- $N \geq 2$: infinite sequence of real, positive, eigenvalues
- $1 < N < 2$: finite number of real positive + infinite sequence of complex conjugate pairs
- $N \leq 1$: no real eigenvalues

(C M Bender & S Boettchner, PRL **80** 5243 (1998))

The necessary condition for the entirely real spectrum is $\mathcal{PT}$ symmetry:

$$U^*(-x) = U(x), \quad \Rightarrow V(-x) = V(x), \; W(-x) = -W(x)$$

Parity-Time (PT) Symmetry

Parity operator: P

$\hat{p} \to -\hat{p},$ $\quad \hat{x} \to -\hat{x}$

Time operator: T

$\hat{p} \to -\hat{p},$ $\quad \hat{x} \to \hat{x},$ $\quad i \to -i,$

Hamiltonian: H

$$\hat{H} = \hat{p}^2 / 2 + V(x)$$

Requirement: $V(x) = V^*(-x)$

- Quantum field theory
- Complex Lie algebra
- Complex crystals
- Condensed matter system
- Population biology
- Optics

VOLUME 80, NUMBER 24 PHYSICAL REVIEW LETTERS 15 JUNE 1998

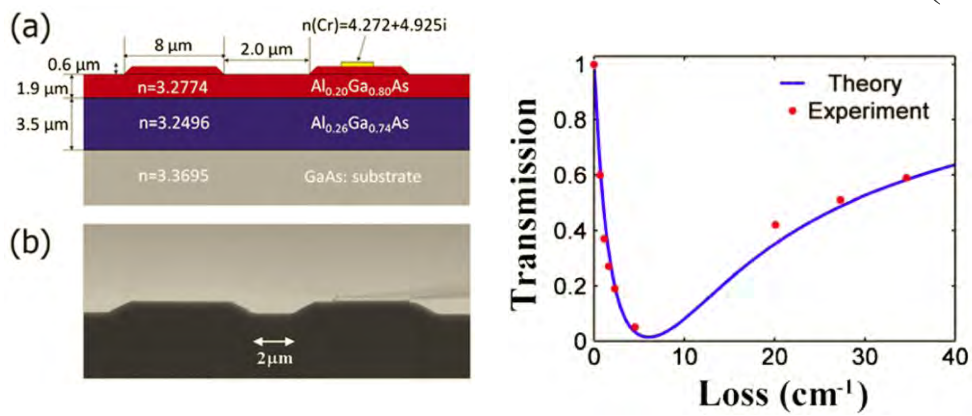
Real Spectra in Non-Hermitian Hamiltonians Having $\mathcal{PT}$ Symmetry

Carl M. Bender[1] and Stefan Boettcher[2,3]
[1] *Department of Physics, Washington University, St. Louis, Missouri 63130*
[2] *Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[3] *CTSPS, Clark Atlanta University, Atlanta, Georgia 30314*

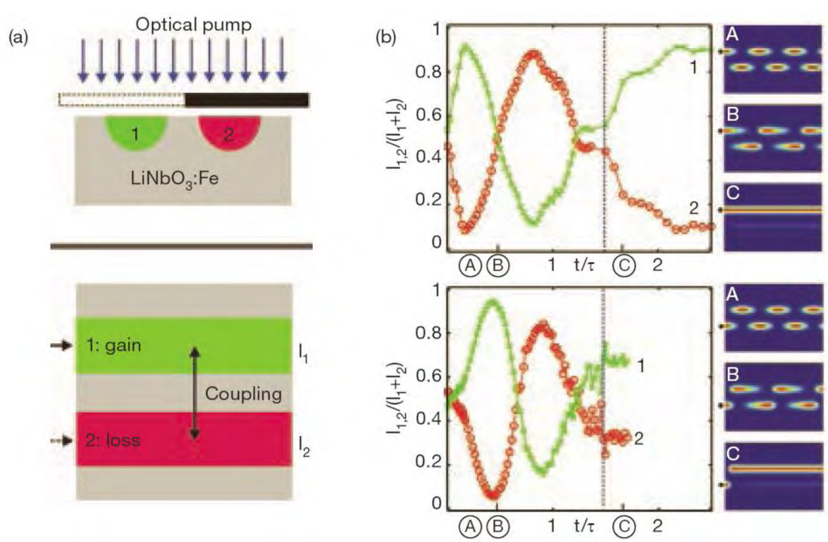
Linear PT-symmetric optical couplers

Observation of PT-symmetry breaking in complex optical potentials

Guo et al. PRL (2009)



Observation of parity-time symmetry in optics

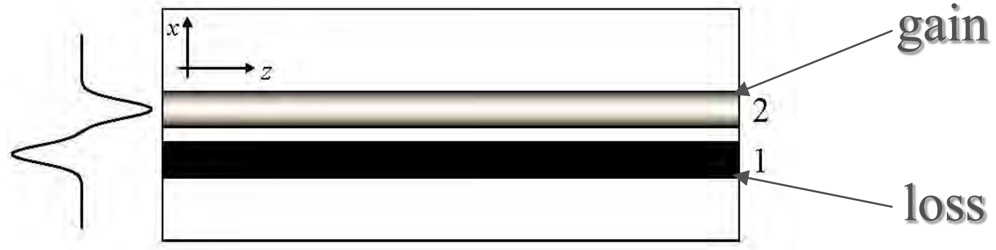


- PT symmetry: supermodes do not experience gain or loss; zero gain/loss on average for arbitrary inputs
- Broken PT symmetry (unbalanced gain and loss): mode confinement and/or amplification in the waveguide with gain

Ruter et al., Nature Physics (2010)

Nonlinear PT-symmetric coupler—a dimer

Optical coupler



Model

$$\boxed{\begin{aligned} i\frac{da_1}{dz} + i\rho a_1 + Ca_2 + \gamma |a_1|^2 a_1 = 0 \\ i\frac{da_2}{dz} - i\rho a_2 + Ca_1 + \gamma |a_2|^2 a_2 = 0 \end{aligned}}$$

PHYSICAL REVIEW A **82**, 043818 (2010)

Nonlinear suppression of time reversals in $\mathcal{PT}$-symmetric optical couplers

Andrey A. Sukhorukov, Zhiyong Xu, and Yuri S. Kivshar

Properties of nonlinear modes

Stationary states:

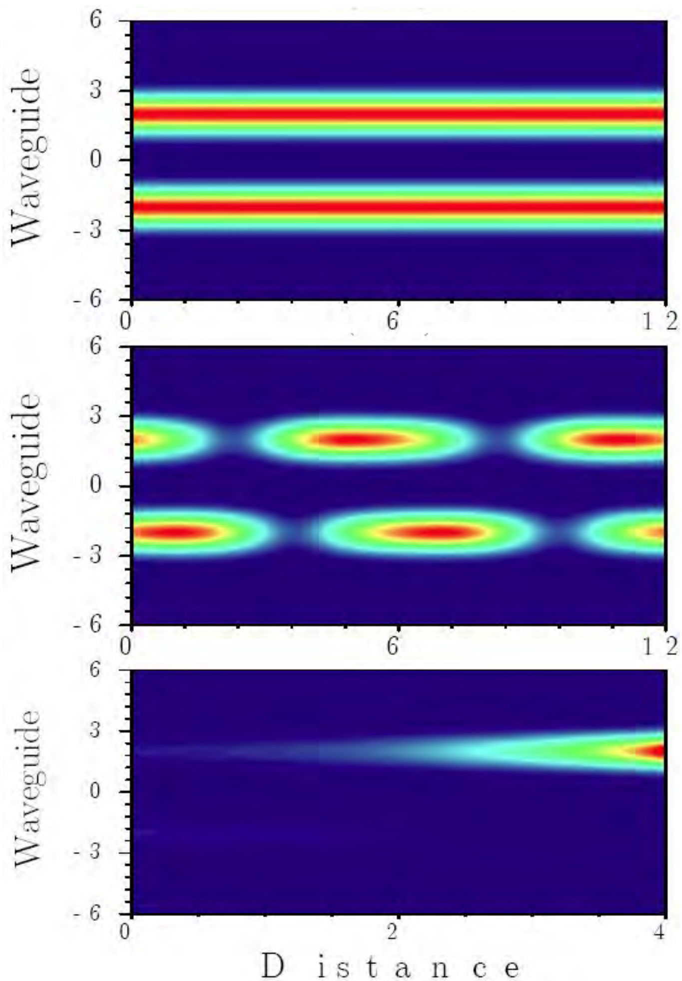
$$a_1 = \sqrt{I}\cos[\theta(z)]\exp[+i\varphi(z)/2 + i\beta(z)]$$
$$a_2 = \sqrt{I}\sin[\theta(z)]\exp[-i\varphi(z)/2 + i\beta(z)]$$

$$\beta_\pm = \gamma I_0/2 + C\cos(\varphi_\pm),$$

$$I = I_0, \theta = \pi/4, \beta = \beta_\pm z,$$

$$\sin(\varphi_\pm) = \rho/C,$$

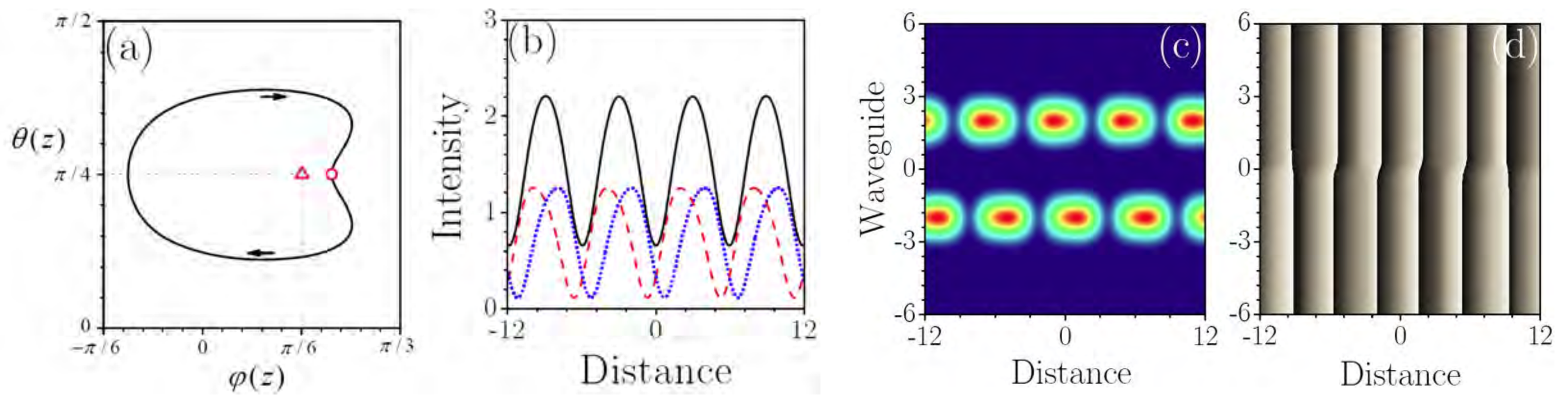
$$\cos(\varphi_\pm) = \mp\sqrt{1-(\rho/C)^2}$$



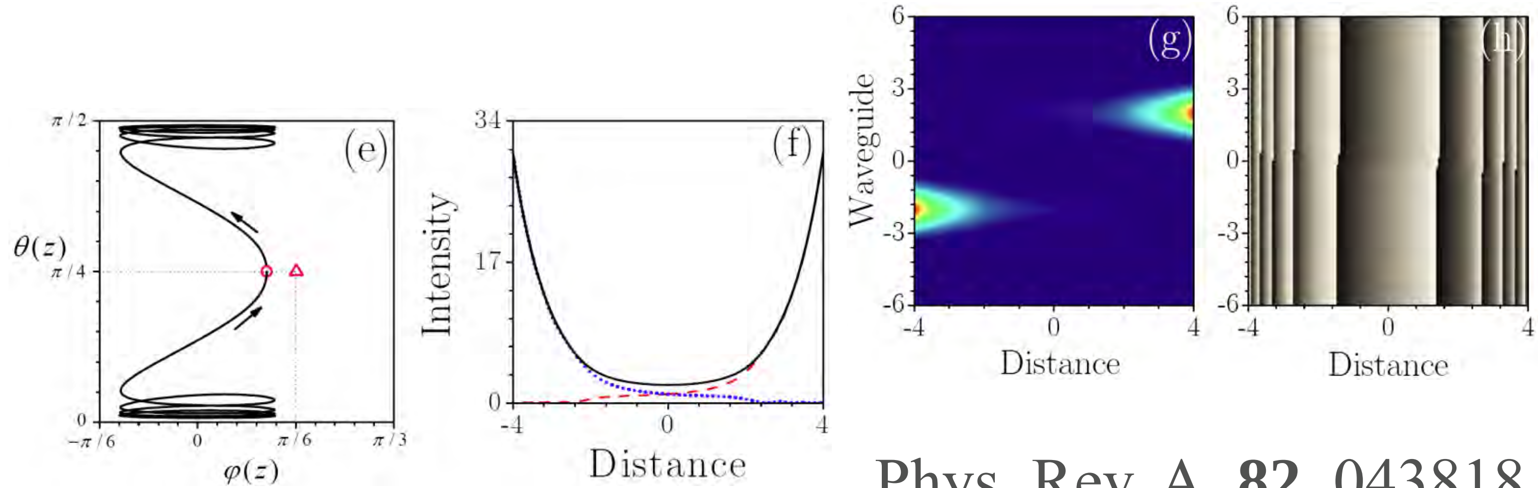
Phys. Rev. A, **82**, 043818 (2010)

Propagation dynamics of nonlinear modes

Periodic evolution of nonlinear modes



Nonlinearity-induced symmetry breaking



Phys. Rev. A, **82**, 043818 (2010)

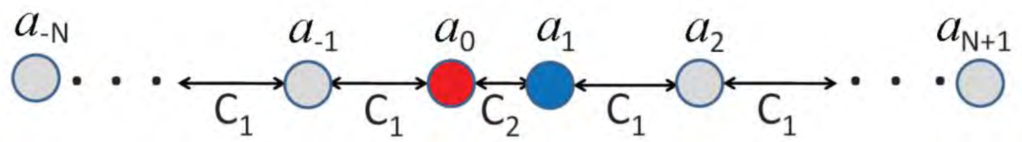
PT-symmetric dimer in a linear chain

$$i\frac{da_j}{dz} + C_1 a_{j-1} + C_1 a_{j+1} = 0, \;\; j \neq 0, 1$$

$$i\frac{da_0}{dz} + i\rho a_0 + C_1 a_{-1} + C_2 a_1 = 0$$

$$i\frac{da_1}{dz} - i\rho a_1 + C_2 a_0 + C_1 a_2 = 0$$

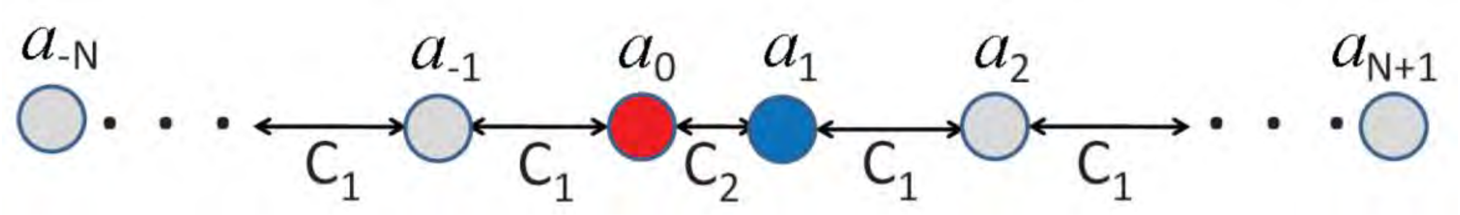
- a_j – mode amplitudes at waveguides
- C – coupling coefficient between the waveguide modes
- ρ – coefficient of gain/loss in waveguides 0,1



Opt. Lett. **37**, 2148 (2012)

PT symmetry breaking for planar lattice

- Boundary conditions $a_{N+2} \equiv 0, \quad a_{-N-1} \equiv 0$



- Consider eigenmodes: $a_n = A_n \exp(i\phi_n + i\beta z)$
- PT symmetry: $\mathrm{Im}(\beta) \equiv 0 \qquad d|a_n|/dz = 0$
- For $n \neq 0,1 \quad C_1|a_n|\left[\sin(\phi_{n+1} - \phi_n)|a_{n+1}| + \sin(\phi_{n-1} - \phi_n)|a_{n-1}|\right] = 0$
- For $|n| \geq 1 \quad |a_n a_{n+1}| \sin(\phi_{n+1} - \phi_n) = 0$
- Consider $n = 0, 1$

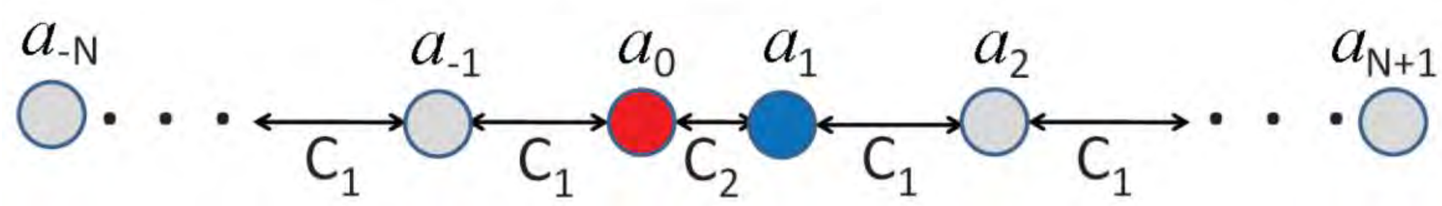
$$|a_0| = |a_1| \text{ and } C_2 \sin(\phi_1 - \phi_0) + \rho = 0$$

- Solvability of last relation defines PT symmetry

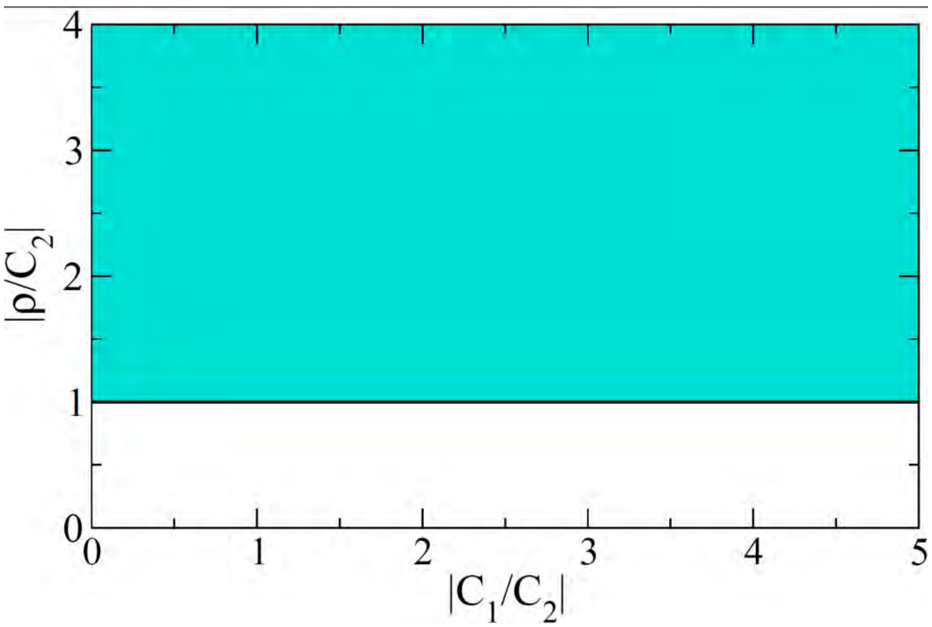
PT symmetry breaking for a straight array

- Stability condition

$$|C_2| > |\rho|$$



- Same stability condition as for isolated PT coupler!
- Does not depend on lattice coupling outside the active region

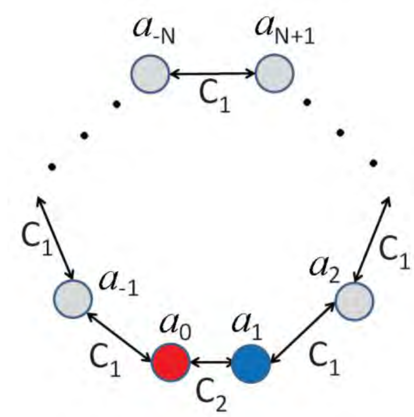


Opt. Lett. **37**, 2148 (2012)

PT symmetry breaking for a circular array

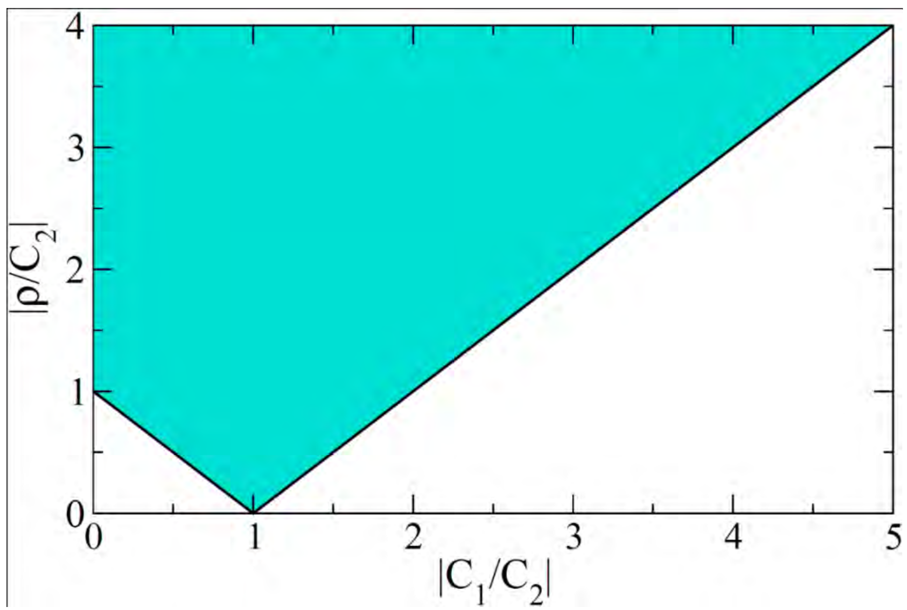
- Consider ratio $J = B_+ \exp(-ik)/F_-$

$$|J|^2 = \frac{C_1^2 - C_2^2 + \rho^2 - 2C_1[\rho - 2C_2 \mathrm{Im}(J)]\sin(k)}{C_1^2 - C_2^2 + \rho^2 + 2C_1\rho\sin(k)}$$



- PT symmetry breaking occurs at a given *k* when solutions disappear
- Threshold corresponds to real *k*
- Stability condition:

$$||C_1| - |C_2|| \geq |\rho|$$

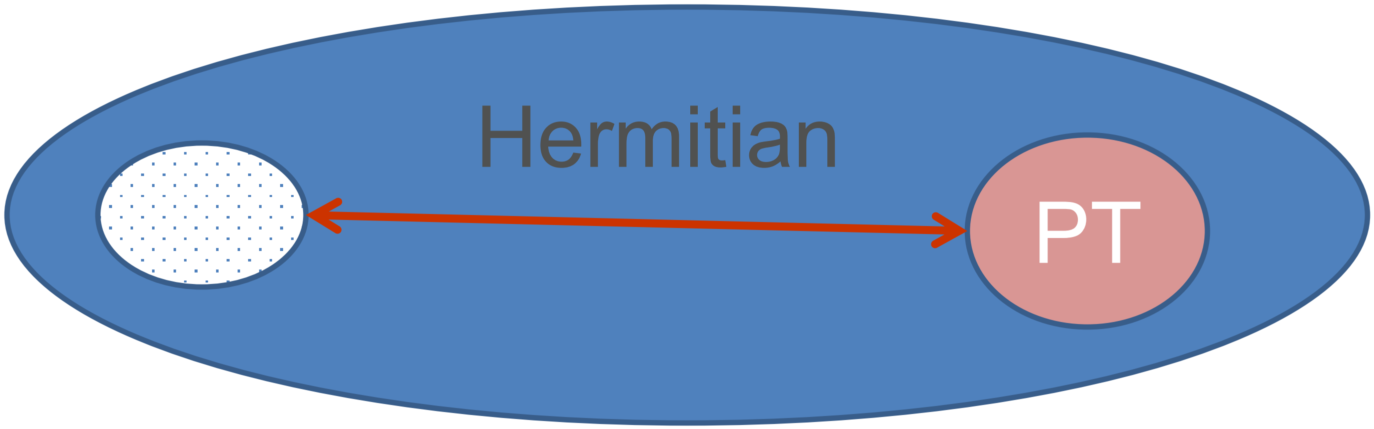


- Threshold depends on all lattice parameters

Opt. Lett. **37**, 2148 (2012)

Nonlocal effects

- PT-defect – non-Hermitian
- Quantum-mechanical context: interaction of a non-Hermitian system with the Hermitian world

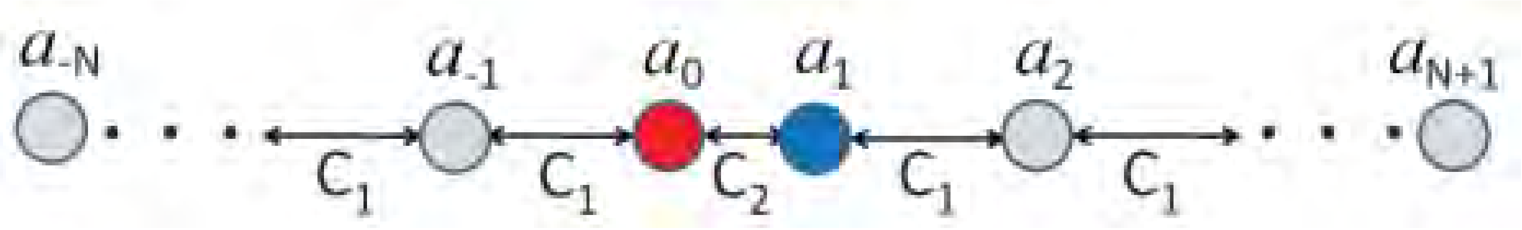


- Dynamics can be sensitive to a potential at distant locations
- Continuing debate on the meaning of nonlocality and relevance to real physical systems

H. F. Jones, Phys. Rev. D 76, 125003 (2007); M. Znojil, Phys. Rev. D 80, 045009 (2009); …

PT-symmetric dimer in a nonlinear chain

- Distant boundaries (infinite lattice limit)



- Kerr-type nonlinearity

$$i\frac{da_j}{dz} + C_1 a_{j-1} + C_1 a_{j+1} + \gamma |a_j|^2 a_j = 0, \;\; j \neq 0, 1,$$

$$i\frac{da_0}{dz} + i\rho a_0 + C_1 a_{-1} + C_2 a_1 + \gamma |a_0|^2 a_0 = 0,$$

$$i\frac{da_1}{dz} - i\rho a_1 + C_2 a_0 + C_1 a_2 + \gamma |a_1|^2 a_1 = 0,$$

- Conservative solitons exist on either sides of PT coupler

$$a_j = A\mathrm{sech}[\delta(j - j_0 - 2C_1 vz)] e^{i[v(j-j_0)+(\delta^2 - v^2)C_1 z + \alpha]}$$

Soliton scattering by a PT-symmetric dimer

Soliton scattering

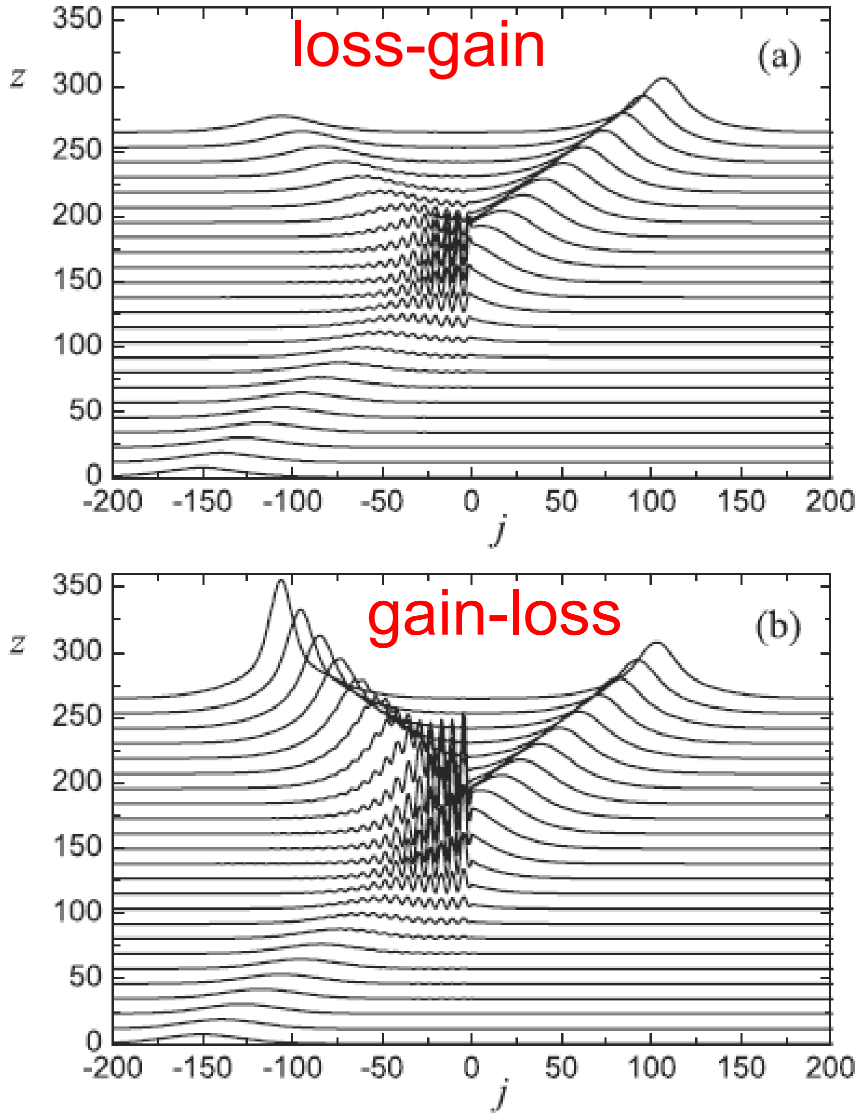
- both reflected and transmitted waves are amplified

Scattering nonreciprocity

- Transmitted waves do not change but reflection depends of the position of gain and loss waveguides

Localized modes

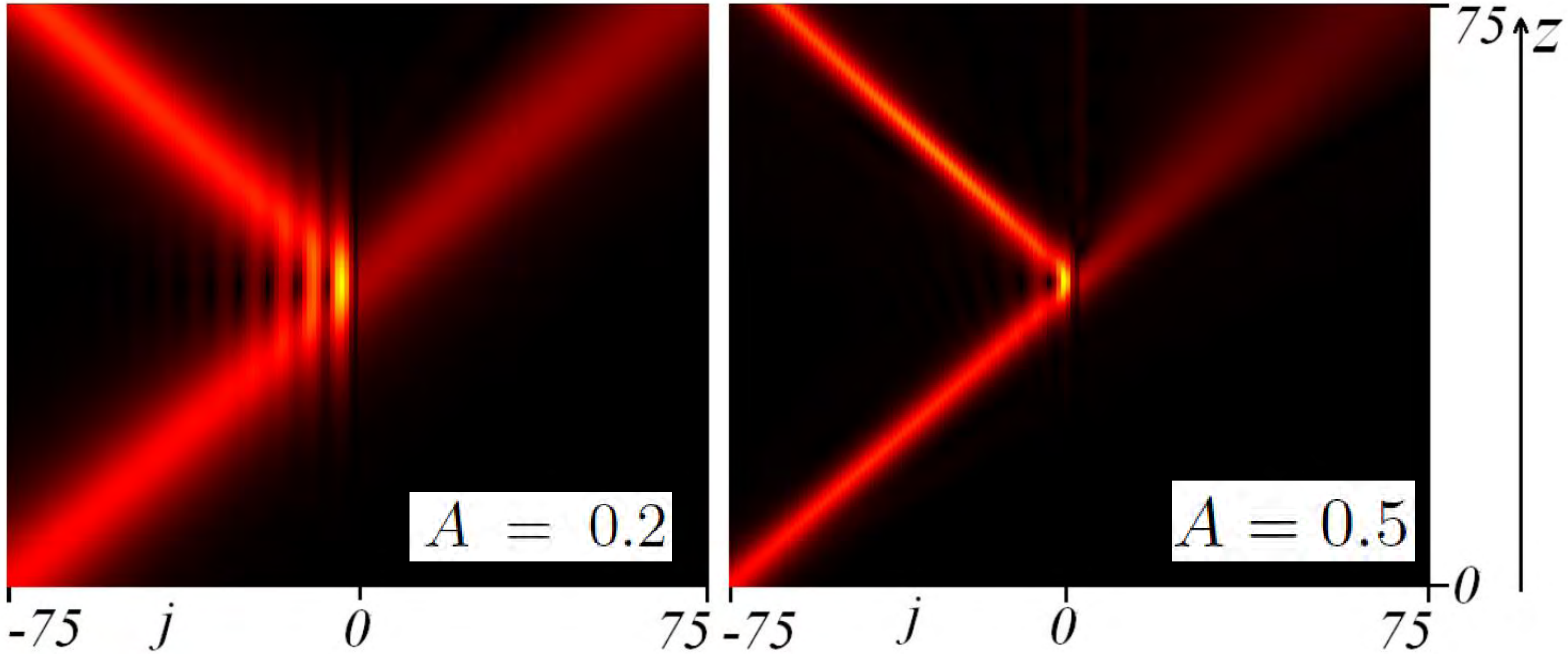
- PT symmetric defect supports a localized mode



Phys. Rev. A **82**, 013833 (2011)

Soliton scattering by PT coupler

- Lattice parameters
$$C_1 = 2,\ C_2 = 4,\ \rho = 1.5,\ \gamma = 1$$
- Soliton velocity $v = 0.5$
- Localized mode at PT coupler is excited when soliton amplitude is increased (right)



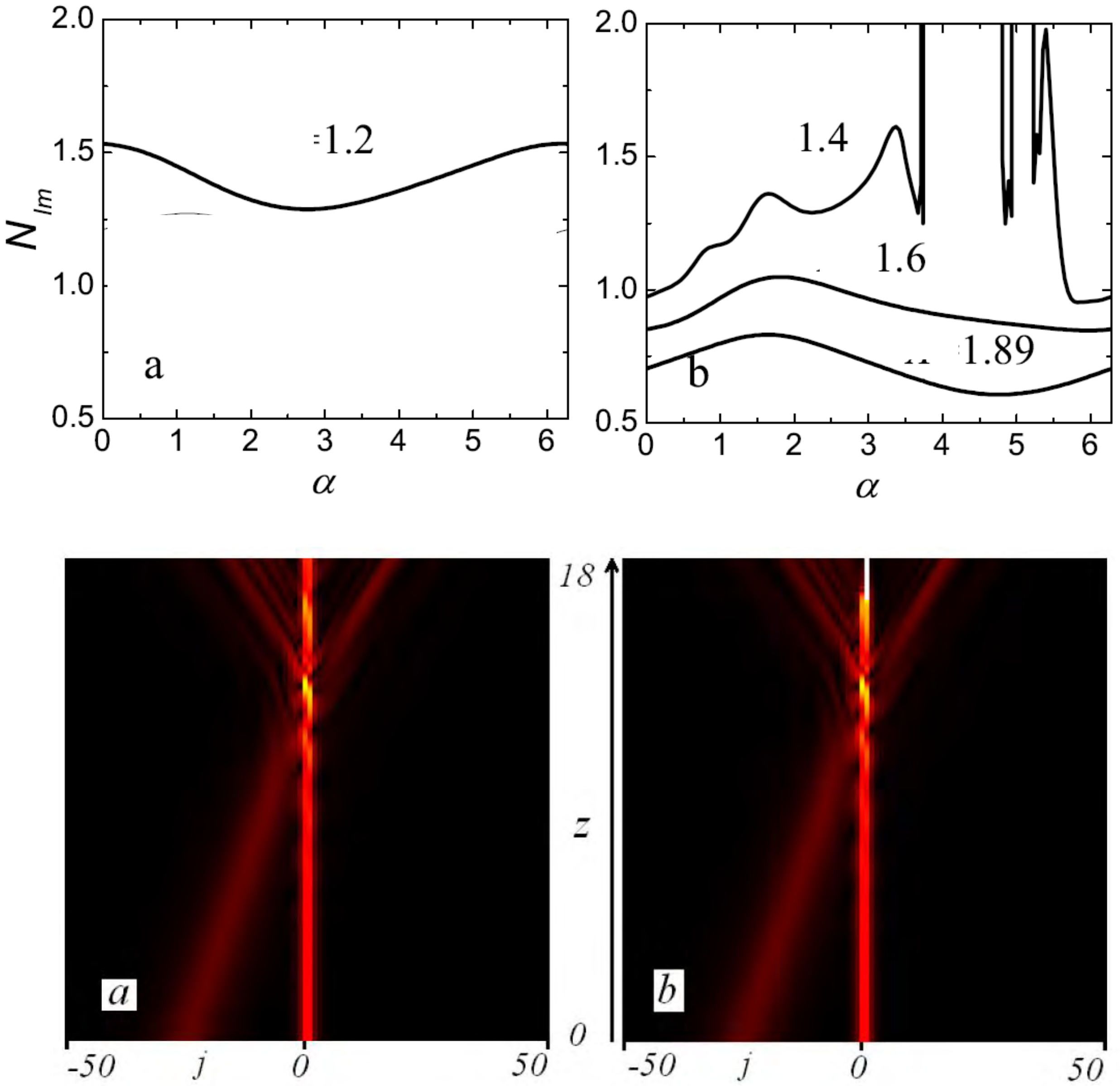
Controlling soliton scattering with localized PT modes

Soliton scattering

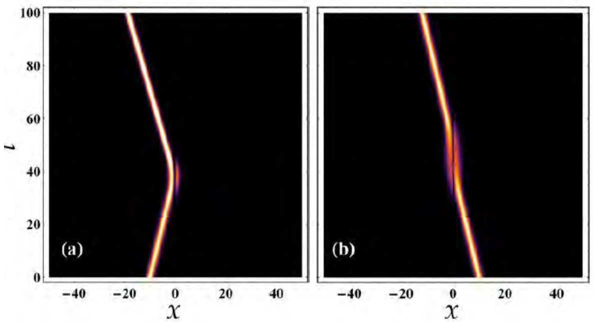
- α - soliton phase
- Labels – localized PT mode amplitude

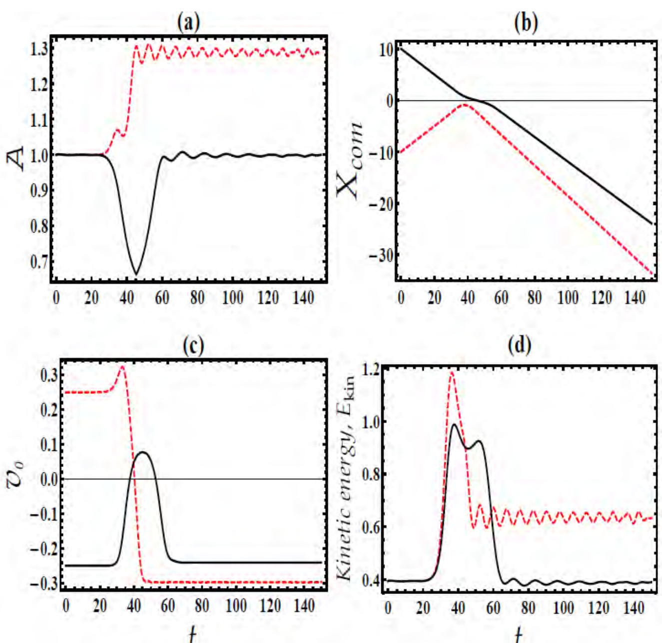
PT symmetry breaking

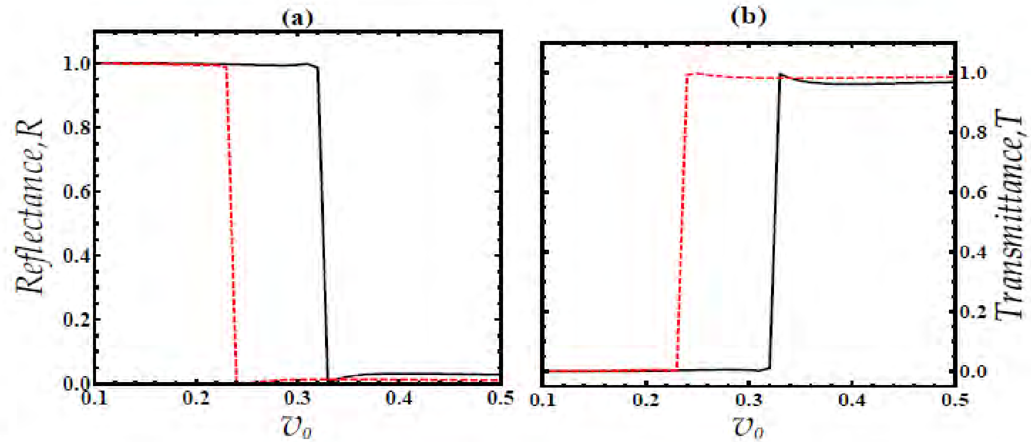
- Mode amplitude 1.4
- Left: α =3.67
  PT symmetry preserved
- Right: α =3.75
  nonlinear PT symmetry breaking



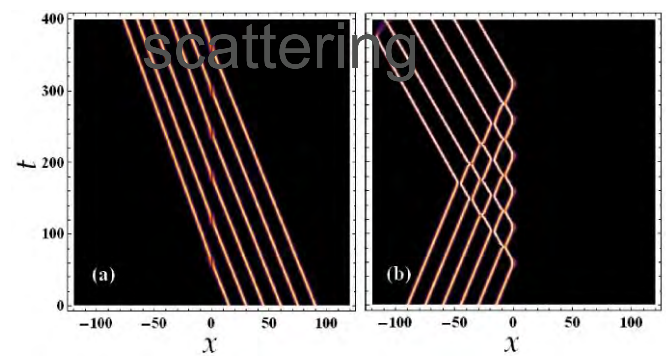
Unidirectional soliton scattering







Multi-soliton scattering



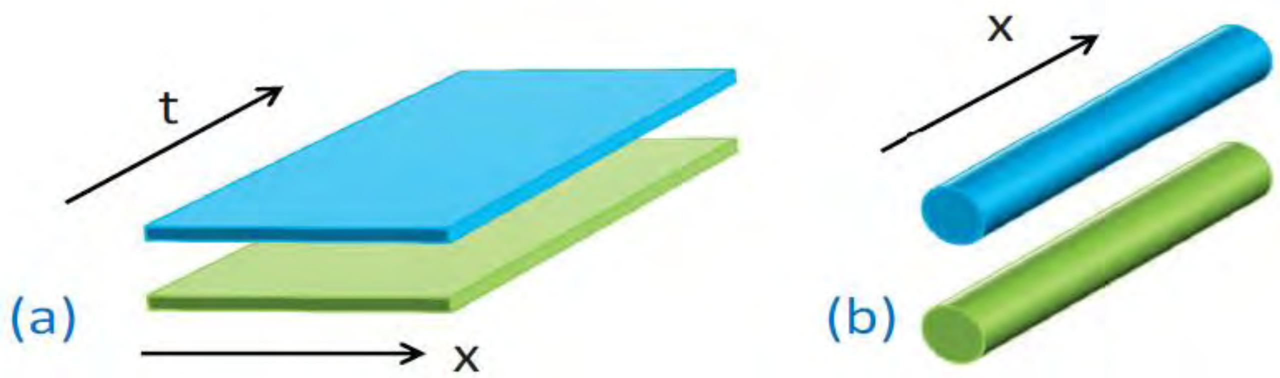
Unidirectional soliton flows in PT-symmetric potentials

U. Al Khawaja[1], S. M. Al-Marzoug[2,3], H. Bahlouli[2,3], and Yuri S. Kivshar[4]

PT symmetric waveguide arrays

$$i\dot{u}_n + C(u_{n+1} - 2u_n + u_{n-1}) + 2|u_n|^2 u_n = -v_n + i\gamma u_n,$$
$$i\dot{v}_n + C(v_{n+1} - 2v_n + v_{n-1}) + 2|v_n|^2 v_n = -u_n - i\gamma v_n$$

S Suchkov, B Malomed, S Dmitriev, and Y Kivshar, PRE **84** (2011)



$$iu_t + u_{xx} + 2|u|^2 u = -v + i\gamma u,$$
$$iv_t + v_{xx} + 2|v|^2 v = -u - i\gamma v.$$

Invariant manifolds and solitons

$$iu_t + u_{xx} + 2|u|^2 u = -v + i\gamma u,$$
$$iv_t + v_{xx} + 2|v|^2 v = -u - i\gamma v.$$

Change of variables:

$$u(x,t) = e^{i(\Omega t - \theta)} U(x,t), \quad v(x,t) = e^{i\Omega t} V(x,t),$$

$$\sin\theta = \gamma,$$

casts the $\mathcal{PT}$-system to

$$iU_t + U_{xx} - \Omega U + 2|U|^2 U = -\cos\theta\, V + i\gamma(U - V)$$
$$iV_t + V_{xx} - \Omega V + 2|V|^2 V = -\cos\theta\, U + i\gamma(U - V),$$

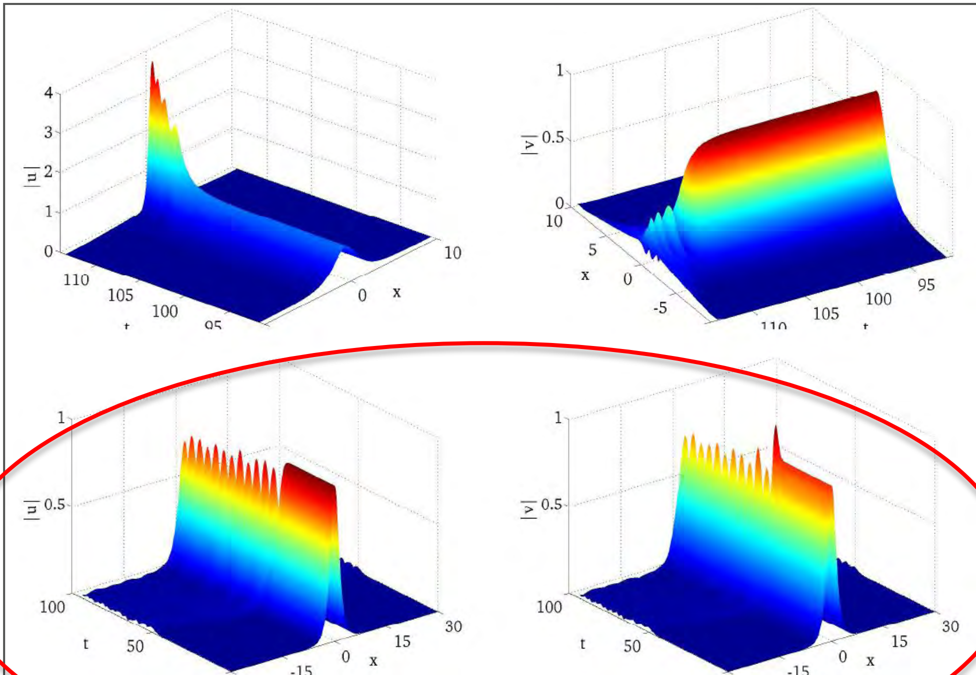
whose $U = V$ reduction is

$$i\phi_t + \phi_{xx} - a^2\phi + 2|\phi|^2\phi = 0, \quad \Omega = a^2 + \cos\theta$$

$$\phi(x) = a\,\mathrm{sech}(ax)$$

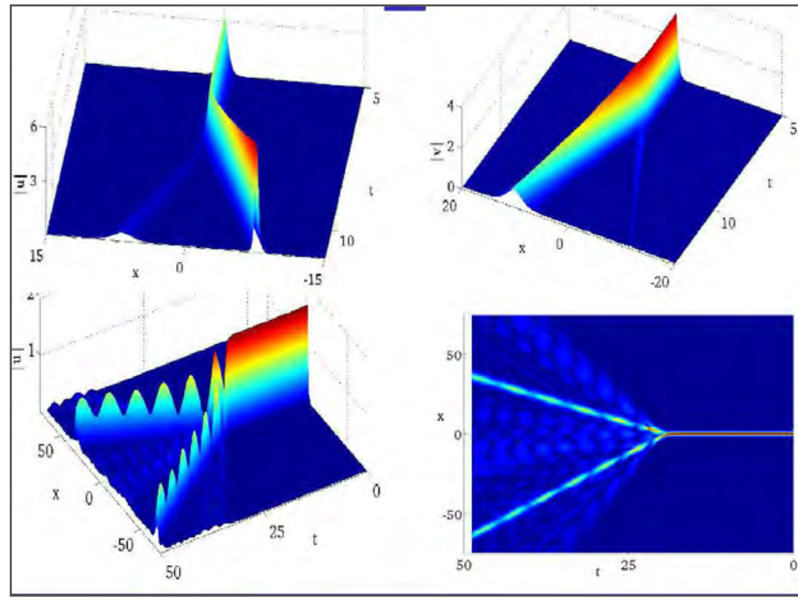
Soliton dynamics and instabilities

High-frequency solitons



A new type of breathers

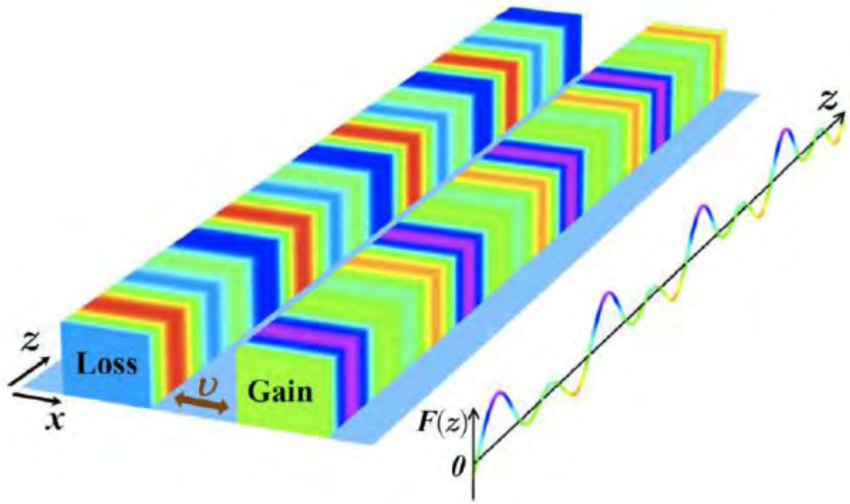
Low-frequency solitons

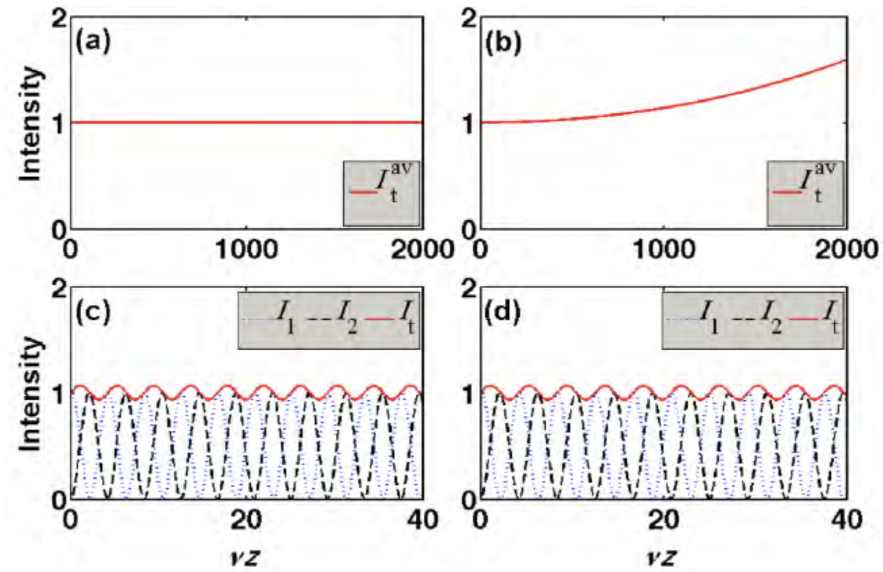


Related publications from our group

Z. Xu, A. Sukhorukov, and Yu.S. Kivshar, Phys. Rev. A **82**, 043818 (2010)
S. Dmitriev, S. Suchkov, A. Sukhorukov, and Yu. Kivshar, Phys. Rev. A **82**, 013833 (2011)
A. Sukhorukov, S. Dmitriev, S. Suchkov, and Yu. Kivshar, Opt. Lett. **37**, 2148 (2012)
N. Alexeeva, I. Barashenkov, A. Sukhorukov, and Yu. Kivshar, Phys Rev A **85**, 063837 (2012)

Pseudo-PT symmetric systems





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Pseudo-Parity-Time Symmetry in Optical Systems

Xiaobing Luo (罗小兵),[1,2] Jiahao Huang (黄嘉豪),[1] Honghua Zhong (钟宏华),[1] Xizhou Qin (秦锡洲),[1] Qiongtao Xie (谢琼涛),[1,3] Yuri S. Kivshar,[4] and Chaohong Lee (李朝红)[1,4,*]