# Self-trapped optical beams: From solitons to vortices

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NATURAM PRIMUM COGNOSCERE RERUM

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# Outline of today's talk

- Solitons: historical remarks
- Recent advances: fields and concepts
- Optical solitons in periodic structures
- Multi-colour optical solitons
- Solitons in Bose-Einstein condensates
- Self-trapped states
- Vortex solitons

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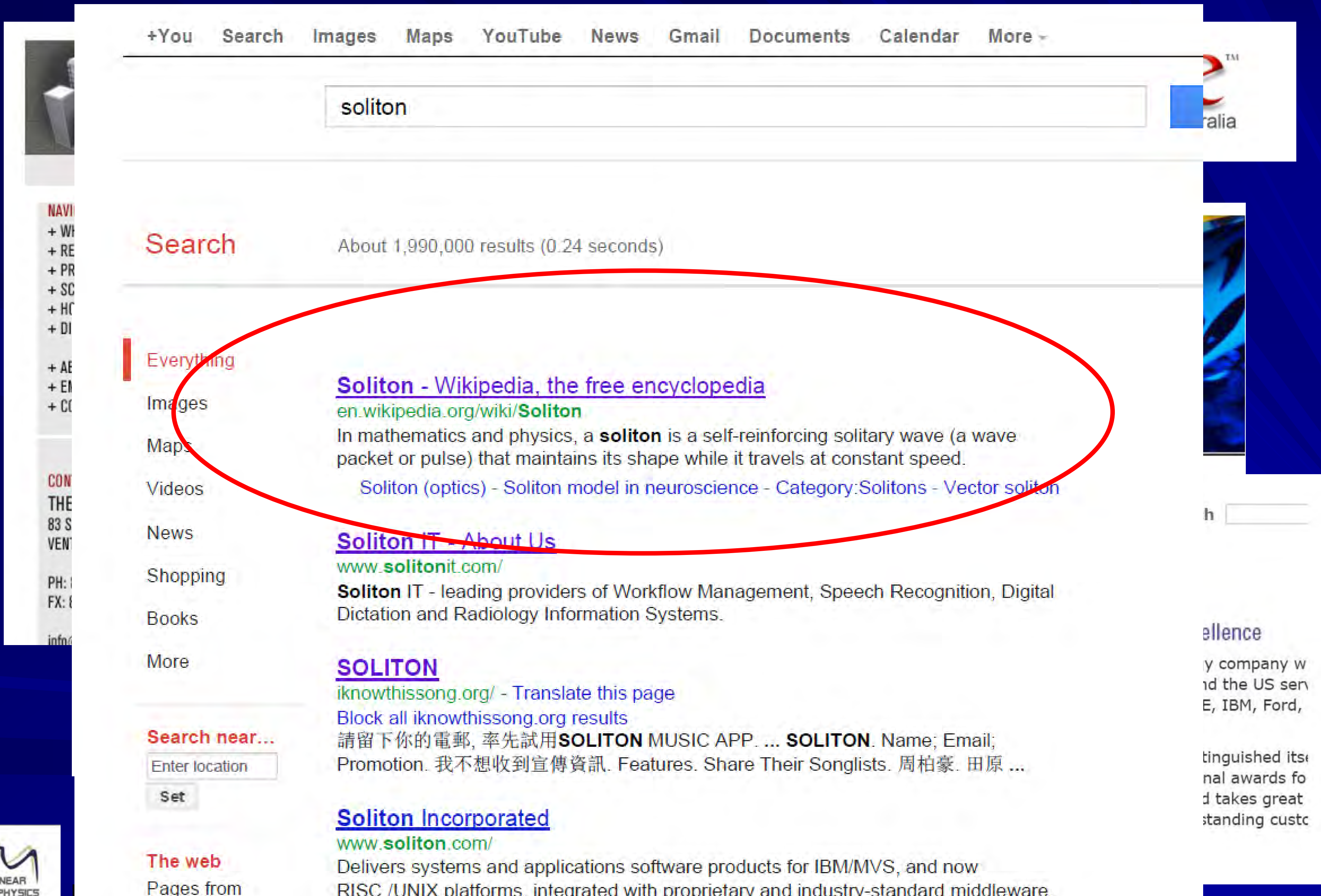
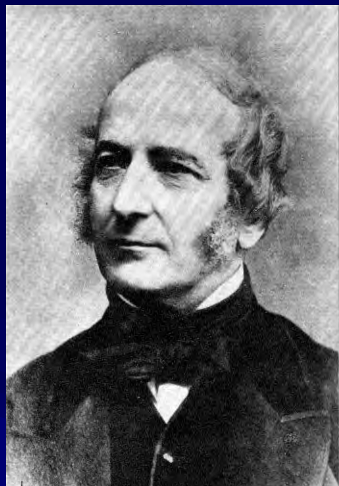
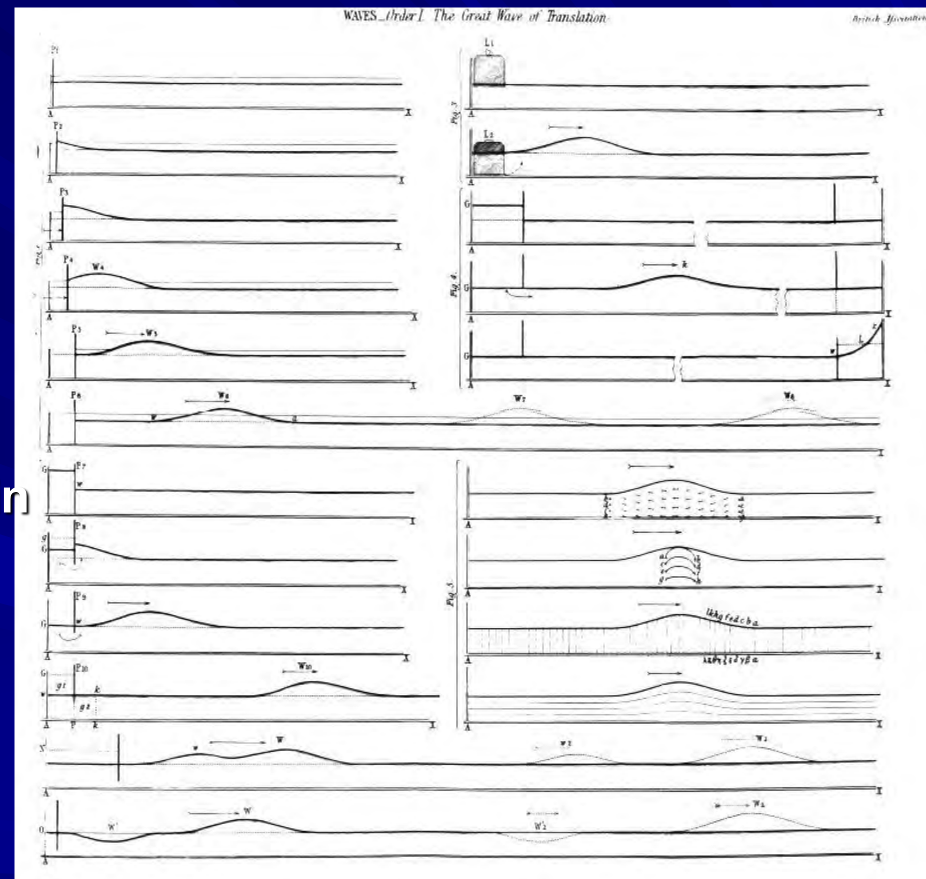
# What is “soliton” ?



# John Scott Russell (1808-1882)



***1834: "... large solitary elevation, .... without change of form or diminution of speed."***




**Very bright engineer: invented an improved *steam-driven road carriage* in 1833. ``Union Canal Society'' of Edinburgh asked him to set up a navigation system with steam boats**

**Russell, *Report on Waves*.**
**Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844.**

# Solitons started in Scotland

**Coast (BBC2 24/04/13)**

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# “Solitons”

VOLUME 15, NUMBER 6 PHYSICAL REVIEW LETTERS 9 AUGUST 1965

## INTERACTION OF “SOLITONS” IN A COLLISIONLESS PLASMA AND THE RECURRENCE OF INITIAL STATES

N. J. Zabusky

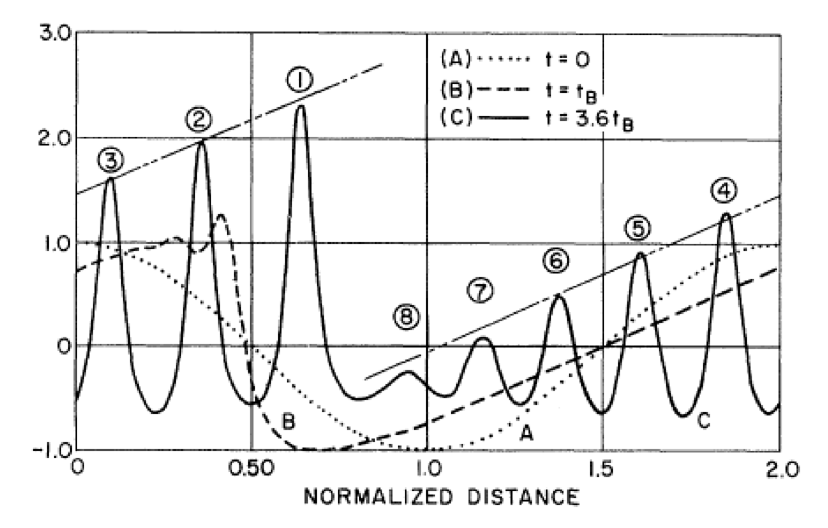
Bell Telephone Laboratories, Whippany, New Jersey

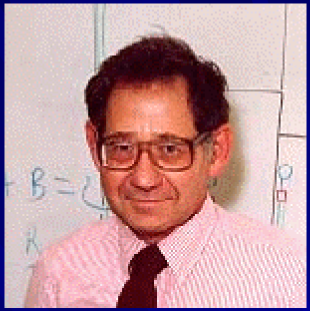
and

M. D. Kruskal

Princeton University Plasma Physics Laboratory, Princeton, New Jersey

(Received 3 May 1965)



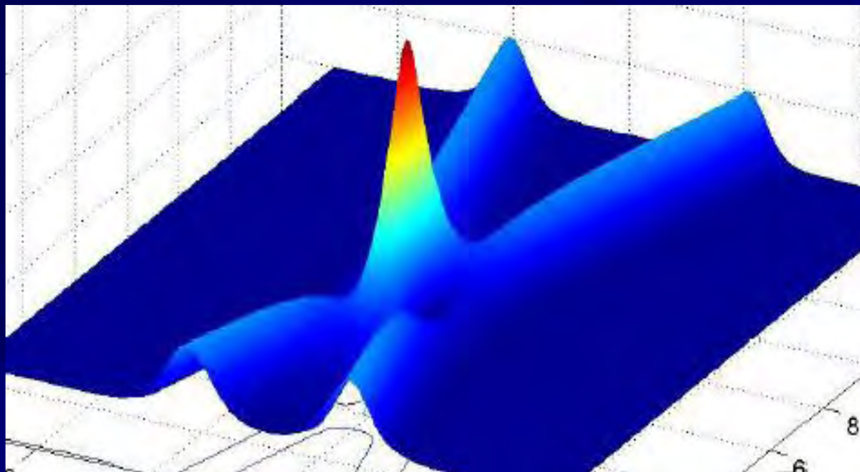



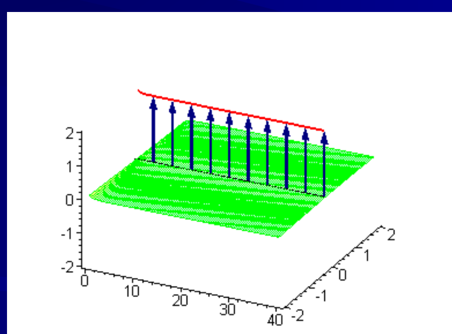
(1925-2006)

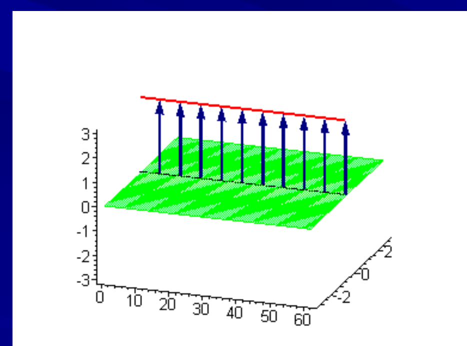
$$U_t + \frac{1}{24} U_{xxx} + \alpha U U_x = 0$$

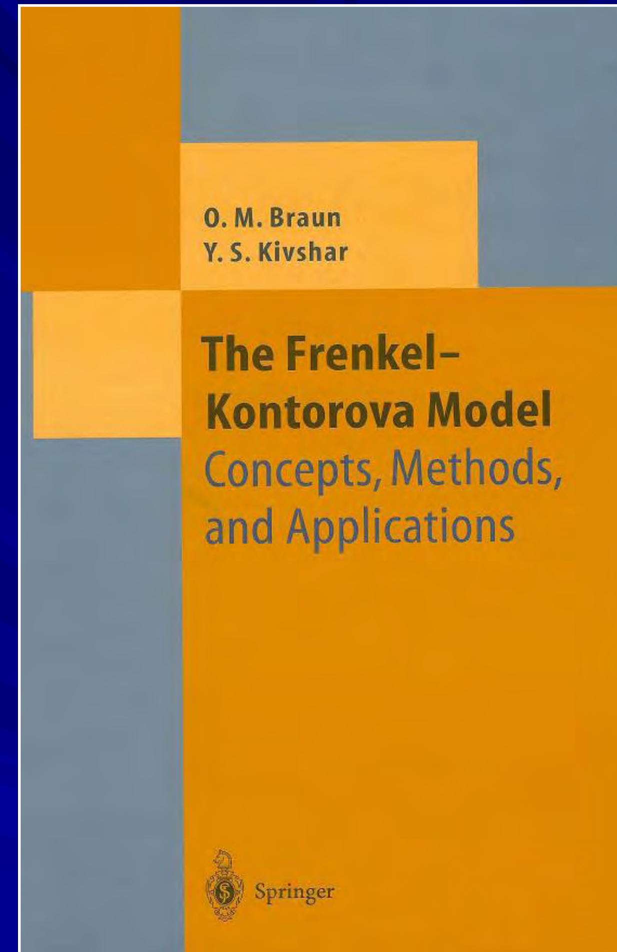
[5]E. Fermi, J. R. Pasta, and S. Ulam, Los Alamos Scientific Laboratory Report No. LA-1940, May 1955 (unpublished). See reference 3 for a review of the problem.

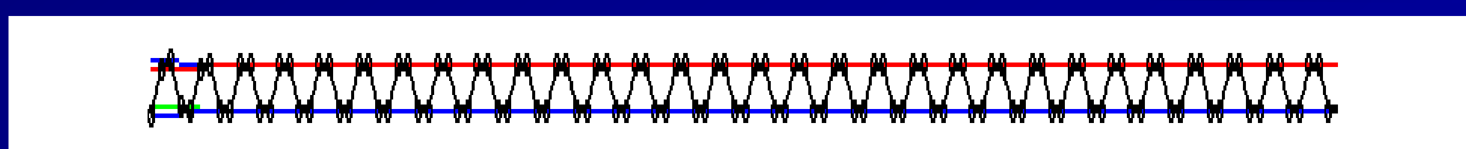
# Other solitons



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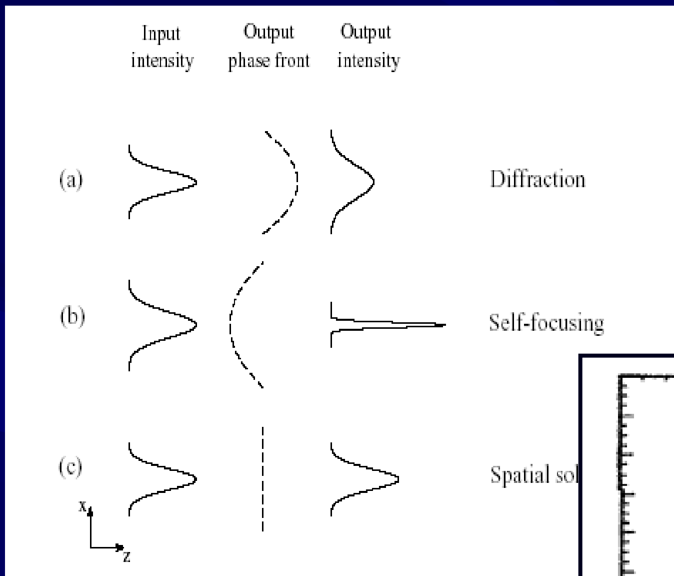
# Recent advances

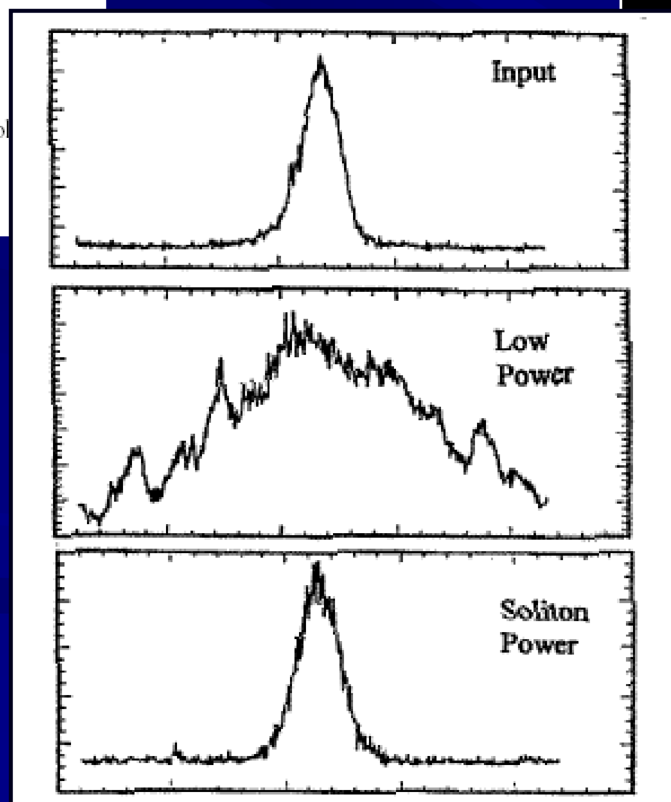
## Importance of nonintergrable models

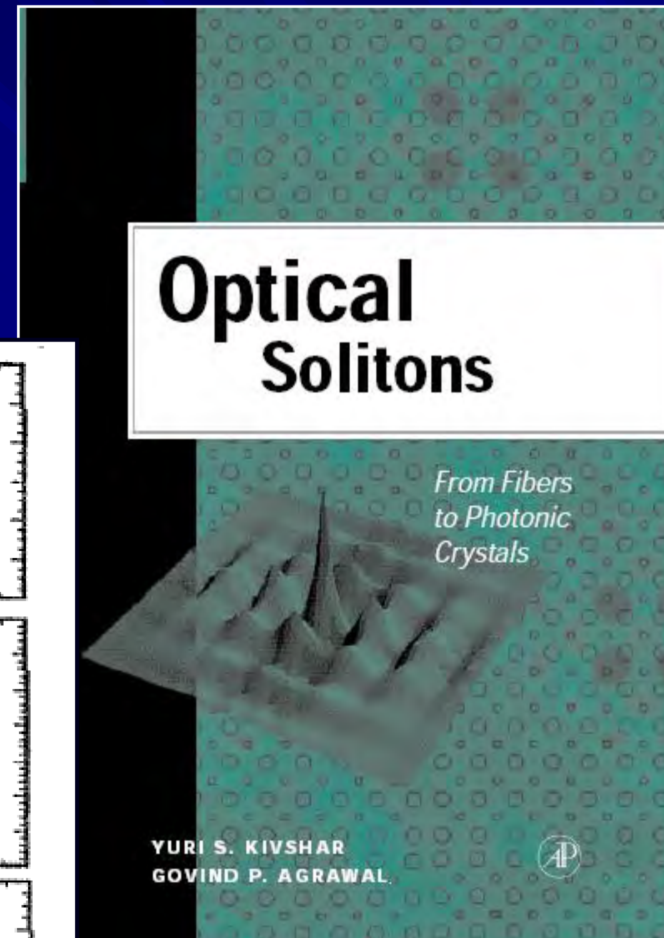
**New types of localized modes**: gap solitons, discrete breathers, compactons, self-trapped modes, azimuthons, etc

**New media and materials**:
nonlinear optics: nonlocal media, discrete and subwavelength structures, slow light
BEC: nonlinearity management
nanostructures: graphene, carbon nanotubes

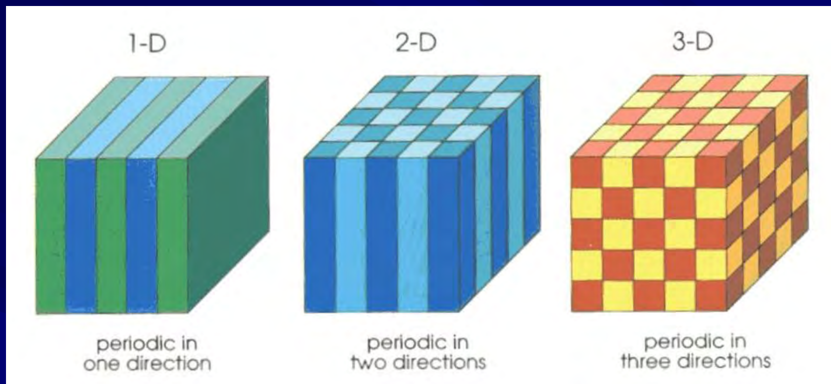
# Self-focusing and spatial optical solitons



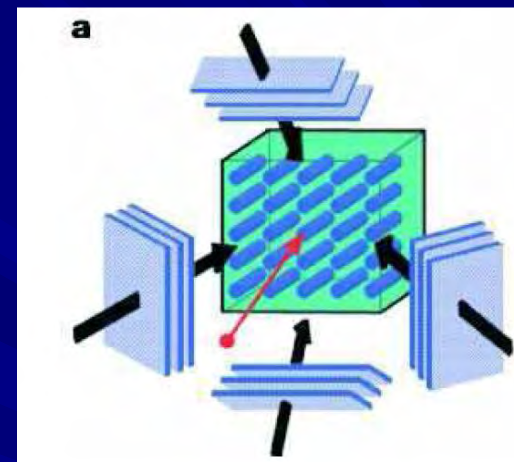



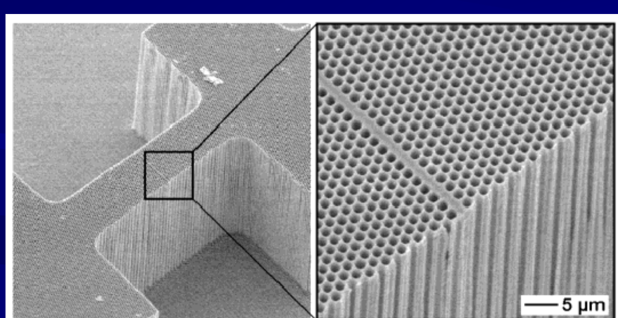
# Photonic crystals and lattices

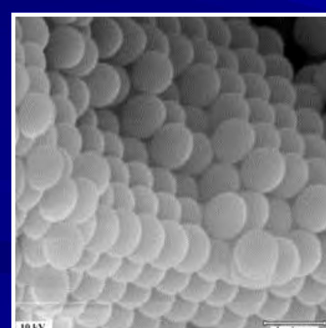


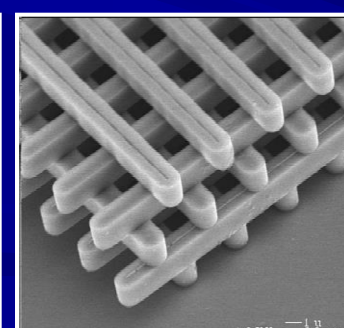

## Optically induced



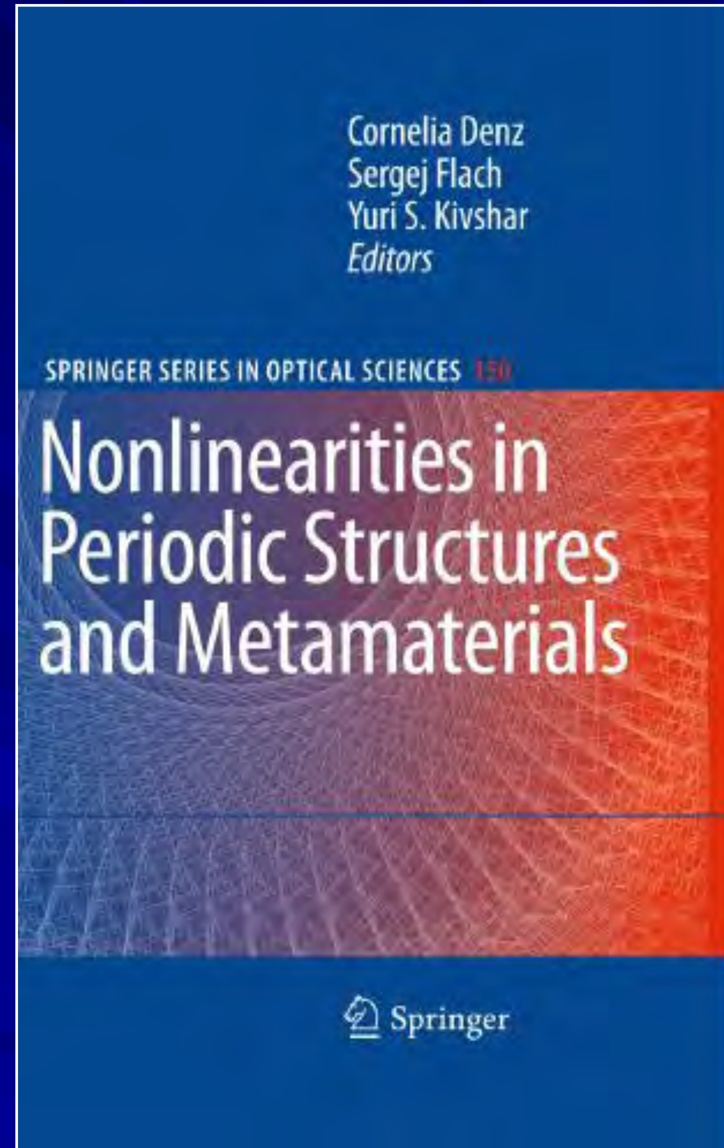

## fabricated



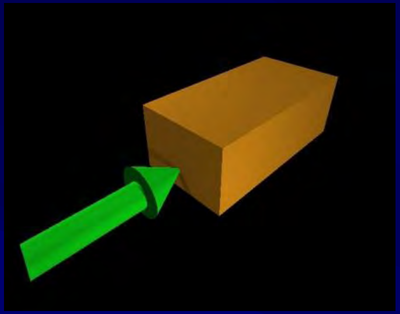



# How does periodicity affect solitons ?

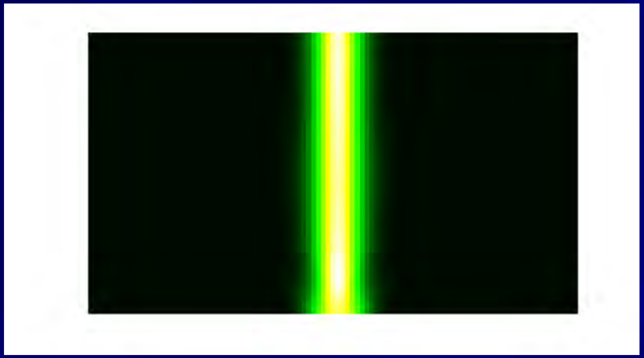


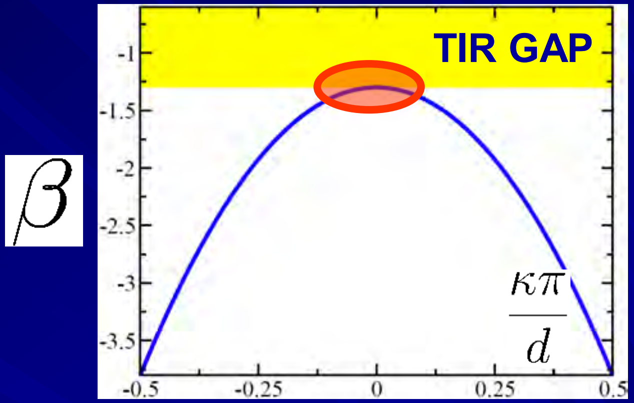
# Spatial dispersion and solitons

**Bulk media**

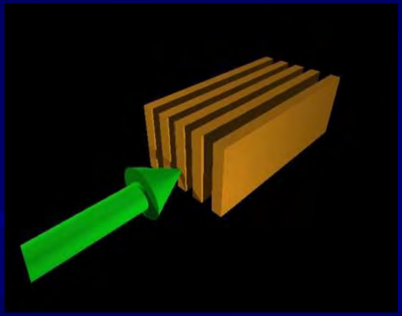


**SPATIAL SOLITON**

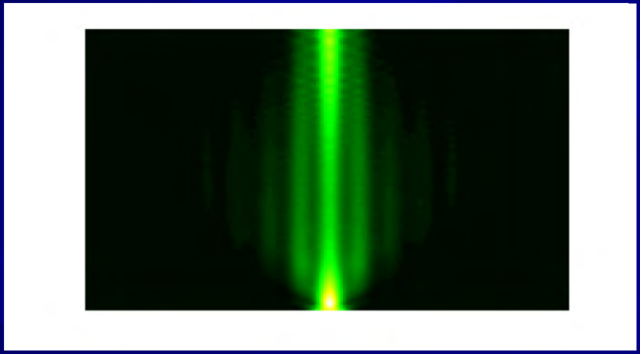


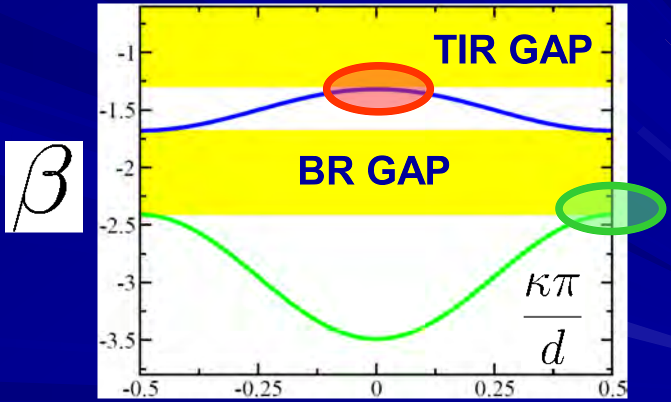



**Waveguide array**



**LATTICE SOLITON**

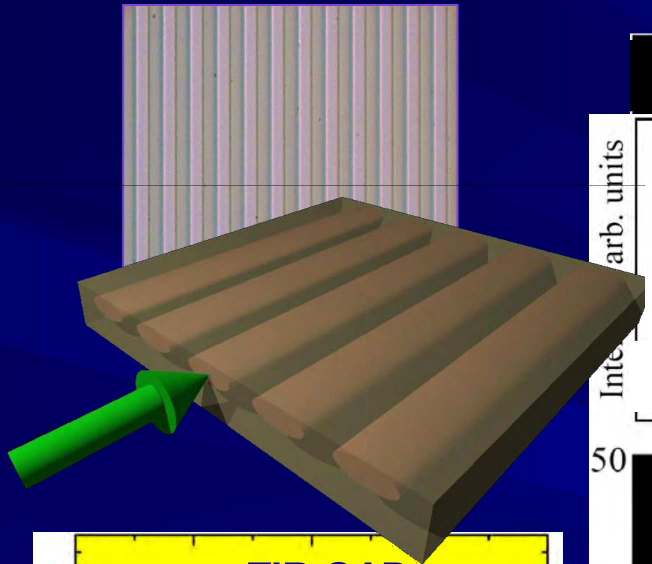


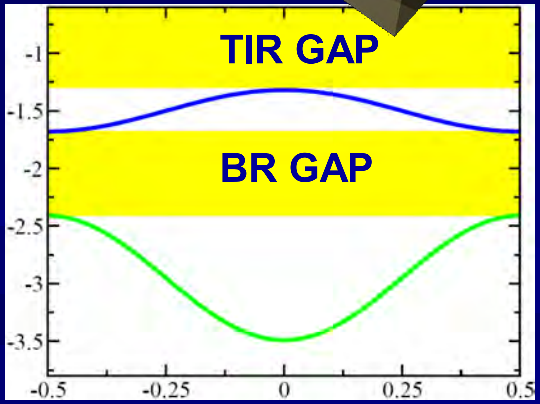



Theory: Christodoulides & Joseph (1988), Kivshar (1993)
Experiments: Eisenberg (1998), Fleischer (2003), Neshev (2003), Martin (2004)

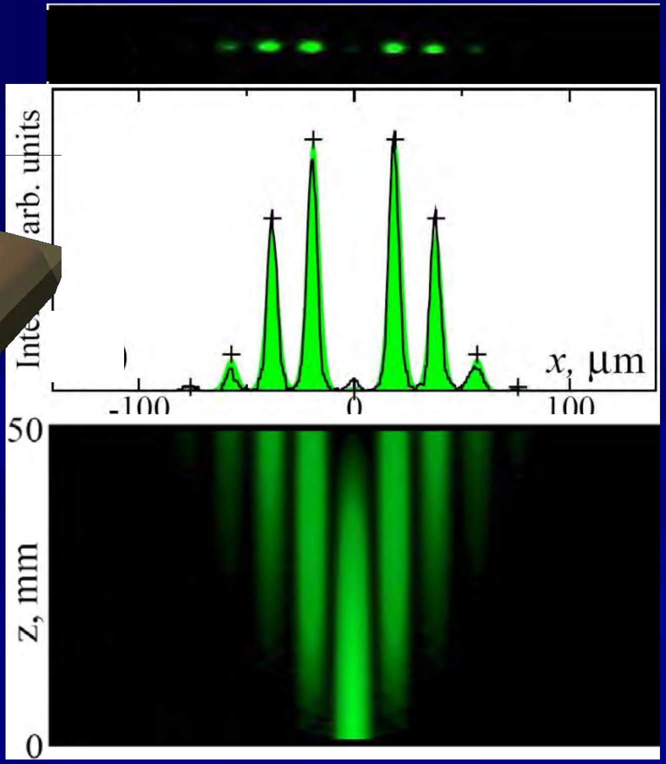
# Gap Solitons - defocusing case

$LiNbO_3$ waveguide array

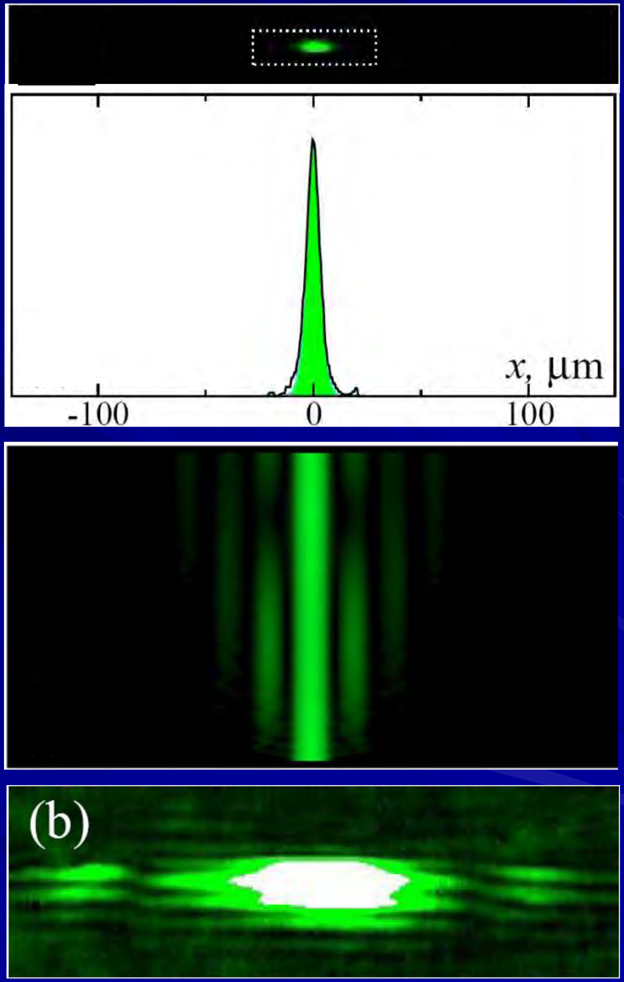





low power 10nW




high power 100μW





Opt. Exp. **14**, 254 (2006)

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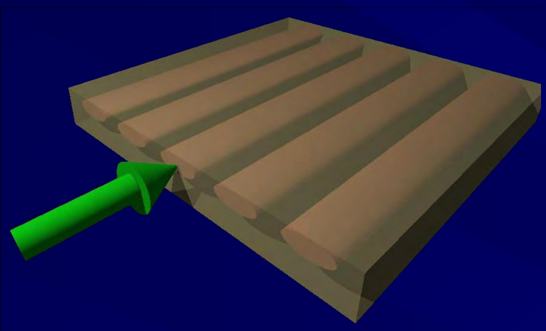
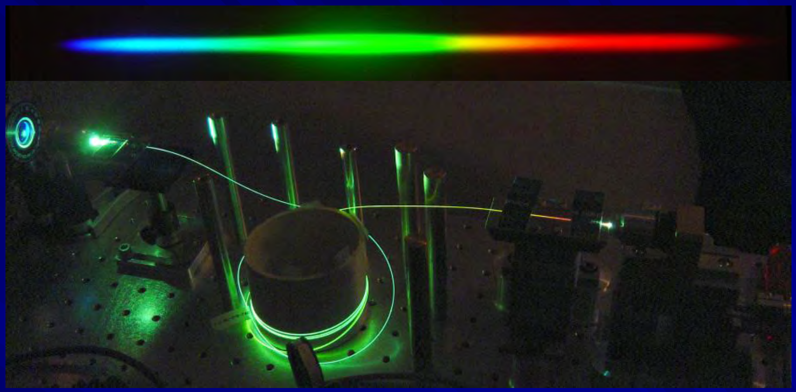
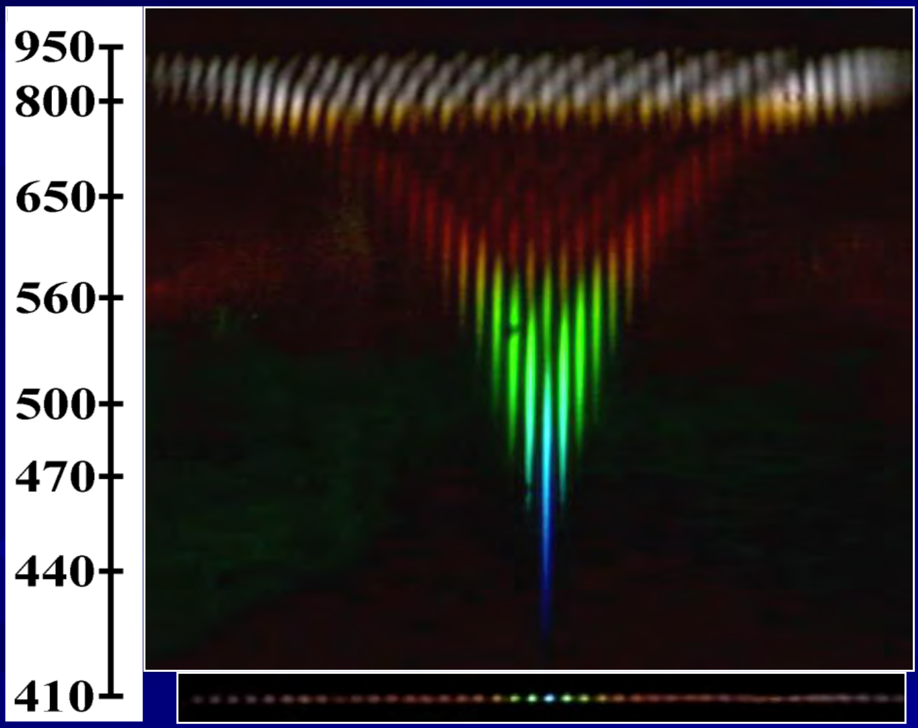
# Polychromatic solitons

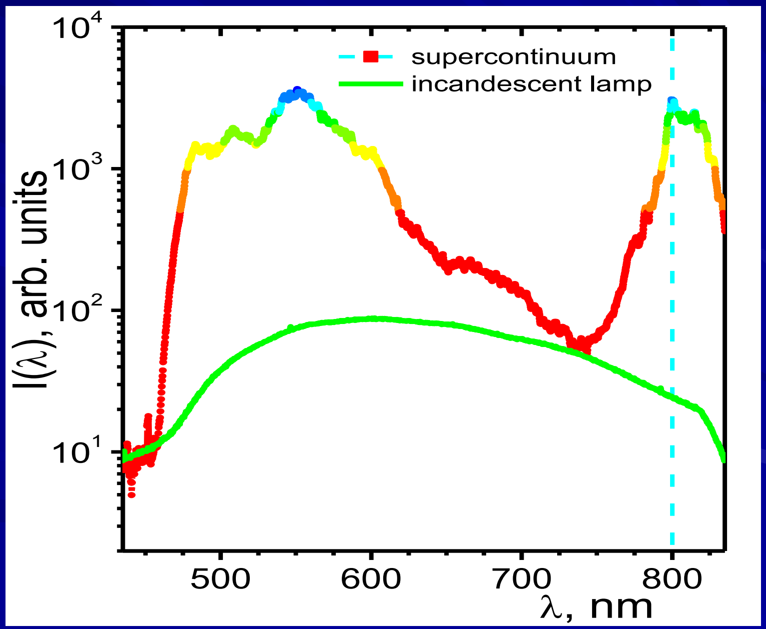
# Diffraction of polychromatic light








waveguides

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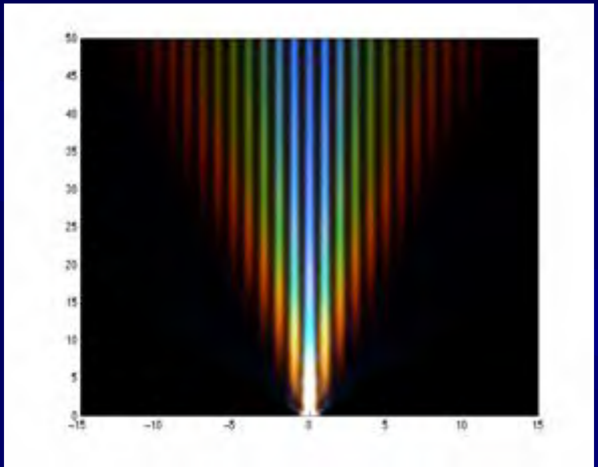
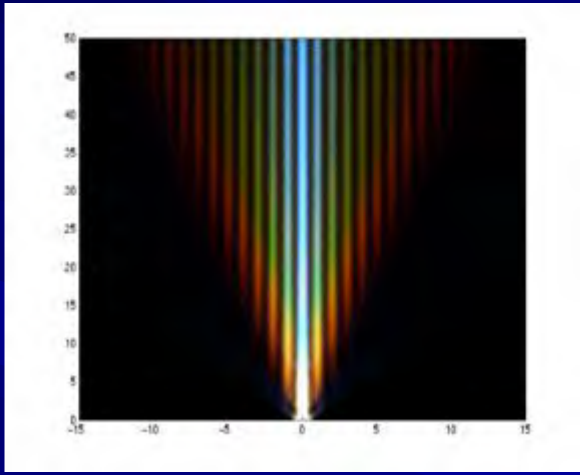
# Theory: coupled NLS equations

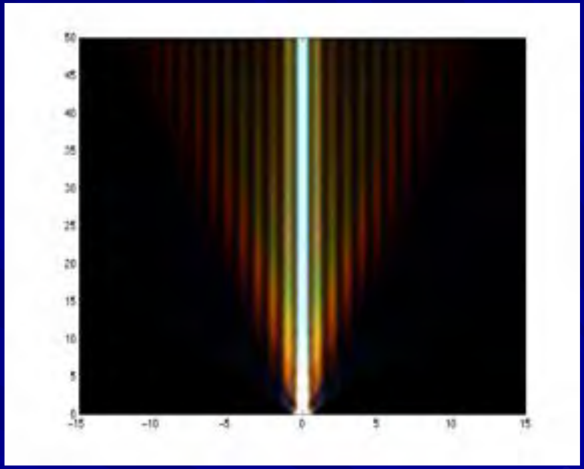
Micro-scale prism



Filtering of red



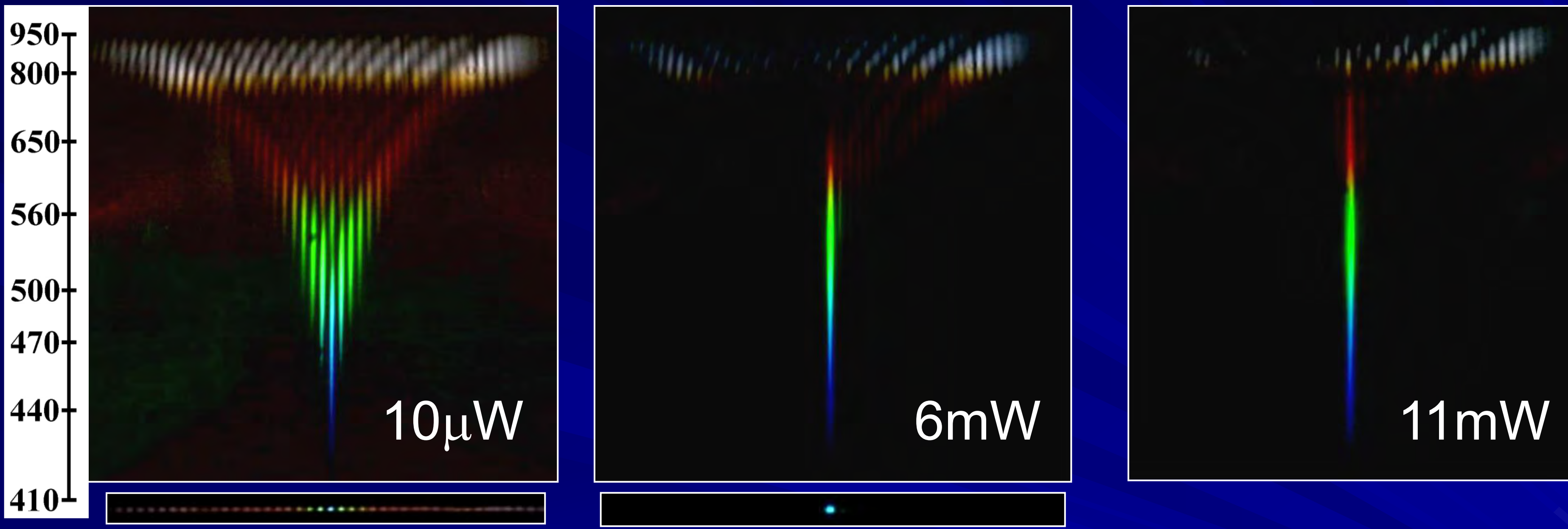
White-light input and output

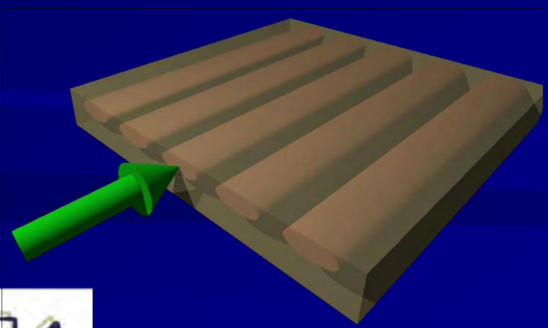


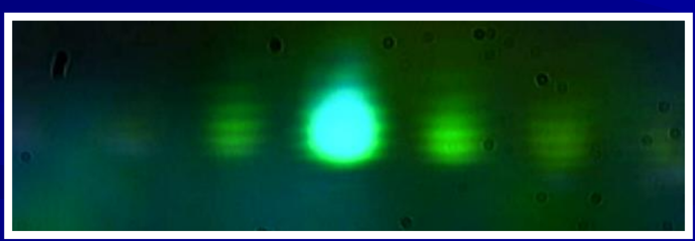
Power

- Optically-controlled separation and mixing of colors

# Experiment: polychromatic gap soliton





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# Nonlinear plasmonic structures

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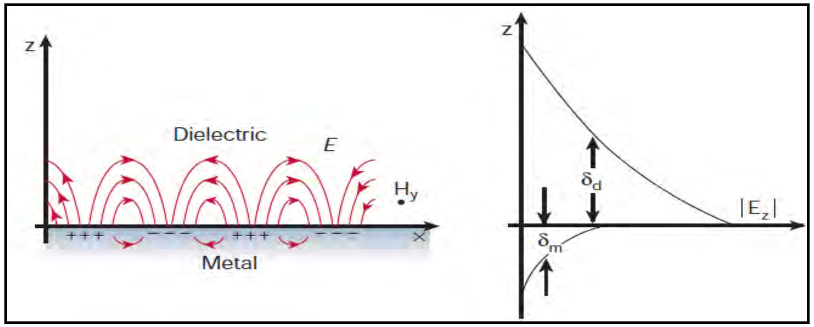
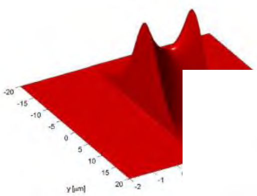
# Plasmon solitons




Nonlinear Kerr-type dielectric

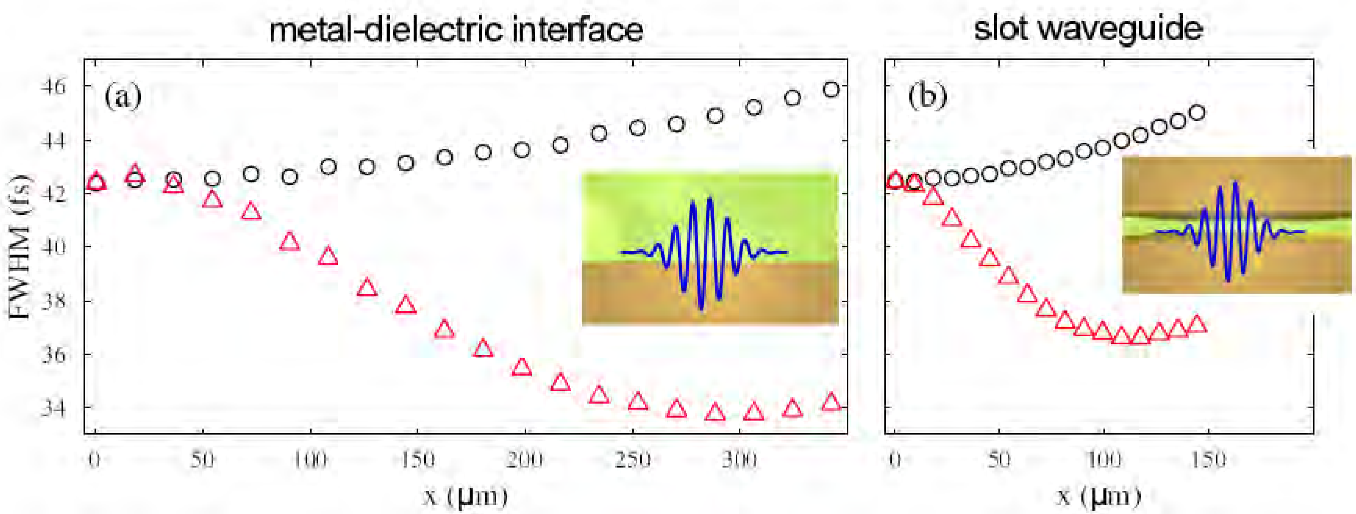
$$\varepsilon = \varepsilon_{linear} + \alpha |E|^2$$

## Plasmon solitons



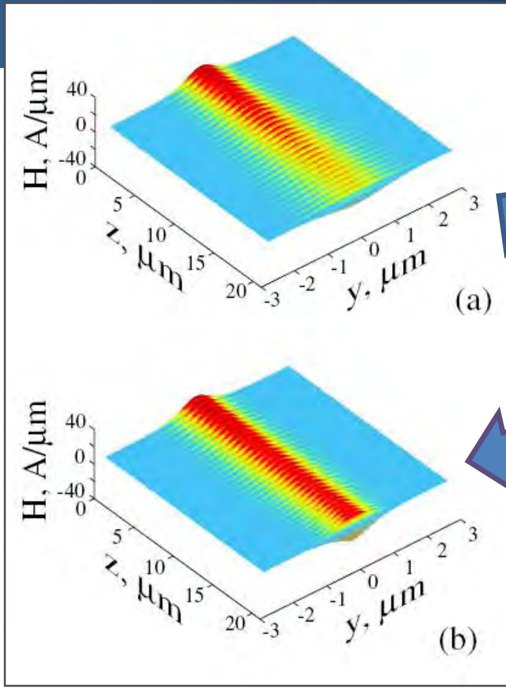
Temporal: A.D. Boardman et al. (1986), A. Pusch et al (2012)

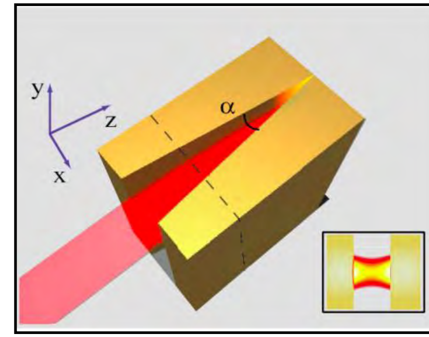
Spatial: M. Orenstein et al (2007); A. Davoyan et al (2009)




A. Pusch, I. Shadrivov, O. Hess, and Yu. Kivshar, Opt. Exp (2013)

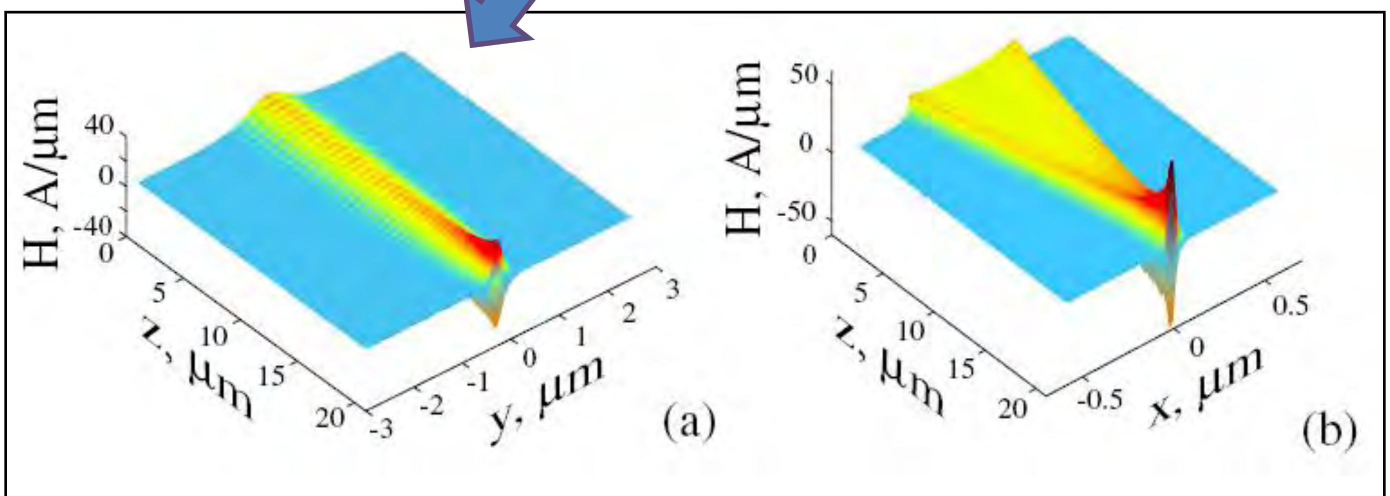
# Spatial solitons and nanofocusing



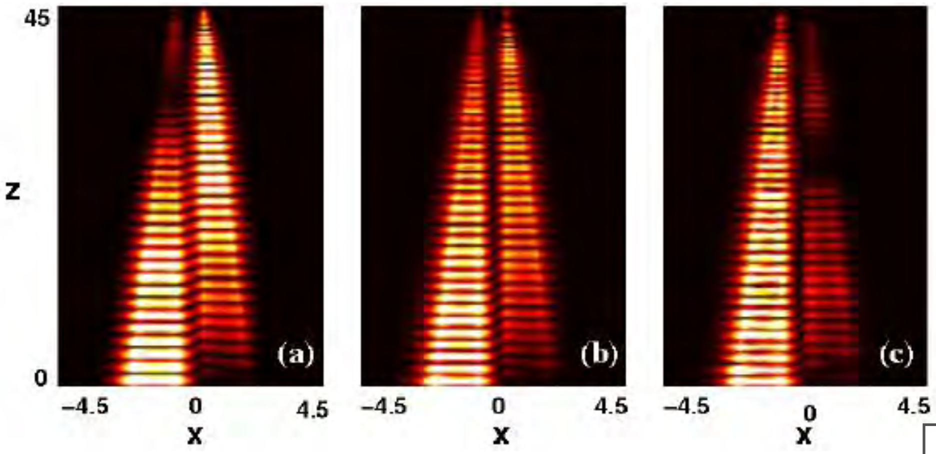

- Beam decay (no taper)
- Stable soliton (optimal taper)
- Soliton focusing (large taper)

$$2j\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial y^2} I + jA\left(\frac{\partial S}{\partial z} + \Gamma\right) + A|A|^2 N_{nln} = 0,$$



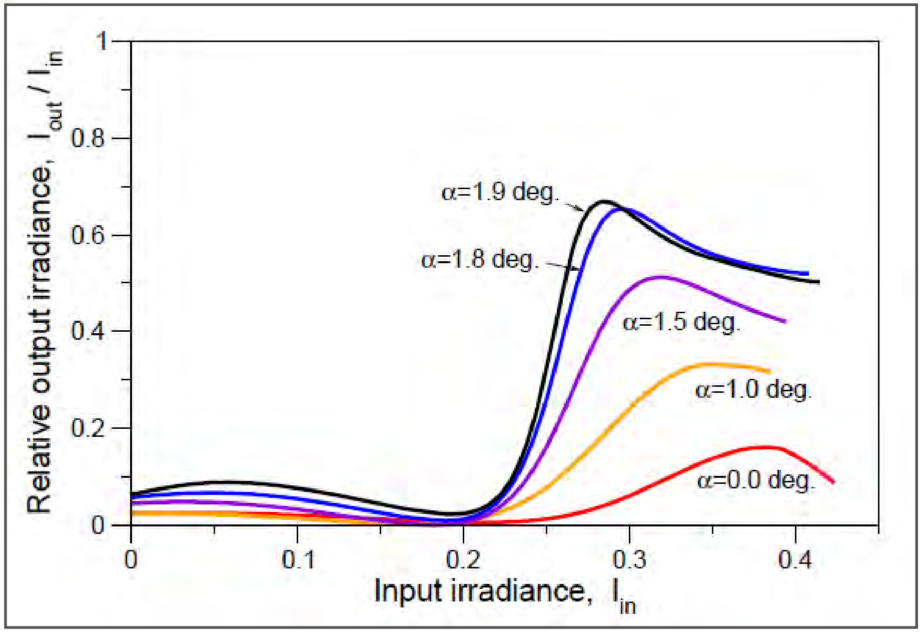

A. Davoyan at al, Phys. Rev. Lett. **105**, 116804 (2010)

# Applications: nonlinear plasmonic couplers



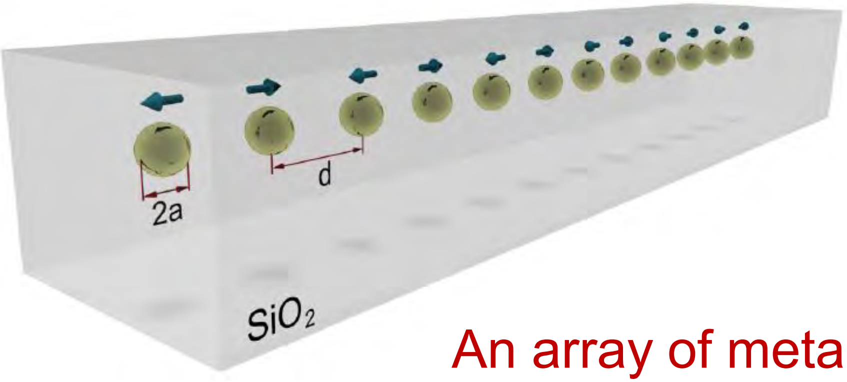

Coupled tapered waveguides can be used for compensating amplitude decay of SPP in plasmonic couplers

By changing the taper angle, we can achieve the effective power transfer in both linear and nonlinear regimes




Salgueiro, Kivshar, Appl Phys Lett (2011); Optics Express (2012)

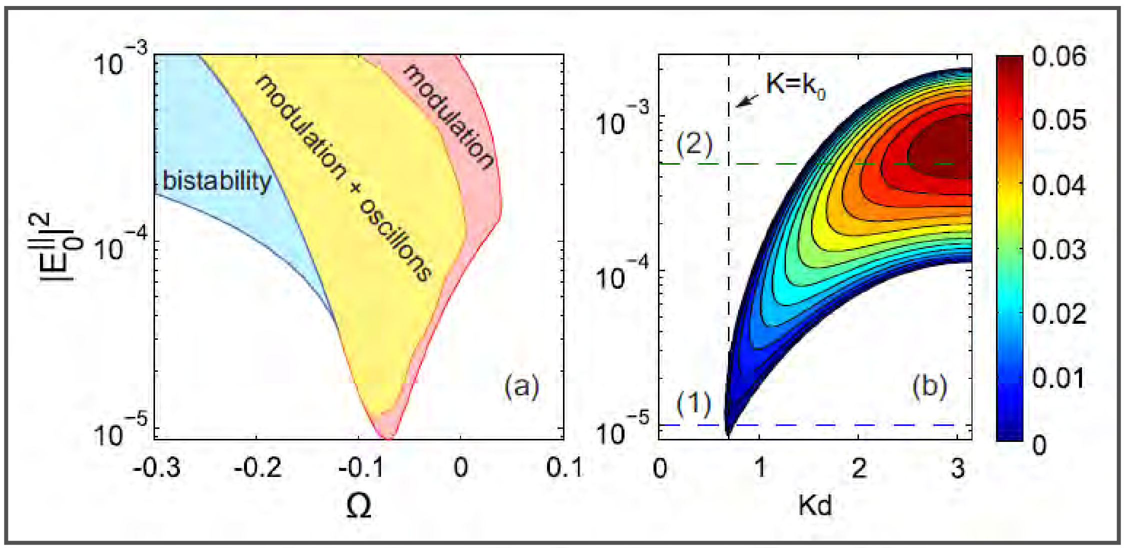
# Arrays of nonlinear metal particles




$$-i\frac{dP_n^{\perp}}{d\tau} + \left(-i\gamma + \Omega + |\mathbf{P}_n|^2\right) P_n^{\perp} + \sum_{m\neq n} G_{n,m}^{\perp} P_m^{\perp} = E_n^{\perp},$$

$$-i\frac{dP_n^{\parallel}}{d\tau} + \left(-i\gamma + \Omega + |\mathbf{P}_n|^2\right) P_n^{\parallel} + \sum_{m\neq n} G_{n,m}^{\parallel} P_m^{\parallel} = E_n^{\parallel},$$

An array of metal nanoparticles in an external driving field




PRL 108, 093901 (2012) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2012

**Subwavelength Modulational Instability and Plasmon Oscillons in Nanoparticle Arrays**

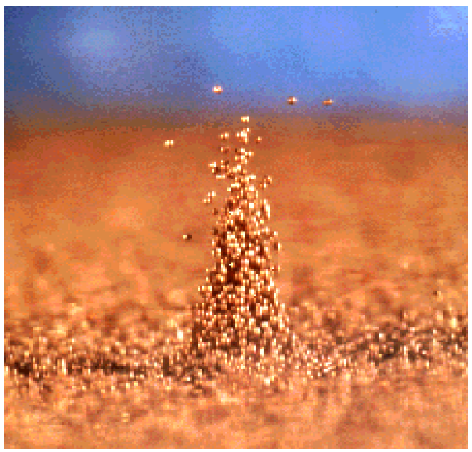
Roman E. Noskov,[1] Pavel A. Belov,[1] and Yuri S. Kivshar[1,2]

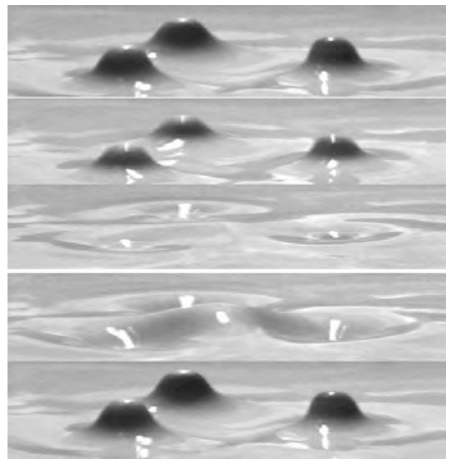
# Modulational instability and oscillons

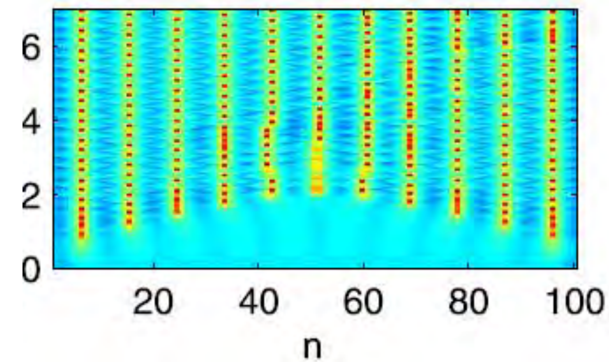
"oscillons" –nonlinear localized modes in externally driven systems

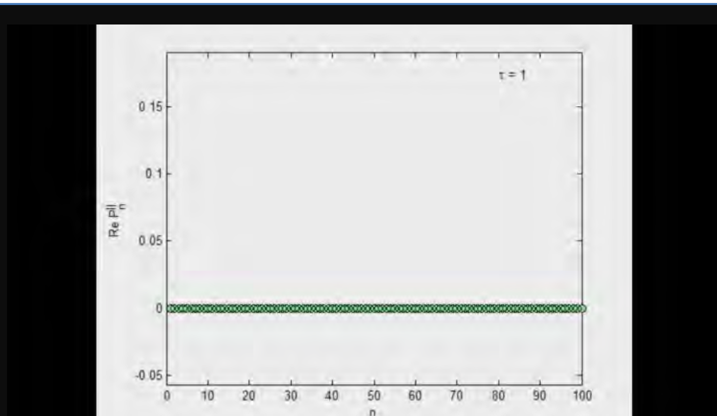
H. Swinney et al, Nature 382, 793 (1996)

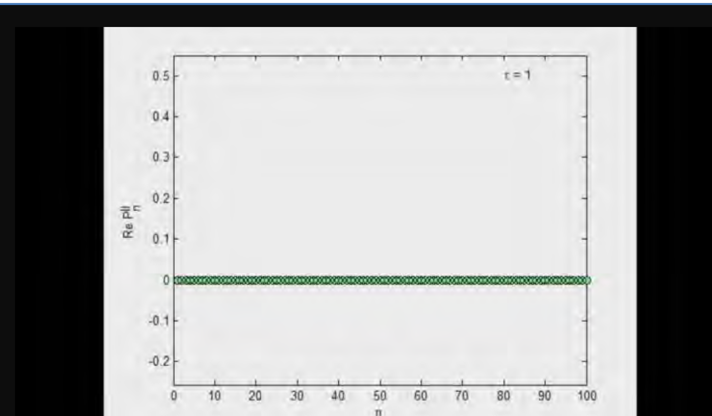
**H. Arbell et al, Phys. Rev. Lett. 85, 756 (2000)**





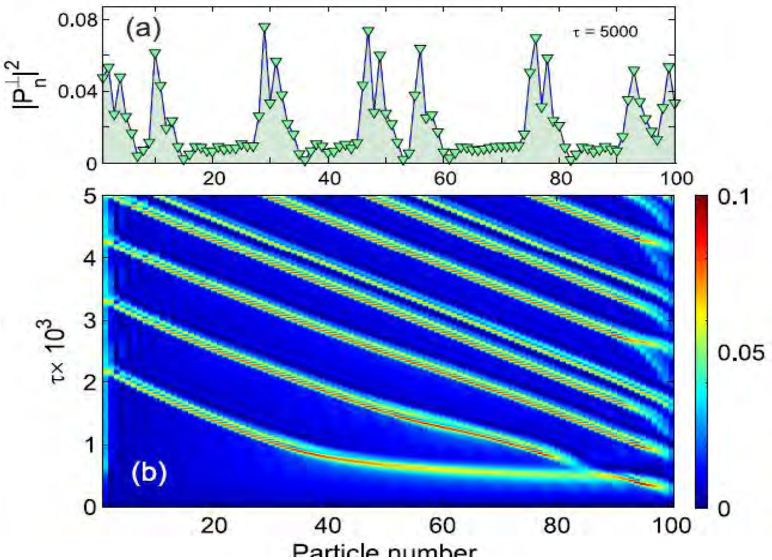


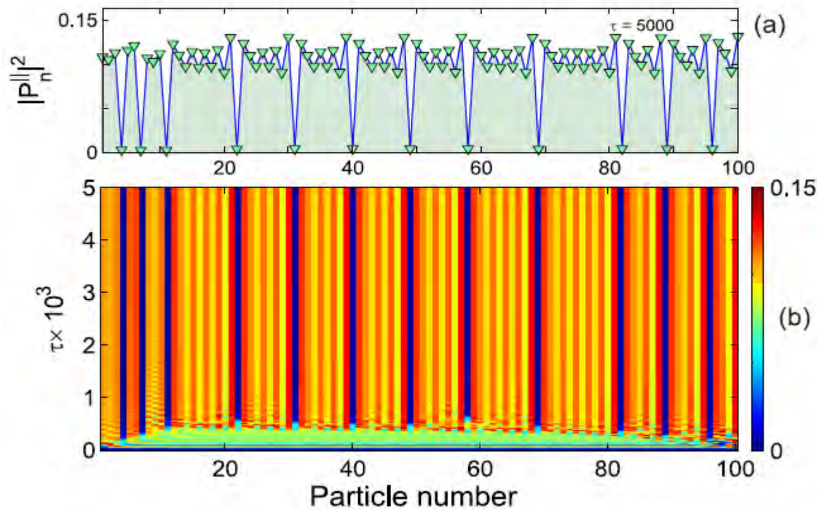



# Zoo of nonlinear modes and their dynamics

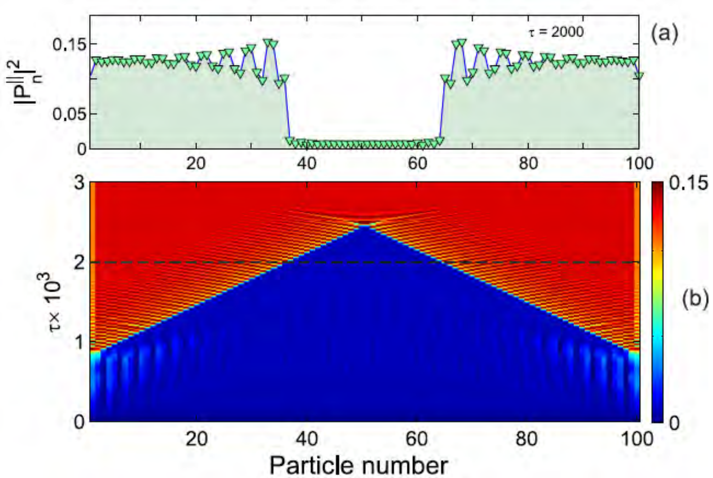
## Bright solitons



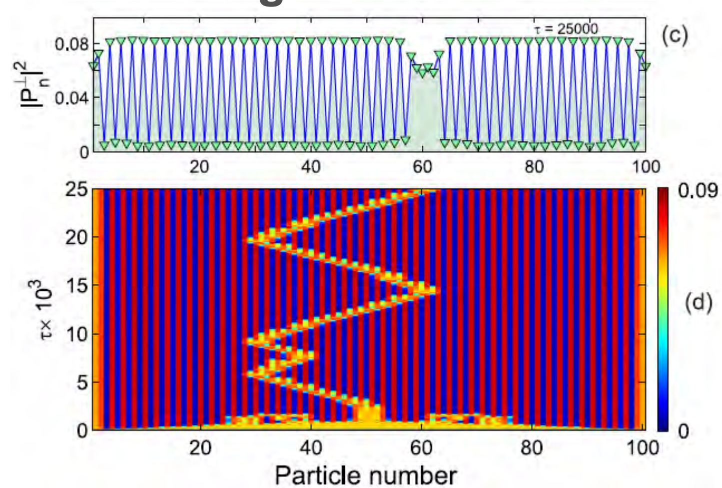

## Dark solitons




## Kinks—domain walls



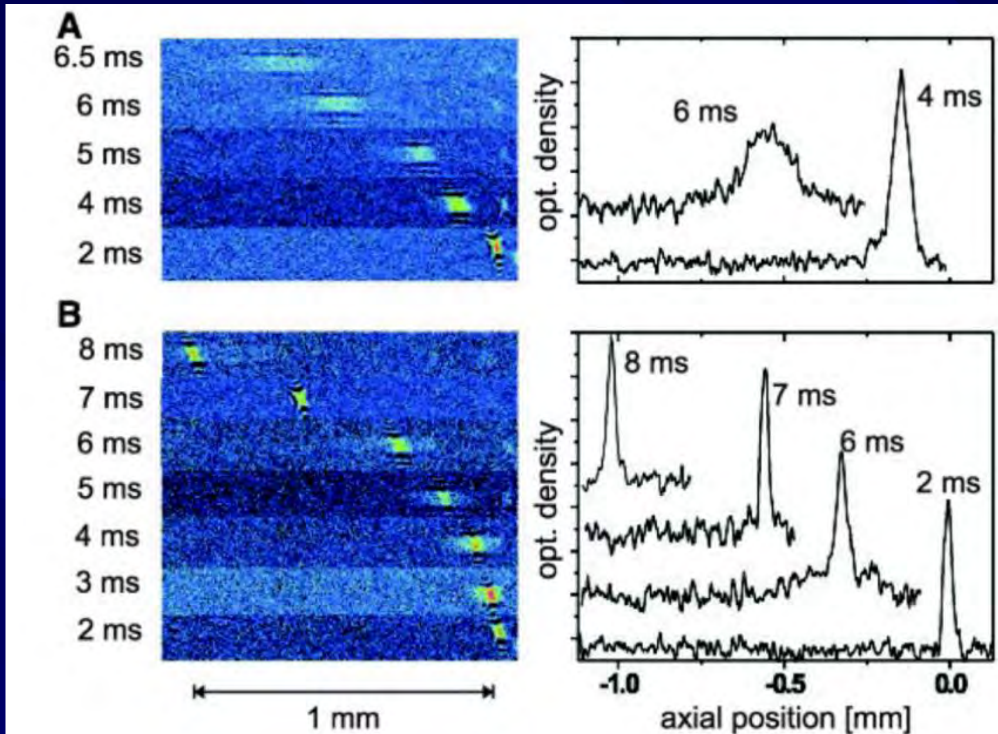

## Drifting dark oscillons

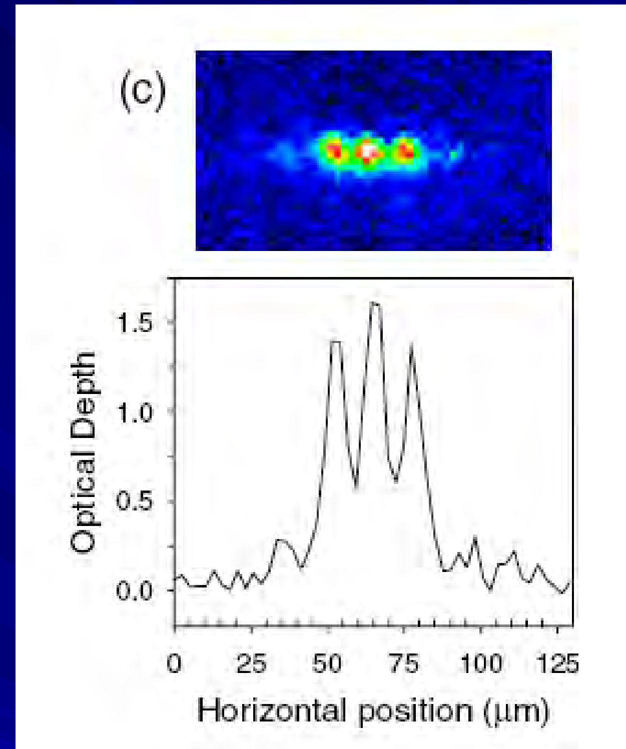



**R. Noskov et al, Sci Rep (2012)**

# Matter waves and BEC

# Solitons in Bose-Einstein condensates



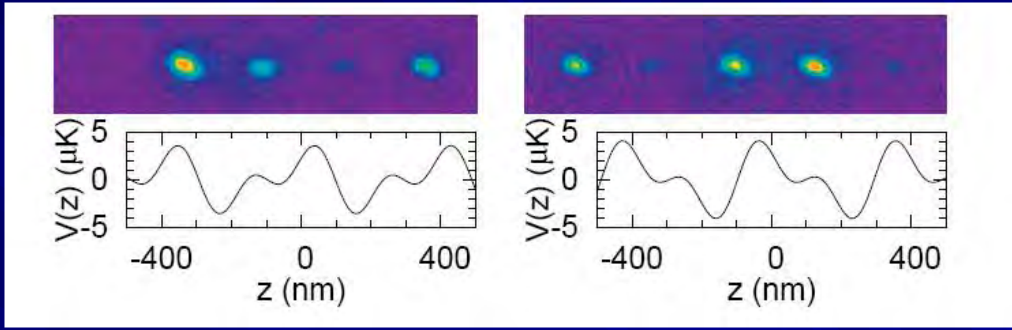




Bright solitons – attractive interaction, negative scattering length
Achieved through self focusing, modulational instability, collapse

L. Khaykovich et al., Science **296**, 1290 (2002);
K. E. Strecker et al., Nature **417**, 150 (2002); S. Cornish et al., PRL **96**, 170401 (2006)

# Driven model for BEC

T. Salger et al., PRL **99**, 190405 (2007)




3D to 1D reduction due to trapping geometry

Normalization using typical scales of the system

$$i\frac{\partial \Psi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \Psi}{\partial x^2} + |\Psi|^2\Psi + V(x,t)\Psi$$

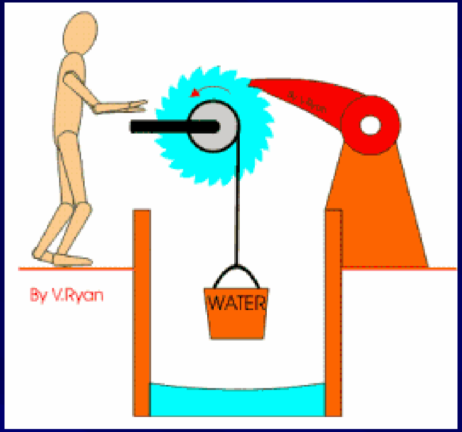
$$V(x,t) = V_0 f(t)\left[\cos(x) + \cos(2x+\phi)\right]$$

$$f(t) = \sin(\omega t) + \sin(2\omega t)$$

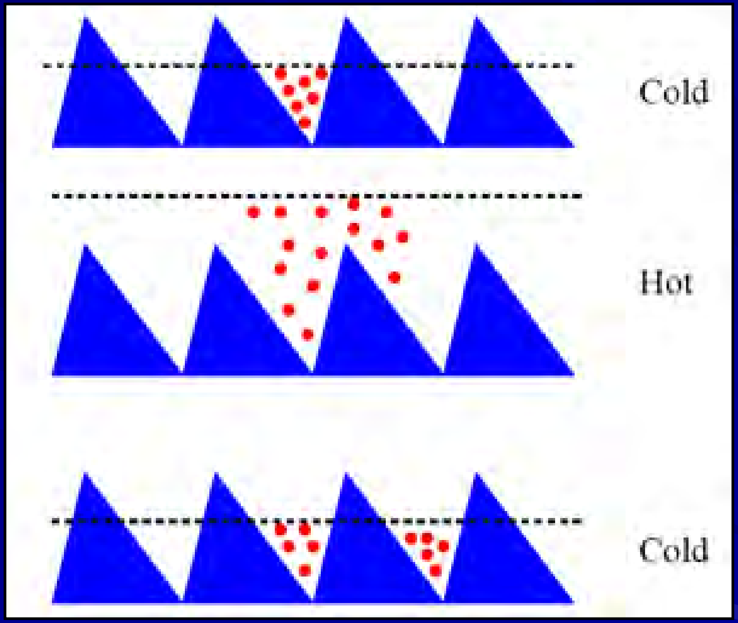
All symmetries are broken, no damping

$x \longrightarrow -x + \tilde{x}$ $\quad t \longrightarrow -t + \tilde{t}$

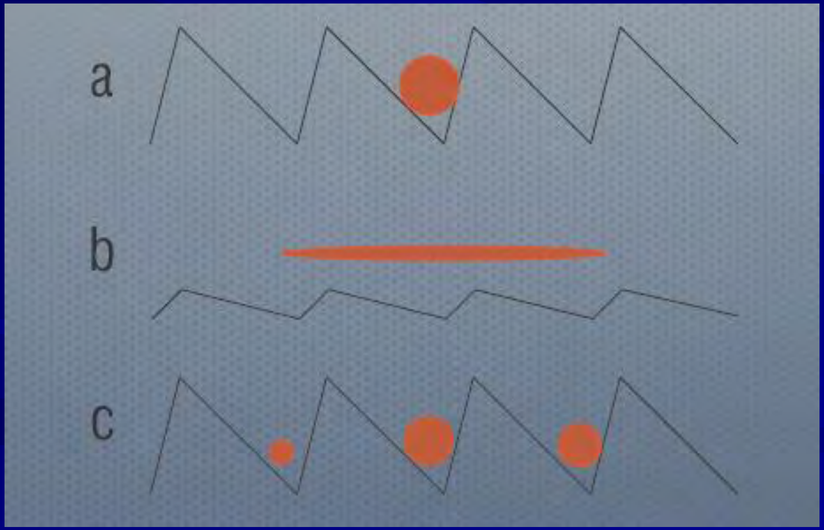
# Physics of ratchets




## Thermal ratchet

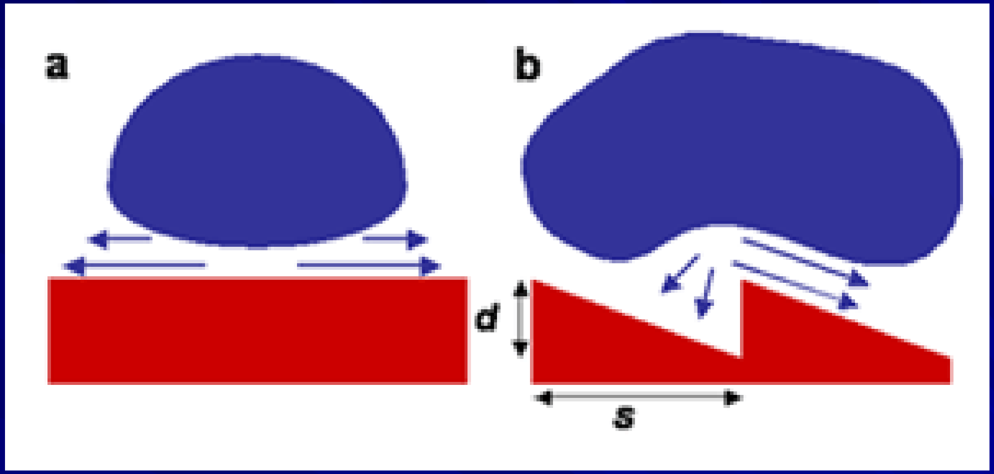



## Flashing ratchet



# Ratchet transport

Self-propelled
Leidenfrost droplets




**H. Linke et al, Phys. Rev. Lett. 96, 154502 (2006)**

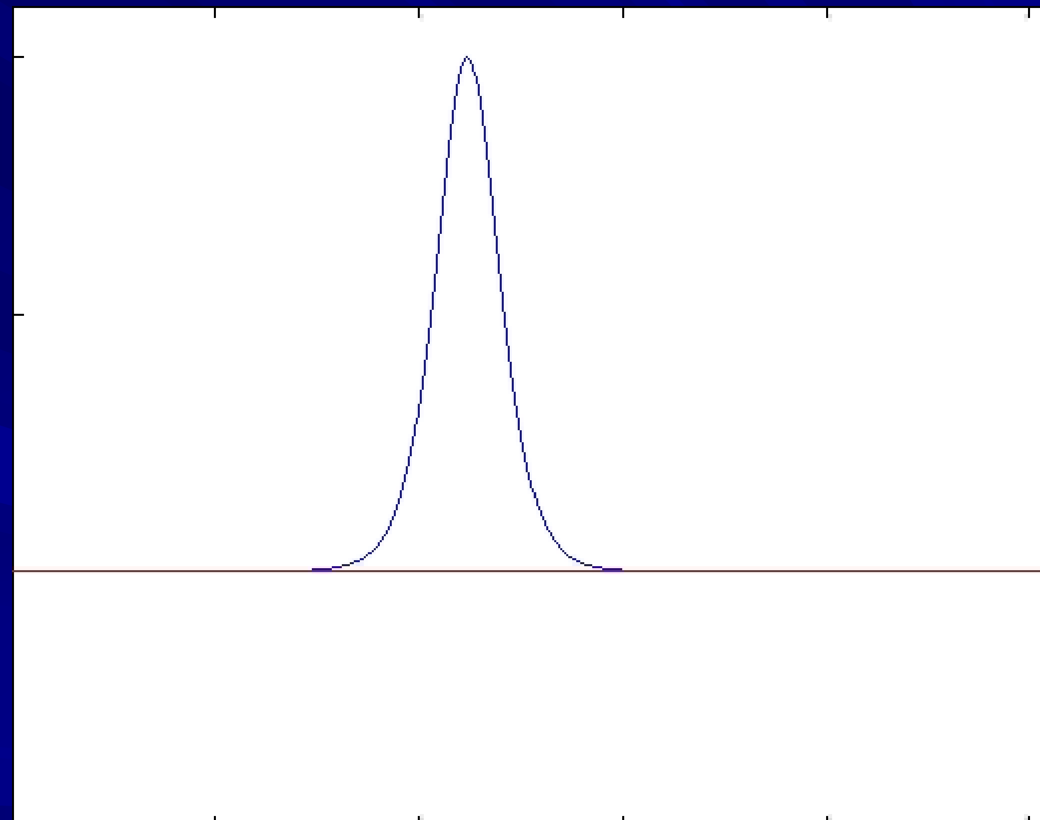
PRL **101,** 150403 (2008) PHYSICAL REVIEW LETTERS week ending 10 OCTOBER 2008

**Dynamics of Matter-Wave Solitons in a Ratchet Potential**

Dario Poletti,[1,2] Tristram J. Alexander,[2] Elena A. Ostrovskaya,[2] Baowen Li,[1,3] and Yuri S. Kivshar[2]

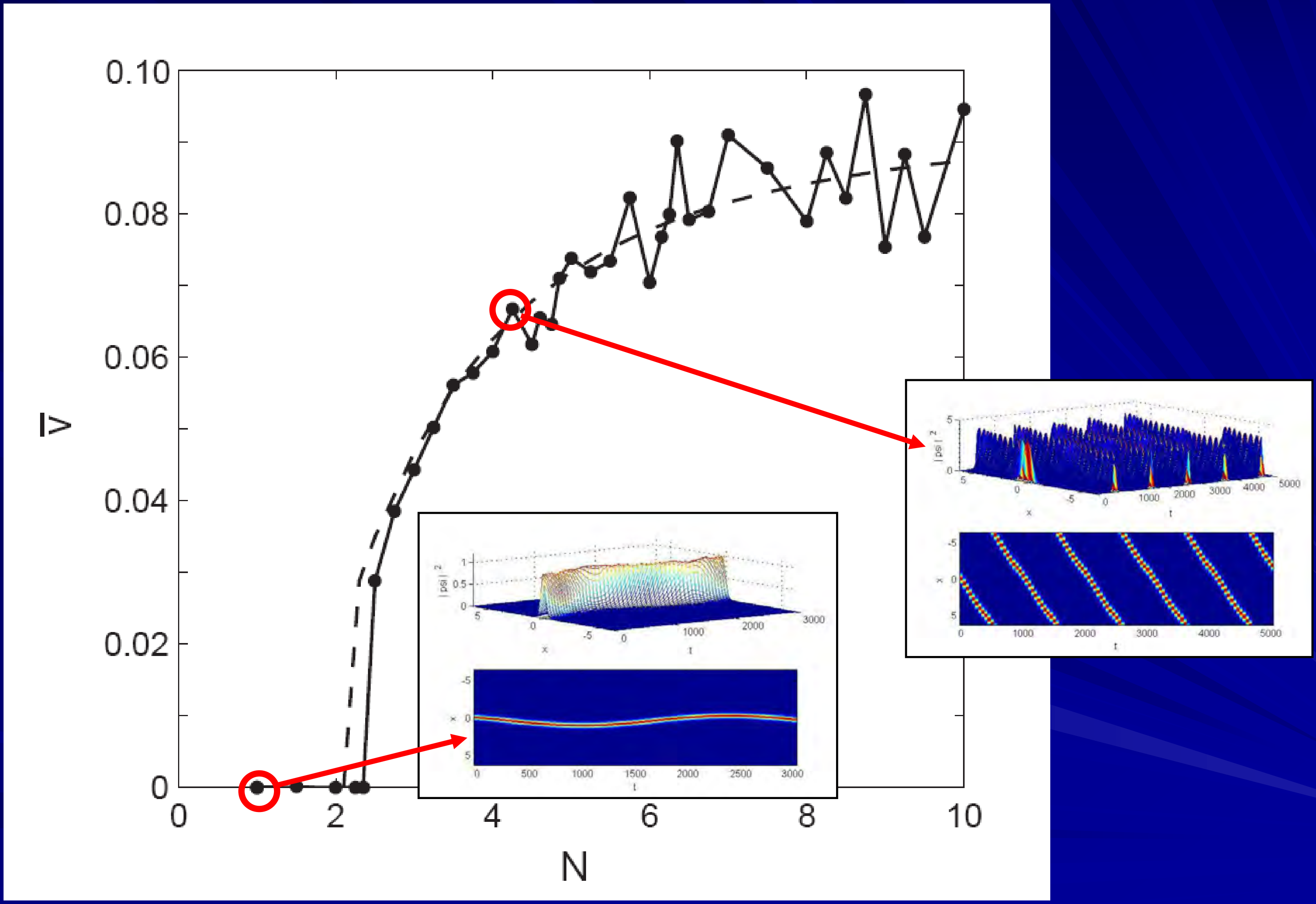
- Being initially at rest, the soliton starts moving provided N larger than a certain critical value

- Cumulative velocity depends on the soliton mass (particle number); this effect can be explained by the effective particle approximation



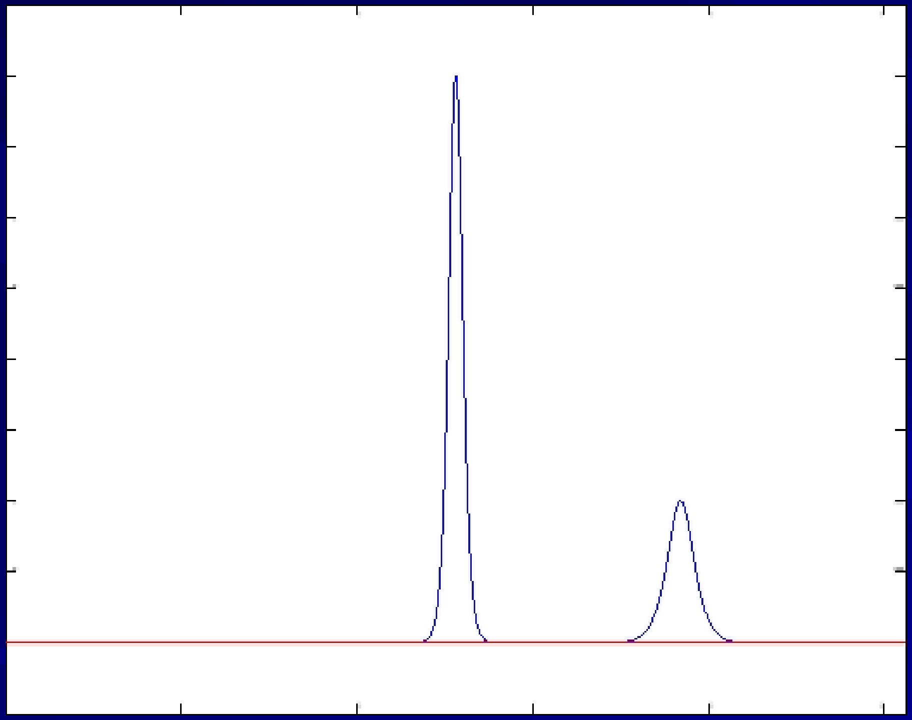
The first example of the mass-dependent soliton ratchet

# Averaged velocity

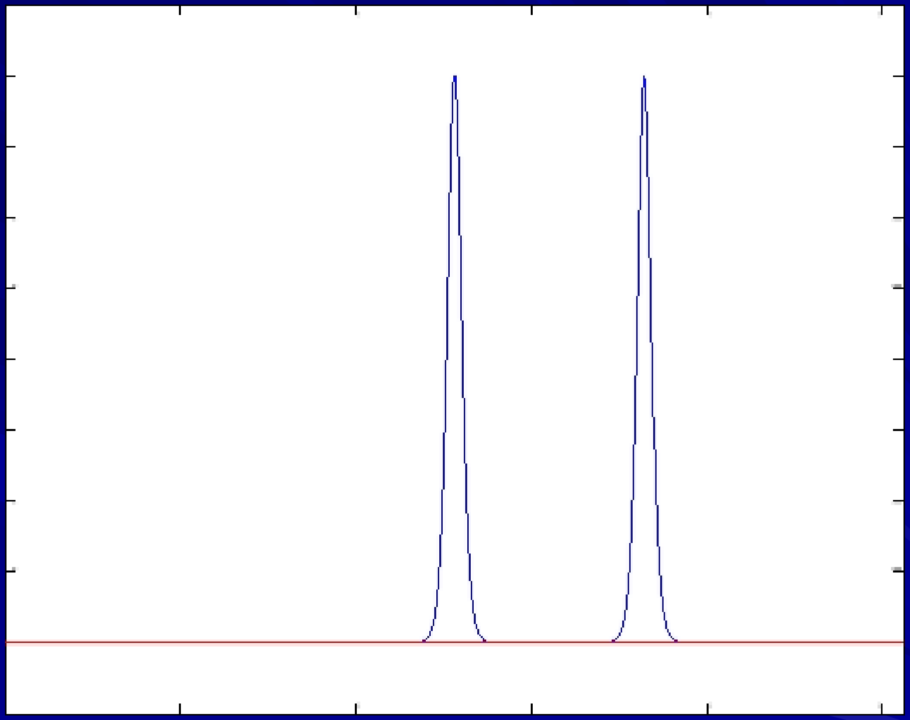



$\bar{v} = 1$ corresponds to 3.5 mm/s

# Collisions of driven solitons



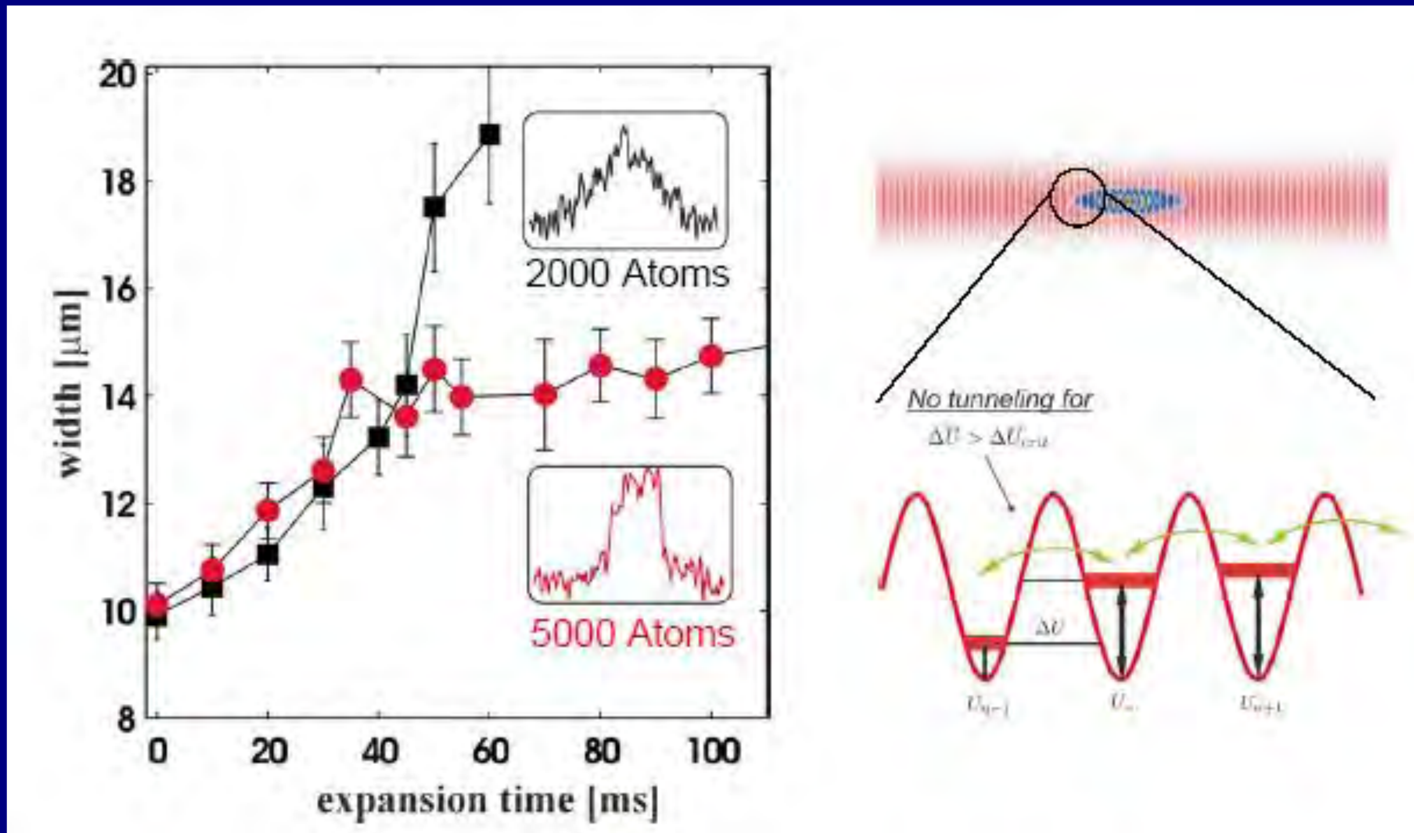
Initially different values of N



Initially equal values of N

# Nonlinear self-trapped states

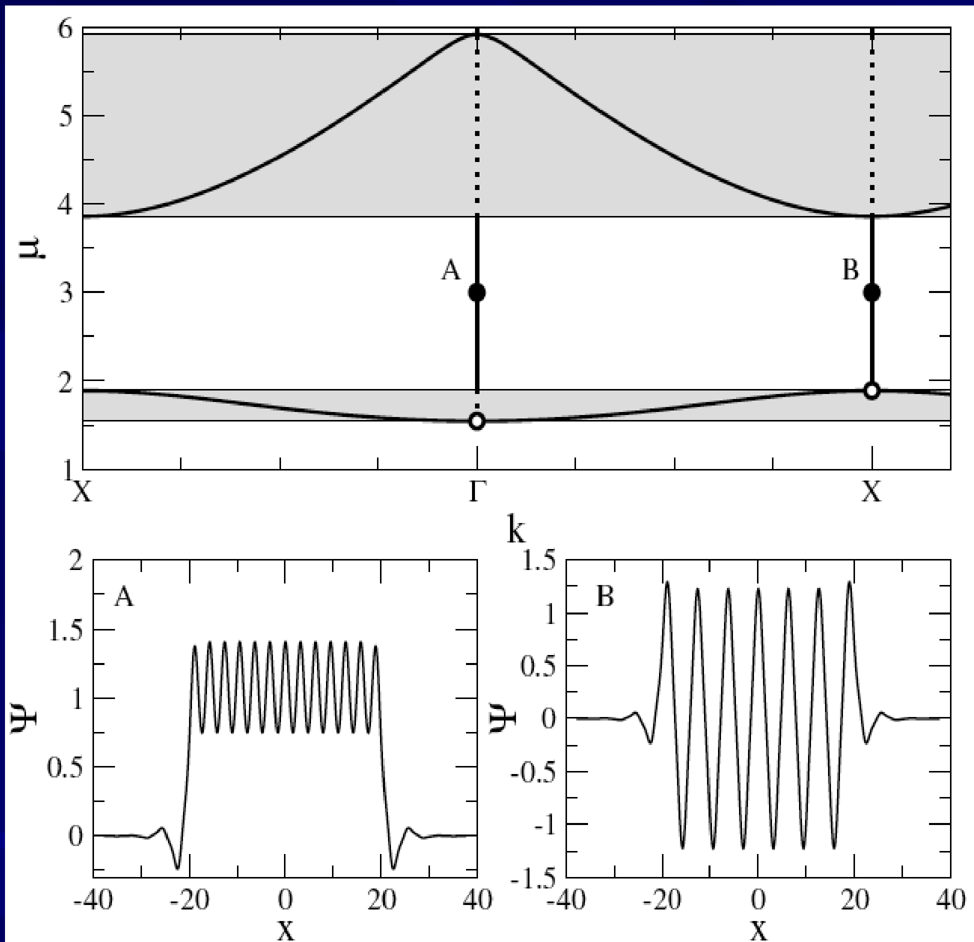
# Self-trapping in BEC




Th. Anker et al, PRL **94**, 020403 (2005)

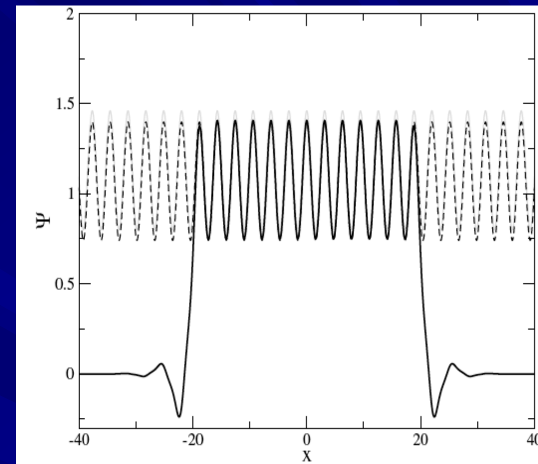
# Novel ‘broad’ gap states

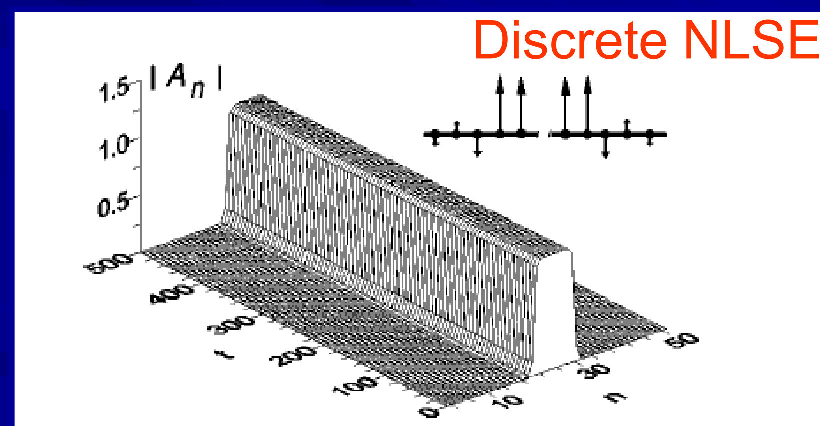
T.J. Alexander *et al*, Phys. Rev. Lett. **96**, 140401 (2006)




two types of modes

truncated nonlinear Bloch modes



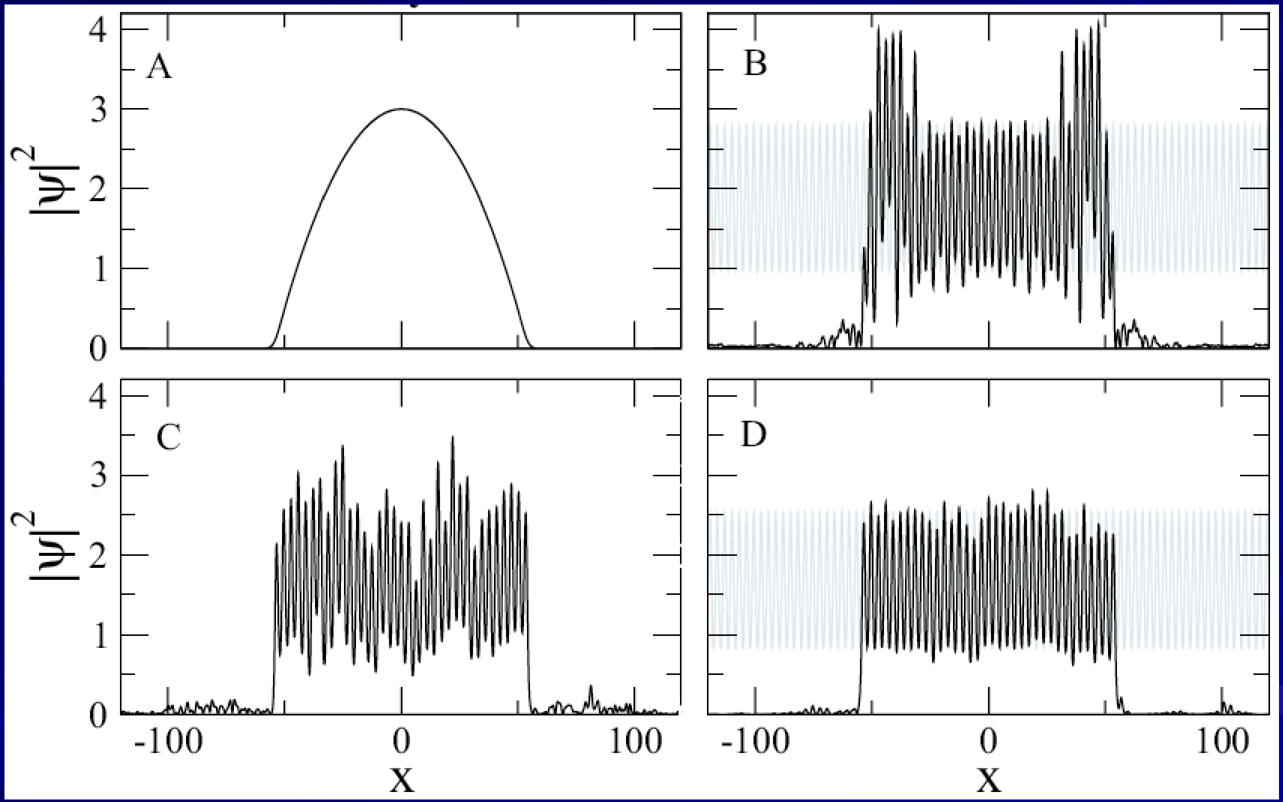




Darmanyan et al, 1999

# Nonadiabatic generation

- Nonadiabatic loading into a 1D optical lattice produces broad states

t=0




t=25 ms

$$V_0 = 4E_R; \quad N \sim 10^3$$

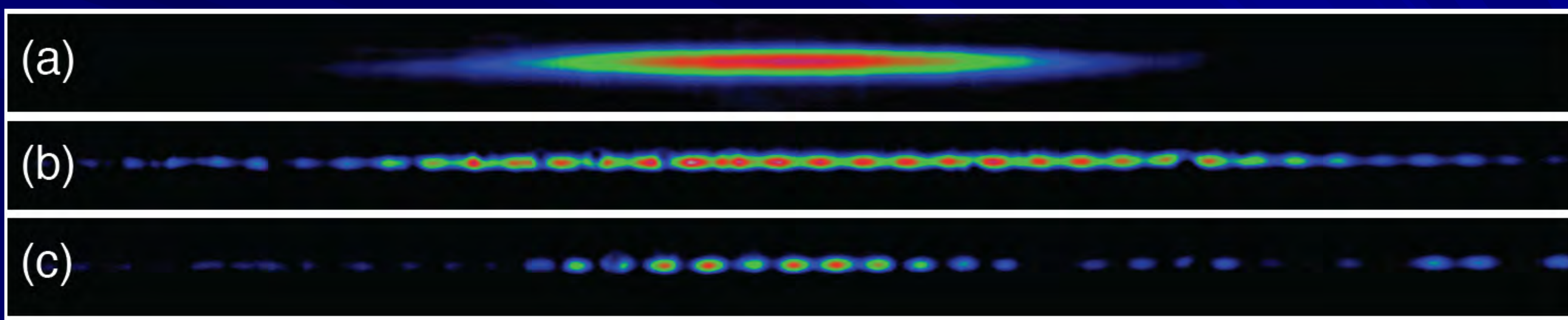
$$\mu < V_0$$

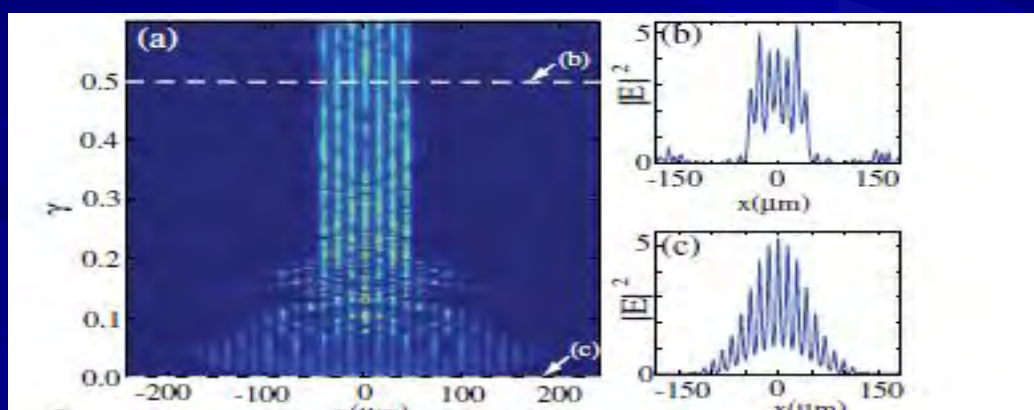
# Experimental observation in optics

PRL 106, 093901 (2011) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2011

**Observation of Nonlinear Self-Trapping of Broad Beams in Defocusing Waveguide Arrays**

Francis H. Bennet,[1] Tristram J. Alexander,[1,2] Franz Haslinger,[1] Arnan Mitchell,[3]
Dragomir N. Neshev,[1] and Yuri S. Kivshar[1]



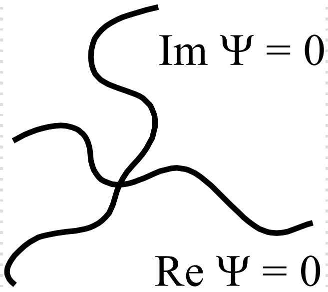



# Vortices and azimuthons

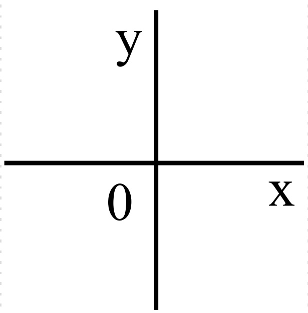
# Optical vortex
# Quantization in complex fields

Wave-function: $\Psi(\mathbf{r}, t) = \mathrm{Re}\,\Psi + i\,\mathrm{Im}\,\Psi$

Field zero's: $\mathrm{Re}\,\Psi = \mathrm{Im}\,\Psi = 0$




$\Psi = \mathrm{x} + i\,\mathrm{y} \;= \rho\; e^{i\varphi}$

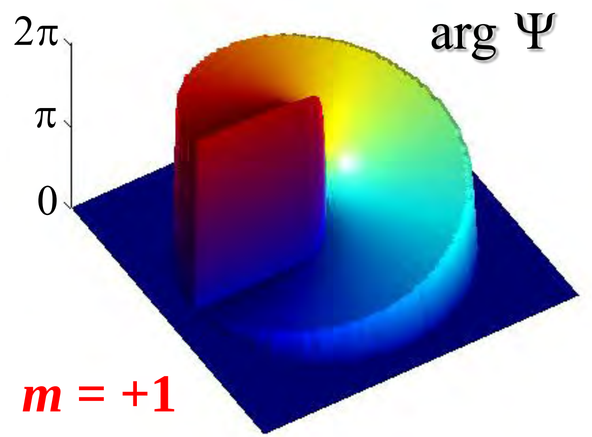



$\rho = \sqrt{x^2 + y^2}$

$\varphi = \tan^{-1}\dfrac{y}{x}$

$x = \rho\cos\varphi$

$y = \rho\sin\phi$

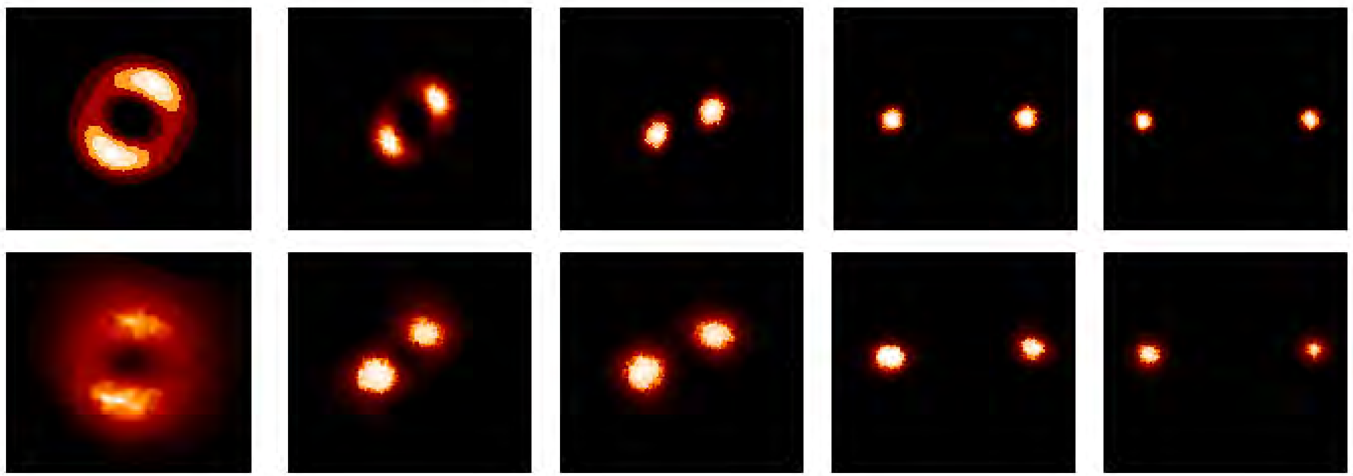



$\arg\Psi(\varphi + 2\pi) = \arg\Psi(\varphi) + 2\pi\, m$

$\Psi \sim (\mathrm{x} + i\,\mathrm{y})^{m}$

*m* – topological index, topological charge, winding number, etc…

# Self-focusing media: vortex break-up



Upper row: Numerical simulations
Lower row: Experimental images

**Tikhonenko et al (1996)**

PRL 95, 203904 (2005) PHYSICAL REVIEW LETTERS week ending 11 NOVEMBER 2005

# Azimuthons: Spatially Modulated Vortex Solitons

Anton S. Desyatnikov, Andrey A. Sukhorukov, and Yuri S. Kivshar

*Nonlinear Physics Centre, Research School of Physical Sciences and Engineering, Australian National University, Canberra, ACT 0200, Australia*

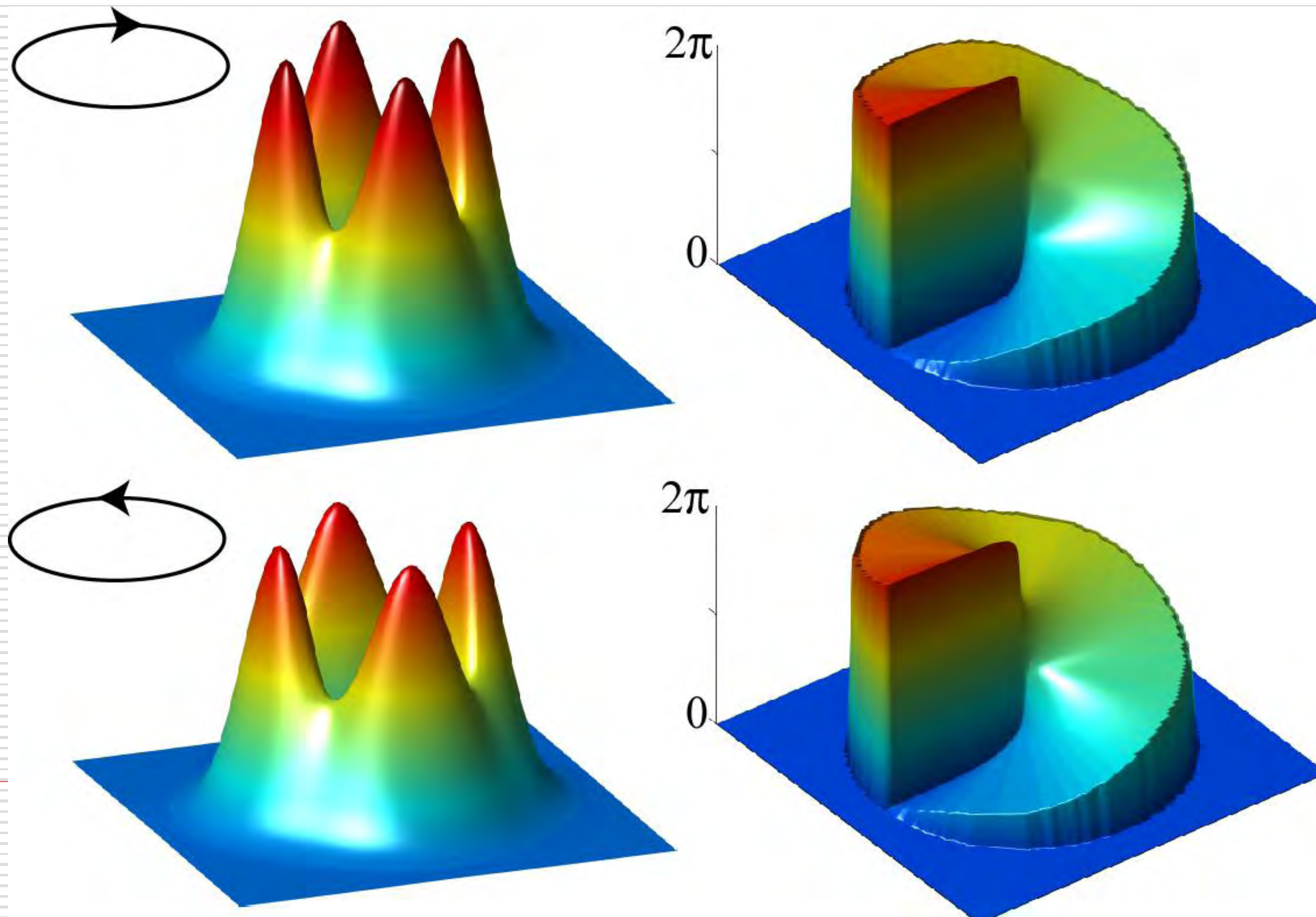
$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} - i\omega\frac{\partial V}{\partial \theta} - kV + F(|V|^2)V = 0$$

$S = +0.91$

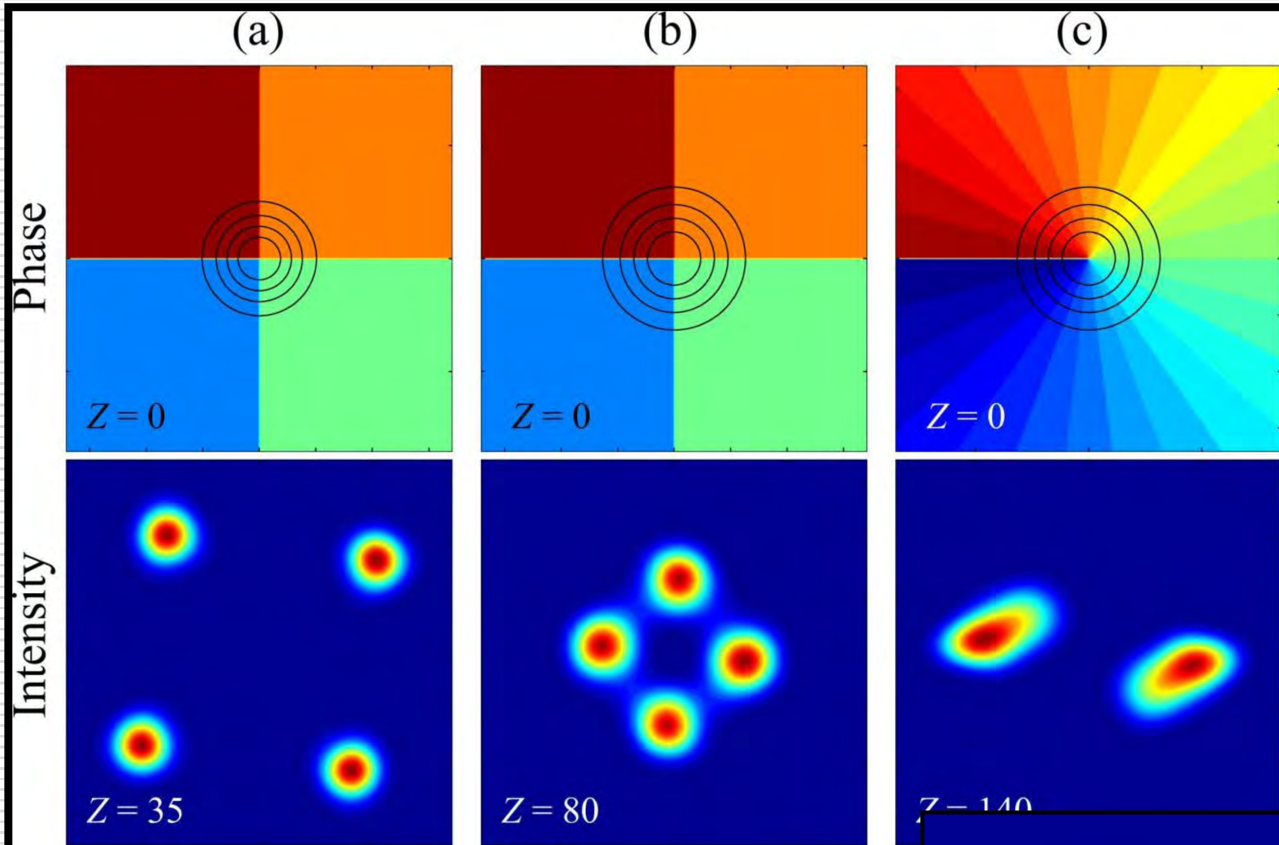
$\omega = -0.087$

$S = +1.09$

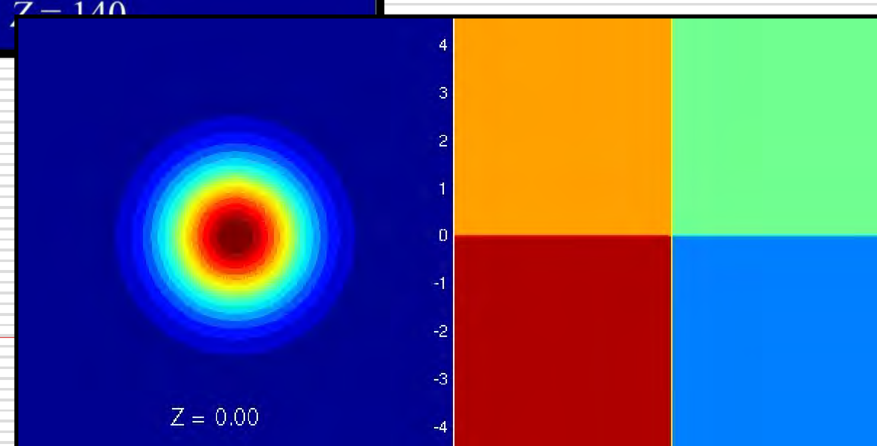
$\omega = +0.279$



# Generation of azimuthons

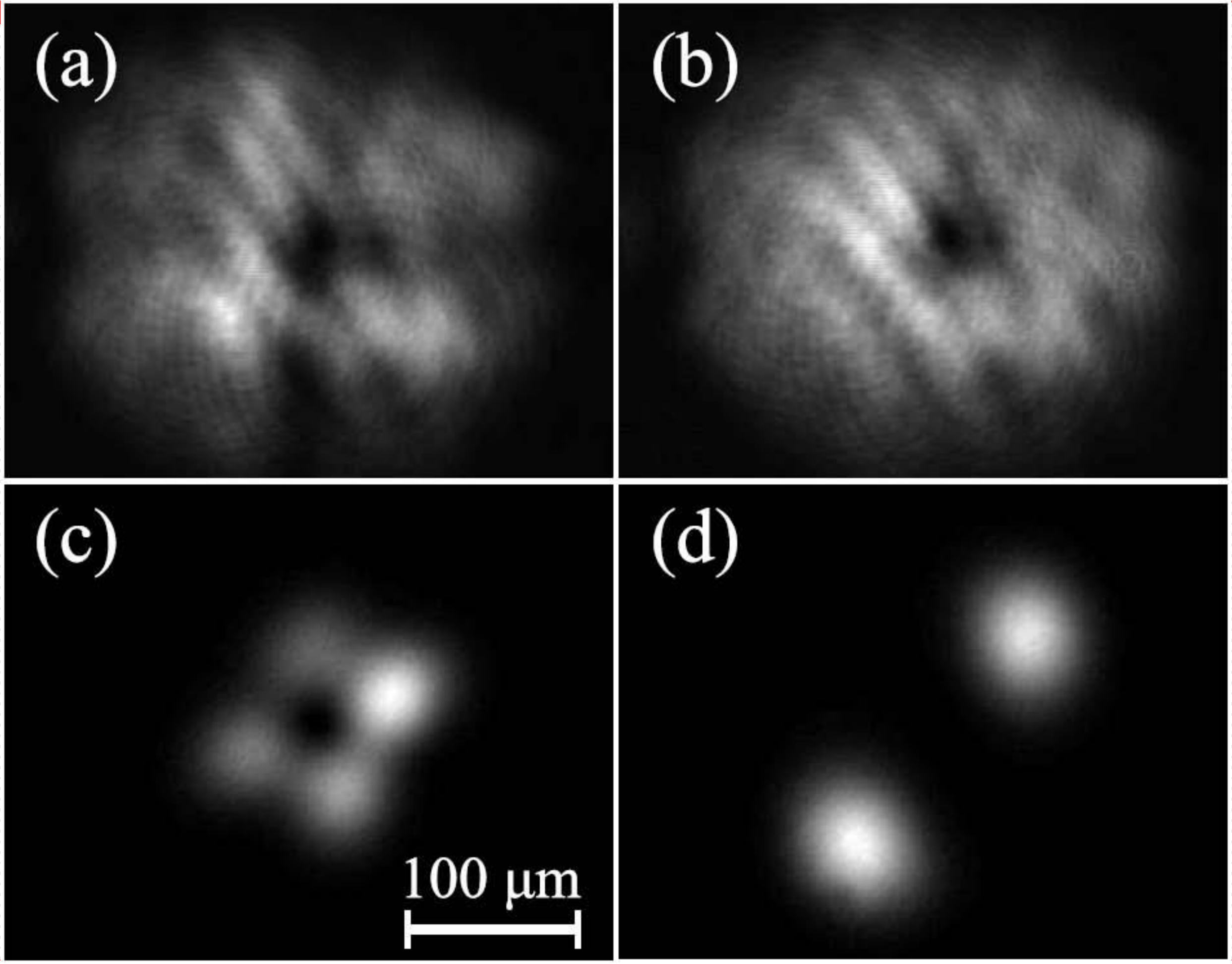



different input powers in (a) and (b)



# Recent experiments

**Step-like phase** **Vortex phase**




**Input**

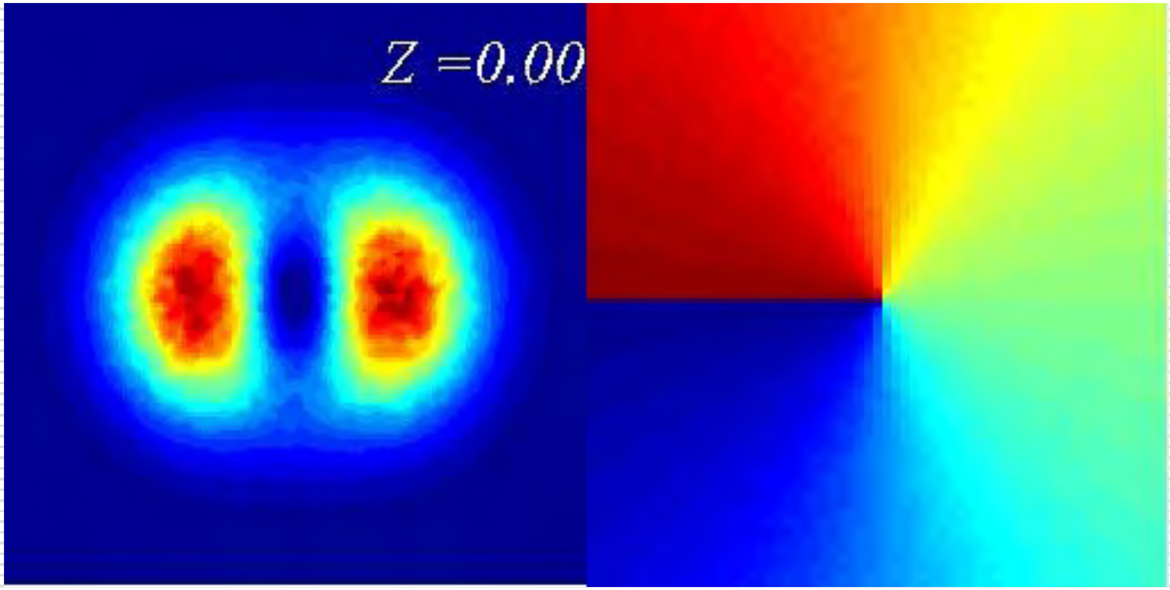
**Output**

Rb cell T= 166 C, detuning 0.66 GHz, power 830 μW

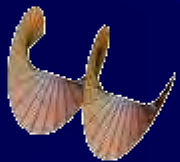
# Stable azimuthons in nonlocal media

A. I. Yakimenko, Yu. A. Zaliznyak, and Yu. S. Kivshar, Phys. Rev. E. **71**, 065603 (2005).
D. Briedis, D. E. Petersen, D. Edmundson, W. Krolikowski, O. Bang, Opt. Express **13**, 435 (2005).

$$i\partial_z E + \nabla^2 E + n\left(I,\vec{r}\right)E = 0 \quad n(I,\vec{r}) = \int R\left(|\vec{r}-\vec{\rho}|\right) I\left(\vec{\rho}\right) d\vec{\rho} \quad R(r) = \sigma^{-2}\pi^{-1}\exp\left(-r^2/\sigma^2\right)$$

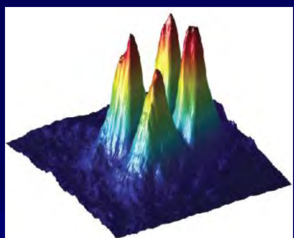


# Singular photonics in Canberra

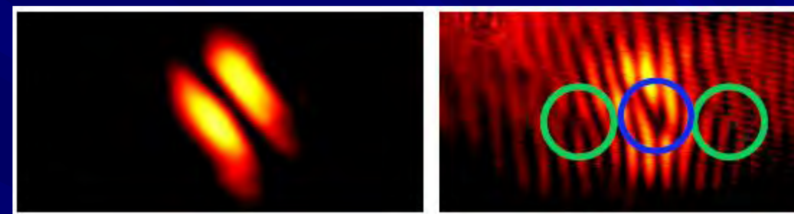


twisted light, vortices

- Experiments in nematic liquid crystals

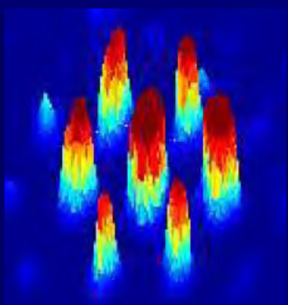


Opt Lett 2011

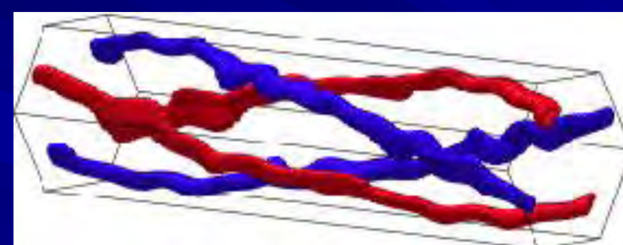


Opt Exp 2011

- Theory and experiments in photonic lattices:



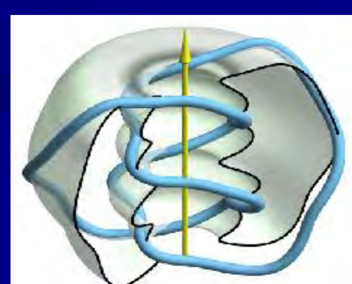
Phys Rev Lett 2010



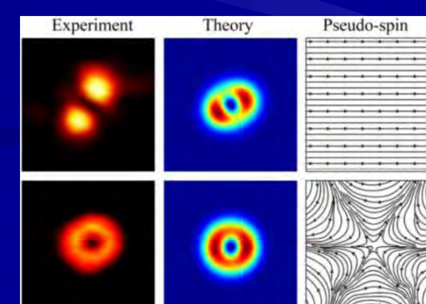
Phys Rev A 2011
Opt Lett 2011

- **NEW** "Topological Optics": vortex knots, knotted fields, and spin textures

Phys Rev Lett 2012



Opt Lett 2012

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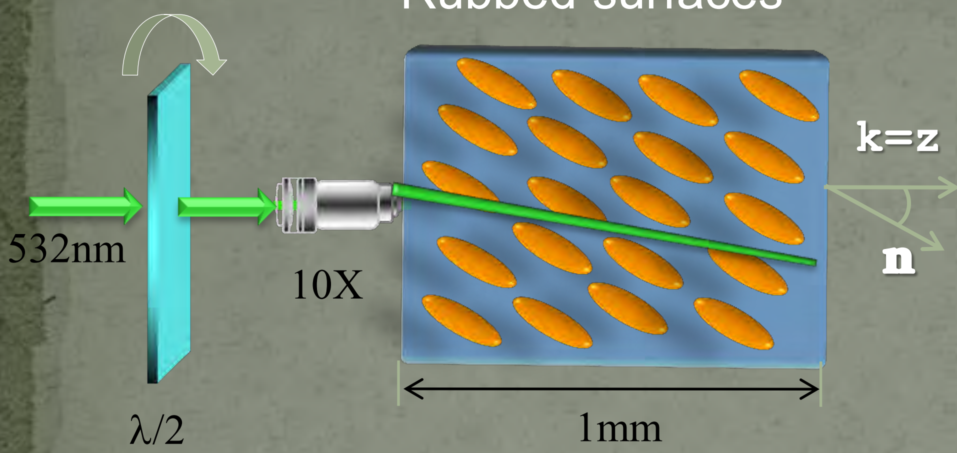
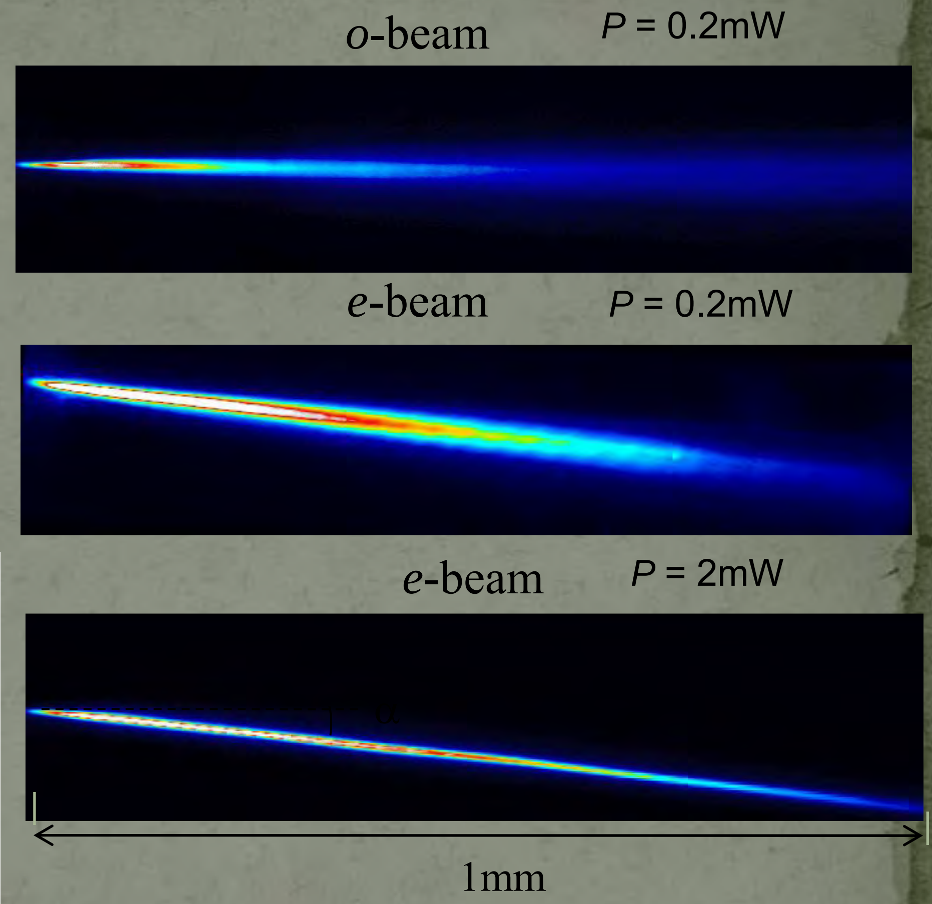
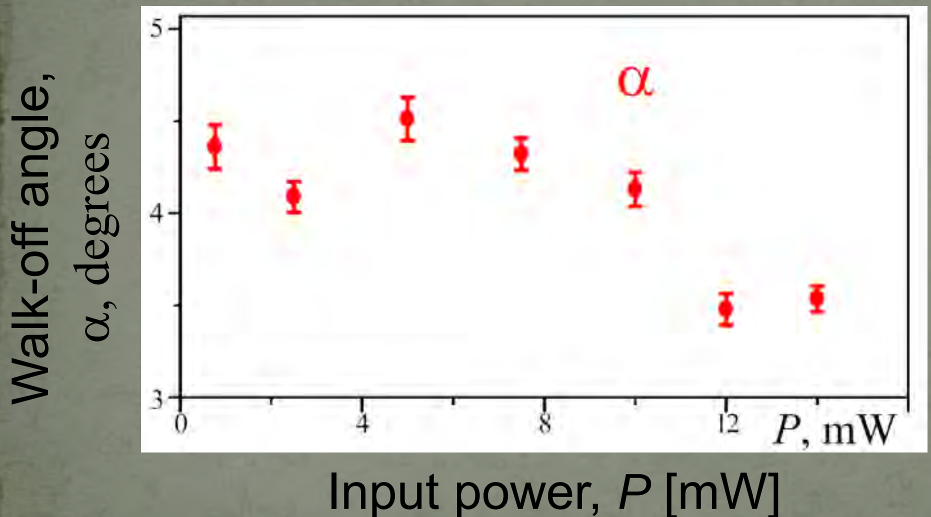
www.rsphysse.anu.edu.au/nonlinear

# Solitons in bias-free nematic liquid crystals



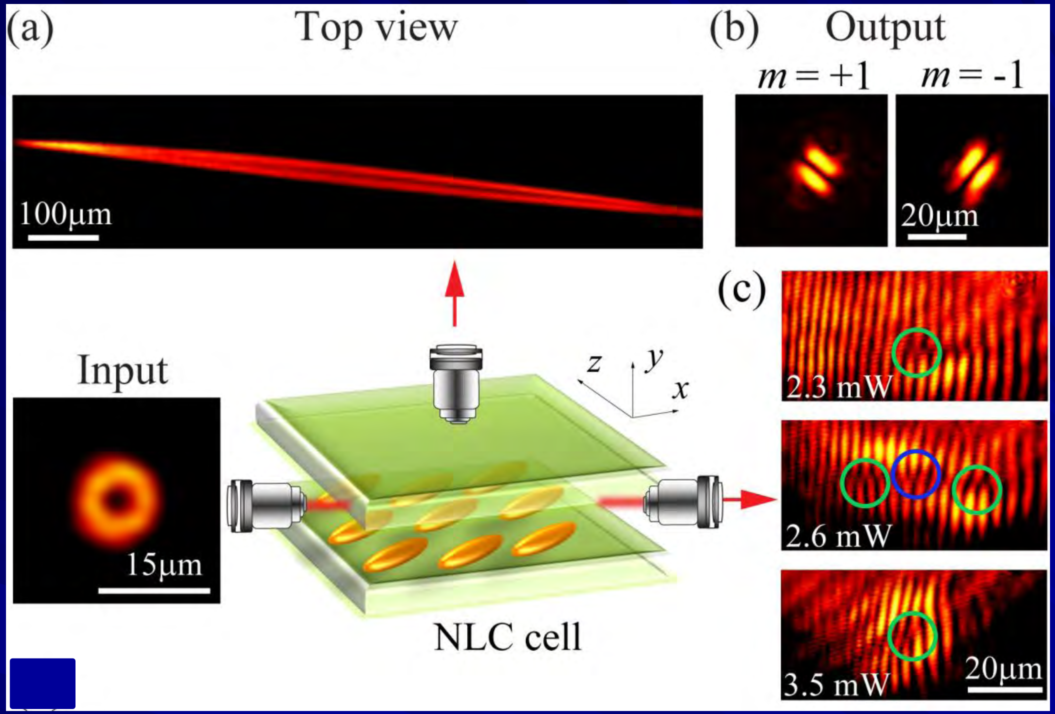



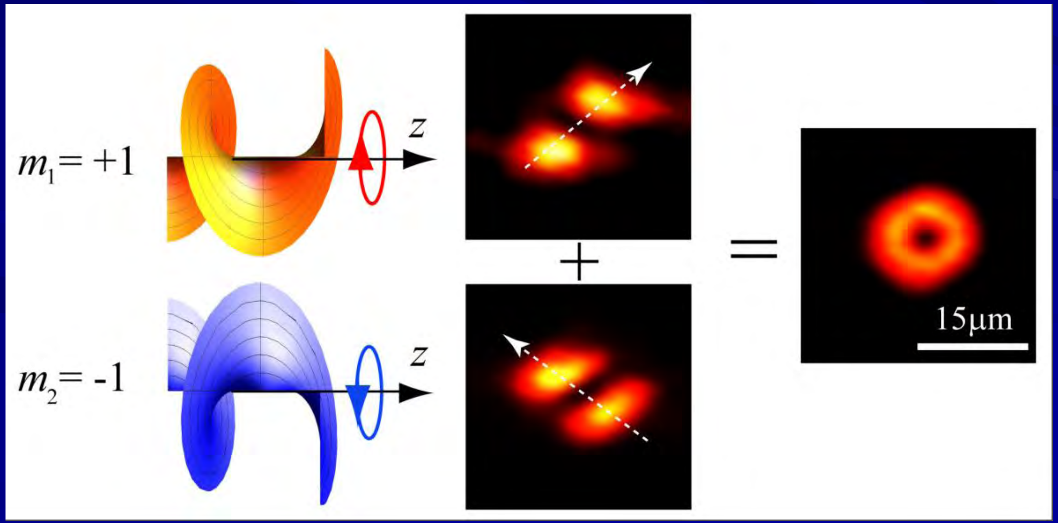

Ya. Izdebskaya, V. Shvedov, A. Desyatnikov, M. Belic, G. Assanto, and Yu. Kivshar, Opt. Express **18**, 3258 (2010).

# Optical vortices in nematic liquid crystals

Ya. Izdebskaya, A. Desyatnikov, G. Assanto and Y. S. Kivshar, Optics Express 19, 21457 (2011).




Y. Izdebskaya, J. Rebling, A. Desyatnikov, Y. S. Kivshar, Optics Letters 37, 767 (2012).

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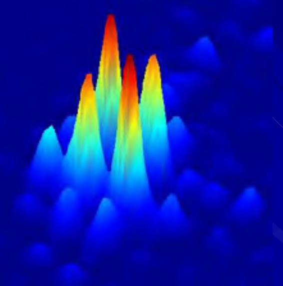
# Conclusions

- Optical systems offer a simple ground for the study of many fundamental effects in physics of nonlinear waves

- Many novel types of localized waves discovered: compactons, vortices, azimuthons, etc

- Generalized concepts: nonlinearity management, self-localized states



- Optical systems allows to observe and study many different types of nonlinear phenomena