Quantum Gravity & the Very Early Universe

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Will summarize the work of many researchers; especially:

Agullo, Barrau, Bojowald, Cailleatau, Campiglia, Corichi, Grain, Kaminski, Lewandowski,

Mielczarek, Mena, **Nelson**, Olmedo, Pawlowski, Singh, Sloan, Velhinho ...

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Setting the Stage

- The Very Early Universe provides a natural arena to test quantum gravity: Guidance for formulating the theoretical framework *and* confronting theories with observations.

- Standard tools used currently: FLRW solutions to Einstein's equations and quantum field theory of linear perturbations on it, ignoring the back reaction. Checked by self-consistency in the very early universe.

- Challenge to quantum gravity theories: Extend this theory the Planck regime where general relativity breaks down. Do theoretically self consistent extensions exist? Do they pass the current observational tests? Are there new predictions for future observations?
Goal: Probe these issues using Loop Quantum Gravity.

- For concreteness, I will focus on the *inflationary scenario* although the framework is general.

For summary, see: AA, Agullo & Nelson *PRL* 109, 251301 (2012);
Viewpoint article, *Physics: Spotlighting Exceptional Research*, 5, 142 (2012.)

Inflationary Paradigm: Incompleteness

Particle Physics Issues:

- Where from the inflaton? A single inflaton or multi-inflatons? Interactions between inflatons? How are particles/fields of the standard model created during 'reheating' at the end of inflation? ...

Quantum Gravity Issues: (Brandenberger, Martin, ...)

- Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped since the 1970's? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?
- Does the slow-roll inflation used to explain the WMAP/PLANCK data arise from natural initial conditions 'at the Beginning' that replaces the big bang in quantum gravity?
- In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?
- Can one arrive at the Bunch-Davies vacuum (at the onset of the WMAP slow roll) from more fundamental considerations?

2. Singularity Resolution

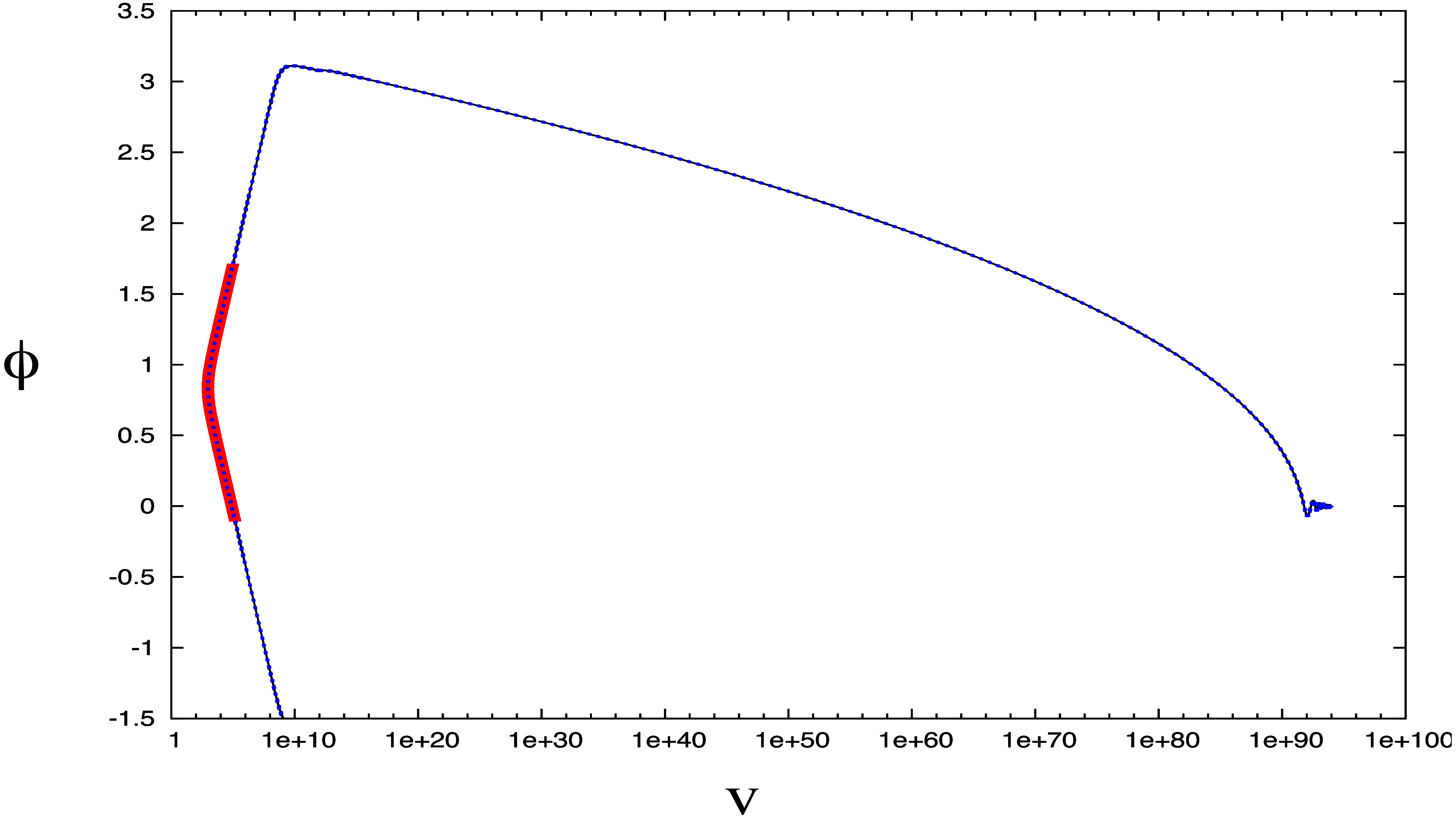
The First Quantum Gravity Issue:

- Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped? Nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

> "One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense." A. Einstein, 1945

- In Loop Quantum Gravity, singularities have been resolved in a large number of cosmological models including the (Flat & Closed) FLRW models, Zero and non-zero Λ, Anisotropic Bianchi models, & Gowdy models that have simplest types of inhomogeneities. Mechanism: Quantum Geometry underlying Loop Quantum Gravity. (Bojowald, AA, Lewadowski, Pawlowski, Singh; Wilson-Ewing; D. Brizuela, Martin-Benito, Mena-Marugan).

Singularity Resolution: $(1/2)m^2\phi^2$ Potential



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh). The Big Bang is replaced by a Big Bounce.

What is behind this singularity resolution?

- Recall: In full LQG, we have a mathematically rigorous kinematical framework uniquely selected by the requirement of background independence (Lewandowski, Okolow, Sahlmann, Thiemann; Fleishchhack). This descends to LQC in a well defined manner (AA, Campiglia).

- This kinematics is distinct from the Schrödinger representation used in the WDW theory. In particular, the differential operator of the WDW equation, $\partial^2 \Psi_o(v,\phi)/\partial v^2 = \ell_P^2 \hat{H}_\phi \Psi_o(v,\phi)$ fails to be well-defined on the LQC Hilbert space and is naturally replaced by a difference operator:

$$C^+(v)\,\Psi_o(v+4,\,\phi) + C^o(v)\,\Psi_o(v,\phi) + C^-(v)\Psi_o(v-4,\,\phi) \;=\; \ell_P^2\;\hat{H}_\phi \Psi_o(v,\phi)$$

(The Step size is determined by the area gap of Riemannian quantum geometry underlying LQG)

- Singularity Resolution: Not because one 'jumps over' the singularity. Finiteness of physical observables. The matter density operator $\hat{\rho}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved. (Discussion within the consistent histories framework (Criag and Singh)).

Features of LQC Dynamics

- No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects (discreteness of eigenvalues of geometric operators.)

- Good agreement with the WDW equation at low curvatures but drastic departures in the Planck regime precisely because the WDW theory ignores quantum geometry (the area gap).

- Effective Equations: To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then:

  $$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where} \quad \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

  Big Bang replaced by a quantum bounce. *Effective equations are surprisingly effective even in the Planck regime.* Simplifies the analysis.

- Mechanism: Quantum geometry creates a brand new repulsive force in the Planck regime, neatly encoded in the difference equation. Replaces the big-bang by a quantum bounce. LQG also resolves black hole singularity (Gambini and Pullin (2013)).

3. Beyond Homogeneity: Our Approach

- The mainstream strategy in LQG: We do not have a complete quantum gravity theory. Progress has occurred by first truncating the classical theory to the physical problem under consideration and then passing to quantum theory using LQG techniques. Success in the study of quantum horizons, obtaining the graviton propagator, and simple cosmological models.

- For inflation, the sector of physical interest: FLRW background with an inflation ϕ in a suitable potential as matter, together with first order perturbations.

- Our Approach: Use the truncation provided by this cosmological sector. (In numerical simulations, $V(\phi) = (1/2)m^2\phi^2$.) The sector includes inhomogeneities, but as perturbations. Thus, quantum fields representing scalar and tensor perturbations now propagate on a quantum FLRW geometry. 'Trans-Planckian' issues are faced squarely (AA, Lewandowski, Kaminski). Caveat: Have to check self-consistency of this truncation! Is the back reaction on quantum geometry negligible even in the Planck regime? If so, there we would have a self consistent extension of the inflationary paradigm to the Planck regime. Thus, two key issues have to be resolved.

Truncated theory

- Truncated Phase Space $\ni\ \{(v, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and their conjugate momenta$\}$
Quantum Theory: Start with $\mathbf{\Psi}(v, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and proceed to the quantum theory using LQG techniques.

- Test field approximation: $\mathbf{\Psi} = \Psi_o(v, \phi) \otimes \psi(\delta h_{ab}, \phi)$, Ψ_o a physical quantum state in the homogeneous sector. Provides QUANTUM background geometry.
- Linearized constraints $\Rightarrow \psi(\delta h_{ab}, \phi) = \psi(T^{(1)}, T^{(2)}, \mathcal{R};\, \phi)$, where $T^{(1)}, T^{(2)}$ are the tensor modes and $\mathcal{R}$ the scalar mode. In the Planck regime of interest, ϕ serves as the ‘internal/relational time’. ψ propagates on the quantum geometry determined by Ψ_o.

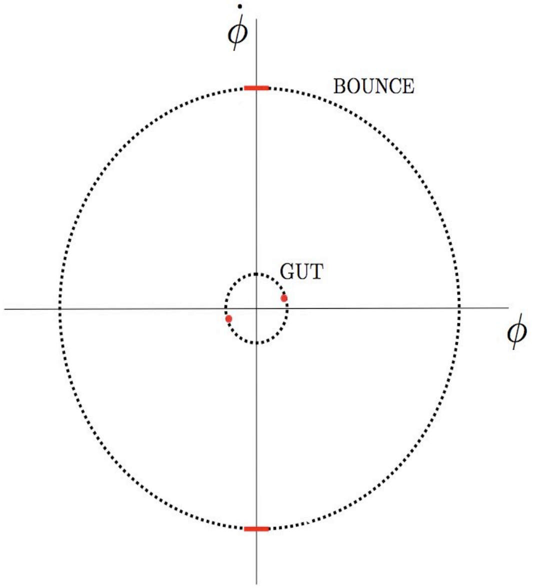
- Idea: Choose $\Psi_o(v, \phi)$ to be sharply peaked at an effective LQC solution g^o_{ab}. Such ‘coherent states’ exist. **First question:** Does the required inflationary phase occur generically in such quantum geometries Ψ_o?

Quantum FLRW Background Geometry Ψ_o

- Let us start with generic data at the bounce **in the effective theory** and evolve. Will the solution enter slow roll at energy scale $\rho \approx 7.32 \times 10^{-12} m_{\rm Pl}^4$ determined from the 7 year WMAP data ? Note: 11 orders of magnitude from the bounce to the onset of the desired slow roll!

- Answer: **YES**. In LQC, $|\phi_{\rm B}| \in (0, 7.47 \times 10^5)$. If $\phi_{\rm B} \geq 0.93$, the data evolves to a solution that encounters the slow roll compatible with the 7 year WMAP data sometime in the future. In this sense, 'almost every' initial data at the bounce evolves to a solution that encounters the desired slow roll sometime in the future.

(AA & Sloan, Corichi & Karami)



- Hence, for the background quantum geometry, we can choose a 'coherent' state Ψ_o sharply peaked at an effective trajectory with $\phi_{\rm B} > 0.93$ and evolve using LQC. WMAP slow roll phase ensured! Choice of $\Psi_o \sim \phi_B$; Free parameter in LQC.

4. Extracting Physics

First, thanks to the background quantum geometry, trans-Planckian modes pose no problem, provided the test field approximation holds: $\rho_{\rm Pert} \ll \rho_{\rm BG}$ all the way from the bounce to the onset of slow roll.

- Second, surprisingly, truncated dynamics of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of Ψ_o is mathematically equivalent to that of $\hat{T}^{(1)}, \hat{T}^{(2)}$, $\hat{\mathcal{R}}$ as quantum fields on a smooth space-time with a ‘dressed’ effective, c-number metric $\bar{g}_{ab}$ (whose coefficients depend on $\hbar$):

$$\bar{g}_{ab} dx^a dx^b = \bar{a}^2 (-d\bar{\eta}^2 + d\vec{x}^2)$$

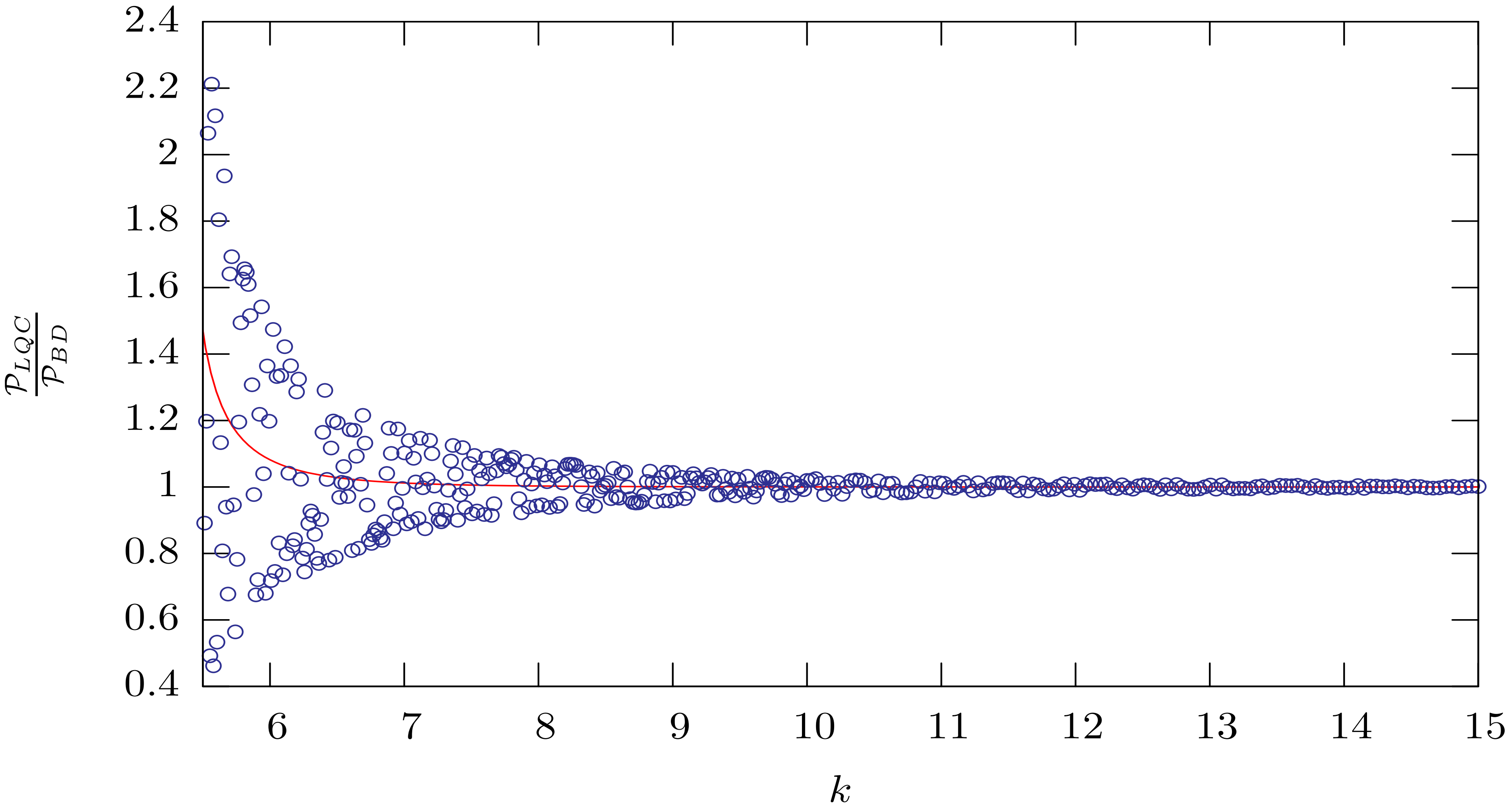
with

$$d\bar{\eta} = \langle \hat{H}_o^{-1/2} \rangle \, [\langle \hat{H}_o^{-1/2} \hat{a}^4 \hat{H}_o^{-1/2} \rangle]^{1/2} \, d\phi; \qquad \bar{a}^4 = (\langle \hat{H}_o^{-1/2} \, \hat{a}^4 \hat{H}_o^{-1/2} \rangle) / \langle \hat{H}_o^{-1} \rangle$$

where H_o is the Hamiltonian governing dynamics of Ψ_o. Analogy with light propagating in a medium.

- Because of this, the mathematical machinery of adiabatic states, regularization and renormalization of the Hamiltonian can be lifted to the QFT on cosmological QSTs under consideration. Result: Full mathematical control on dynamics for computation of the CMB power spectrum, and spectral indices starting from the bounce.

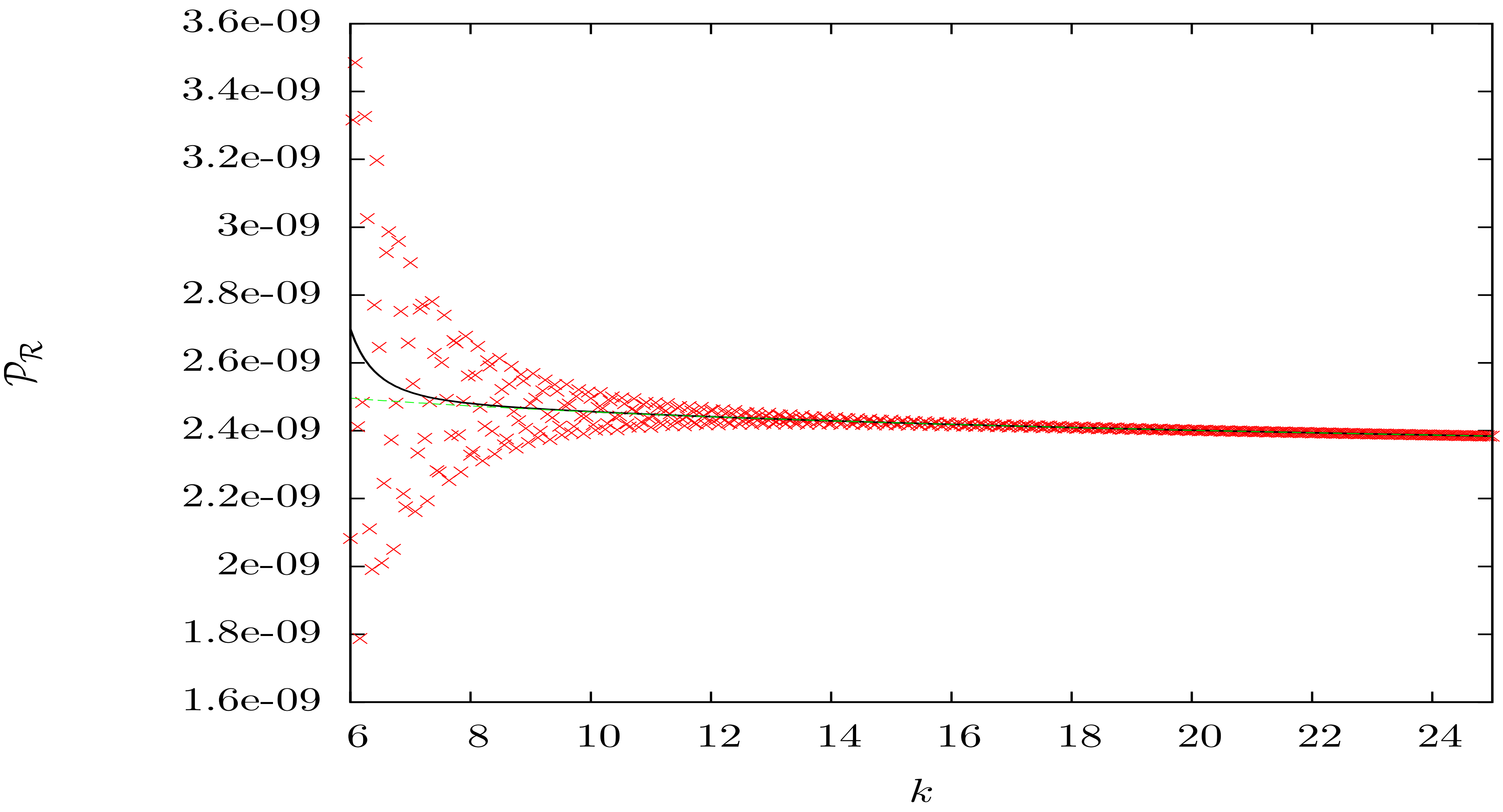
The Scalar Power spectrum: Ratio $(\mathcal{P}_{\rm LQG}/\mathcal{P}_{\rm BD})$



Ratio of the LQC and the standard BD power spectrum for the scalar mode. Blue: Raw data. Red: Binned average. LQC prediction is compatible with observations for $\phi_{\rm B} \geq 1.14 m_{\rm Pl}$.

For $\phi_{\rm B} = 1.2 m_{\rm Pl}$, WMAP $k_{\rm min} = 9 m_{\rm Pl}$. Agreement with standard predictions for $\phi_{\rm B} \geq 1.2 m_{\rm Pl}$. For $\phi_{\rm B} < 1.2 m_{\rm Pl}$: Deviations for future observations.

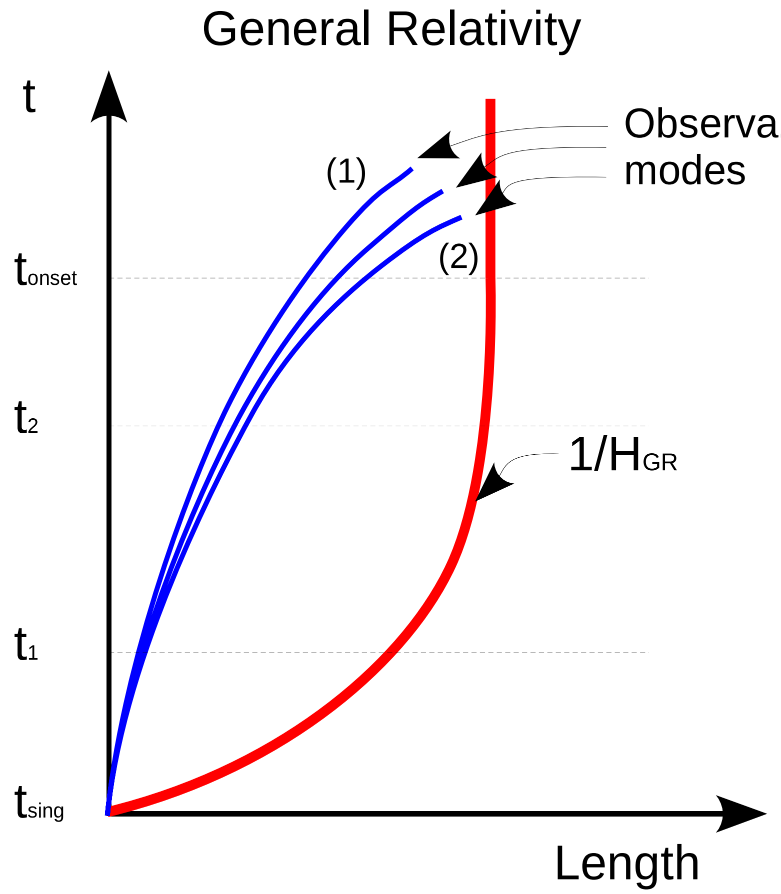
The LQC Tensor Power spectrum

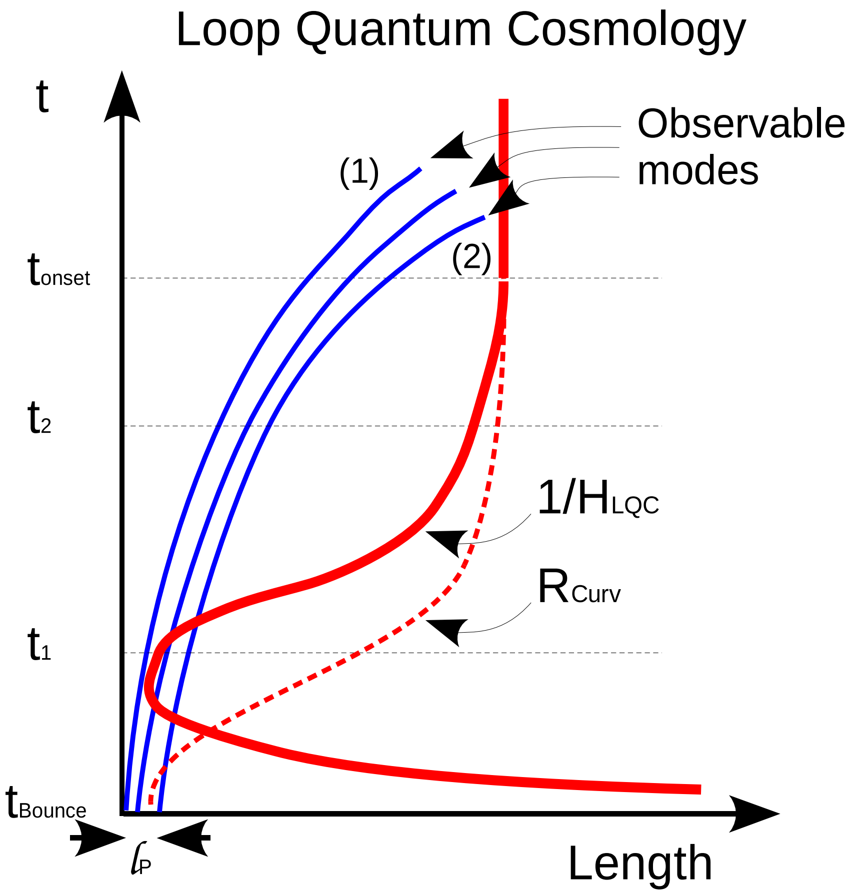


Predicted power spectra for the tensor mode. Black: Binned average. Red: Raw data points.

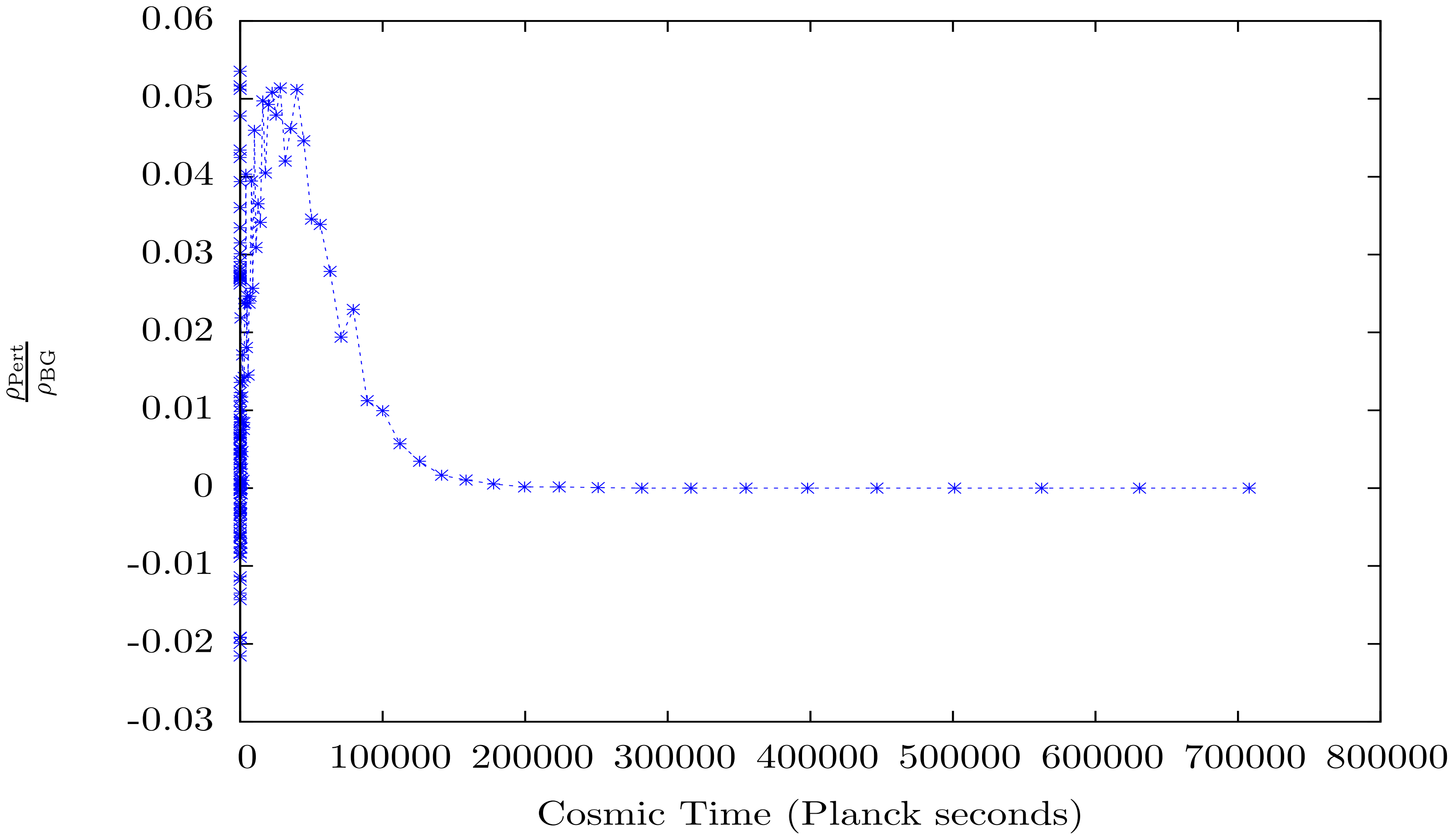
Understanding the Power Spectrum

Only modes with $\lambda > R_{\rm curv}$, the curvature radius, in the pre-inflationary era are excited and populated at the onset of inflation. Can occur in a narrow window for $\phi_{\rm B} \leq 1.2 m_{\rm Pl}$.





Self-consistency of Truncation: $\rho_{\rm Pert}/\rho_{\rm BG}$ vs time.



Renormalized energy density in ψ is negligible compared to that in Ψ_o all the way from the bounce to the onset of slow roll. Here $\phi_{\rm B} = 1.15 m_{\rm Pl}$.

5. Summary: Framework

- The early universe provides an ideal setting to test quantum gravity ideas. Can one obtain a quantum gravity extension of cosmological scenarios to the Planck regime? We focused on inflationary paradigm because it has been extremely successful with structure formation.

The standard theory can be extended using LQG:

- *Background geometry:* Singularity Resolution and precise quantum geometry for the Planck regime. $\surd$

- *Perturbations:* Since they propagate on quantum geometry, using QFT on cosmological quantum geometries (AA, Lewandowski, Kaminski), trans-Planckian issues can be handled systematically provided the test field approximation holds. Analyzed in detail using the renormalized stress-energy of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of Ψ_o. Detailed numerics show that the approximation does hold in most of the parameter space. (Agullo, AA, Nelson). $\surd$

Detailed calculations in the inflationary paradigm but framework is much more general.

Summary: Implications for Inflation

- Extension: For most of the parameter (ϕ_B) space, modes of observational interest are essentially in the Bunch Davies vacuum at the onset of the WMAP slow roll $\Rightarrow$ Predictions of the standard inflationary scenario for the power spectra, spectral indices & ratio of tensor to scalar modes are recovered starting from Planck era. (Agullo, AA, Nelson) $\surd$

- New Effects: There is a small window in the parameter space, for which some observable modes have excitations over the Bunch-Davies vacuum at the onset of inflation. These give rise certain deviations from standard inflation for future observations (e.g., $r \neq -8n_t$; 3-point functions that will be observed in SDSS;...) A window to probe the Planck era around the LQC bounce. (Agullo, AA, Nelson, Shandera, Ganc, Komatsu) $\surd$

- Note: LQG does not imply that inflation must have occurred because it does not address particle physics issues. The analysis simply assumes that there is an inflaton with a suitable potential. But it does show concretely that many of the standard criticisms of inflation can be addressed in LQG by facing the Planck regime squarely.

The framework is general; can be applied to other viable scenarios.

Main References for this talk

- For a summary, see:

AA, Agullo & Nelson PRL 109, 251301 (2012);

- More complete references:

AA, Agullo & Nelson, PRD 87, 043507 (2013); CQG 30, 085014 (2013)

AA & Sloan, GRG (2011), PLB (2009); Corichi & Karami, PRD

AA, Corichi & Singh, PRD (2008); AA, Pawlowski, Singh, PRL & PRD (2006).

Other Results Referred to in the Talk:

- Future Observations:

Agullo & Parker PRD & GRG (2011); Agullo & Shandera JCAP (2012); Ganc & Koamtzu PRD (2012).

- A recent detailed Review of Loop Quantum Cosmology

AA & Singh, CQG (2011).

Supplementary Material

The two slides that follow represent supplementary material, which was not included in the main talk because of the time limitation. They address general questions.

Merits and Limitations of QC

One's first reaction to Quantum Cosmology is often: Symmetry reduction gives only toy models! Full theory much richer and much more complicated.

But examples can be powerful.

- Full QED versus Dirac's hydrogen atom.
- Singularity Theorems versus first discoveries in simple models.
- BKL behavior: homogeneous Bianchi models.

Do *not* imply that behavior found in examples is necessarily generic. Rather, they can reveal important aspects of the full theory and should not be dismissed a priori.

One can work one's way up by considering more and more complicated cases. (e.g. recent work of the Madrid group on Gowdy models which have infinite degrees of freedom). At each step, models provide important physical checks well beyond formal mathematics. Can have strong lessons for the full theory.

Initial conditions on ψ

- Ψ_o: special because assumed to be a ‘coherent state’ at the bounce. But peaked at a generic effective trajectory.

- ψ: Hilbert space $\mathcal{H}$ of perturbations on the quantum geometry Ψ_o is spanned by 4th adiabatic order states on the smooth Friedmann metric $\bar{g}_{ab}$. Excellent control.

- Initial conditions: Since Ψ_o (and hence $\bar{g}_{ab}$) is homogeneous and isotropic, demand that: (i) $\psi \in \mathcal{H}$ also invariant under translations and rotations; (ii) Back reaction of the perturbation ψ on the background Ψ_o can be ignored at the bounce; and (iii) ‘Fundamental uncertainties minimized at the bounce’.

- Intuitive Physical Meaning: Demanding initial quantum homogeneity and isotropy. Heuristic justification: Because of inflation, the observable universe has size of $\leq 10\ell_{\rm Pl}$ at the bounce. The repulsive force of quantum geometry dilutes all inhomogeneities at this scale. So universe is as homogeneous and isotropic as the uncertainty principle allows it to be! Uniqueness issue still being explored. So far the emphasis is on existence of initial conditions that provide a satisfactory extension of the inflationary scenario.