The continuum limit of spin foams and spin nets

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[BD, Martin-Benito, Schnetter 1306.2987]
[BD, Kaminski, to appear]
[BD, Martin-Benito, Steinhaus, to appear]

Quantum Gravity in the Southern Cone, Maresias Sep 2013

Overview

A. Refinement limit of spin foams: Can this work at all?

B. What has been achieved?

C. Some details and results

D. Conclusions

What are spin foams?

Spin foams: path integral approach to quantum gravity.

•path integral defined via regularization = discretization
•not Wick rotated
•no (background) lattice constant
•based on loop quantum gravity structures: generalized lattice gauge theory

•reproduces Regge (discrete gravity) action for single large building blocks:
[Barrett and many others for different models]
[Recently: issues pointed out by Hellmann, Kaminski '12, Han '13: this limit not necessarily semiclassical]
• single building blocks: "sand grains of space time" [as coined by S. Speziale]

Main open question: Refinement limit?

Do we get a smooth beach?

Can this work at all?

Can this work at all?

- Not Wick rotated?
  This will never give anything well defined!

  - Counter example: Ponzano Regge model for 3D gravity

$$Z \sim \delta(\mathrm{curv}) \sim \sum_j \exp(\mathbf{i}\, j\, \mathrm{curv})$$

  - need to take (discrete notion of) diffeomorphism symmetry into account [Freidel, Louapre '04] [BD et al '08-'12]

  - circumvents conformal factor problem

- cannot apply Monte Carlo methods
  - use tensor network renormalization algorithms: nicely adapted to loop quantum gravity concepts (projective measures) [BD '12]

Can this work at all?

•Should you not sum over all triangulations (discretizations)?

•Not if we take discrete notion of diffeomorphism symmetry into account! [Bahr, BD, Steinhaus, ... 08-12]

•conjecture: diffeomorphism symmetry = triangulation invariance;

Although diffeomorphism symmetry is broken in the discrete [Bahr, BD 09]
we can hope to regain diffeomorphism symmetry by coarse graining:
perfect action or perfect discretizations defined by refinement limit.
[Bahr, BD, Steinhaus, ... 08-12: confirmed in examples]

Refinement limit also addresses the issue of diffeomorphism symmetry and triangulation invarinace. Expected to give strong conditions, that might possibly give a unique theory.
[BD '12]

Can this work at all?

- Loop Quantum Gravity / Spin Foams can never work ...

Coarse graining will provide a (easy to fail) test for spin foams.
If it does not work we will learn why.

What have we achieved so far?

What has been achieved?

•Spin foams are much to complicated theories for coarse graining ...

•strategy: two simplifications that nevertheless keep key dynamical mechanism

1.dimension reduction: 4D to 2D (inspired by lattice gauge theory)
defines spin net models [BD, Eckert, Martin-Benito 11]

2. simplification of algebraic data: replace SU(2)xSU(2) by some finite group
[Bahr, BD, Hellmann, Kaminski 12]
Now: replaced by quantum group, which comes up also in the full models.
[BD, Martin-Benito, Steinhaus to appear]
For analytical work can also go back to SU(2)xSU(2).
[BD, Kaminski to appear]

So here we are (almost) back to full models.

What has been achieved?

•Spin foams are generalized lattice gauge theories:
expect two phases:
confined (giving degenerate geometries) and deconfined (topological BF theory)

Most discussion so far involved just these two phases.
Is there a phase where simplicity constraints are realized and which can be interpreted as 4D geometry?

Numerical and analytical results in quantum group spin net models:

Found unexpected large number of fixed points in enlarged phase space.
[BD, Martin-Benito, Schnetter 13, BD, Kaminski to appear, BD, Martin-Benito, Steinhaus to appear]

Some of these can be interpreted to respect simplicity constraints.

In particular: factorizing Barrett Crane model inspired by [Reisenberger '98]

Fixed points are related to anyon models, describing (scale free) dynamics of intertwiners.
These support massless excitations.
Opens up new perspectives of what to expect in refinement limit!

What has been achieved?

•What are relevant parameters for spin foams? How should we choose truncation?

Based on numerical simulations (for spin nets) the following picture emerged:
[BD, Martin-Benito, Schnetter 13]

Fine tuning (of face weights) is necessary to escape the two dominating phases of lattice gauge theory.

This allows to flow into an enlarged phase space, describing dynamics of intertwiners.

Conjecture:
A suitable truncation is provided by restricting to one degree of freedom per intertwiner channel.
Relevant parameters describe which intertwiner channels are allowed and which not.

These conjectures can be tested systematically with the tensor network algorithm methods.

We have an explicit (renormalization) flow equation based implementing a truncation that keeps the flow inside the given phase space.

Summary

- We can actually do something!

- There is a rich fixed point structure: potential large scale limits of the theory.

- conjecture: intertwiner degrees of freedom are the relevant degrees of freedom

- working on fixed lattice is sufficient:
  fixed points describe fully triangulation invariant models: confirms strategy to define models via coarse graining [Bahr, BD '09, BD '12]

Fixed points are related to anyon models, describing (scale free) dynamics of intertwiners.
These support massless excitations.
Opens up new perspectives of what to expect in refinement limit!

Main future task: lift the results to spin foams.

A very few details.

Spin foams and spin nets: generalized lattice theories

3d and 4d first order actions

$$\text{geometry} \sim \text{metric, extrinsic curvature} \sim (\text{n-bein}\, e,\ \text{connection}\ A)$$

first order action in 3d

$$S_{3d} = \int B \wedge F \quad , \qquad B \sim e$$

(Lie algebra valued) d-2 form

curvature of A

BF theory in any dim

$$S_{BF} = \int B \wedge F \qquad F = 0 \quad , \qquad D_A B = 0$$

topological field theory
UV/zero coupling fixed point of lattice gauge theory

Yang Mills in first order

$$S_{YM} = \int B \wedge F + g^2 B \wedge \star B$$

uses background metric

Plebanski action in 4d

$$S_{4d} = \int B \wedge F \quad + \quad \phi B \wedge B \quad , \qquad B \sim *(e \wedge e)$$

Lagrange multiplier

simplicity constraints

Lattice gauge theory

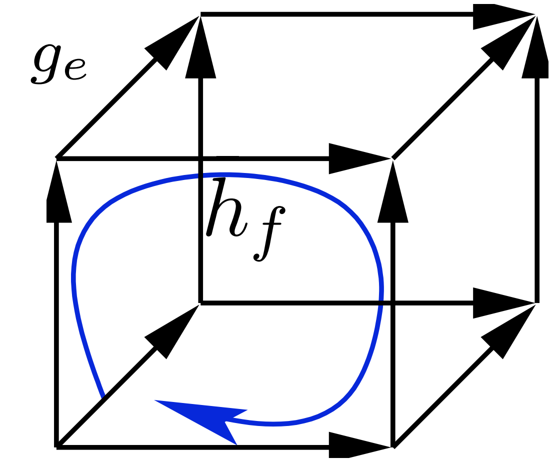
lattice gauge theory

$$Z \sim \sum_{g_e} \prod_f w_f(h_f)$$

group variables at edges

face weight (class function)

face holonomy



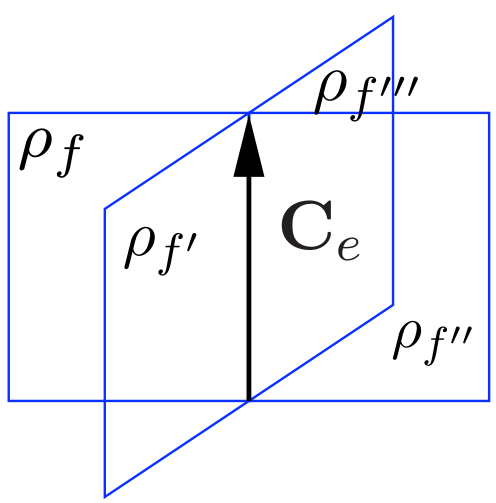
in dual variables
(strong coupling expansion)

$$Z \sim \sum_{\rho_f} \left(\prod_f \tilde{w}_f(\rho_f) \right) \prod_e \mathbf{C}_e(\{\rho_f\}_{f \supset e})$$

representation labels at faces

dual face weight

Haar projector (intertwiner)
on invariant subspace in tensor product of representations meeting at the edge



Lattice gauge theory

Spin foams

partition function:

$$Z \sim \sum_{\rho_f} \left(\prod_f \tilde{w}_f(\rho_f) \right) \prod_e \mathbf{C}_e(\{\rho_f\}_{f \supset e})$$

•parameter space: **face weights**

$$\tilde{w}_f(\rho_f)$$

•phases / fixed points:

- BF /weak coupling / deconfining
- degenerate / strong coupling /confining
- BF on normal subgroups

[Bahr, BD, Hellmann, Kaminski '12]

•parameter space: replace **Haar projector**

$$\mathbf{C}_e(\{\rho_f\}_{f \supset e})$$

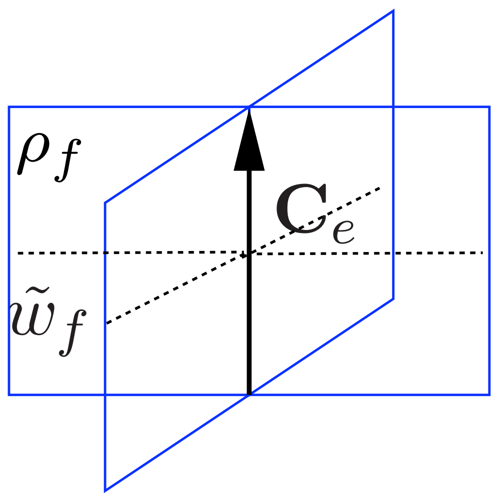
by projector on some smaller subspace

•phases / fixed points:

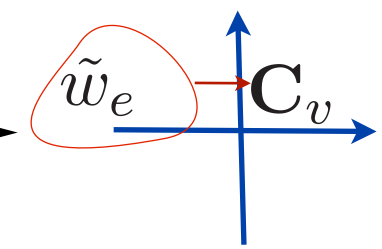
•same phases as for lattice gauge theory

Are there additional phases in spin foams?

Simplification: dimensional reduction



weights



labels/variables

ρ_e

m_e, n_e

$L_2(G) = \oplus_\rho V_\rho \otimes V_\rho^*$

lattice gauge theory
in dual variables

vertex model
in dual variables

gauge symmetry

global symmetry

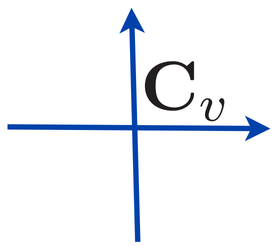
SPIN FOAM MODEL

SPIN NET MODEL

Statistical properties of 4d gauge theories and associated 2d theories are similar.

Examples: Ising model, QCD, ...

Spin net models: intertwiners



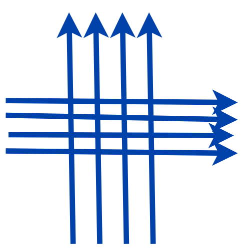
ρ_e

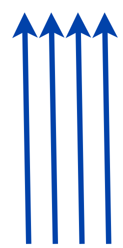
m_e, n_e

$L_2(G) = \oplus_\rho V_\rho \otimes V_{\rho^*}$

under coarse graining:

effective edge

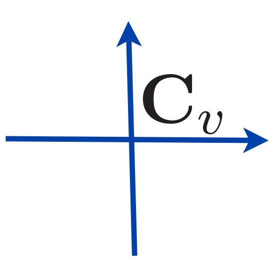


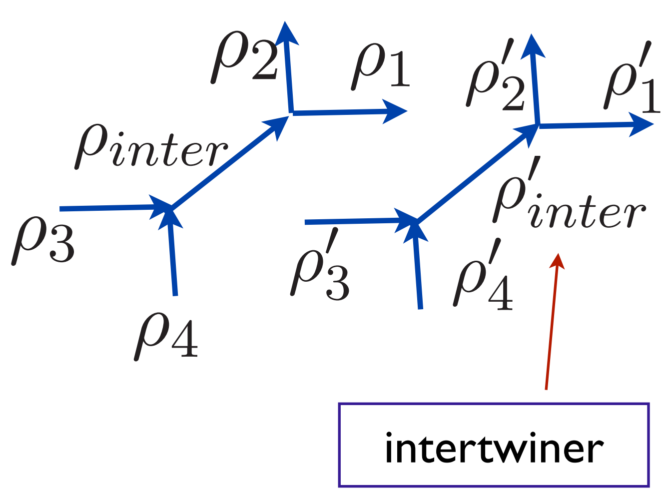


$\rho_e, \rho'_e, \mu_{\rho,\rho'}$

m_e, n_e

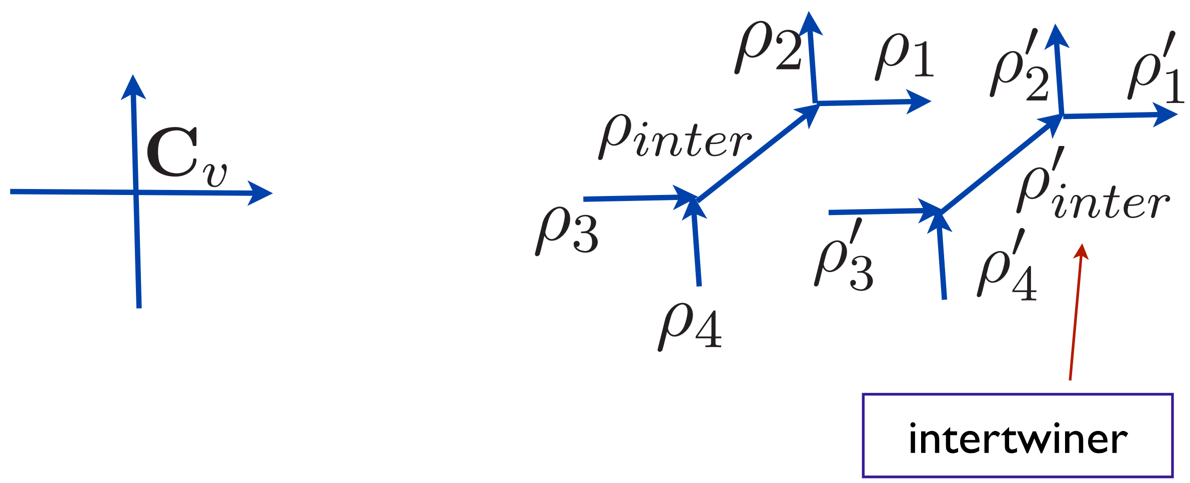
$\bigoplus \mu_{\rho\rho'} V_\rho \otimes V_{\rho'}$





Doubling of representation labels: Under coarse graining we obtain an enlarged space of models (more couplings)

Simplicity constraints and intertwiner dynamics



Conjectur/ Experience:
Excitations of intertwiner channels give the relevant information and determine the phase.

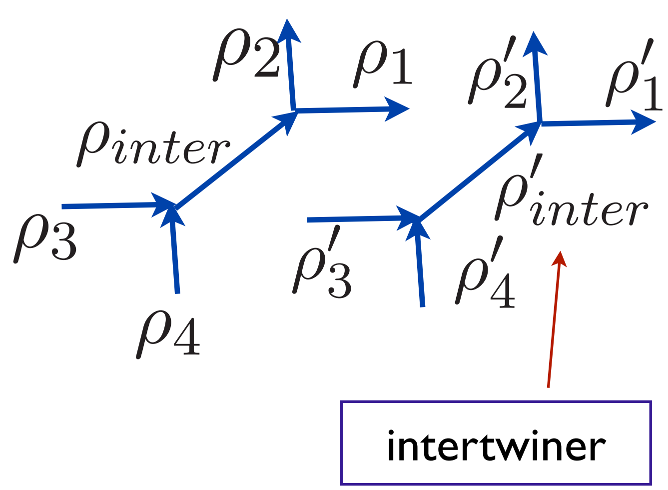
standard (lattice gauge) models	$\rho'_{inter} = \rho^*_{inter}$
simplicity constraints: some rep labels are forbidden	$\rho'_{inter} \neq \rho^*_{inter}$
factorizing models	$\rho_{inter},\ \rho'_{inter}$

only perturbations with equal rep labels are relevant

Fixed points

lattice gauge theory type	BF and strong coupling/ degenerate geometry
factorizing type	A large number of fixed points has been found with quantum group SU(2)_k for general k (numerically and analytically). These generalize to SU(2). Some can be interpreted to respect simplicity constraints, e.g. Barrett-Crane factorizing model.
mixed type	Such fixed points occur at least in S3 and SU(2)_4

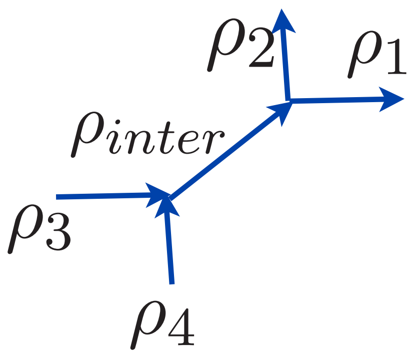
[BD, Kaminski, to appear]



All these fixed points define (2D) topological field theories.

Relation to anyon models

Factorizing models: can consider the `square root'



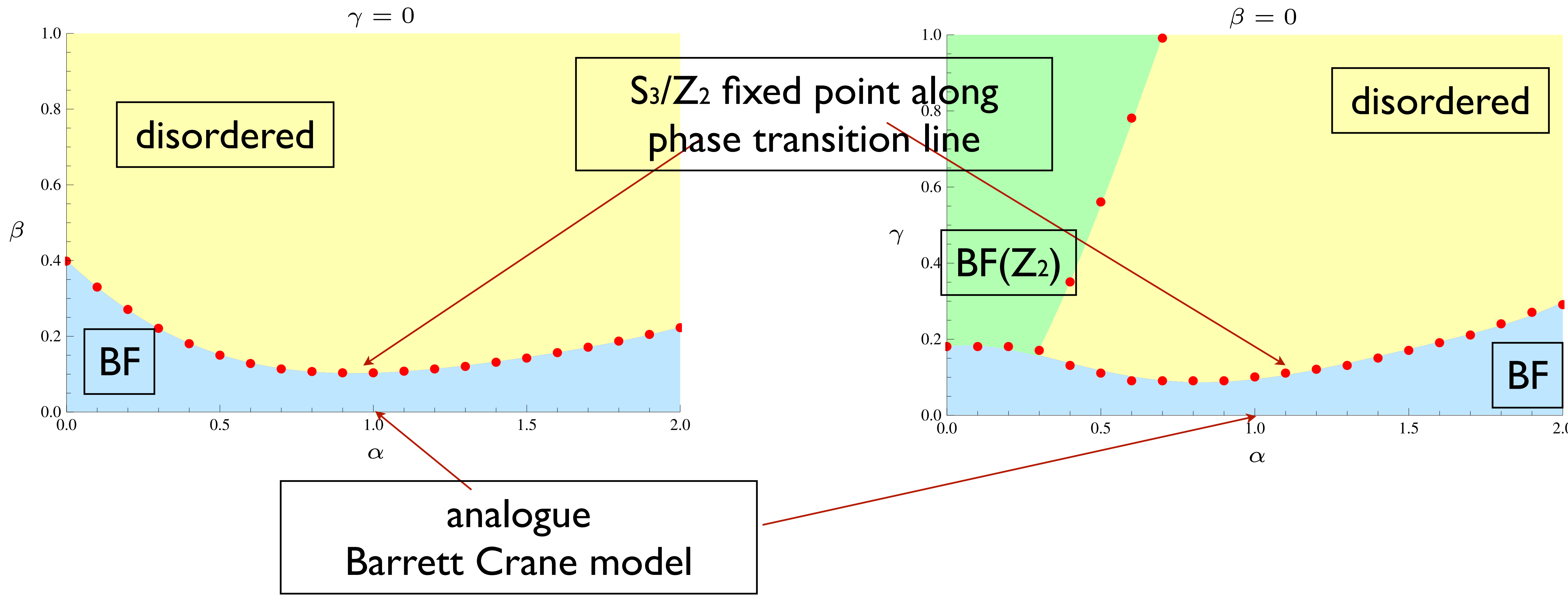
- related to anyon (fusion tree) models:
  particles coupled to Chern Simons theory (no lattice scale!)

- describe merging of these particles: dynamics of intertwiners

- continuum limit (infinite many particles) defines
  critical model (massless excitations) / conformal field theory

Example: S_3 phase diagram

[BD, Martin-Benito, Schnetter '13]



•fine tuning to phase transition line required to avoid fast convergence to BF or 'degenerate geometry' phase

•along phase transition line encountered new fixed point (factorizing model) (for lower accuracy of the coarse graining algorithm)

➡expect more structure along phase transition line

Conclusions

- introduced new class of (scale independent) models: analogue models to spin foams

- tensor network coarse graining can be successfully applied, gives lots of structural information compared to Monte Carlo simulations

- conjecture:
  intertwiner degrees of freedom are the relevant degrees of freedom
  - holds also for spin foam models

- based on this we developed approximation method that describes flow of models and tracks the intertwiner degrees of freedom
  (approximate flow leaves space of models invariant) [BD, Martin-Benito, Steinhaus to appear]

- fine tuning necessary to avoid fast convergence to frozen or disordered phase / escape phases of standard lattice gauge theory models [also: Christensen, Khavkine: BC model]

- interesting phases / fixed points beyond standard lattice gauge theory:
  mass less excitations (expected from background independence / scale freeness)

- although working on fixed lattice the fixed points describe fully triangulation invariant models: confirms strategy to define models via coarse graining [Bahr, BD '09, BD '12]

Outlook

- based on tensor network method developed approximation method that describes flow of models and tracks the intertwiner degrees of freedom ⇨
  - systematic study of quantum group models
  - flow of simplicity constraints: relaxed or strengthened under coarse graining?
  - apply to Barrett Crane and EPRL (analogue) models with q-group [Christensen, Khavkine, Fairbairn, Meusburger, Han]

- relevance for spin foams ⇨
  - same statistical properties for 4D lattice gauge / 2D edge models
  - simplicial 4D models correspond to four-valent vertex models
  - coarse graining in one direction corresponds to 2D coarse graining (anisotropic algorithms coarse grain one direction at a time)

Continuum limit and phase diagram for spin foams in reach!

Additional material

Coarse graining with tensor network methods

[Levin & Nave, Gu & Wen, Vidal ...'00's+]

[BD, Eckert, Martin-Benito, New. J. Phys. '11]

[BD, Martin-Benito, Schnetter '13]

Continuum limit via coarse graining

•gives effective dynamics at different scales / refinement steps

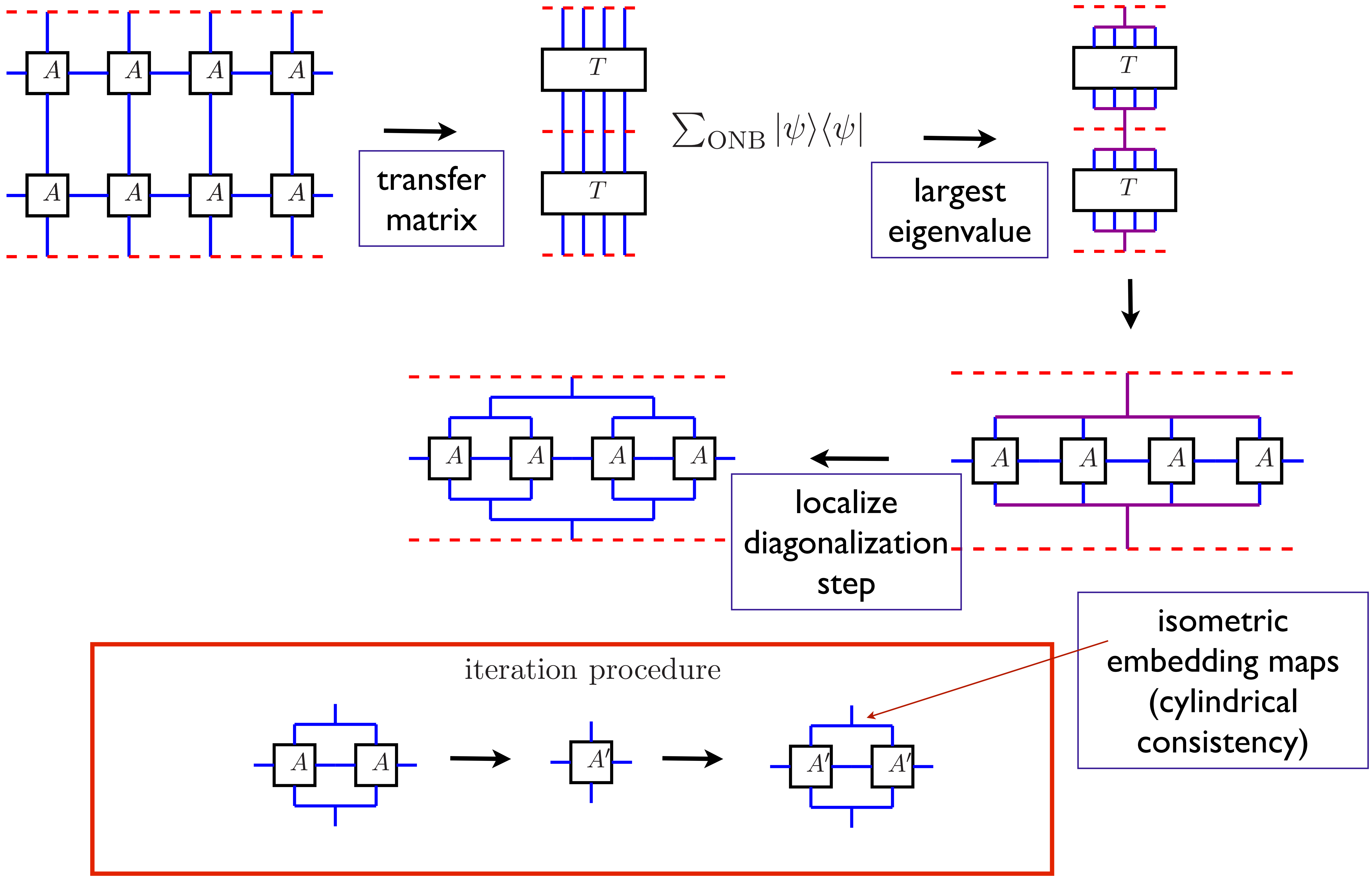
•infinite refinement (fixed point) gives continuum limit

•problem: real space renormalization methods have been very restricted [Migdal-Kadanoff 70's]
- •proliferation of non-local couplings
- •truncations not under control

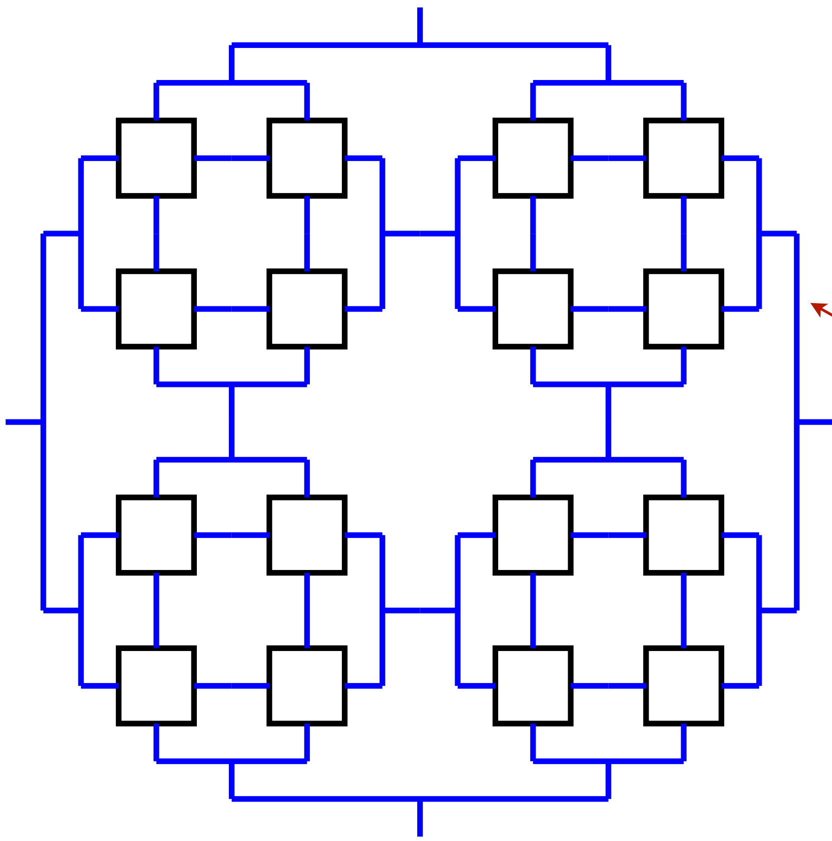
•in the last years new developments in condensed matter/ quantum information
- •density matrix renormalization [White '92,...]
- •matrix product states [Cirac, Verstraete,... 04+]
- •tensor network renormalization [Levin, Nave '06, Gu, Wen '09]
- •entanglement renormalization [Vidal 07+]

•nice correspondence with loop quantum gravity techniques:
dynamical notion of cylindrical consistency defines continuum limit and the structure of physical vacuum [BD '12]

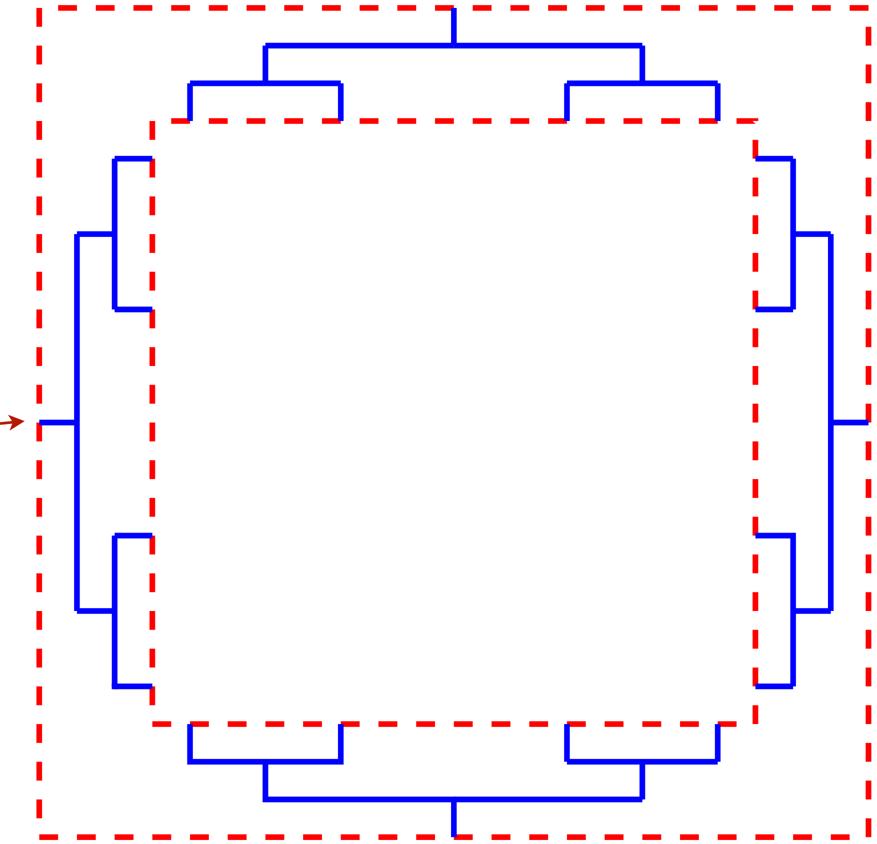
Motivation: transfer operator technique



The procedure for 2D state sum

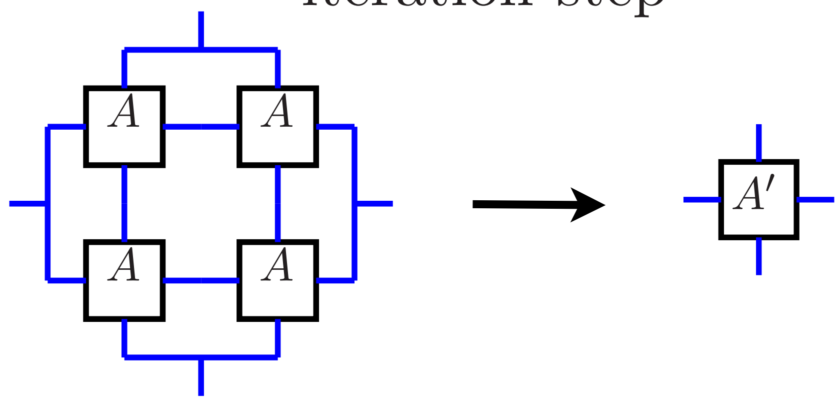


embedding maps: essential for truncation



embedding maps:connect different refinement steps

iteration step

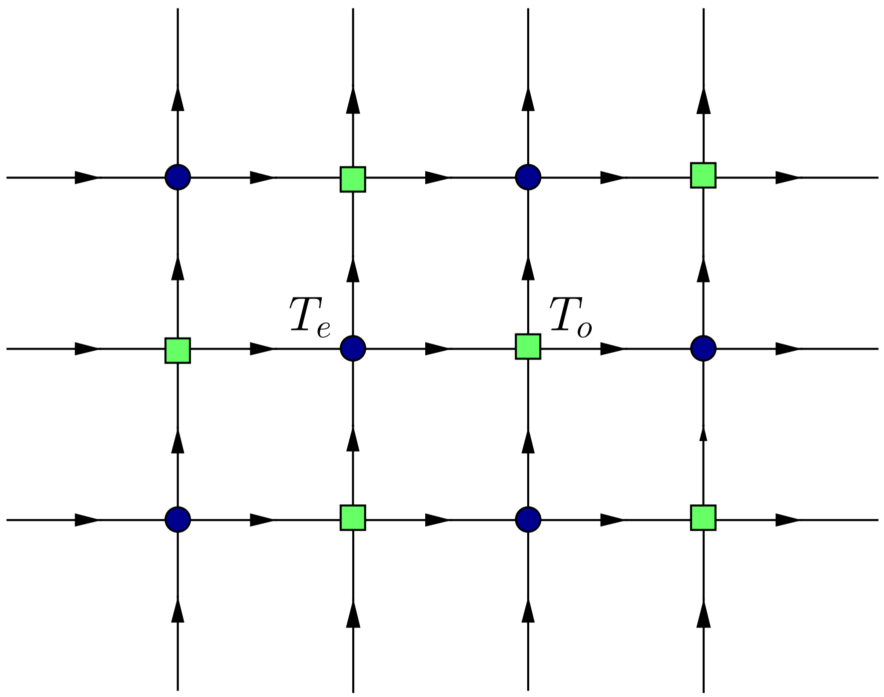


Fixed point defines continuum limit.
Embedding maps at fixed point : encode scale invariance.
(Define triangulation invariant model.)

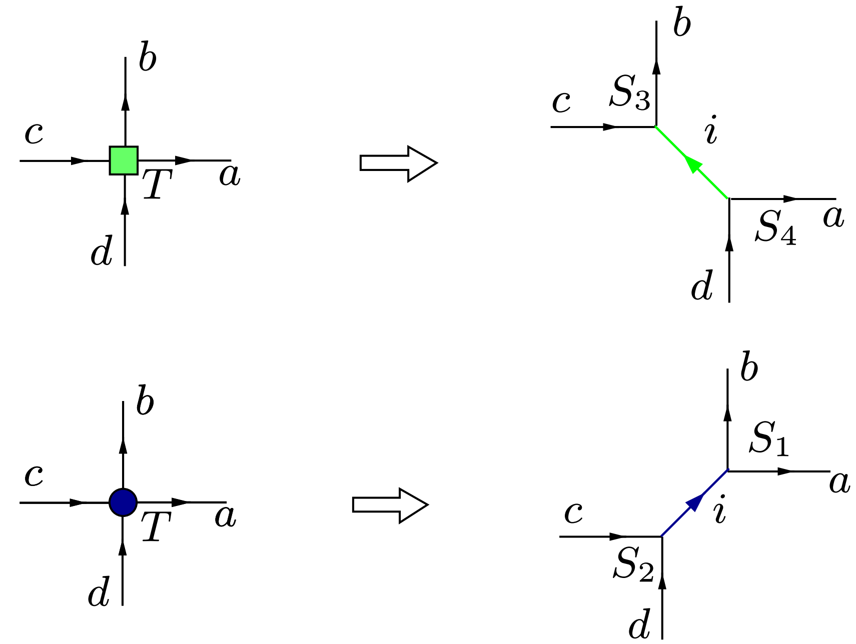
The actual algorithm

[Levin, Nave '07 , Gu, Wen '09]

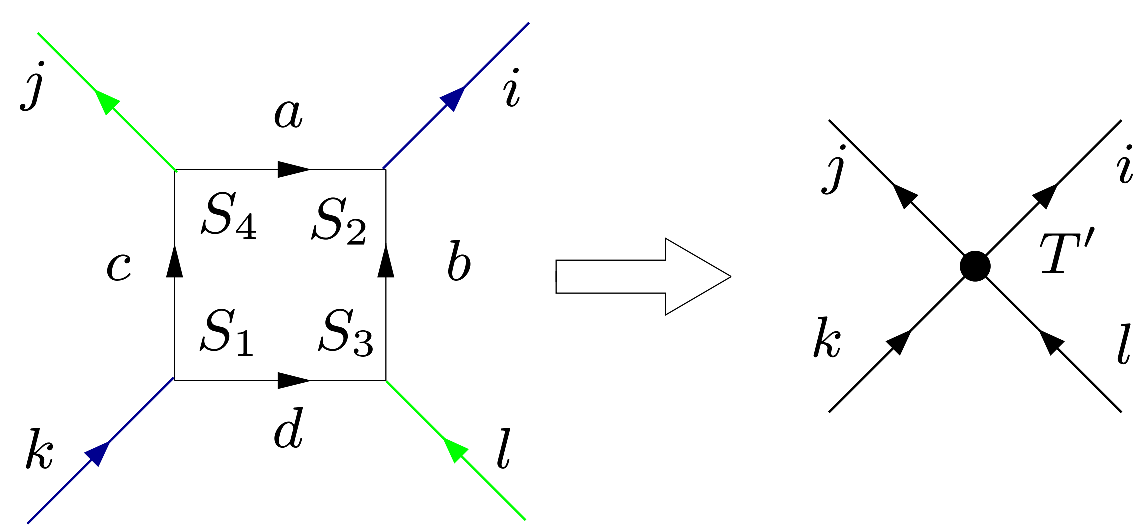
symmetry preserving version:
[BD, Martin-Benito Schnetter 13]



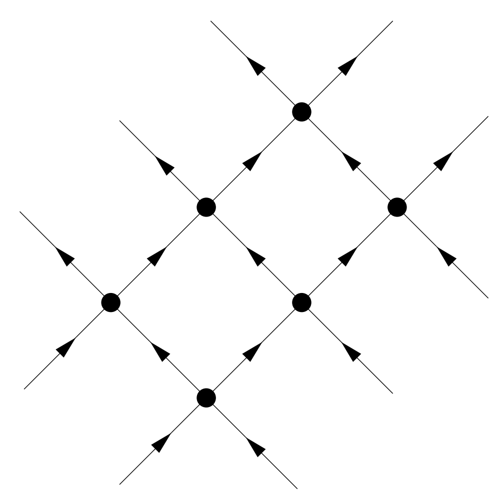
(a) square lattice



(b) splitting of vertices

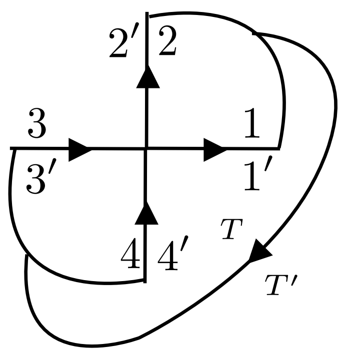


(a) contraction



(b) coarse grained lattice

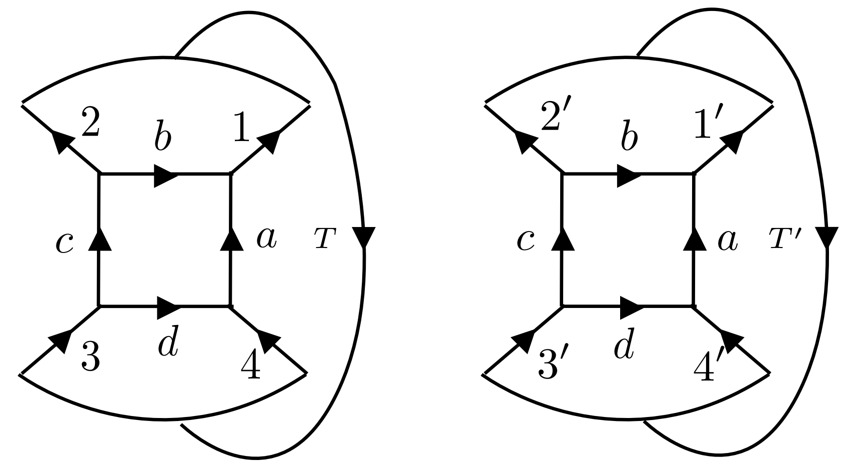
Flow equation



$$(\hat{C}^1_v)^{eff}_{\rho_T,\rho_{T'}}(\rho_1,\rho'_1,m1,\cdots,\rho_4^*,\rho_4'^*,m_4)$$

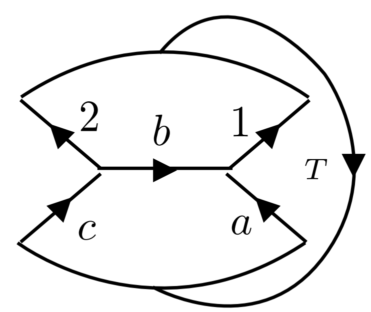
$$1,\cdots,\chi_1(\rho_1,\rho'_1)$$

$$=\sum_{a,b,c,d}$$

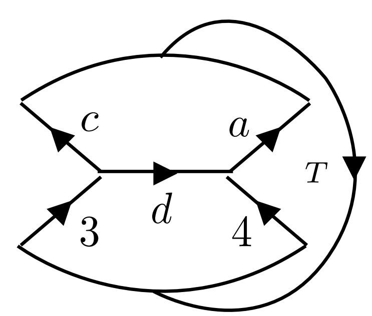


$$\times\ S_2(b,a;m_1)$$
$$\times\ S_4(c,b;m_2)$$
$$\times\ S_1(d,c;m_3)$$
$$\times\ S_3(a,d;m_4)$$

$$\|$$



$$\sim\ \begin{Bmatrix}\rho_T & \rho_c & \rho_a\\ \rho_b & \rho_1 & \rho_2\end{Bmatrix}$$



$$\sim\ \begin{Bmatrix}\rho_T & \rho_3 & \rho_4\\ \rho_d & \rho_a & \rho_c\end{Bmatrix}$$