Resolutions of Singularities in F-Theory

Daniel Vieira Lopes
CIFMC - Universidade de Brasília

Quantum Gravity In The Southern Cone
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Motivation

- String Theory: Candidate for Unification
- D-Branes (90’s)
  - Phenomenologically attractive
  - Non-Perturbative
- F-Theory: Useful tool

Centro Internacional de Física da Matéria Condensada - UnB

Bethe Center for Theoretical Physics
University of Bonn

Nana Cabo Bizet

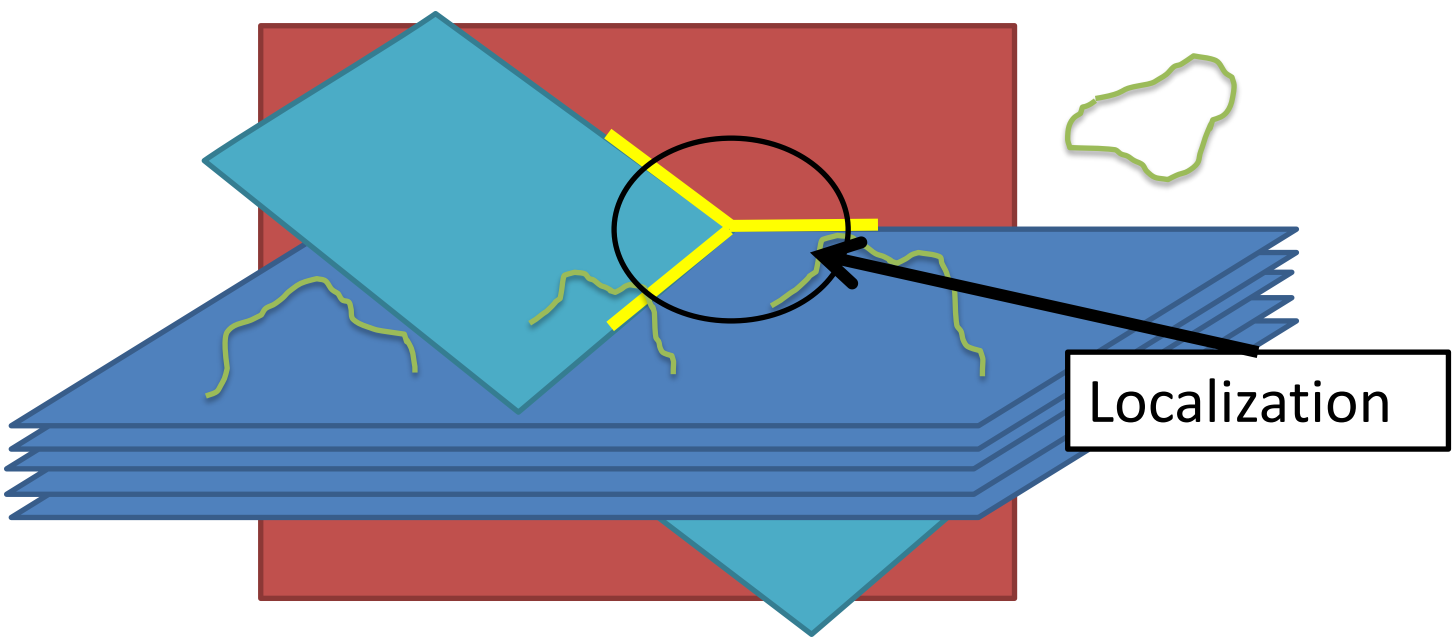
Albrecht Klemm

Outline

1. Models with D-Branes
   i. Review
   ii. Strong Coupling of Type IIB
2. F-Theory
   i. What is it? Why do we need it?
   ii. General Construction
3. An SU(5) Model with an E8 Yukawa Point

Models with D-Branes

- Open strings can end in higher dim. objects: **D-Branes**
- Stack of D-Branes ➔ strings in adjoint rep. (gauge fields)
- Intersecting D-Branes ➔ bi-fundamental rep. (matter)
- Triple Intersection ➔ Yukawa couplings



The Low Energy Limit

- The scale of the massive excitations is close to the Planck scale.
  - Pheno: consider only massless modes
- 10D Superstring → 10D Supergravity
- Type IIB - Bosonic content (closed strings):
  - NS-NS $g_{\mu\nu}, \phi, B_2$ and R-R C_p, p even.

D-Branes

- D*p*-Branes are sources for the Ramond-Ramond C_n fields:
  - Magnetic: C_{7-p}
  - Electric: C_{p+1}
- Gauss Law:
  - $\Delta\Phi_{\mathrm{e}} = \oint * dC_{p+1}$ or $\Delta\Phi_m = \oint dC_{7-p}$

Symmetry of the Type IIB action

- Low Energy Action of Type IIB:

$$S = \int R * 1 - \frac{1}{(\mathrm{Im}\,\tau)^2} d\tau \wedge * d\bar{\tau} - \frac{1}{Im\,\tau} G_3 \wedge * \bar{G}_3 - \\ - \frac{1}{4} \tilde{F}_5 \wedge * \tilde{F}_5 - \frac{1}{2} C_4 \wedge H_3 \wedge F_3 \,,$$

g_s

Where $\tau = C_0 + i e^{-\phi}$, $G_3 = F_3 - \tau H_3$, $H_3 = dB_2$

$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$, $F_{p+1} = dC_p$

- Invariant under the $SL(2,\mathbb{R})$ transformations:

$ad - bc = 1$

$$\tau' = \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} H_3' \\ F_3' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} H_3 \\ F_3 \end{pmatrix}, \quad \tilde{F}_5 \text{ invariant.}$$

Strong-Weak Duality

- String (Fund.-String) is magnetically charged under B_2 $(H_3 = dB_2)$
- D1-Brane (D-String) electrically charged under C_2 $(F_3 = dC_2)$
- $\begin{pmatrix} H_3' \\ F_3' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} H_3 \\ F_3 \end{pmatrix}$ Mag-El. Duality (F $\leftrightarrow$ D)

Introduce (p,q)-strings

- More concretely, starting with Fund.-string,

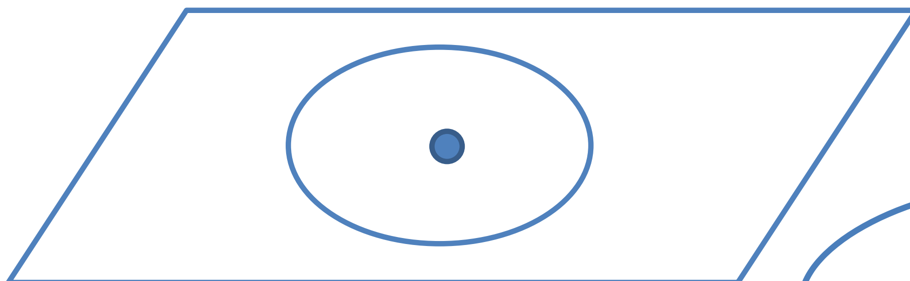
$$b = -c = 1, a = d = 0 \Rightarrow H_3 \to -F_3 \Rightarrow \tau = -\frac{1}{\tau}$$

$Fund \to D$

$g_s \to 1/g_s$

D7-Brane backreaction

- D7-brane mag. charged under C_0
- Gauss law: $\oint dC_0 = 1 \Rightarrow \tau = \frac{1}{2\pi i} \ln z + \tau_0$



$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Transverse space directions (9-7)

$\Rightarrow$ Monodromy $\tau \to \tau + 1 \;\; (C_0 \to C_0 + 1)$

- In general, one can define a (p,q)-7-brane (with monodromy given by the $SL(2,\mathbb{Z})$) on which a (p,q) string attaches.
  - A particular combination of (p,q)-7-branes corresponds to an Orientifold plane.

Sen arXiv:hep-th/9605150

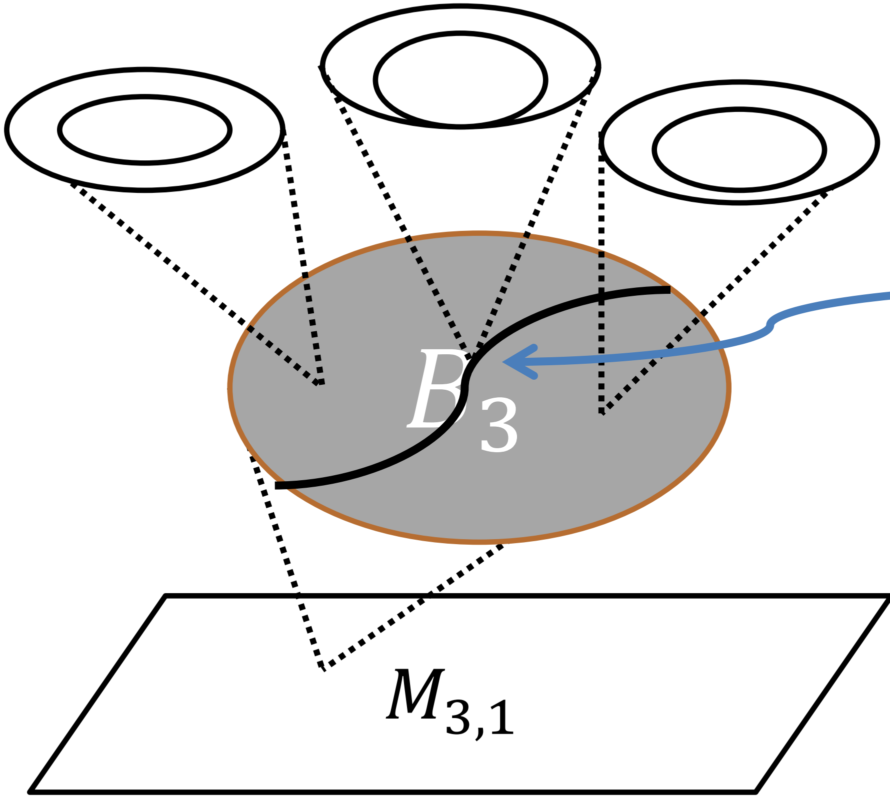
F-Theory

- F-Theory is a geometrical way of encoding this intrinsically non-perturbative aspect of IIB

hep-th/9602022

- Explore $SL(2,\mathbb{Z})$ and turn τ into the complex structure of an auxiliary torus (elliptic fibration)
- $\mathcal{N}=1$ SUSY in 4D $\Rightarrow$ Calabi-Yau 4-fold
- Singularity of torus $\Rightarrow$ Location of the 7-brane
- (Can be constructed from M-Theory on an elliptically fibered Calabi-Yau 4-fold)

F-Theory



$$\tau = C_0 + ie^{-\phi}$$

Recall: Close to D7-Brane ($z = 0$) $\tau \sim \dfrac{\ln z}{2\pi i}$

F-Theory

- Elliptic curve embedded in $\mathbb{P}^2$
$$y^2 = x^3 + fxz^4 + gz^6$$
- Fiber it over base ➔ f, g vary on the base

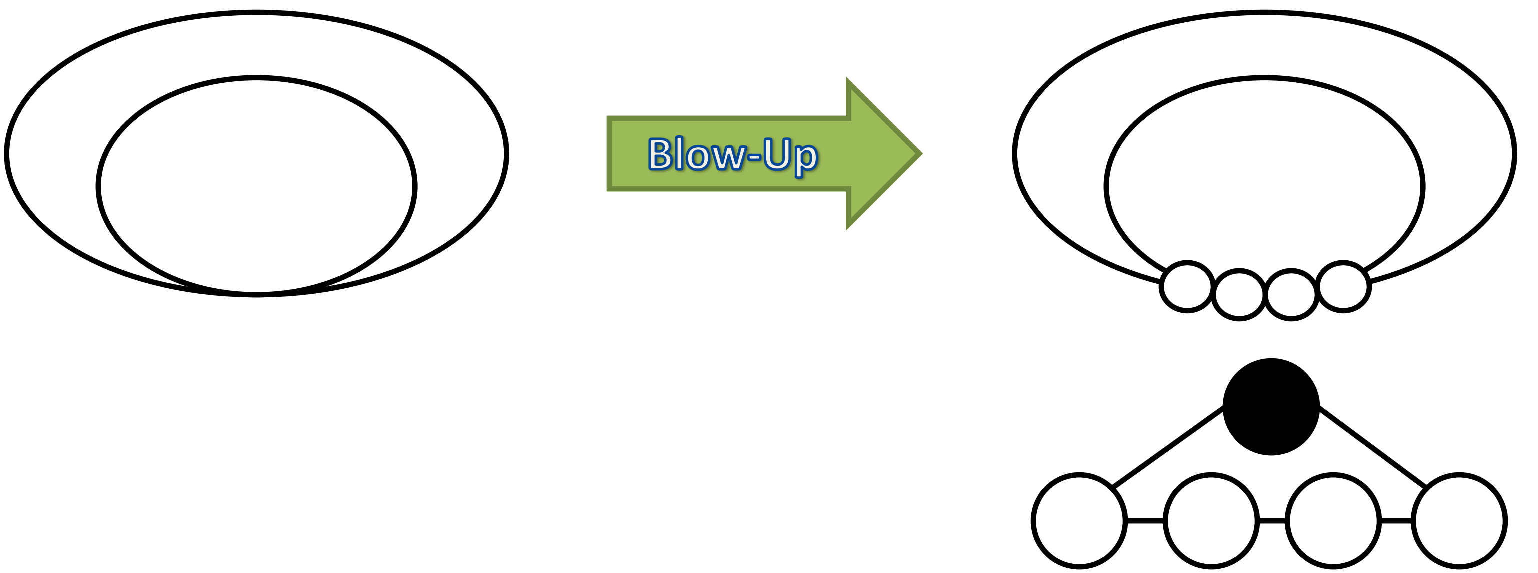
F-Theory

- Elliptic curve embedded in $\mathbb{P}^2$
$$y^2 = x^3 + f(u_i)xz^4 + g(u_i)z^6$$
- Fiber it over base ➔ $f(u_i), g(u_i)$
- Singular when the discriminant
$$\Delta = 27g^2 + 4f^3 = 0$$
- Stack of branes ➔ Stronger singularities
- Kodaira Classification

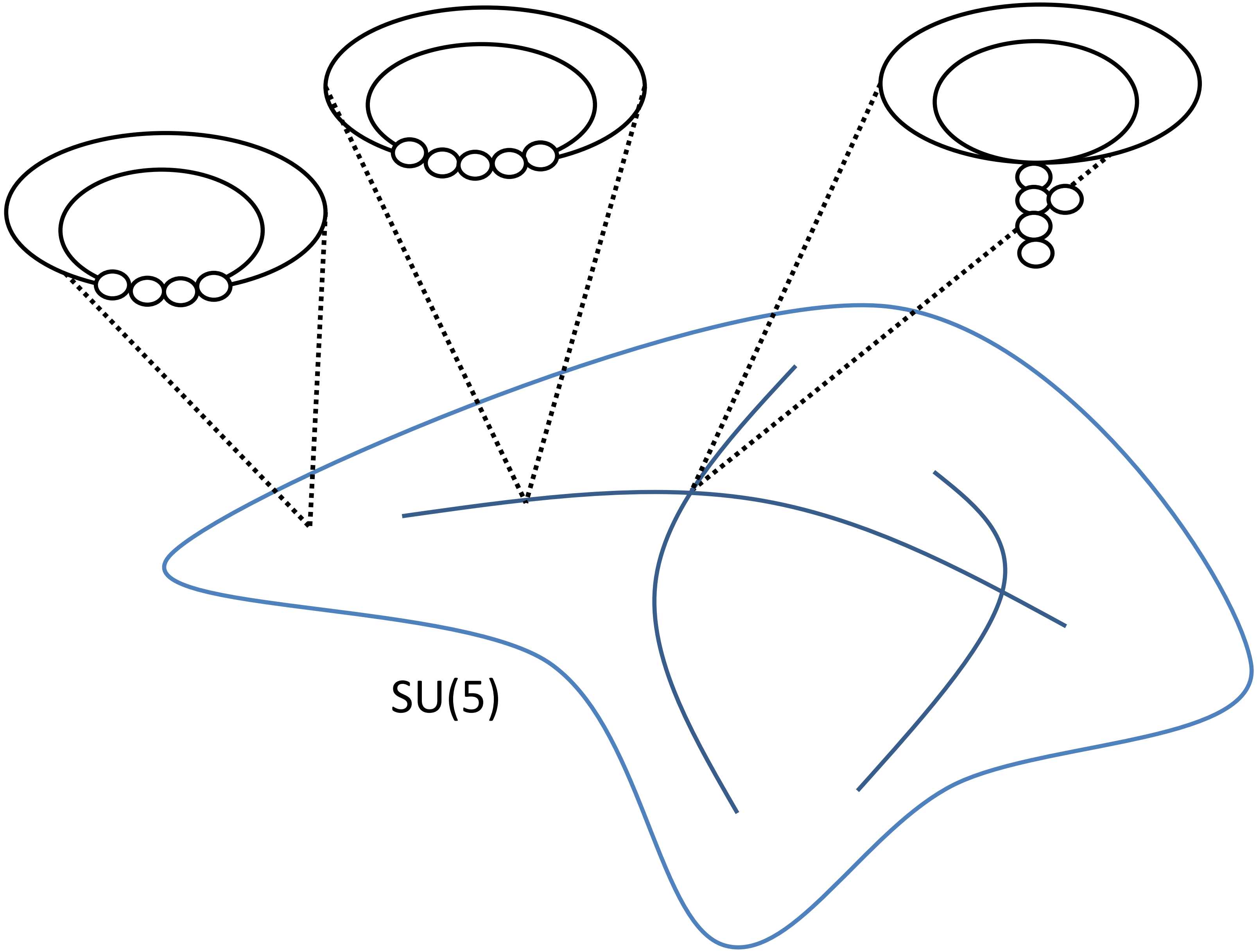
Type	ord(f)	ord(g)	ord(Δ)	$j(\tau)$	Group	Monodromy
I_0	≥ 0	≥ 0	0	$\mathbb{R}$	—	$\mathbf{1}_{2\times 2}$
I_1	0	0	1	∞	$U(1)$	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
I_n	0	0	$n > 1$	∞	A_{n-1}	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$
II	≥ 1	1	2	0	—	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$
III	1	≥ 2	3	1	A_1	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
IV	≥ 2	2	4	0	A_2	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$
I_n^*	2	≥ 3	$n+6$	∞	D_{n+4}	$\begin{pmatrix} -1 & -b \\ 0 & -1 \end{pmatrix}$
	≥ 2	3				
IV^*	≥ 3	4	8	0	E_6	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$
III^*	3	≥ 5	9	1	E_7	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
II^*	≥ 4	5	10	0	E_8	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$

F-Theory

- No fundamental description
- Bring in M-Theory compactified to 3D



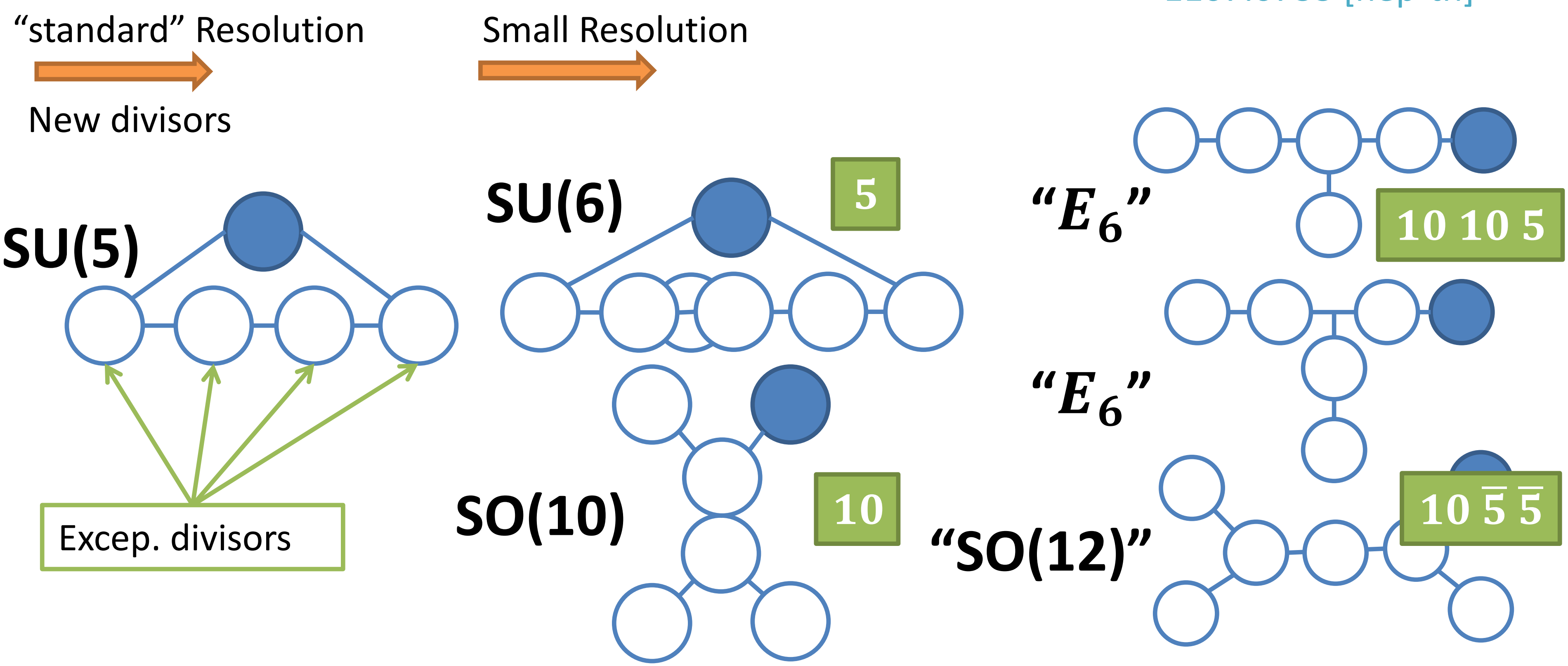
M2-Branes and C_3 form ➔ Gauge bosons
Massless in the Blow-down limit


SU(5)

F-Theory Models

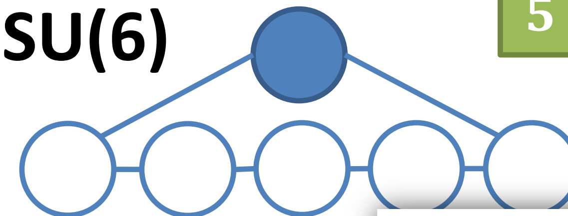
- The Kodaira classification holds only for **codimension 1**
- Some early works studied collisions in **codimension 2**
- Esole-Yau: explicit resolution up to **codimension 3**

1107.0733 [hep-th]



Marsano, Schafer-Nameki
1108.1794 [hep-th]

SU(6)



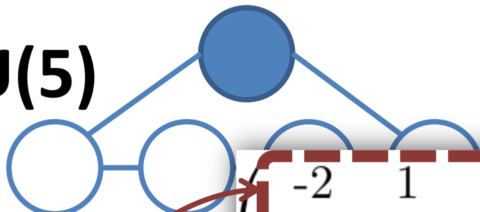
5

$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

$-(\mu_5 - \alpha_1)$

$\mu_5 - \alpha_1 - \alpha_2$

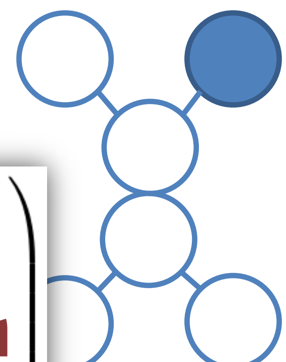
SU(5)



$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

$-\alpha_1$

SO(10)



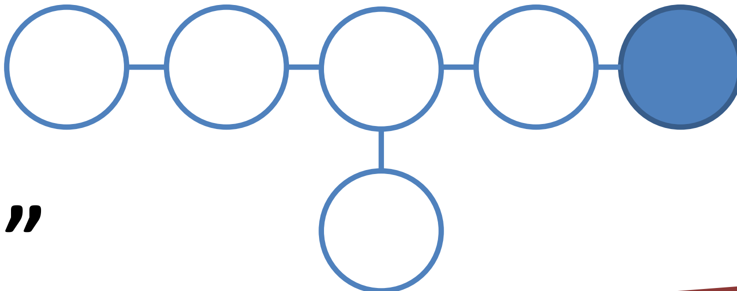
10

$[\mu_{10}]$

$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 1 & -1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 1 & -1 \end{pmatrix}$$

Marsano, Schafer-Nameki
1108.1794 [hep-th]

Top Yukawa coupling



"E_6"

$[\mu_{10}] + [\mu_{10}] + [\mu_5] \rightarrow$ Invariant

An E_8 Yukawa point?

Why? Proposed to explain flavor hierarchy

Heckman, Tavanfar, Vafa
0906.0581 [hep-th]

The resolution could be (and is) more complicated and bring more insights into the phenomenological and mathematical aspects of F-Theory

Our Approach

- We construct by hand (specifying particular relations of the elliptic fibration) such that we get the desired singularity
- We resolve it
  - (there are many ways to do it, but there are common features in all of them)

Some Results

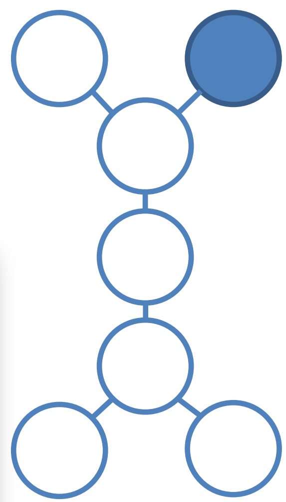
- Matter curves of higher degree singularities lead to combined representations:

SO(12)

$[\mu_{10}]$

$[\mu_5]$

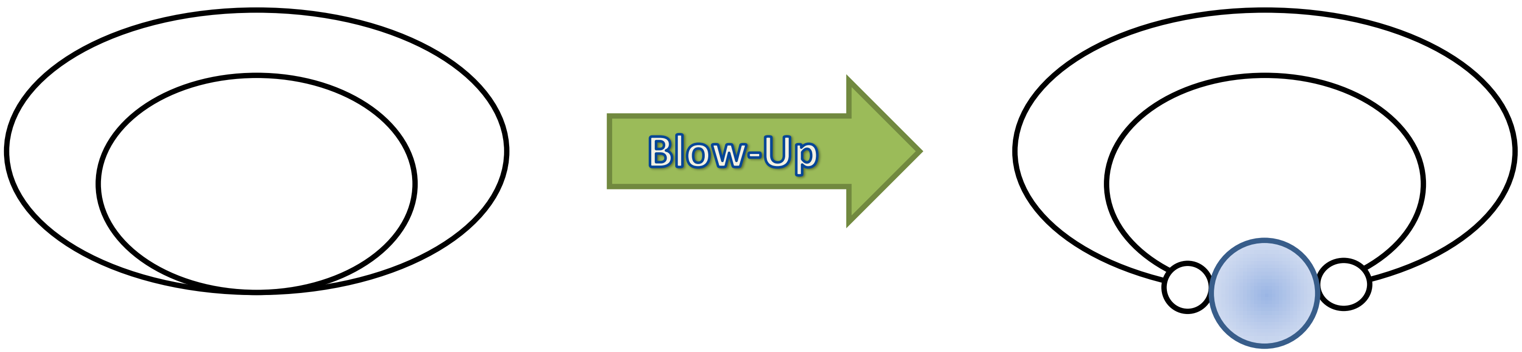
$$\begin{pmatrix} -1 & 1 & -1 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$



$\mathbf{5+10}$

Some More Results

- The resolution often requires the introduction of higher dimensional spaces on the fiber



- This higher dim. object was antecipated in some previous works by other authors Candelas 0009228 [hep-th]
- M5-Branes ➔ Tensionless strings?

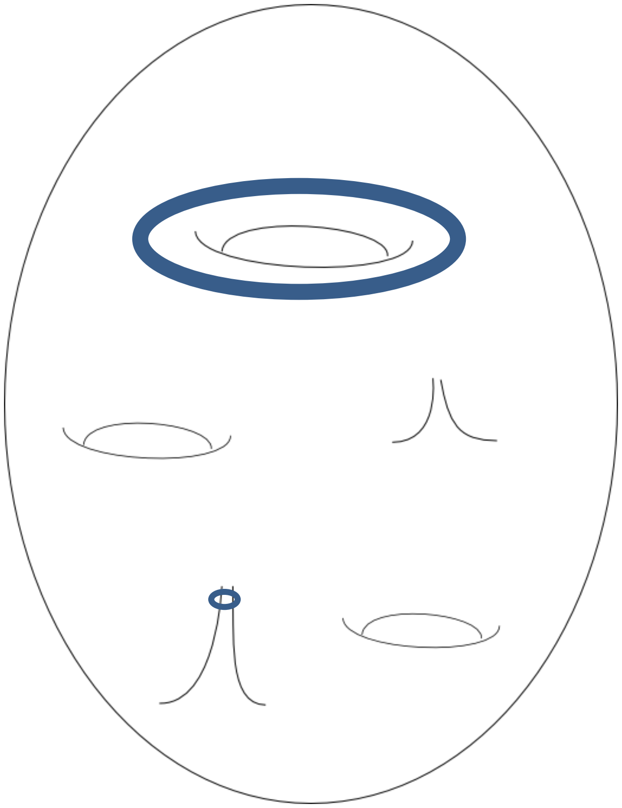
Conclusions

- D-Branes provide a nice framework to construct phenomenology;
- However, not completely satisfying;
- Strong coupled regime;
  - Apart from the problem of knowing just small part of moduli space;
- F-Theory: interesting framework to construct GUTs
- Explicit resolution of higher codim not trivial: higher dimensional fiber

“Decoupling” from Gravity

- The “Standard Model” will live in subsets of the compactified space, of volume much smaller than the full Calabi-Yau

$$\alpha_{GUT}^2 \sim \frac{1}{M_{pl}^2} \frac{Vol(B_3)}{Vol(S_2)^2}$$



An E_8 Yukawa point?

Why? Proposed to explain flavor hierarchy Heckman, Tavanfar, Vafa 0906.0581 [hep-th]

"Tate Model" for SU(5)

$$\boxed{-y^2 + x^3 + \beta_0 w^5} + \beta_2 x w^3 + \beta_3 y w^2 + \beta_4 x^2 w + \beta_5 x y = 0$$

$$(y^2 = x^3 + f x z^4 + g z^6)$$

$w \to 0$ corresponds to the SU(5) brane.

For example, "E_6" obtained from $w \to 0,\ \beta_5 \to 0,\ \beta_4 \to 0$.

The resolution of

$$-y^2 + x^3 + \beta_0 w^5 = 0$$

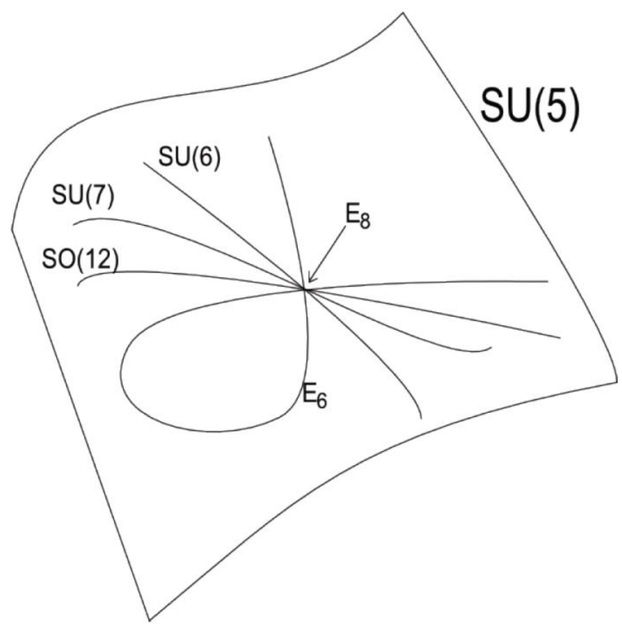
leads to affine E_8. Naively we expect $\{w, \beta_2, \beta_3, \beta_4, \beta_5\} \to 0$.

But should be codimension 3.

An E_8 Yukawa point?

- Impose relations among the β_is, so $\{w, \beta_2, \beta_3, \beta_4, \beta_5\} \rightarrow 0$ with just three restrictions, $\{w, p, q\} \rightarrow 0$.
- Import results from spectral cover construction of Heterotic String, via the Heterotic / F-Theory duality.
- The β_is are related to higgsings of the original heterotic E_8 that is deformed to the SU(5).

$$\beta_2 = \beta_0(3p^2 + pq + 3q^2),$$
$$\beta_3 = \beta_0(p+q)(2p^2 + 3pq + 2q^2),$$
$$\beta_4 = 2\beta_0 pq(p^2 + 4pq + q^2),$$
$$\beta_5 = 4\beta_0 p^2 q^2 (p+q)$$



Resolution of the E_8 model

- In codim >1 small resolutions are not enough
- Blow-up divisors on top of the curves!
- Higher dimensional fiber!