Spontaneous generation of geometry in four dimensions

BY

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References

- Spontaneous generation of geometry in four dimensions. JA, D. Espriu, D. Puigdomenech,Phys.Rev. D86 (2012) 025015
- The emergence of geometry: a two-dimensional toy model.JA, D. Espriu, D. Puigdomenech,Phys.Rev82 (2010) 045018.

Introduction

- General Relativity is non-renormalizable
- The low energy theory of strong interactions(chiral lagrangian) is non-renormalizable as well.
- But QCD is renormalizable.
- It produces the chiral lagrangian by spontaneous symmetry breaking of the chiral symmetry, which is a symmetry of QCD for massless quarks.
- Is there an analogous mechanism for gravity?

The model

- The free Lagrangian density is

$$\mathcal{L}_0 = i\,\bar{\psi}_a\,\gamma^a\,(\partial_\mu + i\,w_\mu^{bc}\,\sigma_{bc})\,\chi^\mu + i\,\bar{\chi}^\mu\,\gamma^a\,(\partial_\mu + i\,w_\mu^{bc}\,\sigma_{bc})\,\psi_a, \qquad (1)$$

ψ_a and χ^μ are two species of fermions transforming, respectively, under Lorentz ($a, b...$ are tangent space indices) and Diffeomorphisms ($\mu, \nu...$ are world indices).

- A spin connection w_μ is included.

- It is important to notice that there is no metric to start with; there is no need of one as long as χ^μ transforms as a spinorial density.

- Interaction term

$$\mathcal{L}_I = i\,B_\mu^a\,(\bar{\psi}_a\,\chi^\mu + \bar{\chi}^\mu\,\psi_a) + c\,\det(B_\mu^a), \qquad (2)$$

The object of the interaction (2) is to trigger the spontaneous breaking of the global symmetry via fermion condensation.

Upon use of the equations of motion for the auxiliary field B^a_μ

$$\bar{\psi}_a \chi^\mu + \bar{\chi}^\mu \psi_a = - \tag{3}$$

$$i\, c \frac{1}{(D-1)!} \epsilon_{a a_2 \dots a_D} \epsilon^{\mu \mu_2 \dots \mu_D} B^{a_2}_{\mu_2} \dots B^{a_D}_{\mu_D}$$

and thus

$$\langle \bar{\psi}_a \chi^\mu + \bar{\chi}^\mu \psi_a \rangle \neq 0 \longleftrightarrow B^a_\mu \neq 0. \tag{4}$$

If a non-zero value for the fermion condensate appears the field B^a_μ acquires an expectation value.

On translational invariance grounds, for $w^{ab}_\mu = 0$ the vacuum of the theory is obtained from the gap equation for the potential.

This equation has a general non-trivial solution corresponding to $B^a_\mu = M\, \delta^a_\mu$ In 4D

$$\begin{gathered} c\, M^3 - 2\, N \int \frac{d^D k}{(2\,\pi)^D} \frac{M}{k^2 + M^2} = 0 \\ c\, M^3 + N \frac{M^3}{8\,\pi^2} \left(\frac{2}{\epsilon} - \log \frac{M^2}{4\,\pi\,\mu^2} - \gamma + 1 \right) = 0, \end{gathered} \tag{5}$$

whose formal solution is

$$M^2 = \mu^2\, e^{8\pi^2 c(\mu)/N}, \tag{6}$$

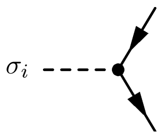
where $\mu \frac{d\,c}{d\,\mu} = -\frac{N}{4\,\pi^2}$, making M a renormalization-group invariant. In the previous, we introduced the usual mass scale μ to preserve the correct dimensionality of the D-dimensional integral as dimensional regularization is used. For the solution to actually exist we have to require $c > 0$ if $M > \mu$. If $\mu > M$ the solution exists only if $c < 0$. Therefore $c > 0$ will be the case we are interested in on physical grounds.

B_μ^a has the right structure to be identified as the vierbein.

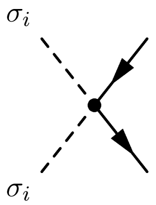
The free fermion propagator of the theory in the broken phase can then be easily found after replacing B_μ^a by its vacuum expectation value. With a 4D matrix notation

$$\Delta^{-1}(k)^i_{\ j} = \frac{-i}{M}\left(\delta^i_j - \frac{\gamma^i\,(\not{k} - i\,M)\,k_j}{k^2 + M^2}\right). \tag{7}$$

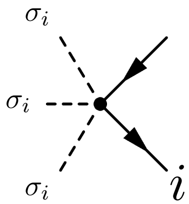
Feynman Rules



$$i\frac{1}{2}M\delta^i_\mu$$

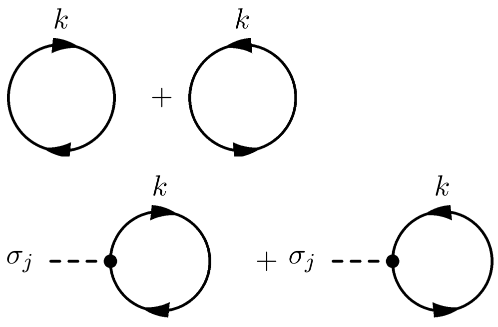


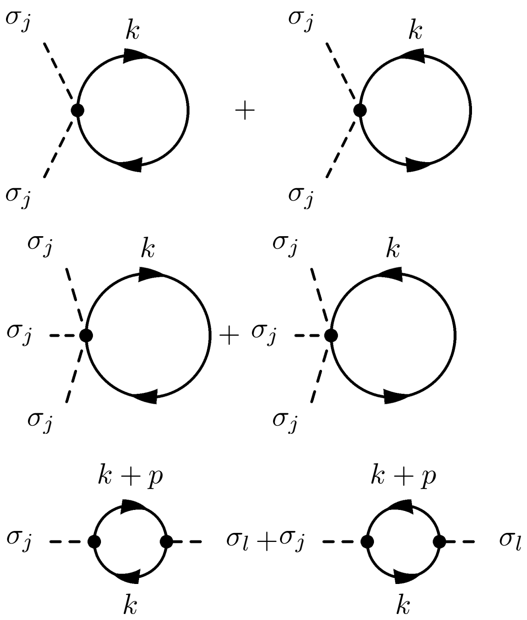
$$-\, i\frac{1}{8}M\delta^i_\mu$$



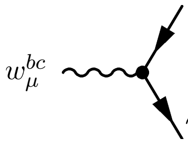
$$i\frac{1}{48}M\delta^i_\mu$$

Effective Action

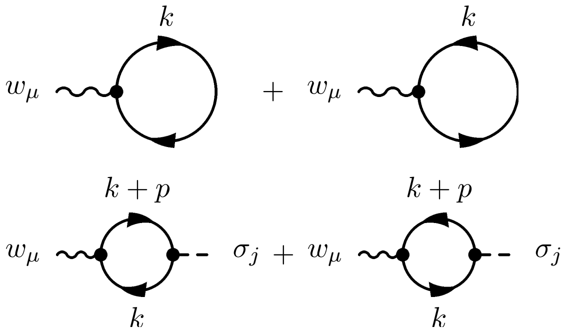


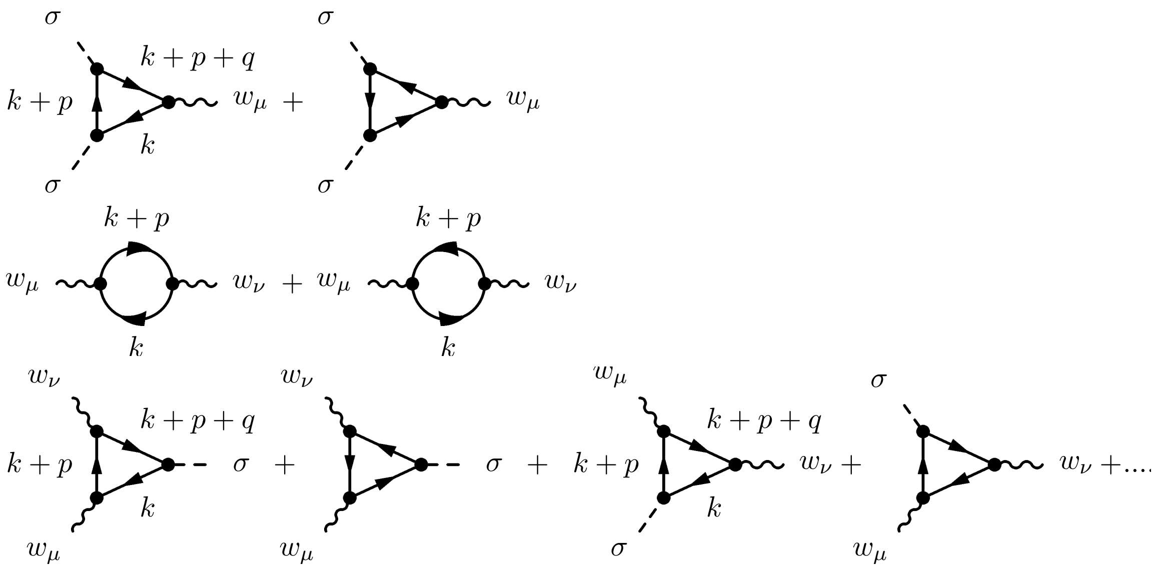


Diagrams with w_μ^{ab}



$$i\gamma^a w_\mu^{\mathrm{bc}} \sigma_{\mathrm{bc}} = \frac{\gamma^a}{4}[\gamma_b, \gamma_c].$$




σ
k + p + q
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k + p + q
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σ +
w_μ
k + p
w_ν +
w_ν +....

Summary of divergences

The full calculation at order $M^2\, p^2$ resums to the following term in the effective action

$$\frac{M^2\,\Box\,\sigma\, e^{-\sigma}}{32\,\pi^2}\left(\frac{2}{\epsilon}-\log\left(\frac{M^2}{4\,\pi\,\mu^2}\right)-\gamma+\frac{1}{3}\right). \tag{8}$$

Note that this term has the same structure that $\sqrt{g}\, R$ for a conformally flat metric. This divergence can be absorbed by redefining $\mathcal{L}_R$ and using the equations of motion. This is already telling us that the theory is renormalizable only on shell. Namely when the spin connection w_μ^{ab} corresponds to the Levi-Civita one. In our approach this identification is forced by the use of the equations of motion.

Effective action and physical constants

We recall our conventions. We have used euclidean conventions so that the (emerging) metric has signature (+,+,+,+). The effective action at long distances is defined by the functional integral

$$\int [d\,g]\exp\left(-S[g]\right), \tag{9}$$

where

$$g_{\mu\nu} = \eta_{ab}\, e^a_\mu\, e^b_\nu = \frac{1}{M^2}\,\eta_{ab}\, B^a_\mu\, B^b_\nu \tag{10}$$

The effective action obtained after the diagrammatic calculation of the previous sections is

$$\begin{aligned} S_{eff} = \int d^4 x \Bigg(& c'\, M^4\, e^{-2\sigma} - N\,\frac{M^4}{8\,\pi^2}\, e^{-2\sigma}\left(\log\left(\frac{M^2}{\mu^2}\right) - \frac{3}{2}\right) \\ & + A'\, M^2\, \Box\, \sigma\, e^{-\sigma} - N\,\frac{M^2}{32\,\pi^2}\, \Box\, \sigma\, e^{-\sigma}\left(\log\left(\frac{M^2}{\mu^2}\right) - \frac{28}{3}\right)\Bigg) + \ldots, \end{aligned} \tag{11}$$

where $c' = c + \frac{N}{8\,\pi^2}\left(\frac{2}{\epsilon} + \log 4\,\pi\, - \,\gamma\right)$ and $A' = A + \frac{N}{8\,\pi^2}\left(\frac{2}{\epsilon} + \log 4\,\pi\, - \,\gamma\right)$ are renormalized coupling constants that have absorbed the divergences. The $\overline{MS}$ subtraction scheme is assumed. Note that the finite part of the term proportional to M^2 has received a contribution from the diagrams containing only w_μ^{ab} fields.

Making use of the gap equation, we can write the previous expression as

$$S_{eff} = \int d^4\, x \left(N\, \frac{M^4}{16\,\pi^2}\, e^{-2\sigma} + A'\, \Box\, \sigma\, e^{-\sigma}\, M^2 - N\, \frac{M^2}{32\,\pi^2}\, \Box\, \sigma\, e^{-\sigma} \left(\log\left(\frac{M^2}{\mu^2}\right) - \frac{28}{3}\right)\right) + ..., \tag{12}$$

The resulting effective theory thus describes a geometry with a cosmological term. Appealing to covariance arguments we can now express (11) in terms of invariants

$$S_{eff} = \int d^4\, x \left[\frac{N}{16\,\pi^2}\, M^4\, \sqrt{g} + \left(A' - \frac{N}{48\,\pi^2} \left(\log\left(\frac{M^2}{\mu^2}\right) - \frac{28}{3}\right)\right) M^2\, \sqrt{g}\; R + \ldots \right]. \tag{13}$$

Next we recall that the classical Einstein action corresponding to the euclidean conventions is:

$$S = -\frac{M_P^2}{32\,\pi}\int d^4\,x\sqrt{g}\,(R - 2\,\Lambda). \tag{14}$$

Now identifying

$$\begin{aligned} \frac{N}{16\,\pi^2}\,M^4 &= 2\,\Lambda\frac{M_P^2}{32\,\pi} \\ M^2\left(A' - \frac{N}{48\,\pi^2}\left(\log\left(\frac{M^2}{\mu^2}\right) - \frac{28}{3}\right)\right) &= -\frac{M_P^2}{32\,\pi}, \end{aligned} \tag{15}$$

we indeed obtain

$$S_{eff} = -\frac{M_p^2}{32\,\pi}\int d^4\,x\,\sqrt{g}\,(R - 2\,\Lambda) + \mathcal{O}(p^4). \tag{16}$$

As we see from the previous discussion, the integration of the fermions (assumed to be the fundamental degrees of freedom in the theory) yields a positive cosmological constant. As for the value of M_P^2, the Planck mass squared, the sign is not really automatically defined.

Fine-tuning and running of the constants

To ensure that the action is renormalization group invariant, thus observable, the following beta function for each free constant in the theory must be obeyed

$$\mu \frac{d\,c'}{d\,\mu} = -\frac{N}{4\,\pi^2}$$
$$\mu \frac{d\,A'}{d\,\mu} = -\frac{N}{24\,\pi^2} \tag{17}$$

At scales $\mu \gg M$ the relevant degrees of freedom are not gravitons, but the $2\,N$ fermions appearing in the microscopic Lagrangian. On the other hand, at the moment that fermions become the relevant degrees of freedom, geometry loses its meaning. There is then no 'shorter' distance than M^{-1}, or at the very least this regime cannot be probed.

The 'graviton' loops are not included here; they are suppressed by one power of N if N is large. To see this last statement we recall that the usual power counting rules show that the exchange of the vierbein degrees of freedom would be accompanied by a factor of M_P^{-2}, suppressed by $1/N$.

To understand the issue it is probably useful to appeal to the QCD analogy.

– At long distances strong interactions are well described by the pion chiral Lagrangian, parametrized by f_π or the $O(p^4)$ coefficients, generically named low energy constants (LEC). The LEC are a complicated function of α_s, the coupling constant of QCD.

– The microscopic theory proposed in this paper is the analogous of QCD, while the resulting effective theory (16) is the counterpart of the chiral effective Lagrangian. Then M_P and Λ are the LEC of the present theory.

– At some scale, $q \sim M$ the effective theory stops making sense. At that moment the relevant degrees of freedom change and, as a result, the metric disappears. Exactly in the same way as for large momentum transfers we do not see pions but quarks.

– If there is no metric there is no geometry and, in particular, the notion of distance disappears altogether at length scales below M^{-1}.

Summary

- We have proposed a model where 4D gravity emerges from a theory without any predefined metric. The minimal input is provided by assuming a differential manifold structure endowed with an affine connection. Nothing more.

- The Lagrangian can be defined without having to appeal to a particular metric or vierbein.

- Gravity and distance are induced rather than fundamental concepts in this proposal. At sufficiently short scales, when the effective action does not make sense anymore, the physical degrees of freedom are fermionic. At such short scales there is not even the notion of a smaller scale.

- A very important aspect of the model is the improvement of the ultraviolet behavior. After integration of the fundamental degrees of freedom all the divergences can be absorbed in the redefinition of the cosmological constant and the Planck mass.

- With the running, dictated by the corresponding beta functions, both quantities are renormalization group invariant. In addition the Gauss-Bonnet invariant is renormalized too. This happens in spite of the bad ultraviolet behavior of the propagator and the ultimate reason, we think, is that these are the *only* counterterms that can be written without having to assume an underlying metric that does not exist before spontaneous symmetry breaking takes place.

- At long distances the fluctuations around the broken vacuum are the relevant degrees of freedom and are described by an effective theory whose lowest dimensional operators are just those of ordinary 4D gravity.

- They of course exhibit the usual divergences of quantum gravity but this now poses in principle no problem as we know that at very high energies this is not the right theory. For $q \sim M$ one starts seeing the fundamental degrees of freedom. Gravitons are the Goldstone bosons of a broken global symmetry.

- In a sense the fundamental fermions resolve the point-like 3-graviton, 4-graviton, etc. interactions into extended form factors and this is the reason for the mitigation of the terrible ultraviolet behavior of quantum gravity. However this is only part of the story, because this could be equally achieved by using Dirac fermions coupled to gravity (or any other field for that matter). This would in fact be just a reproduction of the old program of induced gravity (ADLER) and therefore not that interesting.

- The really novel point in this proposal is that the microscopic fermion action does not contain any metric tensor at all. Then not only is the metric and its fluctuations –the gravitons– spontaneously generated, but the possible counterterms are severely limited in number.

→ A number of extensions and applications come to our mind. Perhaps the most intriguing one from a physical point of view would be to investigate in this framework singular solutions in GR such as black holes.

→ A more in-depth study of the renormalizability issue is certainly required too as there are issues related to the renormalization group to be addressed in the present setting.

THANK YOU