Recovering General Relativity from Hořava Theory

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Based on:

JB and A. Restuccia, arxiv:1004.0055 [IJMPD 2012]

JB and A. Restuccia, arxiv:1010.5531 [PRD 2011]

JB and A. Restuccia, arxiv:1106.5766 [PRD 2011]

JB, A. Restuccia and A. Sotomayor, arxiv:1205.2284 [PRD 2012]

JB, A. Restuccia and A. Sotomayor, arxiv:1302.1357 [PRD 2013]

Outline

1. Hořava theory
   - Hořava's proposal
   - The complete nonprojectable theory
   - Preliminary discussion
2. The degrees of freedom at $\lambda = 1/3$
   - Generalities
   - The theory with $\lambda = 1/3$
3. The $\alpha \to 0$ limit
   - Emerging of general relativity
   - Spherically symmetric solutions
4. The linearized theory

Relativistic higher curvature theories

- No renormalization of the perturbative quantization of general relativity.
  ➜ Look for alternatives.
- **Relativistic high-curvature models**: adding terms

$$(R_{\alpha\beta\mu}{}^{\nu})^n$$

  improves divergences, but the theories generically have ghosts.

  [Stelle, 70's]
- The ghosts can be seen in the propagator ➜ Additional poles with wrong sign.

High order time-derivatives ➜ Ghosts.

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Horava's proposal

- How can we add high-curvature terms without adding poles?

Hořava's proposal: suppose that we are not forced to use space-time covariant terms.

- Change the symmetry of general covariance by a smaller group:

Foliation-preserving diffeomorphisms (FDiff)

There exists a preferred time direction, the space-time is foliated in spatial submanifolds of constant time. The symmetries are coordinate transformations that preserve this foliation.

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Improved propagator

- One may include spatial high-curvature terms $(R_{ijk}{}^{l})^{z}$.
- It is expected that the propagator is modified

$$\frac{1}{\omega^2 - |\vec{k}|^2 - G(|\vec{k}|^2)^z}$$

- At UV, the propagator is dominated by $1/G(|\vec{k}|^2)^z$.
- large enough z ➜ renormalizable (or even finite!).

★ Hořava's prototipe for quantum gravity: $z = 3$.

Features of Hořava theory

- Very likeable assumptions:
  - Quantum gravity as a standard quantum field theory.
  - Gravitational field ➜ space-time metric.
  - Four dimensions.
- Very challenging assumption:
  - Abandon general covariance as a fundamental symmetry.
  - Instead, use as fundamental symmetry:

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Foliation-preserving diffeomorphisms

- Start with a given time coordinate t.
- Space-time is foliated by space-like hypersurfaces Σ of constant t.
- The gauge symmetry must preserve the foliation.

Foliation-preserving diffeomorphisms (FDiff)

$$\tilde{t} = \tilde{t}(t)\,, \qquad \tilde{x}^i = \tilde{x}^i(t, x^j)$$

Contrast with general space-time diffeomorphisms:

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Gravitational field variables

- Space-time metric in ADM variables

$$ds^2 = (-N^2 + N_i N^i)\, dt^2 + 2N_i\, dt\, dx^i + g_{ij}\, dx^i\, dx^j$$

- FDiff preserve $N = N(t)$ ➜ **Projectable version of the theory.**
- We take $N = N(t, x^i)$ ➜ **Nonprojectable version.** We choose this version because of its connection with GR (...as we are going to see...).

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Hořava action

General Relativity in ADM variables

$$S = \int dt d^3x \sqrt{g} N (K_{ij} K^{ij} - K^2 + R) \,,$$

[Arnowitt, Deser and Misner, 1959-1962]

$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - 2\nabla_{(i}N_{j)})$ ➜ Extrinsic curvature of Σ.

R ➜ Ricci scalar of the 3D metric g_{ij}.

$K = g^{ij} K_{ij}$

Hořava action

Original Hořava action

$$S = \int dt d^3x \sqrt{g} N \left(K_{ij}K^{ij} - \lambda K^2 - \mathcal{G}_{ijkl} E^{ij} E^{kl} \right) ,$$

$$\begin{aligned} E^{ij} &= \frac{1}{w^2} C^{ij} - \frac{\mu}{2} \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \\ &= \text{derivative of a 3d spatial functional} \end{aligned}$$

➜ Detailed balance principle

- C^{ij} : 3d Cotton tensor.

[Hořava, 2009]

Hořava action

- **UV dominant term**: (Cotton)^2
  ➜ sixth-order in spatial derivatives ($z = 3$).
- Why undetermined constant λ?
  ➜ Both $K_{ij}K^{ij}$ and K^2 are invariant under FDiff.
- Symmetry of general space-time diffeomorphisms requires $\lambda = 1$.

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Generalizations

- **Abandon the detailed balance principle**: It is not a symmetry principle
- The potential may be any scalar under FDiff

Terms of Blas, Pujolás and Sibiryakov

The spatial vector $a_i = \partial_i \ln N$ is covariant under FDiff. It can enter into the potential:

$$a_i a^i \quad , \quad \nabla_i a^i \quad , \quad a^i a^j R_{ij} \quad \ldots$$

[Blas, Pujolàs and Sibiryakov, 2009]

- These terms are induced by quantum corrections
  ➜ complete actions should include them.

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Complete theory

Complete, nonprojectable Hořava theory

$$S = \int dt d^3x \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V} \right) ,$$

- $\mathcal{V} = \mathcal{V}[g_{ij}, a_i]$ ➜ Most general $z = 3$ potential invariant under FDiff.

[Blas, Pujolàs and Sibiryakov, 2009]

Some terms:

- $z = 1$ ➜ R, $a_i a^i$ (only these)
- $z = 2$ ➜ R^2, $a^i a^j R_{ij}$, $(a_i a^i)^2$, $\nabla_i a^i R \dots$
- $z = 3$ ➜ $C^{ij} C_{ij}$, $(a_{ij} a^{ij})^3$, $(a_i a^i)^2 R$, $a_i \nabla^4 a^i \dots$

➜ near 100 terms in total!!

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What does one expect at first sight?

- The theory seems to improve renormalization at UV...but general relativity has a solid phenomenology at large distances...

Can general relativity be recovered at large distances?

- **An extra mode?**
  This theory uses the same field variables of GR, but the gauge group is smaller...
  ➜ One would expect the presence of extra modes...but...

Perhaps the dynamics drops the extra modes out!!

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What has been claimed about the Hořava theory?

- As a standard lore many people believe that the limit $\lambda \to 1$ is necessary to recover general relativity.
- Related issue ➜ strong coupling problem of the extra mode:

Strong coupling

If one assume that $\lambda \to 1$ at low energies, then the coupling constants of the self-interactions of the extra mode diverge.

[Charmousis et al., 2009; Papazoglou and Sotiriou, 2010]

- Also: claims about non closure of the algebra of constraints, vanishing lapse function.

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The complete, nonprojectable Hořava theory at $\lambda = 1/3$

Hamiltonian analysis: primary constraints

Why is the $\lambda = 1/3$ value so special?

- Canonical-conjugate pairs (g_{ij}, π^{ij}) , (N, ϕ)
  In Hořava theory N cannot be regarded as a Lagrange multiplier.
- Kinetic term is universal, so we always have

$$\frac{\pi^{ij}}{\sqrt{g}} = G^{ijkl} K_{kl} ,$$

$$G^{ijkl} \equiv \tfrac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl} .$$

- is G^{ijkl} invertible? ➜ The answer splits out in two ways...

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Hamiltonian analysis: primary constraints

1. If $\lambda \neq 1/3$, matrix G^{ijkl} can be inverted.

   ➜ We may solve $\dot{g}_{ij}$ in terms of π^{ij}.

2. If $\lambda = 1/3$,

$$G^{ijkl} g_{kl} = 0 \,,$$

   matrix G^{ijkl} cannot be inverted.

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$$\pi \equiv g_{ij}\pi^{ij} = 0 \,.$$

- In both cases one may get the Hamiltonian.

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$$H = \int d^3x \left(N\mathcal{H} + N_i\mathcal{H}^i + \sigma\phi + \mu\pi \right) + \text{boundary t.}$$

σ, μ ➜ Lagrange multipliers.

$$\mathcal{H} \equiv \frac{1}{\sqrt{g}}\pi^{ij}\pi_{ij} + \sqrt{g}\tilde{\mathcal{V}}$$

Modified potential:

$$\tilde{\mathcal{V}} \equiv \mathcal{V} + \frac{1}{N}\sum_{r=1}(-1)^r \nabla_{i_1\cdots i_r}\left(N \frac{\partial \mathcal{V}}{\partial(\nabla_{i_r\cdots i_2} a_{i_1})} \right) .$$

Momentum constraint: $\mathcal{H}^i \equiv -2\nabla_j\pi^{ij} + \phi\partial^i N$

Preservation of constraints

- At $\lambda = 1/3$ there arises the primary constraint $\pi = 0$.
- Conversely,

$$\boxed{\pi = 0 \text{ protects the value } \lambda = 1/3}$$

➜ Other value of λ would eliminate the constraint!!.

- Preservation of $\pi = 0$ leads to another constraint, $\mathcal{C} = 0$,

$$\mathcal{C} \equiv \frac{3N}{2\sqrt{g}} \pi^{ij}\pi_{ij} - g_{ij}\frac{\delta}{\delta g_{ij}} \int d^3y \sqrt{g} N \tilde{\mathcal{V}} \, .$$

Constraints

- First class ➜ $\mathcal{H}^i = 0$
- Second class ➜ $\mathcal{H} = 0$, $\phi = 0$, $\pi = 0$, $\mathcal{C} = 0$.

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Preservation of constraints

Preservation of 2nd-class constraints ➜ conditions for the multipliers σ, μ.

Equations for the Lagrange multipliers

$$E_1^{(6)}(\sigma, \mu) = 0 \,, \qquad E_2^{(6)}(\sigma, \mu) = 0$$

➜ For $z = 3$ these are 6th-order elliptic diff. eqs. for σ, μ.
(very involved expressions)

★ Dirac's algorithm closes consistently.

Physical degrees of freedom

Physical degrees of freedom

{Canonical variables} $-$ {constraints + gauge symmetries}

||

2 phys. degs. of freedom.

★ There is not any extra mode in this theory!!!

- **Remark**: the $\lambda = 1/3$ theory has two more constraints, $\pi = 0$, $\mathcal{C} = 0$. They eliminate the extra mode of Hořava theory.

Emerging of general relativity: the $\alpha \to 0$ limit

- At large distances neglect the high-order terms, keep the effective action.
- In the lowest-order potential

$$\mathcal{V} = -R - \alpha\, a_i a^i$$

Drop the a^2 term out by sending $\alpha \to 0$.

Extra constraints

$$\pi = 0\,, \qquad \mathcal{C} = (\nabla^2 - R)N = 0$$

➜ Both equations can be seen on the side of general relativity as a gauge fixing procedure!! (maximal slicing gauge).

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Emerging of general relativity: the $\alpha \to 0$ limit

- The Hamiltonian becomes (the $\alpha \to 0$ limit implies $\mu = 0$)

$$
\begin{aligned}
H &= \int d^3x (N\mathcal{H} + N_i\mathcal{H}^i + \sigma\phi) + E_{\text{ADM}} \\
\mathcal{H} &= \frac{1}{\sqrt{g}}\pi^{ij}\pi_{ij} - \sqrt{g}R\,.
\end{aligned}
$$

➜ This Hamiltonian is exactly the one of general relativity in the maximal slicing gauge.

The $\alpha \to 0$ limit leads exactly to the dynamics of general relativity.

Static spherically symmetric solutions

Perturbative solutions

Approximate solutions that are trustable when α is small.

- Static, spherically symmetric ansatz

$$ds^2_{(4)} = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{(2)} \,.$$

- Exact field equations

$$\begin{aligned} rf' + f - 1 - \frac{\alpha}{2} r^2 f \left(\frac{N'}{N}\right)^2 &= 0\,, \\ \frac{f'}{rf} + \frac{N''}{N} + \frac{f'N'}{2fN} - \alpha \left(\frac{N'}{N}\right)^2 &= 0\,, \\ \frac{1}{2} rf' + f - 1 + \frac{rfN'}{N} &= 0\,. \end{aligned}$$

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Static spherically symmetric solutions

- From the first and last equations

$$8(1-h)(rh)' + \alpha\left[\left((rh)' - h\right)^2 + 4(rh)'\right] = 0\,, \qquad h \equiv 1 - f\,.$$

- Perturbations: assume that the perturbative solution have the linear-order form

$$h = h^{(0)} + \alpha h^{(1)}$$

- We may solve the equation iteratively in orders of alpha, starting from the zero order:

$$(1-h^{(0)})(rh^{(0)})' = 0 \quad \Rightarrow \quad f^{(0)} = 1 - \frac{A}{r}$$

- Put $f^{(0)}$ back into the equation, expand up to linear order and obtain $f^{(1)}$.

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Static spherically symmetric solutions

- N is obtained from f using the α-independent field equation

$$\frac{1}{2} r f' + f - 1 + \frac{r f N'}{N} = 0 \,.$$

- We arrive at the perturbative solutions

$$\begin{aligned} N(r) &= \left(1 - \frac{A}{r}\right)^{1/2} \\ &\quad + \frac{\alpha}{8}\left(1 - \frac{A}{r}\right)^{-1/2} \left[\frac{A - 4B}{r} + \left(1 - \frac{A}{2r}\right) \ln\left(1 - \frac{A}{r}\right)\right], \\ f(r) &= 1 - \frac{A + \alpha B}{r} + \frac{\alpha A}{8r} \ln\left(1 - \frac{A}{r}\right). \end{aligned}$$

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Static spherically symmetric solutions

- We have obtained explicitly the smooth deformation of the Schwarzschild solution.
- This perturbative solution coincides with the expansion in α of the exact solution up to linear order [JB, Restuccia and Sotomayor, in preparation]

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Static spherically symmetric solutions

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Asymptotia

- The expansion for large r is

$$N^2 = 1 - \frac{2GM}{r} - \frac{\alpha (GM)^3}{6r^3} + \mathcal{O}\left(\frac{1}{r^4}\right),$$

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The linearized theory with $\lambda = 1/3$

IR effective action

$$S = \int dt d^3x \sqrt{g} N \left(K_{ij} K^{ij} - \tfrac{1}{3} K^2 + R + \alpha a_i a^i \right) ,$$

- Perturbations of Minkowski

$$g_{ij} = \delta_{ij} + h_{ij} , \qquad \pi^{ij} = p_{ij} , \qquad N = 1 + n .$$

- Consequences of constraints/symmetries:

$$\begin{aligned} (\text{3D Diff}) + (\mathcal{H}^i = 0) \quad &\rightarrow \quad h_{ij} \text{ and } p_{ij} \text{ are transverse,} \\ \pi = \mathcal{H} = \mathcal{C} = 0 \quad &\rightarrow \quad p^T = h^T = n = 0 . \end{aligned}$$

Quadratic Hamiltonian

- We are left only with the conjugate pair h_{ij}^{TT}, p_{ij}^{TT}.

Quadratic physical Hamiltonian

$$H = \frac{1}{2} \int d^3x \left(p_{ij}^{TT} p_{ij}^{TT} + \partial_i h_{jk}^{TT} \partial_i h_{jk}^{TT} \right) ,$$

➜ Equal to the quadratic Hamiltonian of GR.

- Notice that there are not α terms in the quadratic Hamiltonian (constraints imply $n = 0$)

Conclusions and remarks

Consistency in general

- Complete, nonprojectable Hořava theory enjoys a consistent Hamiltonian formulation with $\lambda = 1/3$.
- The value $\lambda = 1/3$ is protected by the $\pi = \mathcal{C} = 0$ constraints.
- The theory has good properties of positiveness of the energy [JB, Restuccia and Sotomayor, 2013]

The degrees of freedom

- At $\lambda = 1/3$ there are not extra modes.
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Conclusions and remarks

Connection with general relativity

- General relativity can be recovered smoothly in the $\alpha \to 0$ limit.
- Since both theories share the same degrees of freedom, there are not discontinuities.
- There is not strong coupling problem associated to any extra mode.
- We give further support by looking at the static spherically symmetric solution near the $\alpha \to 0$ limit, obtaining explicitly the smooth deformation of the Schwarzschild solution.
- This coincides with expanding the exact solution in α.

Conclusions and remarks

The linearized theory

- $\lambda = 1/3$ yields general relativity at linearized level.
- The theory propagates gravitational waves as in general relativity.

Theory with $\lambda \neq 1/3$

- For $\lambda \neq 1/3$ the theory ha an extra mode.
- However, the limit $\alpha \to 0$ yields again GR.
- The problem of strong coupling should be re-analyzed carefully, since it is associated to the limit $\lambda \to 1$
- In this case, one must analyze possible discontinuities when decoupling the extra mode.