Quantum Gravity in the Southern Cone VI

# Quarkonium melting by anisotropy

Diego Trancanelli
University of São Paulo

Based on 1208.2672 (JHEP) with Chernicoff, Fernández, and Mateos
and 1105.3472 (PRL), 1106.1637 (JHEP) with Mateos

field theory from classical

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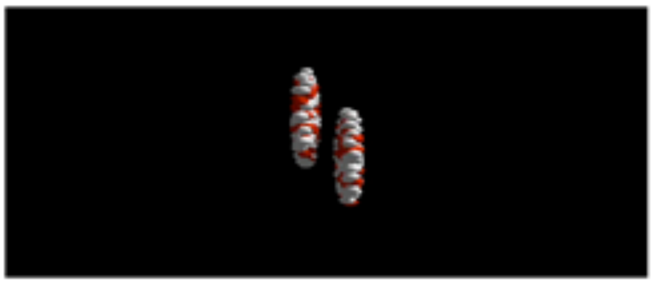
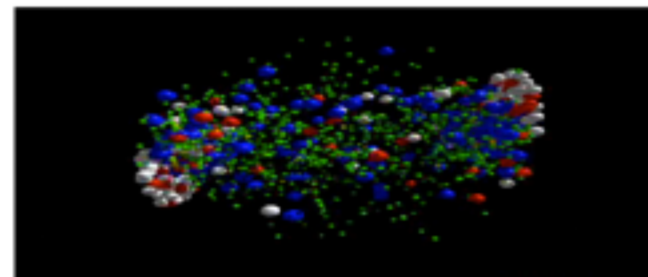
# Motivation

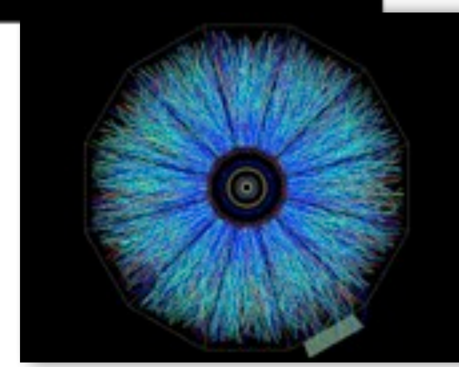
In a heavy ion collision at RHIC and LHC

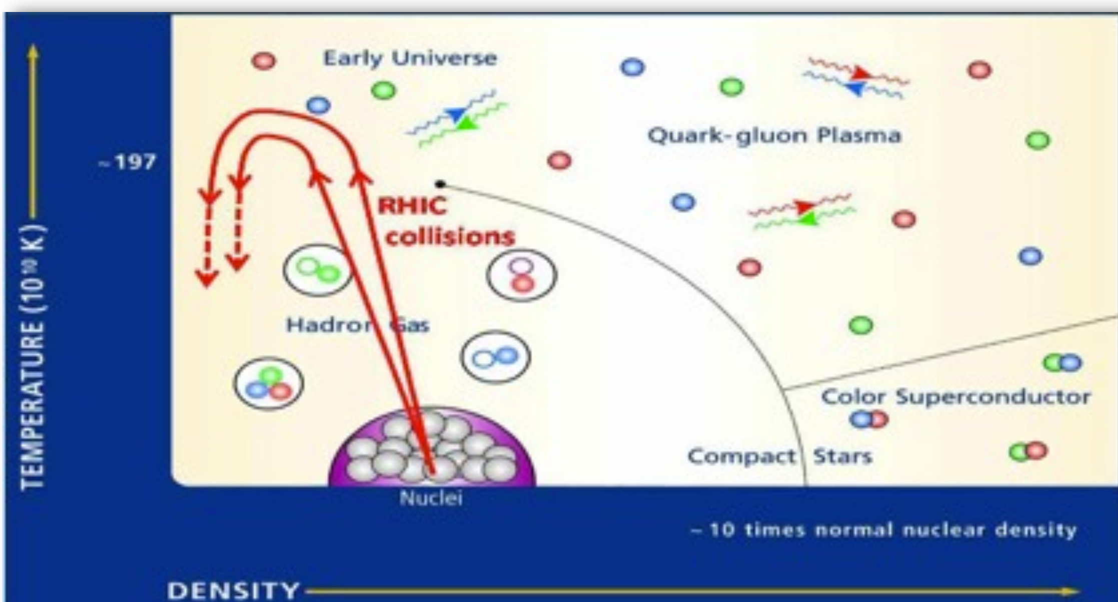
~400 nucleons go in



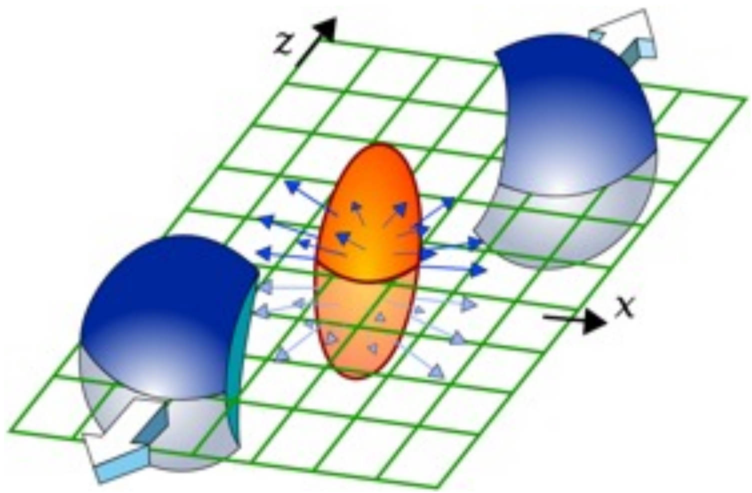
~10^4 hadrons come out







Collisions are generically non-central




$z \equiv$ longitudinal/beam direction

$x, y \equiv$ transverse plane

Collision $\quad \tau_{\text{out}} \quad \tau_{\text{iso}} \lesssim 1\text{fm}$

| Far from equilibrium | Anisotropic hydrodynamics | Isotropic hydrodynamics |
|---|---|---|
| | $P_\perp \neq P_\parallel$ | $P_\perp = P_\parallel$ |

After a short period of time the system is in thermal equilibrium

$$\sim 200 - 400\mathrm{MeV}$$

[Shuryak '03-'04]

It is a strongly coupled system (ideal fluid + small $\eta/s$)!

Some observables are sensitive to the presence of the anisotropy, e.g.:

- Energy loss and momentum broadening
- Photon/dilepton production
- Quarkonium physics

Quarkonium refers to charm-anticharm mesons ( $J/\Psi$, $\Psi'$ , $\chi_c$ , ... ) and bottom-antibottom mesons ( $\Upsilon$, $\Upsilon'$, ... ).

→ $J/\Psi$ mesons survive as bound states in a hot medium up to some dissociation temperature $T_d$ that is higher than the deconfinement temperature $T_c$ (lattice predicts: $T_d(J/\Psi) \simeq 2T_c$ ).

→ RHIC/LHC data: $J/\Psi$ - suppression in nucleus-nucleus collisions when compared to proton-proton or proton/nucleus collisions.

$J/\Psi$ mesons are screened in the quark gluon plasma.

It is interesting to understand this screening.

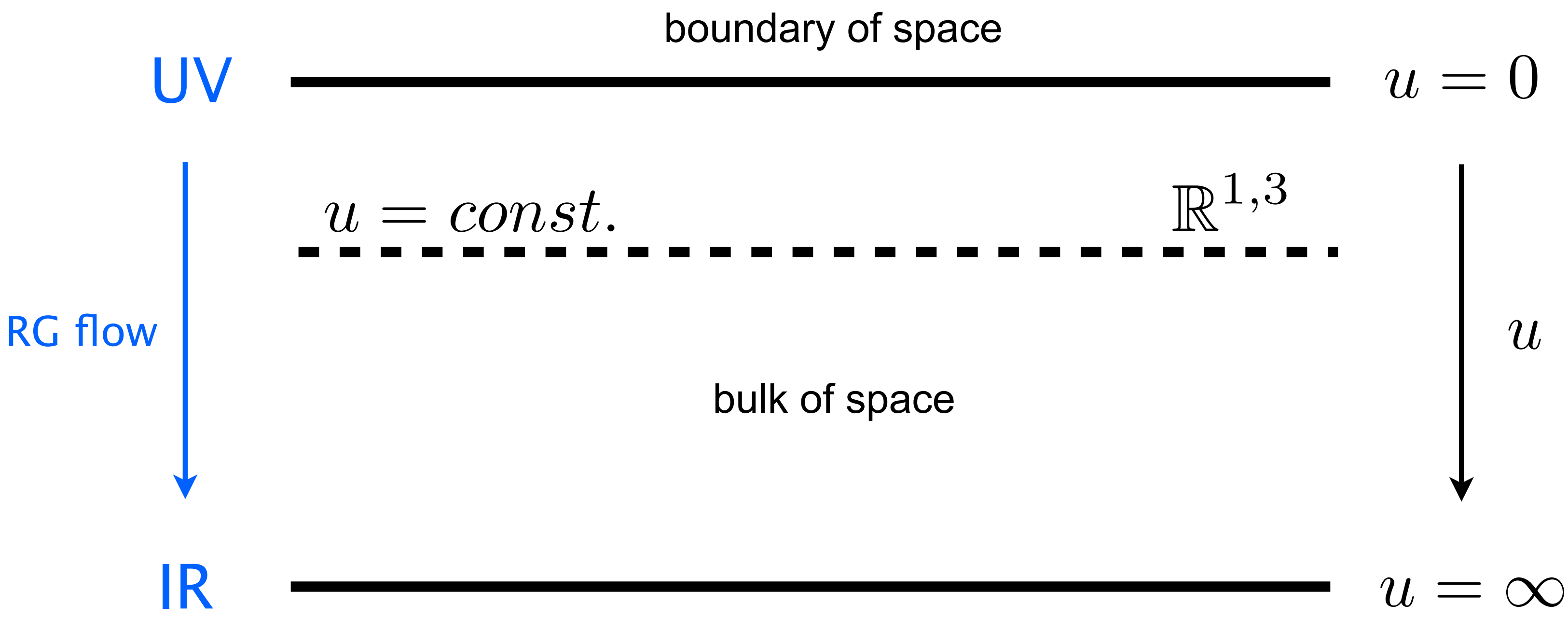
→ What is the effect of the anisotropy?

→ The mesons may move with significant transverse momentum through the hot medium. What is the effect of such “wind”?

Given the strong coupling nature of the system, the application of perturbative methods is problematic.

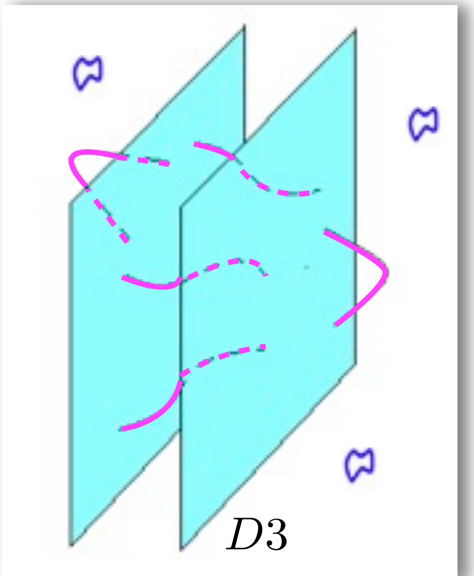
The gauge/string duality (a.k.a. AdS/CFT correspondence) might help to address these questions.

# Lightning review of AdS/CFT




This is a realization of the holographic principle:
a theory of quantum gravity in a region of space should be described by a non-gravitational theory on the boundary.

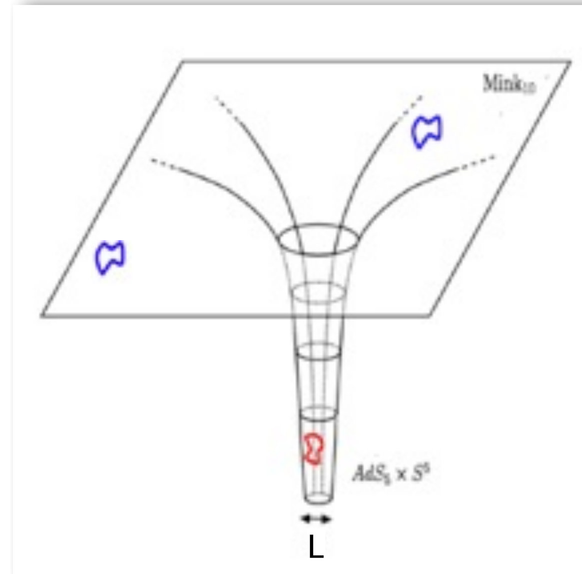
To construct an explicit example of such duality one needs string/brane toolkits: [Maldacena, '97]




$g_s N_c \ll 1$

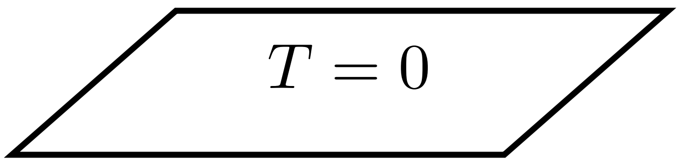
“dual”

$g_s N_c \gg 1$




N=4 SU(N$_c$) SYM = IIB strings in $AdS_5 \times S^5$

$\mathcal{N}=4$ SYM on $\mathbb{R}^{3,1}$ $=$ Type IIB on $AdS_5 \times S^5$

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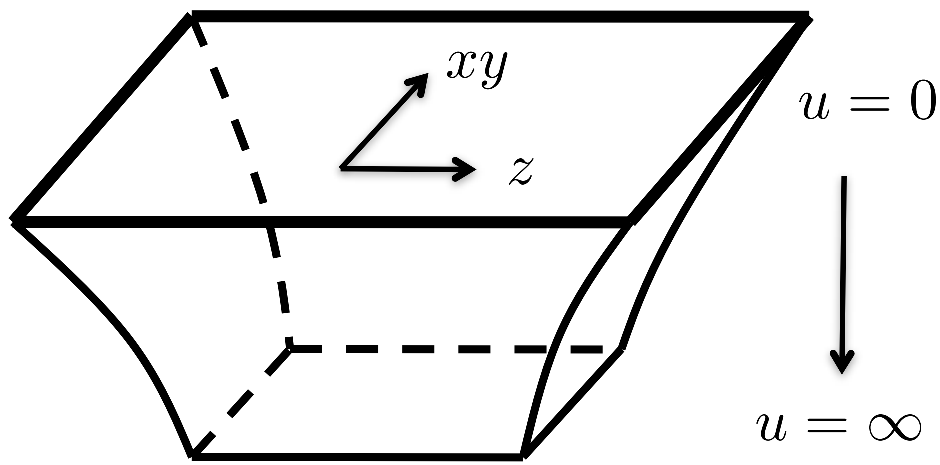



$$ds^2 = \frac{L^2}{u^2}\left[\,-dt^2 + d\vec{x}^2 + du^2\right]$$

Parameters:

$$\overset{\text{string theory}}{\boxed{\frac{g_s^2\ell_s^8}{L^8} \sim \frac{G_N}{L^8} \sim \frac{\ell_P^8}{L^8}}} \propto \overset{\text{gauge theory}}{\boxed{\frac{1}{N_c^2}}}, \qquad \overset{\text{string theory}}{\boxed{\frac{\ell_s^2}{L^2}}} \propto \overset{\text{gauge theory}}{\boxed{\frac{1}{\sqrt{\lambda}}}}$$

The limit in which the quantum string theory reduces to a classical theory of gravity

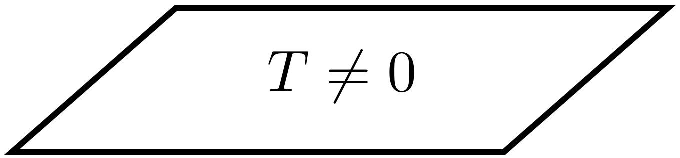
$$\frac{\ell_P^8}{L^8} \ll 1\,, \qquad \frac{\ell_s^2}{L^2} \ll 1$$

corresponds to a large Nc, strongly coupled gauge theory

$$N_c \gg 1\,, \qquad \lambda \gg 1$$

This is the regime relevant for this talk.

$\mathcal{N}=4$ SYM at finite temperature $=$ Schwarzschild AdS black hole

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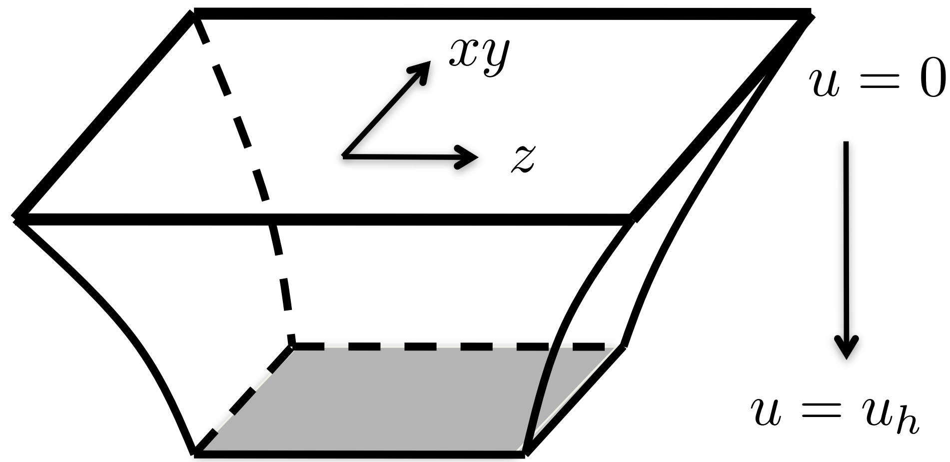



$$ds^2 = \frac{L^2}{u^2}\left[-f(u)dt^2 + d\vec{x}^2 + \frac{du^2}{f(u)}\right]$$

with $$f(u) = 1 - \frac{u^4}{u_h^4}$$

We will refer to this metric as the isotropic case.

Of course, N=4 SYM is not QCD. Nonetheless, at the typical temperatures of the QGP, it is a ‘reasonable’ proxy:

| | QCD | N=4 SYM |
|---|---|---|
| T=0 | Nc=3, confining,... | Nc large, conformal, supersymmetric,... |
| T>Tc | strongly coupled plasma, fundamental matter,... | strongly coupled plasma, fundamental and adjoint matter,... |
| T>>Tc | weakly coupled | strongly coupled |

Moreover, certain observables might be quite universal.

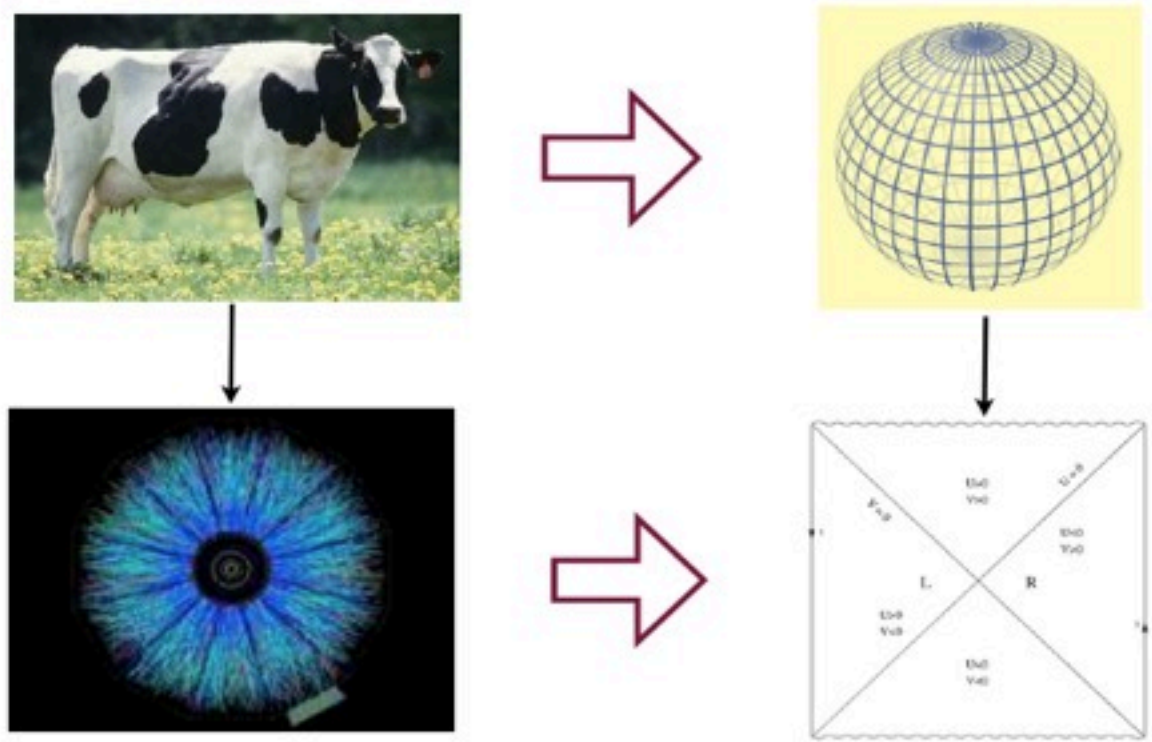
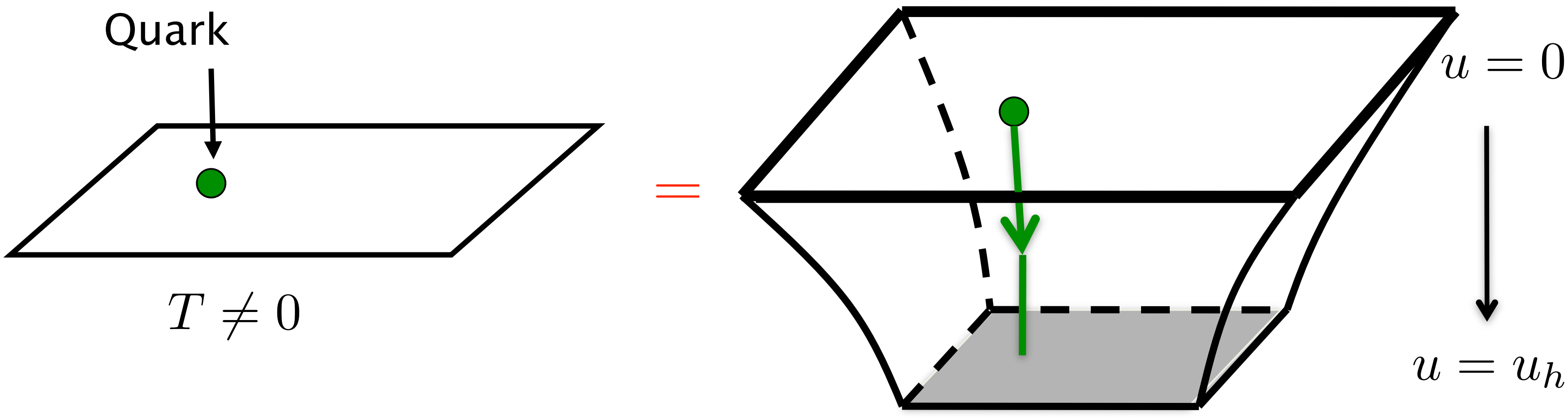



**FIGURE 3.** Nature confronts us sometimes with complicated objects such as the one in the upper left corner. In simple situations, as for example in collisions at ultrarelativistic energies, it is sufficient to replace it by the simpler object in the right upper corner. The same philosophy can be applied to the strongly coupled quark gluon plasma!

[Picture credit: K. Landsteiner]

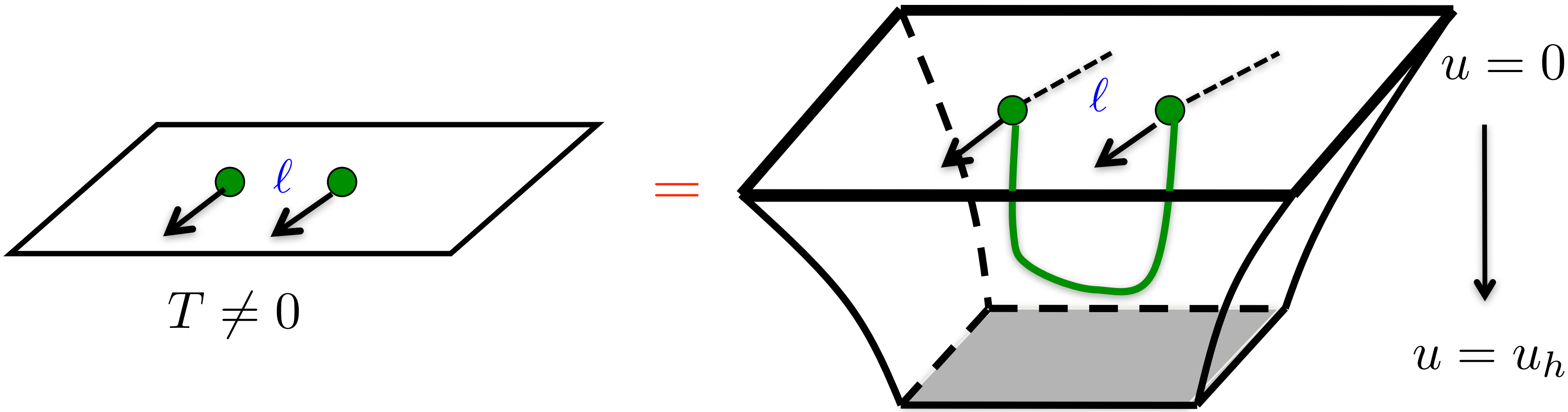
## External quark = Fundamental string




- A fundamental string extending from the boundary at $u = 0$ to the horizon corresponds to an infinitely massive quark.

- The string endpoint represents the quark, while the rest of the string models the gluon field.

meson moving at constant velocity = U-shaped string moving at constant velocity




We are interested in studying an anisotropic strongly coupled plasma.

How can we do that using the AdS/CFT correspondence?

Addressing time evolution and anisotropy at the same time is hard.

As a first step, let's study a static anisotropic plasma: this is a good approximation if $t_{char} << t_{evolution}$ .

We want a gravity solution which is:

* static and spatially anisotropic

* with a horizon and regular on and outside the horizon

* with AdS boundary conditions in the UV

It is hard to find a source of anisotropy that satisfies all the requirements above:

* vector current: dissipates, non static

* anisotropic stress-tensor imposed by hand:
  gives a singularity

[Janik-Witaszczyk '08]

* non-commutativity: destroys AdS in the UV

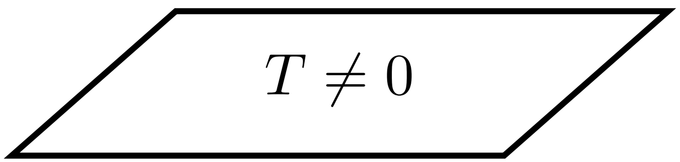
[Hashimoto-Itzhaki '99]
[Maldacena-Russo '99]

[Mateos, DT, '11]

The gauge theory that we will consider is a deformation of $\mathcal{N}=4$ SYM.

$$S = S_{\mathcal{N}=4} + \int \theta(\vec{x}) \mathrm{Tr} F \wedge F$$

with $\theta(\vec{x}) \propto z$

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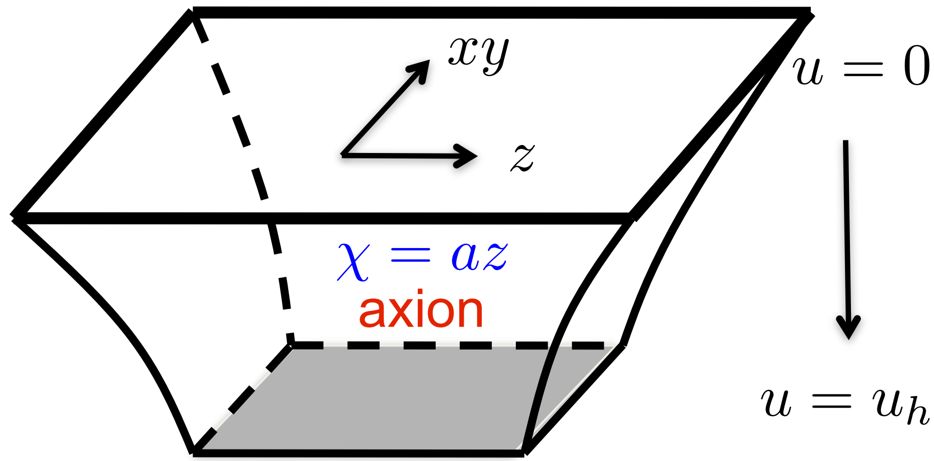



$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2_{\mathrm{YM}}} = \chi + i e^{-\phi}$$

This triggers an RG flow to an IR Lifshitz fixed point:

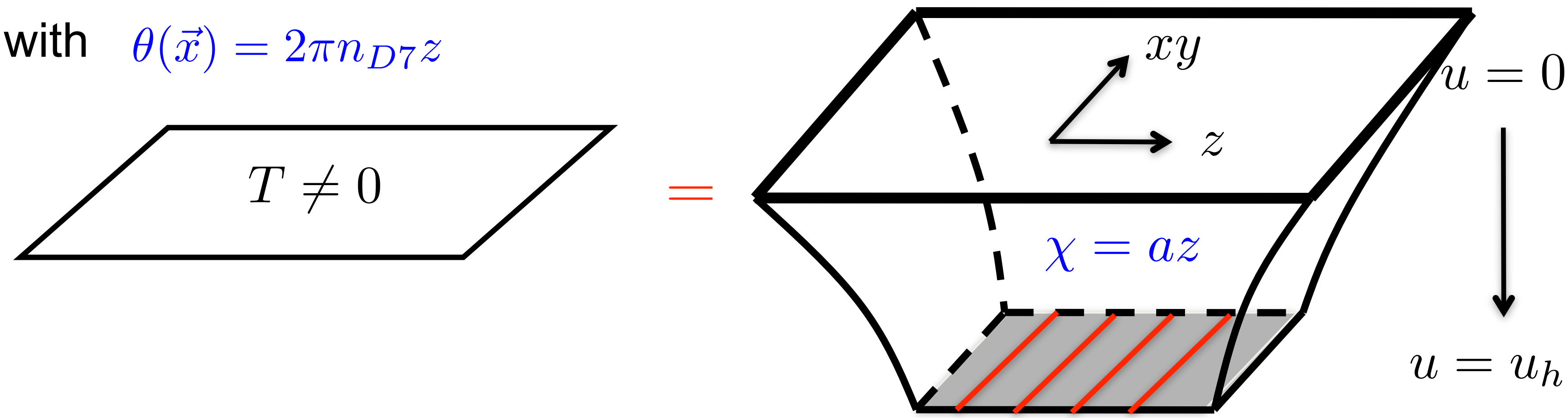
$$(t, x, y, u) \to (kt, kx, ky, ku)\,, \qquad z \to k^{2/3} z$$

[Mateos, DT, '11]

The gauge theory that we will consider is a deformation of $\mathcal{N}=4$ SYM

$$S = S_{\mathcal{N}=4} + \int \theta(\vec{x}) \mathrm{Tr} F \wedge F$$

with $\theta(\vec{x}) = 2\pi n_{D7} z$




The axion is magnetically sourced by D7-branes

| | $t$ | $x$ | $y$ | $z$ | $u$ | $S^5$ |
|---|---|---|---|---|---|---|
| $N_c\ D_3$ | x | x | x | x | | |
| $n_{D_7}\ D_7$ | x | x | x | | | x |

$$a = \frac{\lambda n_{D7}}{4\pi N_c}$$

We take a 10-dim. Ansatz which is a direct product:

$$\mathcal{M}_5 \times S^5$$

(recall that the D7-branes preserve the SO(6) isometry).

The action reduces to 5-dim. axion-dilaton AdS gravity:

$$S = \frac{1}{16\pi G_N} \int \sqrt{-g} \left( \mathcal{R} + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2} e^{2\phi} (\partial\chi)^2 \right)$$

EH term

−ve CC

dilaton (scalar)

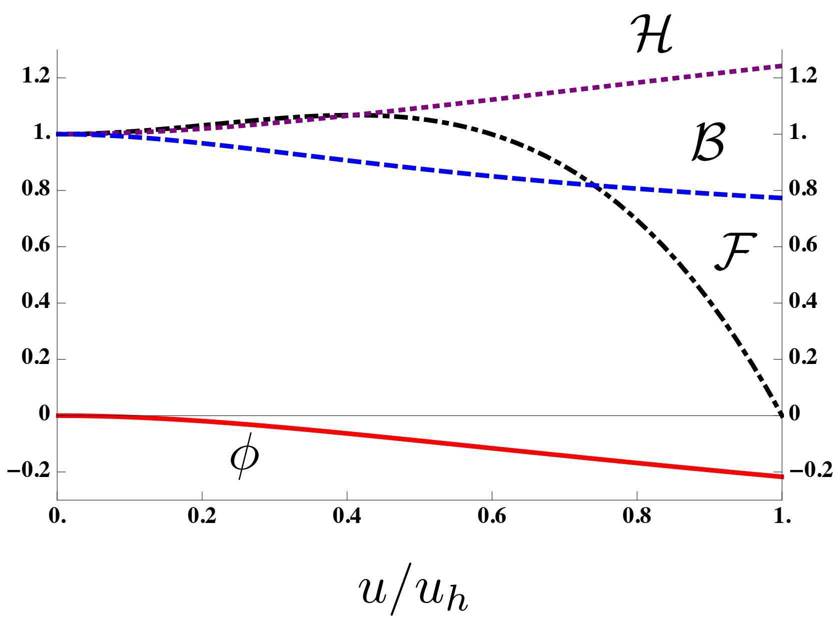
axion (scalar)

Important: this has a full-fledged embedding in string theory, it is not a bottom-up model.

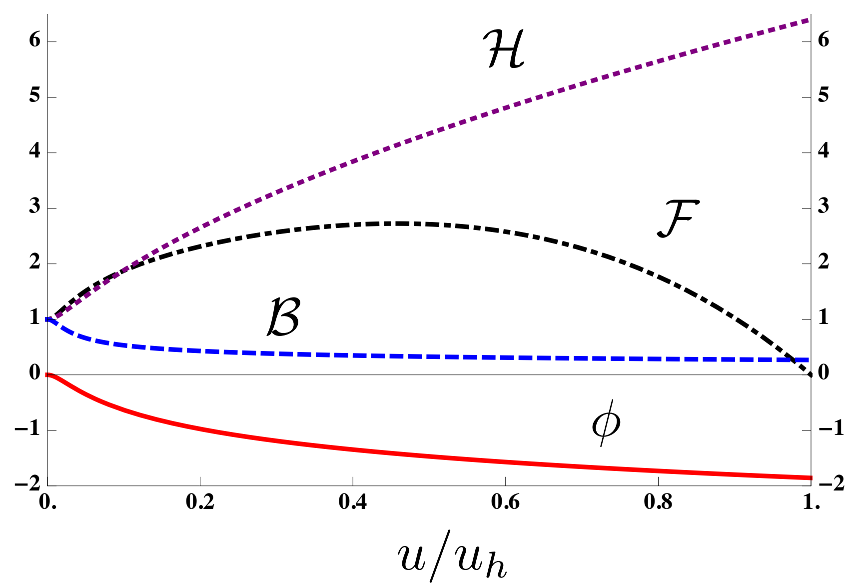
The anisotropic metric is

$$ds^2 = \frac{L^2}{u^2}\left[ -\mathcal{F}(u)\mathcal{B}(u)dt^2 + dx^2 + dy^2 + \mathcal{H}(u)dz^2 + \frac{du^2}{\mathcal{F}(u)} \right]$$

$\chi(z) = az$ and $\phi \equiv \phi(u)$




$a/T = 4.4$



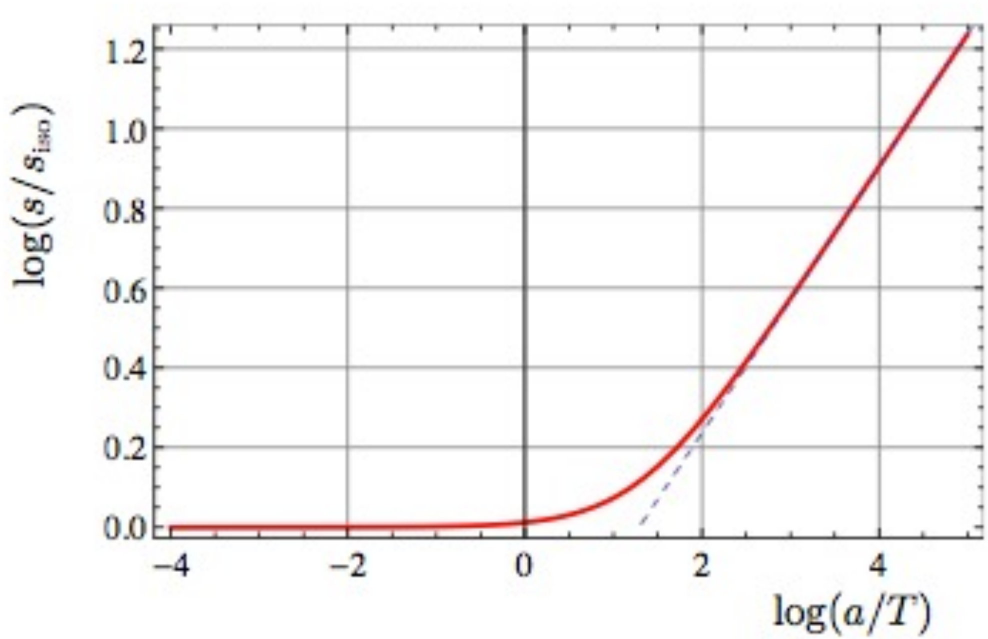

$a/T = 86$

→ Regular on and outside the horizon

→ RG flow between AdS (UV) and Lifshitz type (IR)

→ The entropy density interpolates between

$T \gg a \quad s \sim T^3$ and $T \ll a \quad s \sim a^{1/3} T^{8/3}$



## Holographic stress tensor

The 1-point function of boundary operators (e.g. the stress-tensor of the theory) can be read off from the asymptotic fall-off of the bulk fields, via a procedure called holographic renormalization.

This procedure gives us expressions for the energy and pressures of the plasma:

[Papadimitriou, '11]

$$\langle T_{ij} \rangle = \begin{pmatrix} E & & & \\ & P_x & & \\ & & P_y & \\ & & & P_z \end{pmatrix}$$

The stress tensor is:

* anisotropic $P_x = P_y \equiv P_{xy} \neq P_z$

* conserved (translation invariance is preserved in this state)

* non traceless $\langle T_i^i \rangle = \frac{N_c^2}{48\pi^2} a^4$

The conformal anomaly has important consequences:

$$E(a,T) = a^4 f\left(\frac{T}{a}\right) + \boxed{a^4 \frac{N_c^2}{48\pi^2} \log\left(\frac{a}{\mu}\right)}$$

$\mu$ is an arbitrary scale introduced in the renormalization process.

# Thermodynamics and instabilities

We work in the canonical ensemble with free energy

$$F = E - Ts \qquad\qquad dF = -s dT + \Phi da$$

The anisotropy is responsible for the difference in pressures:

$$P_z - P_{xy} = \Phi a$$

The necessary and sufficient conditions for local thermodynamic stability are:

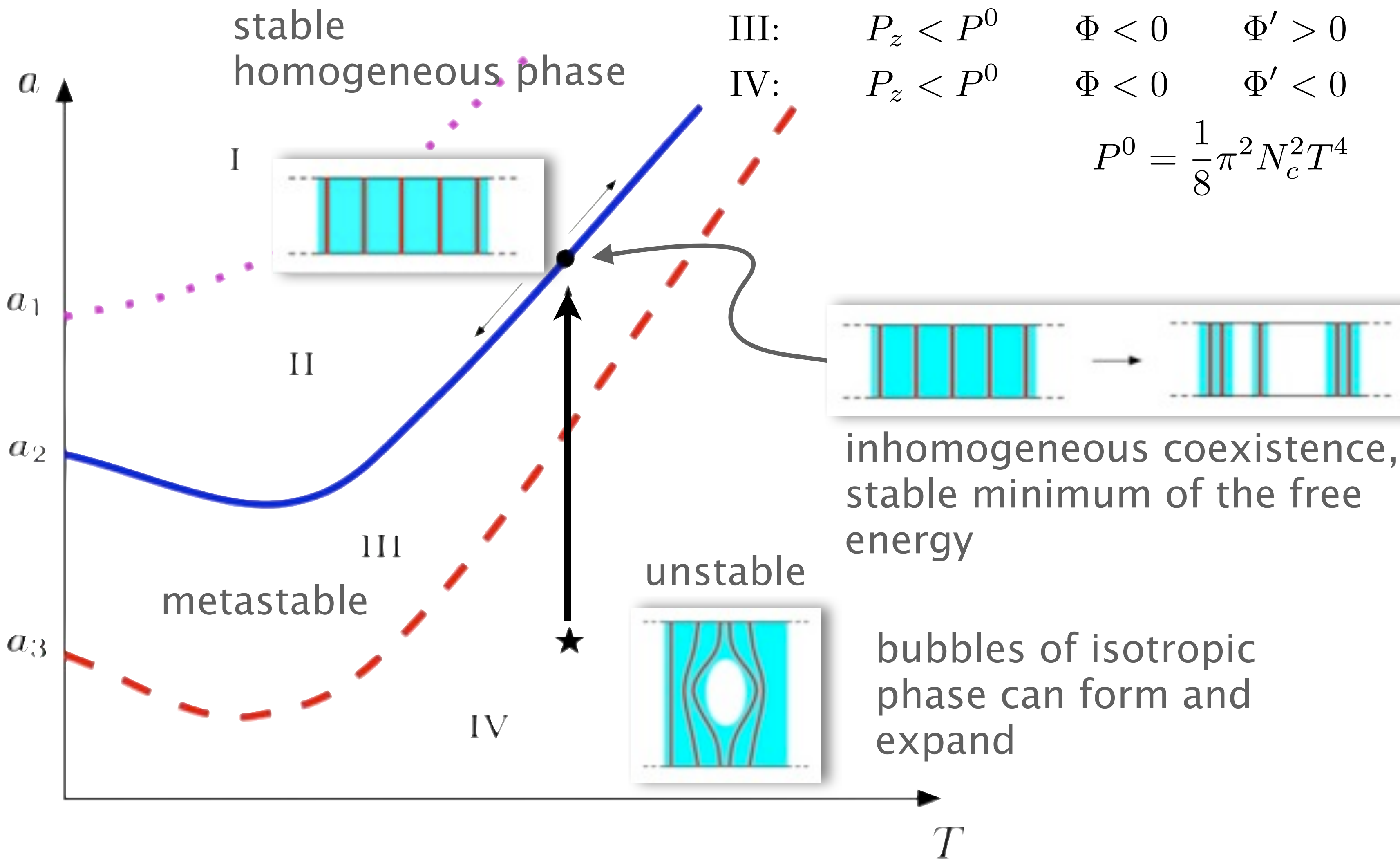
$$\left(\frac{\partial E}{\partial T}\right)_a > 0$$ ✓

$$\left(\frac{\partial \Phi}{\partial a}\right)_T > 0$$ ✗

The canonical ensemble phase diagram is:

| | | | |
|---|---|---|---|
| I: | $P_z > P^0$ | $\Phi > 0$ | $\Phi' > 0$ |
| II: | $P_z > P^0$ | $\Phi < 0$ | $\Phi' > 0$ |
| III: | $P_z < P^0$ | $\Phi < 0$ | $\Phi' > 0$ |
| IV: | $P_z < P^0$ | $\Phi < 0$ | $\Phi' < 0$ |

$$P^0 = \frac{1}{8}\pi^2 N_c^2 T^4$$



# Quarkonium physics

→ The screening length $L_s$ is defined as the separation between a $q\bar{q}$ such that for $\ell < L_s$ $(\ell > L_s)$ it is energetically favorable for the pair to be bound (unbound).

→ We will determine $L_s$ by comparing the action $S(\ell)$ of the $q\bar{q}$ pair to the action $S_{\mathrm{unb}}$ of the unbound system:

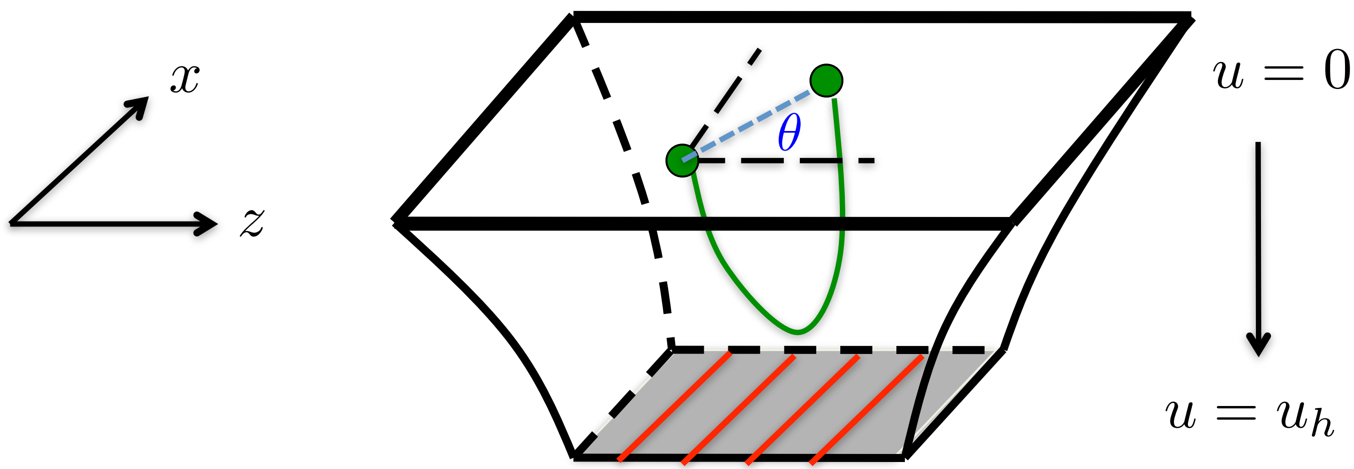
$$\Delta S(\ell) = S(\ell) - S_{\mathrm{unb}}$$

| |
|---|
| The screening length is the maximum value of $\ell$ for which $\Delta S$ is positive |

(In Euclidean signature, this criterion corresponds to determining which configuration has the lowest free energy).

## Warm-up: static case

Given the rotational symmetry in the xy-plane, the most general case is to consider the dipole (= $q\bar{q}$ pair) in the xz-plane.




We choose the static gauge $\tau = t$ , $\sigma = u$ and the embedding

$$Z(u) = z(u)\cos\theta \qquad\qquad X(u) = x(u)\sin\theta$$

The action for the U-shaped string takes the form

$$S = -\frac{L^2}{2\pi\alpha'} 2 \int dt \int_0^{u_{\max}} du \frac{1}{u^2} \sqrt{\mathcal{B}(1 + \mathcal{F}\mathcal{H}\cos^2\theta z'^2 + \mathcal{F}\sin^2\theta x'^2)}$$

Two conserved momenta $\Pi_z$ and $\Pi_x$ associated to translation invariance in the x and z directions. The on-shell action can be written as

$$S = -\frac{L^2}{2\pi\alpha'} 2 \int dt \int_0^{u_{\max}} du \frac{1}{u^2} \frac{\mathcal{B}\sqrt{\mathcal{F}\mathcal{H}}}{\sqrt{\mathcal{F}\mathcal{B}\mathcal{H} - u^4(\Pi_z^2 + \mathcal{H}\Pi_x^2)}}$$

where the turning point $u_{\max}$ is determined by the conditions

$$x'(u_{\max}) = z'(u_{\max}) \to \infty$$

$$\mathcal{F}\mathcal{B}\mathcal{H} - u^4(\Pi_z^2 + \mathcal{H}\Pi_x^2)|_{u_{\max}} = 0 \qquad \Rightarrow \qquad u_{\max} \equiv u_{\max}(a, T, \Pi_i)$$

From the boundary conditions we obtain the relation between the momenta $\Pi_z$, $\Pi_x$ and the quark-antiquark separation $\ell$

$$\frac{\ell}{2} = \int_0^{u_{\max}} du X' = \int_0^{u_{\max}} du Z'$$

Finally, to determine $L_s$, we need to subtract from the U-shaped string action that of the unbound pair (i.e. two straight strings)

$$S_{\mathrm{unb}} = -\frac{L^2}{2\pi\alpha'} 2 \int dt \int_0^{u_{\mathrm{h}}} du \frac{\sqrt{\mathcal{B}}}{u^2}$$

The UV divergences associated to integrating all the way to the boundary cancel out in the difference, and in the dipole rest frame there are no IR divergences.

The isotropic result $a = 0$

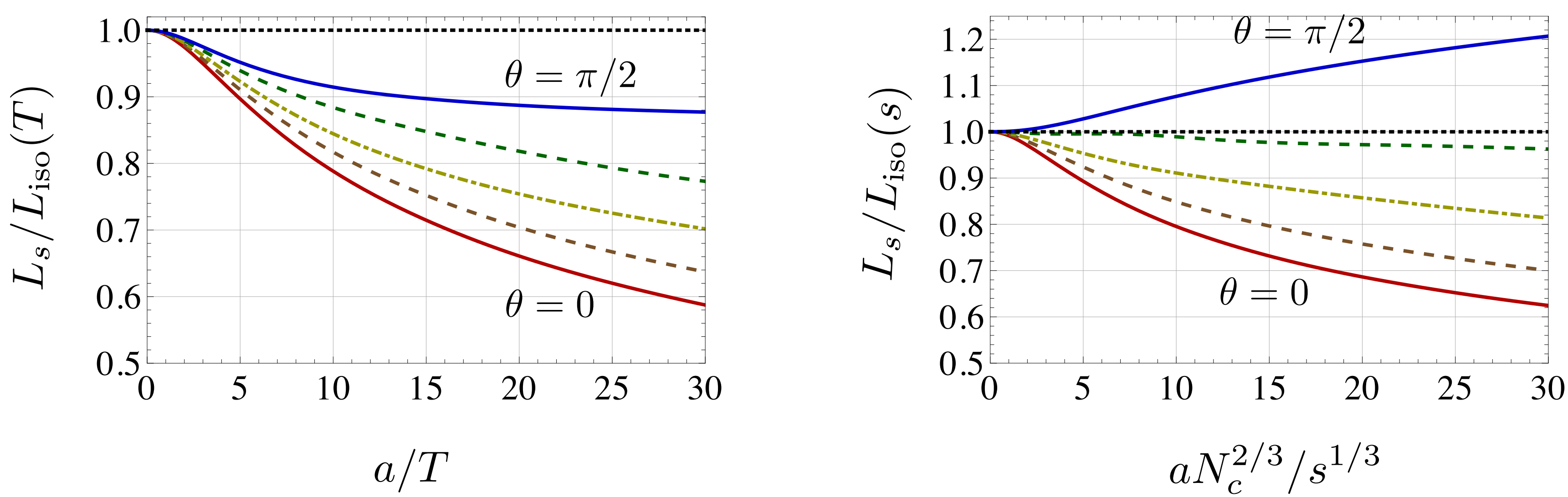
$$L_s \simeq \frac{0.24}{T}$$

or

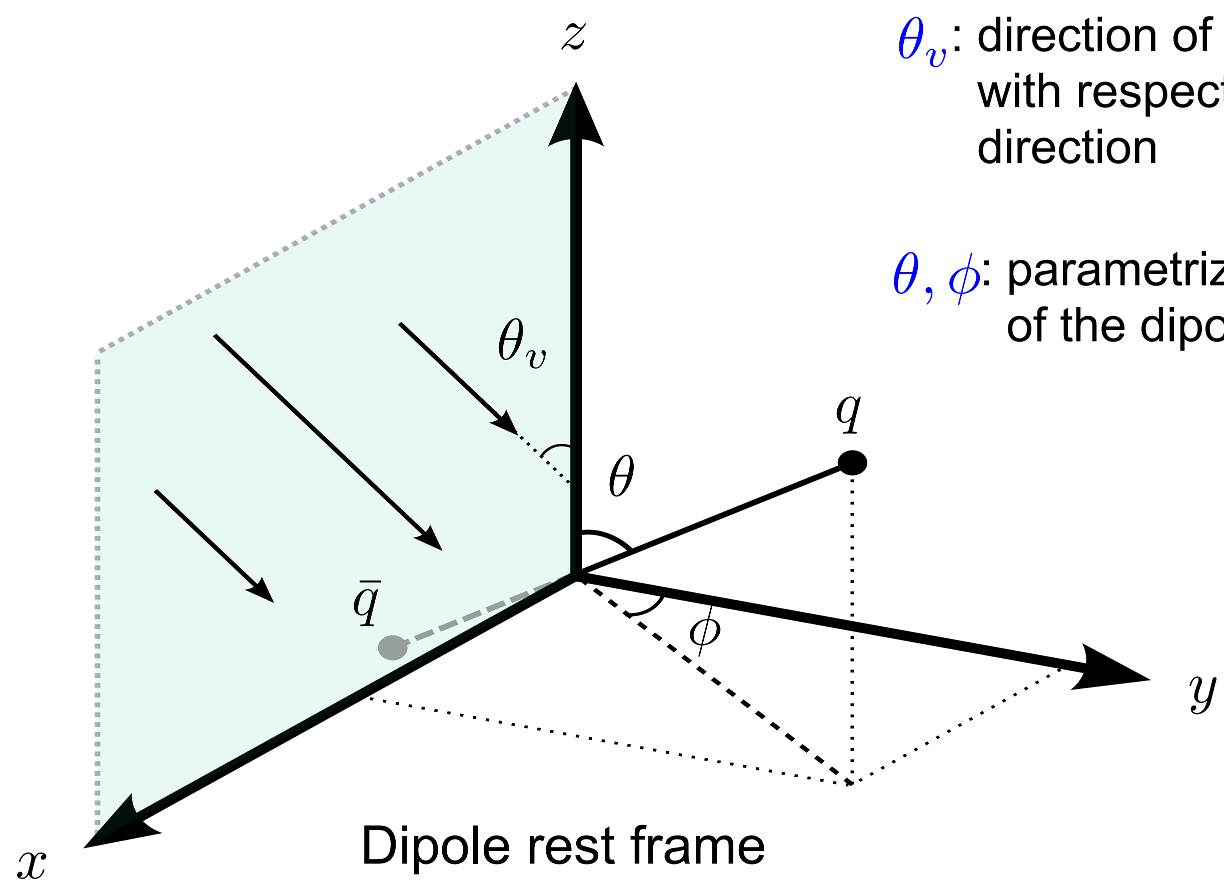
$$L_s \simeq 0.24\left(\frac{\pi^2 N_c^2}{2s}\right)^{1/3}$$

[Rey et al; Brandhuber et al]

Anisotropic results normalized to the isotropic ones:



# Dipole in a plasma wind




Dipole rest frame

$\theta_v$: direction of the wind (velocity) with respect to the anisotropic direction

$\theta, \phi$: parametrize the orientation of the dipole

As in the static case, to determine the screening length we need to compare the actions of a bound and an unbound quark-antiquark pair.

Let us summarize the important steps:

→ The unbound action is that of 2 strings moving with constant velocity

[Chernicoff, Fernandez, Mateos, DT, `12]

→ The position of the turning point is now $u_{\max} \equiv u_{\max}(a, T, \Pi_i, v)$

We will first consider the ultra-relativistic limit.
Two reasons for doing this:

- → It is relevant for the experiments.
- → It can be understood analytically.

It is easy to check that for a fixed separation of the string endpoints

$$\lim_{v \to 1} u_{\max} \to 0$$

Then the dynamics of the string can be determined using the near boundary expansion of the metric (known analytically).

[Mateos, DT, `11]

(We send first the quark mass to infinity and then $v \longrightarrow 1$)

For the isotropic case: $L_s(T, v) \sim (1 - v^2)^{1/4}$ [Liu et al.]

We want to compare the two actions in the ultra-relativistic limit and see how they scale with $(1-v^2)$.

After some algebra:

$$\Delta S(l,v) \sim (1-v^2)^{-1/2} \times (\text{finite integral})$$ motion outside the transverse plane

$$\Delta S(l,v) \sim (1-v^2)^{-1/4} \times (\text{finite integral})$$ motion within the transverse plane

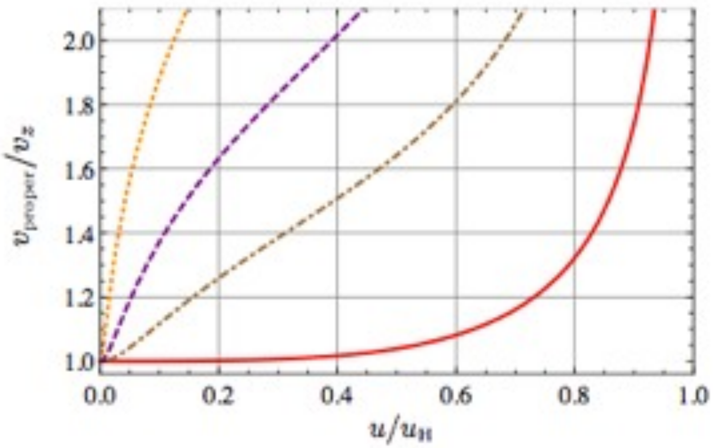
Finally, using the boundary conditions, we obtain how the screening length scales in the ultra-relativistic limit:

$$\boxed{L_s \sim \begin{cases} (1-v^2)^{1/2} \times \mathcal{I}(a,\Pi_i,\mathcal{O}(u^6)) \text{ if } \ \theta_v \neq \pi/2 \\ \\ (1-v^2)^{1/4} \times \mathcal{J}(a,T,\Pi_i,\mathcal{O}(u^6)) \text{ if } \ \theta_v = \pi/2 \end{cases}}$$

Another interesting limit to consider: $a/T \gg 1$

→ The proper velocity along "z" of a point on the string at some $u$

$$v_{\rm proper}(u) = v_z \sqrt{\frac{\mathcal{H}(u)}{\mathcal{F}(u)\mathcal{B}(u)}}$$




$\mathcal{H}(u)$ increases from $u=0$ to $u=u_h$, more steeply as $a/T$ increases, $\mathcal{F}(u)\mathcal{B}(u)$ has the opposite behavior:

→ Maximum value of $u_{\rm max}$ beyond which $v_{\rm proper}$ becomes superluminal

→ We can show that for $v_z \neq 0$, $u_{\rm max}$ decreases as $a/T$ increases

$$\lim_{a/T\gg 1} u_{\rm max} \to 0 \quad \Rightarrow$$

Again, use near boundary metric to study $L_s$. This only depends on a.

→ It is straight-forward (dimensional analysis) to check that

for a fixed value of $v_z \neq 0 \quad \Rightarrow \quad L_s \sim a^{-1}$

The $a/T \gg 1$ limit can be understood as $a \to \infty$ at fixed $T$, or as $T \to 0$ at fixed $a$.

→ Even as $T \to 0$ a meson of size $\ell$ will dissociate above

$$a_{diss} \sim \ell^{-1}$$

For motion within the transverse plane ($v_z = 0$) or static dipoles:

$$L_s \propto T^{-1}$$

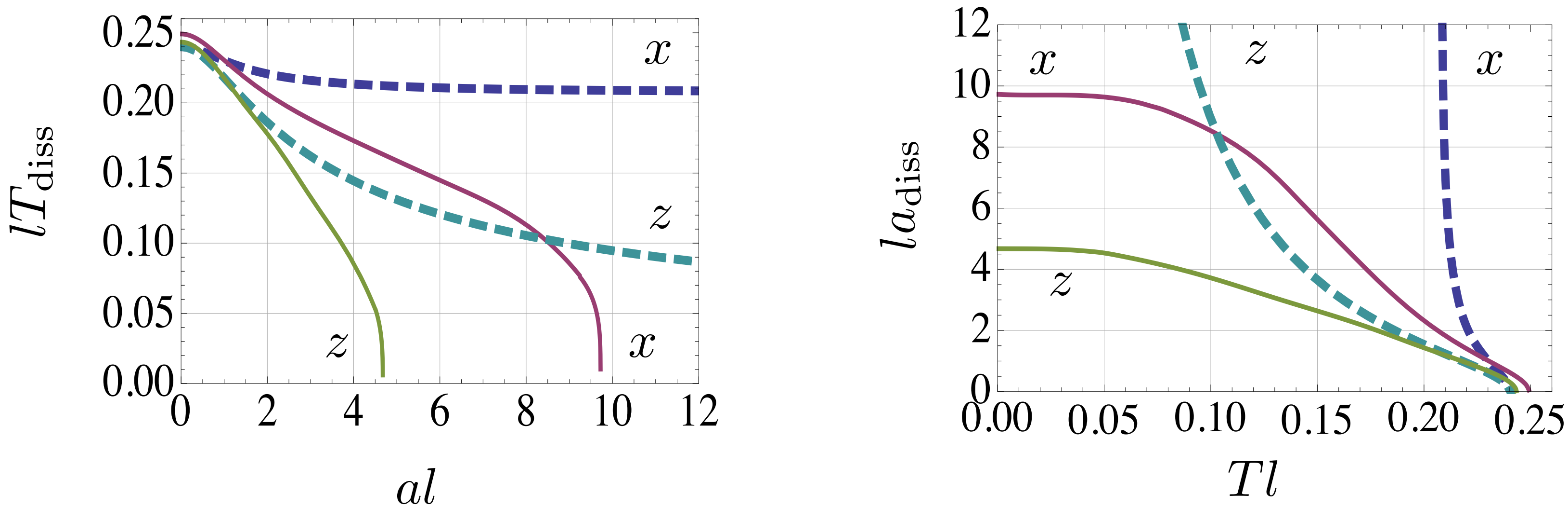
- So far we have studied $L_s(a,T,v)$, but clearly we could also think of $T_{\rm diss}(a,\ell,v)$ and $a_{\rm diss}(T,\ell,v)$

i.e. $T_{\rm diss}(a,\ell,v)$ characterizes the dissociation of a $q\bar{q}$ pair of fixed size $\ell$ in a plasma with a given degree of anisotropy $a$. Analogously for $a_{\rm diss}$

- Using our results for the screening length, we can study the behavior of $T_{\rm diss}(a,\ell,v)$ and $a_{\rm diss}(T,\ell,v)$

Numerical results:

- - - at rest
—— moving along the z-direction ( $v = 0.45$ )




→ As explained before, even at zero temperature a meson of size $\ell$ will dissociate if the anisotropy is increased above

$$a_{\rm diss}(T=0,\ell) \propto 1/\ell$$

and the proportionality "constant" is a decreasing function of the velocity.

# Conclusions

• Complete characterization of the screening length for quarkonia moving with arbitrary velocities and orientations in an anisotropic, strongly coupled plasma.

• Mesons dissociate above a certain critical value of the anisotropy, even at zero temperature.

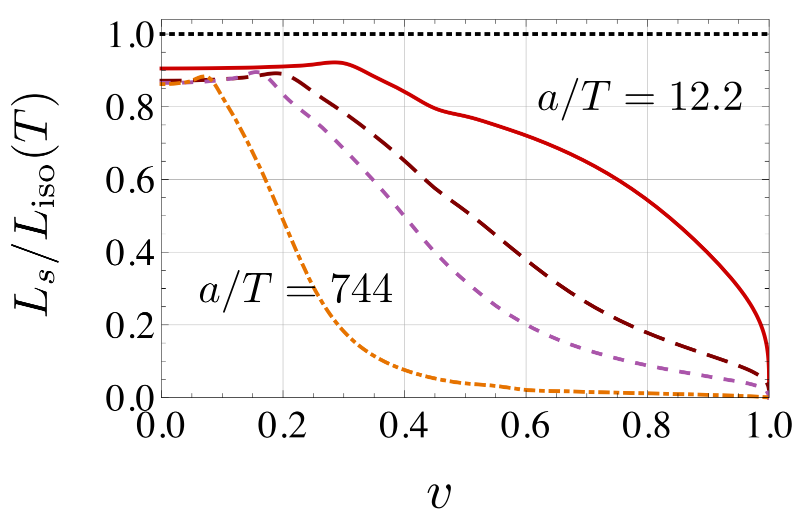
• There is a limiting velocity (<1) for mesons moving through the plasma, even at zero temperature.

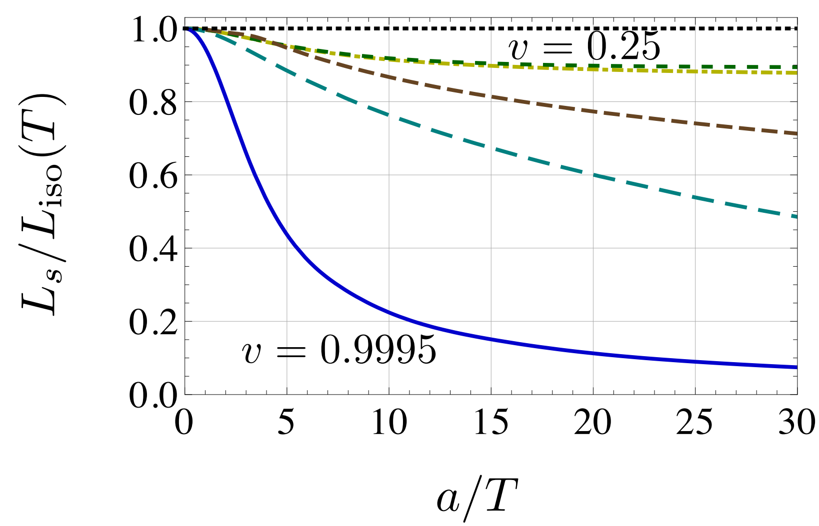
• The gravity calculation involves only the coupling of the string to the background metric, so any anisotropy source that gives rise to a qualitatively similar metric (in particular, a non-boost invariant $O(u^2)$ term in the asymptotic expansion) will yield qualitatively similar results.

# Extra slides

A glimpse of the numerical results:

→ Wind along "z" and dipole along "x"

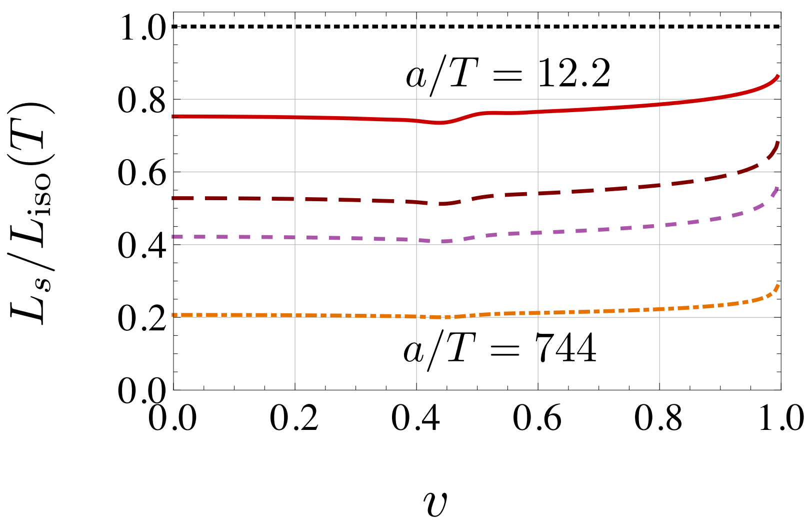


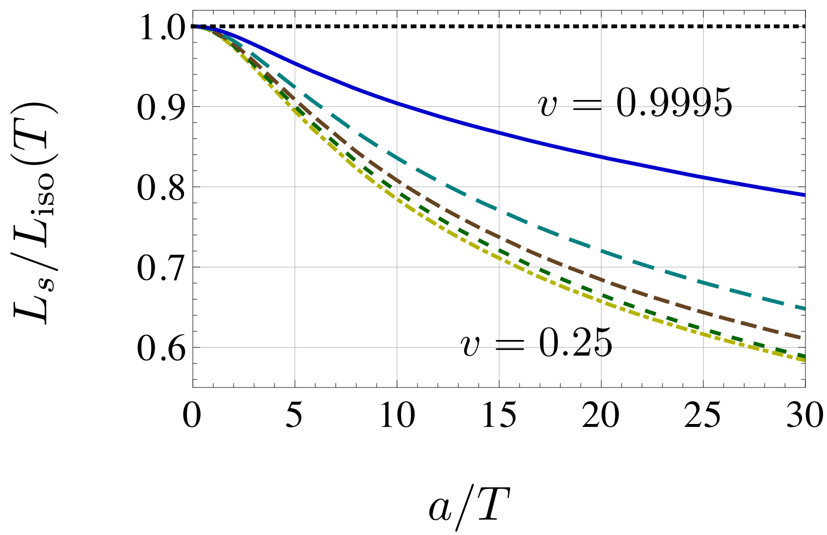




$L_s/L_{\rm iso}$ vanishes as $(1-v^2)^{1/4}$ in the limit $v \to 1$

A glimpse of the numerical results:

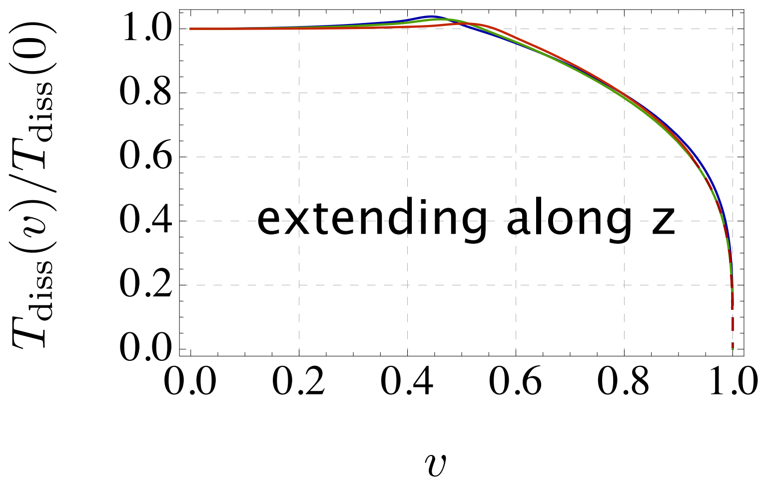
→ Wind along "x" and dipole along "z"

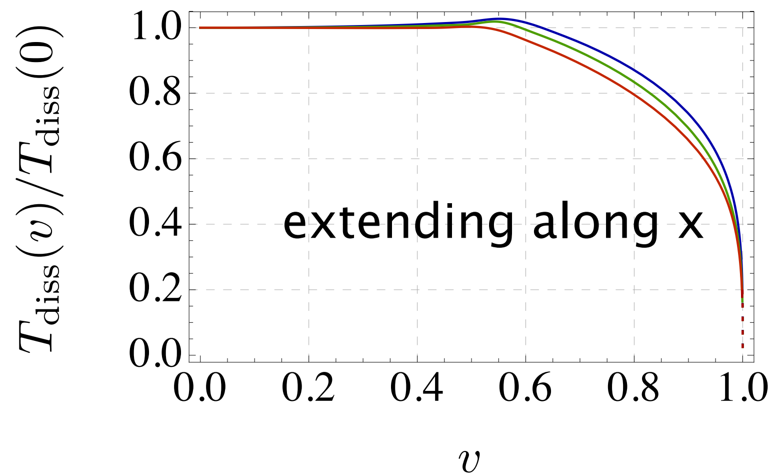



$L_s/L_{\rm iso}$ approaches a finite, non-zero value as $v \to 1$

Numerical results for motion within the transverse plane:



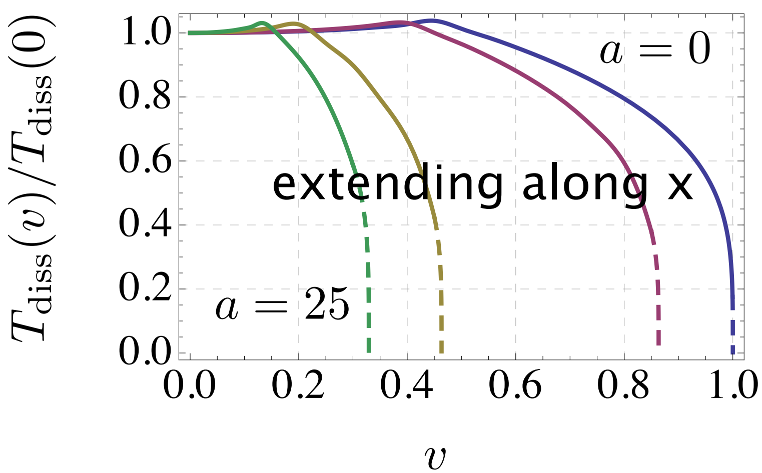



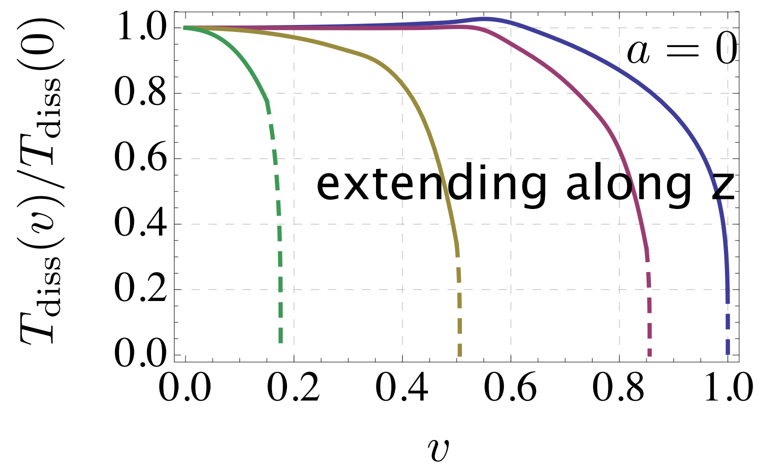

The behavior is qualitatively analogous to that of the isotropic case

$$T_{\mathrm{diss}}(v) \simeq T_{\mathrm{diss}}(0)(1 - v^2)^{1/4}$$

[Liu et al.]

Numerical results for motion outside the transverse plane:

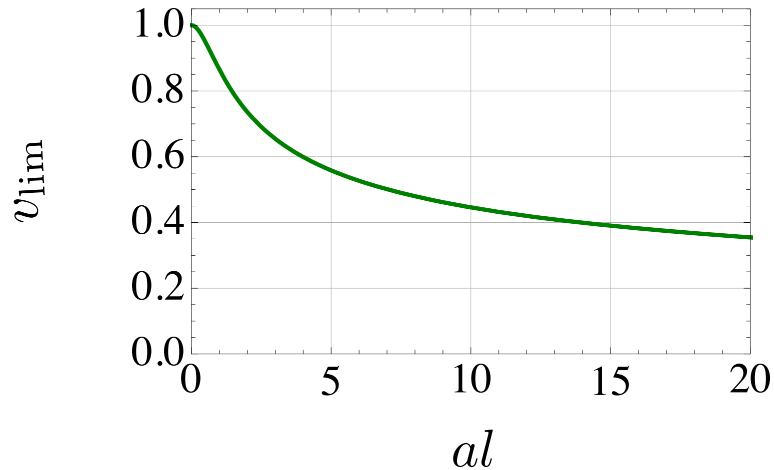



There is a limiting velocity $v_{\rm lim} < 1$ even at zero temperature!

The anisotropy is responsible for the dissociation.

Limiting velocity for a fixed anisotropy and $T = 0$, meson oriented along the x-direction and moving along the z-direction

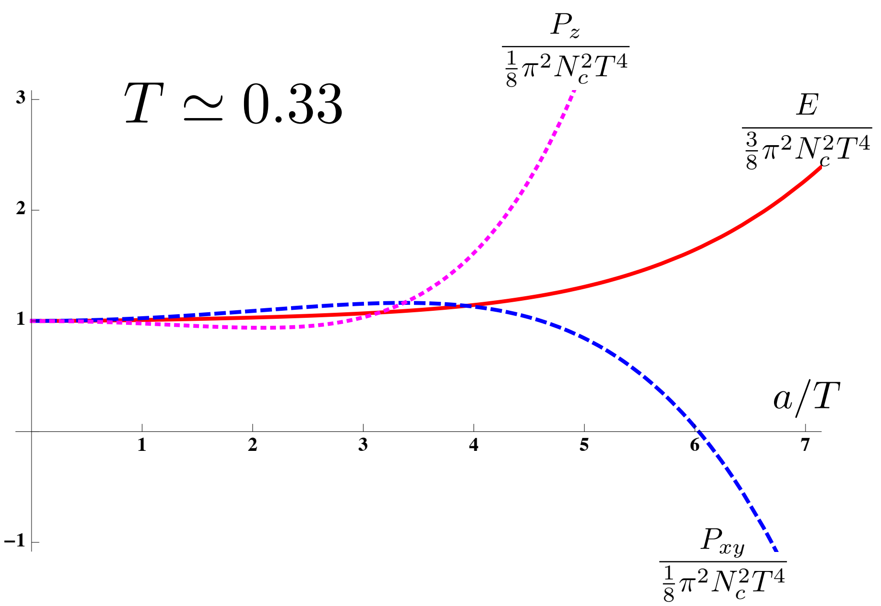


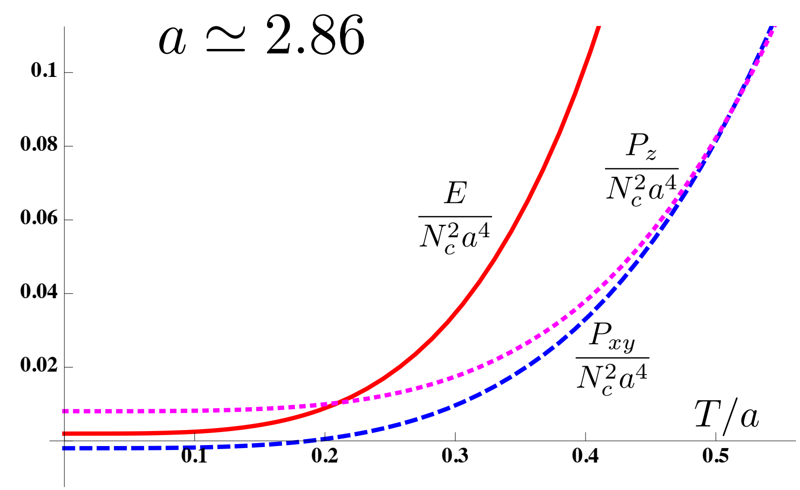

In the case of ultra-relativistic motion and $a/T \gg 1$:

$$a_{\mathrm{diss}} \sim \frac{1}{\ell}(1 - v_{\mathrm{lim}}^2)^{1/2} \text{ if } \theta_v \neq \pi/2$$

$$T_{\mathrm{diss}} \sim \frac{1}{\ell}(1 - v_{\mathrm{lim}}^2)^{1/4} \text{ if } \theta_v = \pi/2$$

Examples:




* as $a \to 0$ energy and pressures approach their isotropic values

* specific heat and speeds of sound are positive

* energy and pressures can become negative, but still bounded from below