Quantum Gravity in the Southern Cone VI

Maresias, Brazil. September 11-14, 2013

From String Fluxes to DOUBLE and EXTENDED Field Theory

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Use

G.A, P. Cámara, A. Font, L.Ibañez (JHEP 0605:070,2006).
G.A , P.Cámara, A. Rosabal (Nucl.Phys.B814:21-52,2009).
G.A, W. Barón, D. Marqués, C.Nuñez, (JHEP 2011)
G.A, D. Marqués, M. Graña, A. Rosabal, (arXiv:1302.5419)
G.A., D. Marqués, C. Nuñez, (CQG,2013)
G.A, D. Marqués, M. Graña, A. Rosabal, in progress

TALK:

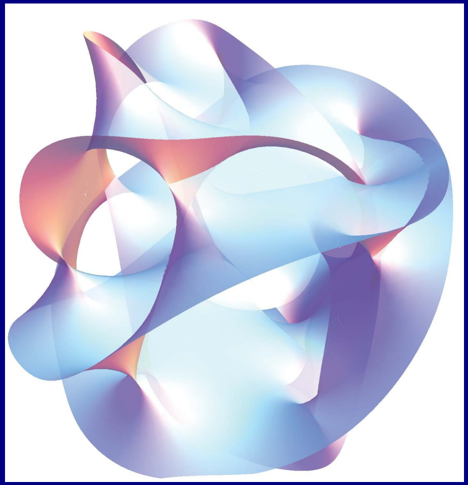
- Sugras, String dualities and fluxes
- Double Field Theory (DFT)
- Dimensional reduction
- DFT Effective action is electric sector of $\mathcal{N}=4$ gauged supergravity
- DFT provides a geometric interpretation for all T-dual fluxes
- Extended Field Theory (EFT)
- Connection with $\mathcal{N}=8$
- EFT provides a geometric interpretation for all U-dual fluxes
- Conclusions

STRINGS

D=10

Top-bottom

Bottom-top



D=4 (broken) **GAUGED SUGRA**

(should contain the **Standard Model**)

FLUXES:

RR 3-form $\mathcal{F}_{mnp} = \partial_{[m} c_{np]}$

NS 3-form $H_{mnp} = \partial_{[m} b_{np]}$

$$\frac{1}{(2\pi\sqrt{\alpha'})^{p-1}} \int_{\Sigma_p} \hat{F}_p = m_p \in \mathbb{Z}$$

$$d * \hat{F} = 0$$ E.O.M (no source)

$$d\hat{F} = 0$$ Bianchi identity

For fixed set of fluxes m_p geometrical moduli cannot be changed arbitrarily

- Moduli " Σ_p " are stabilized
- Avoid non physical long range forces
- Susy breaking mechanism
- New phenomenology

Flux compactifications

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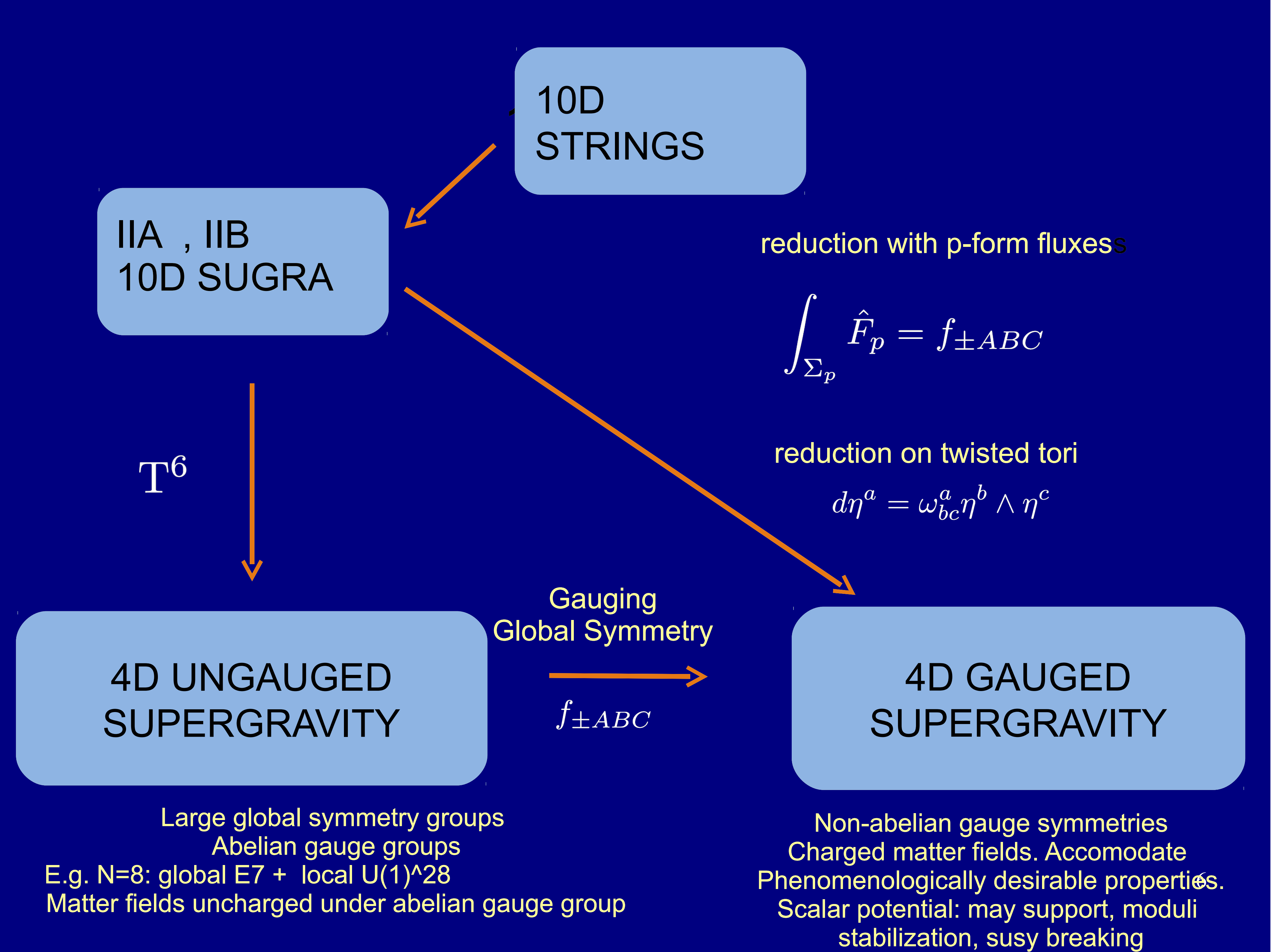
Gauged supergravities

- Deformations of standard abelian sugras.
- Matter fields become charged with respect to (generically) non-abelian gauge fields.
- Deformation parameters are the gaugings ~ quantized fluxes


10D STRINGS
IIA , IIB 10D SUGRA
reduction with p-form fluxes
$\int_{\Sigma_p} \hat{F}_p = f_{\pm ABC}$
reduction on twisted tori
$d\eta^a = \omega^a_{bc}\eta^b \wedge \eta^c$
T^6
Gauging Global Symmetry
$f_{\pm ABC}$
4D UNGAUGED SUPERGRAVITY
4D GAUGED SUPERGRAVITY
Large global symmetry groups
Abelian gauge groups
E.g. N=8: global E7 + local U(1)^28
Matter fields uncharged under abelian gauge group
Non-abelian gauge symmetries
Charged matter fields. Accomodate
Phenomenologically desirable properties.
Scalar potential: may support, moduli
stabilization, susy breaking

$\mathcal{N}=4$ Gauged sugra

Global symmetry $SL(2,\mathbb{Z})_S \times O(6,6)$

gaugings
$$\begin{cases} f_{\alpha ABC} & (\mathbf{2},\mathbf{220}) \\ f_{+ABC} & \text{electric} \\ f_{-ABC} & \text{magnetic} \\ \xi_\alpha^M & (\mathbf{2},\mathbf{12}) \end{cases}$$

J. Schon, M. Weidner, (2006)

$\mathcal{N}=4$ sugra scalar potential

$$V_{\mathcal{N}=4} = \frac{1}{16}\Big[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta}\Big(\frac{1}{3}M^{MQ}M^{NR}M^{PS} + \Big(\frac{2}{3}\eta^{MQ} - M^{MQ}\Big)\eta^{NR}\eta^{PS}\Big) - \frac{4}{9} f_{\alpha MNP} f_{\beta QRS}\epsilon^{\alpha\beta} M^{MNPQRS} + 3\xi_\alpha^M \xi_\beta^N M^{\alpha\beta} M_{MN}\Big],$$

scalars

$M^{\alpha\beta}$ $\quad$ $M_{MN} = (\mathcal{V}\mathcal{V}^T)_{MN}$

$$\frac{SL(2)|_B}{U(1)} \times \frac{SO(6,6)}{SO(6)\times SO(6)}$$

$$M^{\alpha\beta} = \frac{1}{s_x}\begin{pmatrix} 1 & s_y \\ s_y & |S|^2 \end{pmatrix}$$

$66-15-15=36$

$(U^m{}_{\tilde q}\;,\; T^{l\tilde m}\;+\;\ldots)$

HOWEVER:

• Gauged sugras contain more gaugings $f_{\pm ABC}$ than those that can be reached through geometric compactifications of string sugras.

• Type IIA, IIB “dual” string theories lead to non-duality connected effective D=4 gauged sugras, i.e.

$$W_{IIA} \xrightarrow{\text{T-duality}} \neq W_{IIB}$$

Duality information is lost at the level of effective 10 D theories

Gauged sugra contains stringy (duality) information not present at the level of effective 10D theories

Double Field Theory (DFT):

T-duality explicit in field theory

Hull, Zwiebach	0904.4664
	0908.1792
Hohm, Hull, Zwiebach	1003.5027
	1006.4823

W. Siegel (1992)·
A. Tseytlin (1991)
M.Duff
Recently:
Hohm, Kwak,Albertsson, Dai, Kao, Lin, Jeon, Lee, Park,Thompson, G.A, Baron, Marqués, Nuñez, Geissbuhler, Graña, Rosabal, Penas, Andriot, Larfors, Lust, Patalong, Blumenhagen....

DFT

Hull, Zwiebach,2009
Hull, Hohm, Zwiebach, 2010

T-duality explicit in field theory

String on a circle

$$M^2 = (N + \tilde{N} - 2) + p^2 \frac{l_s}{R^2} + \tilde{\omega}^2 \frac{R^2}{l_s}$$

$$\frac{R^2}{l_s} \leftrightarrow \frac{\tilde{R}^2}{l_s} = \frac{l_s}{R^2} \qquad\qquad p \leftrightarrow \tilde{\omega}$$

$O(d,d)$ in d compact dimensions

Double internal space $\quad i = 1, \ldots d$

$$X^M = (x^i, \tilde{x}_i) \quad \leftrightarrow \quad P_M = (p_i, w^i)$$

duals

It proves useful to double the whole space

$D = d + n \qquad X^M = (x^i, \tilde{x}_i) \qquad M = 1, \ldots 2D$

$O(D,D)$ invariance

drop ~~$\tilde{x}_i$~~ $\quad i = 1, \ldots, n$ space-time duals

$O(D,D)$ Provides a unified description for all lower dimensions

DFT

$$D = d + n$$

Double space: $X^M = (x^i, \tilde{x}_i)$ fundamental of $O(D,D)$

Field content: $g_{ij}(x^i, \tilde{x}_i)\ , b_{ij}(x^i, \tilde{x}_i) \quad \phi(x^i, \tilde{x}_i)$

Correspondence with massless bosonic modes of 10D string

Generalized metric:

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix} \in O(D,D) \qquad \mathcal{H}\eta\mathcal{H} = \eta$$

Invariant dilaton:

$$e^{-2d} = \sqrt{g}e^{-2\phi}$$

Local symmetries

$$\xi^M = (\tilde{\varepsilon}_i, \varepsilon^i)$$

generalized Lie derivative

on vectors:

$$\mathcal{L}_\xi V^M = \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P$$

⬇

$$\delta_\xi e^{-2d} = \partial_M \left(\xi^M e^{-2d}\right)$$

$$\delta_\xi \mathcal{H}^{MN} = \xi^P \partial_P \mathcal{H}^{MN} + \left(\partial^M \xi_P - \partial_P \xi^M\right) \mathcal{H}^{PN} + \left(\partial^N \xi_P - \partial_P \xi^N\right) \mathcal{H}^{MP}$$

when $\partial_M = (0, \partial_i)$

$$\delta_\xi g^{ij} = \mathcal{L}_\varepsilon g^{ij}$$

$$\delta_\xi b_{ij} = \mathcal{L}_\varepsilon b_{ij} + \partial_i \tilde{\varepsilon}_j - \partial_j \tilde{\varepsilon}_i$$

GR diffeos+gauge transformations

Action

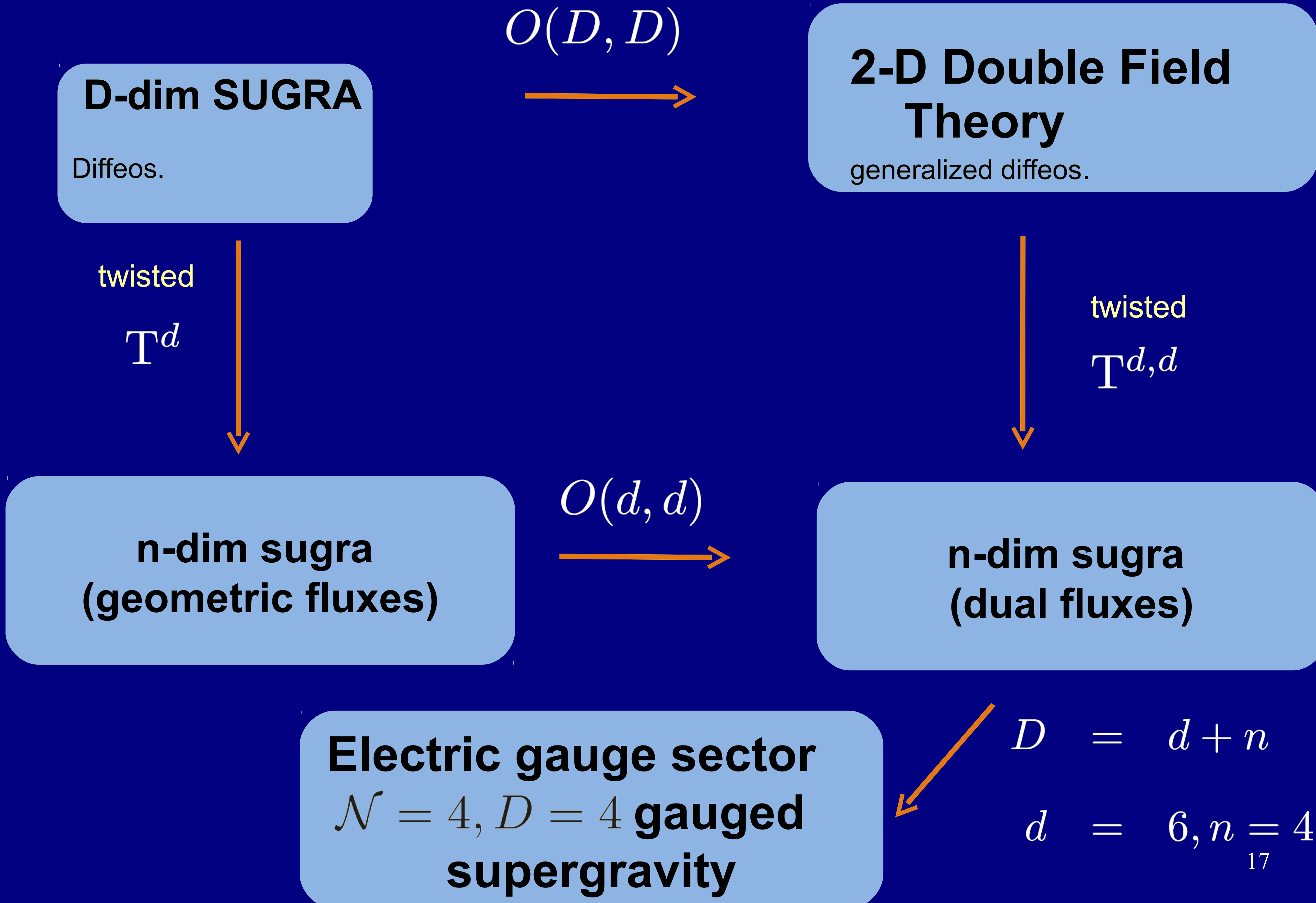
$$S_{DFT} = \int dx d\tilde{x}\ e^{-2d}\ \mathcal{R}$$

$$\mathcal{R} = \frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_N \mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_K \mathcal{H}_{NL} - 4\mathcal{H}^{MN}\partial_M d \partial_N d$$

Recall that when $\partial_M = (0, \partial_i)$

$$S_{DFT} \ \rightarrow \ S_{bosonic} = \int dx \sqrt{g} e^{-2\phi} \left(\mathbf{R} + 4(\partial\phi)^2 - \frac{1}{12} H^2 \right)$$

GR action in D dimensions



$O(D, D)$
2-D Double Field Theory
generalized diffeos.
D-dim SUGRA
Diffeos.
twisted
T^d
twisted
$\mathrm{T}^{d,d}$
$O(d, d)$
n-dim sugra
(geometric fluxes)
n-dim sugra
(dual fluxes)
Electric gauge sector
$\mathcal{N} = 4, D = 4$ gauged supergravity
$D = d + n$
$d = 6, n = 4$

Constraints

Demanding that two successive transformations behave as one transformation

$$[\delta_{\xi_1}, \delta_{\xi_2}] = \delta_{\xi_{12}}$$

defines the C-bracket

$$\xi_{12} = [\xi_1, \xi_2]_{\mathrm{C}}^M = 2\xi_{[1}^N \partial_N \xi_{2]}^M - \xi_{[1}^P \partial^M \xi_{2]P}$$ up to constraints

C-bracket does not satisfy Jacobi identity

$$\delta_{\mathcal{J}_{\mathrm{C}}(\xi_1,\xi_2,\xi_3)} V^M{}_N = \frac{3}{2} \partial^R \left(\xi_{[1}^P \xi_2^Q \partial_P \xi_{3]Q} \right) \partial_R V$$

Original DFT requires the imposition of two constraints

- Weak constraint

$$\partial_M \partial^M A = 0$$

(Level matching condition

$$p.w = 0 \equiv \partial_m \partial_{\tilde{m}} = 0$$

)

- Strong constraint

$$\partial^M A \, \partial_M B = 0$$

where $A(x, \tilde{x}), B(x, \tilde{x})$ generic fields and gauge parameters.

Closure of gauge algebra and invariance of the action is ensured

$O(D,D)$ explicit but not truly doubled

$$\partial^M A(x,\tilde{x})\partial_M B A(x,\tilde{x}) = 0$$

$$\rightarrow A(x,\tilde{x}) = A(x) \qquad \text{or} \qquad \mathrm{A}(\mathrm{x},\tilde{\mathrm{x}}) = \mathrm{A}(\tilde{\mathrm{x}})$$

Weak and strong constraints are suff cient conditions for gauge invariance of DFT and closure of the algebra of gauge symmetries, but they are not necessary

Graña, Marqués, 2011

In terms of a frame

D. Geissbhuler(2011)
D. Geissbhuler, D. Marqués, C.Nuñez, V. Penas (2013)
G.A., D.Marqués, C.Nuñez (2013)

$$\mathcal{H}_{MN} = E^{\bar{A}}{}_M \; S_{\bar{A}\bar{B}} \; E^{\bar{B}}{}_N \,, \qquad \eta_{MN} = E^{\bar{A}}{}_M \; \eta_{\bar{A}\bar{B}} \; E^{\bar{B}}{}_N \,,$$

$$E^{\bar{A}}{}_M = \begin{pmatrix} e_{\bar{a}}{}^i & e_{\bar{a}}{}^j b_{ji} \\ 0 & e^{\bar{a}}{}_i \end{pmatrix} \,, \qquad S_{\bar{A}\bar{B}} = \begin{pmatrix} s^{\bar{a}\bar{b}} & 0 \\ 0 & s_{\bar{a}\bar{b}} \end{pmatrix} \,.$$

Dynamical fluxes

$$\begin{aligned} \mathcal{F}_{\bar{A}\bar{B}\bar{C}} &= E_{\bar{C}M} \mathcal{L}_{E_{\bar{A}}} E_{\bar{B}}{}^M = 3\Omega_{[\bar{A}\bar{B}\bar{C}]} \,, \\ \mathcal{F}_{\bar{A}} &= -e^{2d} \mathcal{L}_{E_{\bar{A}}} e^{-2d} = \Omega^{\bar{B}}{}_{\bar{B}\bar{A}} + 2E_{\bar{A}}{}^M \partial_M d \,, \end{aligned}$$

$$\Omega_{\bar{A}\bar{B}\bar{C}} = E_{\bar{A}}{}^M \partial_M E_{\bar{B}}{}^N E_{\bar{C}N} = -\Omega_{\bar{A}\bar{C}\bar{B}}$$

Action

$$S_{DFT} = \int dx d\tilde{x}\; e^{-2d}\, \mathcal{R}$$

where

$$\begin{aligned}\mathcal{R} \;\; &= \;\; \mathcal{F}_{\bar{A}\bar{B}\bar{C}}\, \mathcal{F}_{\bar{D}\bar{E}\bar{F}} \left[\frac{1}{4} S^{\bar{A}\bar{D}} \eta^{\bar{B}\bar{E}} \eta^{\bar{C}\bar{F}} - \frac{1}{12} S^{\bar{A}\bar{D}} S^{\bar{B}\bar{E}} S^{\bar{C}\bar{F}} - \frac{1}{6} \eta^{\bar{A}\bar{D}} \eta^{\bar{B}\bar{E}} \eta^{\bar{C}\bar{F}}\right] \\ &\quad + \mathcal{F}_{\bar{A}} \mathcal{F}_{\bar{B}} \left[\eta^{\bar{A}\bar{B}} - S^{\bar{A}\bar{B}}\right] .\end{aligned}$$

Scherk-Schwarz dimensional reductions

G.A, W. Barón, D. Marqués, C.Nuñez, (JHEP 2011)

D.Geissbuhler(2011),

Split coordinates $(\mathbb{X}, \mathbb{Y})$

$\mathbb{X}$ space-time

$\mathbb{Y}$ internal

1. Choose reduction ansatz: give explicit dependence $\psi(\mathbb{X}, \mathbb{Y})$
2. Verify that the $\mathbb{Y}$ dependence factorizes out of the gauge transformations
3. Plug the ansatz in the action and integrate the $\mathbb{Y}$ dependence

Effective theory defined over $\mathbb{X}$ coordinates.

Reduction ansatz

$U(\mathbb{Y})$ twist

$$\xi^M(\mathbb{X},\mathbb{Y}) = (U(\mathbb{Y})^{-1})^M{}_A\ \widehat{\xi}^A(\mathbb{X}) \ , \qquad U(\mathbb{Y}) \in O(D,D)$$

$$E^{\bar{A}}{}_M(\mathbb{X},\mathbb{Y}) = \widehat{E}^{\bar{A}}{}_I(\mathbb{X})\ U^I{}_M(\mathbb{Y}) \ , \qquad d(\mathbb{X},\mathbb{Y}) = \widehat{d}(\mathbb{X}) + \lambda(\mathbb{Y}) \, ,$$

$$\mathcal{H}(\mathbb{X},\mathbb{Y}) \to U^T(\mathbb{Y})\ \widehat{\mathcal{H}}(\mathbb{X})\ U(\mathbb{Y}) \ ,$$

$\widehat{\Psi} = \widehat{\Psi}(\mathbb{X})$ hat indicates space time dependence

Split space-time and internal contributions

$$M = (\mu, A) \qquad \bar{M} = (\bar{a}, \bar{A})$$

frame

$$\widehat{E}^{\bar{A}}{}_I = \begin{pmatrix} \widehat{e}_{\bar{a}}{}^{\mu} & -\widehat{e}_{\bar{a}}{}^{\rho}\widehat{c}_{\rho\mu} & -\widehat{e}_{\bar{a}}{}^{\rho}\widehat{A}_{A\rho} \\ 0 & \widehat{e}^{\bar{a}}{}_{\mu} & 0 \\ 0 & \widehat{\Phi}^{\bar{A}}{}_B \widehat{A}^B{}_{\mu} & \widehat{\Phi}^{\bar{A}}{}_A \end{pmatrix}$$

Generalized metric

$$\widehat{\mathcal{H}}_{IJ} = \begin{pmatrix} \widehat{g}^{\mu\nu} & -\widehat{g}^{\mu\rho}\widehat{c}_{\rho\nu} & -\widehat{g}^{\mu\rho}\widehat{A}_{A\rho} \\ -\widehat{g}^{\nu\rho}\widehat{c}_{\rho\mu} & \widehat{g}_{\mu\nu} + \widehat{A}^C{}_{\mu}\widehat{\mathcal{M}}_{CD}\widehat{A}^D{}_{\nu} + \widehat{c}_{\rho\mu}\widehat{g}^{\rho\sigma}\widehat{c}_{\sigma\nu} & \widehat{\mathcal{M}}_{AC}\widehat{A}^C{}_{\mu} + \widehat{A}_{A\rho}\widehat{g}^{\rho\sigma}\widehat{c}_{\sigma\mu} \\ -\widehat{g}^{\nu\rho}\widehat{A}_{B\rho} & \widehat{\mathcal{M}}_{BC}\widehat{A}^C{}_{\nu} + \widehat{A}_{B\rho}\widehat{g}^{\rho\sigma}\widehat{c}_{\sigma\nu} & \widehat{\mathcal{M}}_{AB} + \widehat{A}_{A\rho}\widehat{g}^{\rho\sigma}\widehat{A}_{B\sigma} \end{pmatrix}$$

And replacing SS reduced dynamical fluxes in the action

$$S_{DFT} = \int d\mathbb{X} d\mathbb{Y}\, e^{-2d}\, \mathcal{R} = \int d\mathbb{X} d\mathbb{Y}\, e^{-2d} \times$$

$$\mathcal{F}_{\bar A \bar B \bar C}\, \mathcal{F}_{\bar D \bar E \bar F} \left[\frac{1}{4} S^{\bar A \bar D} \eta^{\bar B \bar E} \eta^{\bar C \bar F} - \frac{1}{12} S^{\bar A \bar D} S^{\bar B \bar E} S^{\bar C \bar F} - \frac{1}{6} \eta^{\bar A \bar D} \eta^{\bar B \bar E} \eta^{\bar C \bar F}\right]$$

$$+ \mathcal{F}_{\bar A} \mathcal{F}_{\bar B} \left[\eta^{\bar A \bar B} - S^{\bar A \bar B}\right] .$$

$$\mathcal{F}_{\bar A \bar B \bar C}(\mathbb{X}, \mathbb{Y})$$

SS

$M = (\mu, A) \qquad \bar{M} = (\bar{a}, \bar{A})$

Dynamical fluxes

$$\mathcal{F}_{\bar{A}\bar{B}\bar{C}}(\mathbb{X}, \mathbb{Y}) = 3 E_{[\bar{A}}{}^{M} \partial_M E_{\bar{B}}{}^{N} E_{\bar{C}]N}$$

$\Downarrow$

$$\begin{aligned}
\mathcal{F}_{\bar{a}\bar{b}\bar{c}}(x) &= \widehat{e}_{\bar{a}}{}^{\mu} \widehat{e}_{\bar{b}}{}^{\nu} \widehat{e}_{\bar{c}}{}^{\rho}\ \mathcal{G}_{\mu\nu\rho}\ , \\
\mathcal{F}_{\bar{a}\bar{b}}{}^{\bar{C}}(x) &= \widehat{e}_{\bar{a}}{}^{\mu} \widehat{e}_{\bar{b}}{}^{\nu} \widehat{\Phi}^{\bar{C}}{}_{C}\ F^{C}{}_{\mu\nu}\ , \\
\mathcal{F}_{\bar{a}\bar{B}}{}^{\bar{C}}(x) &= \widehat{e}_{\bar{a}}{}^{\mu} \widehat{\Phi}^{\bar{C}}{}_{C}\ D_{\mu} \widehat{\Phi}_{\bar{B}}{}^{C}\ ,
\end{aligned}$$

$$D_{\mu} \widehat{\Phi}_{\bar{B}}{}^{C} = \partial_{\mu} \widehat{\Phi}_{\bar{B}}{}^{C} - f_{AB}{}^{C} \widehat{A}_{\mu}{}^{A} \widehat{\Phi}_{\bar{B}}{}^{B}$$ covariant derivative

$f_{ABC}(U(\mathbb{Y}))$ constant gaugings

The effective action

$$S_{eff} = v \int d^n x \sqrt{g} e^{-2\phi} \Big\{ \mathbf{R} + 4\, \partial^\mu \Phi \partial_\mu \Phi - \frac{1}{4} \mathcal{H}_{AB} \mathcal{F}^{A\mu\nu} \mathcal{F}^{B}{}_{\mu\nu} - \frac{1}{12} \mathcal{G}_{\mu\nu\rho} \mathcal{G}^{\mu\nu\rho} + \frac{1}{8} D_\mu \mathcal{H}_{AB} D^\mu \mathcal{H}^{AB} - V \Big\}$$

$v = \int d\mathbb{Y} e^{-2\lambda}$ overall factor

Scalar potential

$$V = \frac{1}{4} f^{C}{}_{DA}\, f^{D}{}_{CB} \mathcal{H}^{AB} + \frac{1}{12} f^{E}{}_{AC}\, f^{F}{}_{BD} \mathcal{H}^{AB}\, \mathcal{H}^{CD}\, \mathcal{H}_{EF}$$

Gauge Transformations

$$\mathcal{L}_\xi V^M = \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P .$$

$$\widehat{\xi} = \left(\epsilon^\mu, \tilde{\epsilon}_\mu, \lambda^A\right) =$$ (diffeos, B transfs, gauge transfs)

$$\begin{aligned}
\delta_{\widehat{\xi}} g_{\mu\nu} &= \mathcal{L}_\epsilon g_{\mu\nu} \\
\delta_{\widehat{\xi}} B_{\mu\nu} &= \mathcal{L}_\epsilon B_{\mu\nu} + (\partial_\mu \tilde{\epsilon}_\nu - \partial_\nu \tilde{\epsilon}_\mu) - \left(A^A{}_\mu \partial_\nu \lambda_A - A^A{}_\nu \partial_\mu \lambda_A\right)/2 \\
\delta_{\widehat{\xi}} A^A{}_\mu &= \mathcal{L}_\epsilon A^A{}_\mu - \partial_\mu \lambda^A + f^A{}_{BC}\ \lambda^B A^C{}_\mu \\
\delta_{\widehat{\xi}} \mathcal{H}_{AB} &= f_{AC}{}^D\ \lambda^C \mathcal{H}_{DB} + f_{BC}{}^D\ \lambda^C \mathcal{H}_{AD}
\end{aligned}$$

closure

$$f^M{}_{N[P} f^N{}_{QR]} = 0$$

Field Strengths

$$\mathcal{F}^{A}{}_{\mu\nu} \quad = \quad \partial_\mu A^{A}{}_{\nu} - \partial_\nu A^{A}{}_{\mu} + f^{A}{}_{BC} A^{B}{}_{\mu}\, A^{C}{}_{\nu}$$

$$\mathcal{G}_{\mu\rho\lambda} \quad = \quad 3\partial_{[\mu} B_{\rho\lambda]} + f_{ABC} A^{A}{}_{\mu} A^{B}{}_{\rho} A^{C}{}_{\lambda} + 3\partial_{[\mu} A^{A}{}_{\rho} A_{\lambda]A}$$

Covariant derivative

$$D_\mu \mathcal{H}_{AB} = \partial_\mu \mathcal{H}_{AB} + f^{C}{}_{AD} A^{D}{}_{\mu} \mathcal{H}_{CB} + f^{C}{}_{BD} A^{D}{}_{\mu} \mathcal{H}_{AC}$$

Gaugings-fluxes encode information about twists

$$f_{ABC}(U(\mathbb{Y}))$$

$$f_{ABC} = 3\Omega_{[ABC]} \qquad (\mathbf{220})$$

$$\Omega_{ABC} \equiv \eta_{CD}(U^{-1})^M{}_A(U^{-1})^N{}_B\partial_M U^D{}_N$$

$$f^M{}_{N[P}f^N{}_{QR]} = 0$$

quadratic constraints

Connection with $\mathcal{N} = 4$ Supergravity

Effective DFT action	Gauged $\mathcal{N} = 4$ sugra
f_{ABC}	f_{+ABC}
$\mathcal{H}_{AB}$	M_{AB}
$e^{-2\phi}$	$2\mathrm{Im}(\tau)$
A_μ^A	A_μ^A
$\mathcal{G}^{\mu\nu\rho}$	$2e^{4\phi}\epsilon^{\sigma\mu\nu\rho}\partial_\sigma \mathrm{Re}(\tau)$
$f_{[AB}{}^E f_{C]DE} = 0$	$f_{[AB}{}^E f_{C]DE} = 0$

Towards a geometry of the generalized space

- Generalized covariant derivative
- Generalized torsion
- Generalized Riemann tensor (not uniquely defined)
- Generalized Ricii tensor
- Generalized Ricci scalar

G.A, D. Marqués, M. Graña, A. Rosabal (2013)

Extended Field Theory:

$E_{7,7}$ U-duality explicit in field theory.

$$X^{\mathcal{M}} = (x^{\mu}, Y^{M}) \qquad M = 1, \ldots 56$$

⇓

56-dimensional internal mega-space

$$E$$

$$\Downarrow$$

$$G_n = nD\,space - time \times E_n$$

Concentrate in

$$\Downarrow$$

$$G = 4D\,space - time \times E_{7,7}$$

extended gauge parameter

$$\Xi^{\mathbb{M}} = (\xi^{\mu}, \xi^{M}, \Sigma_{\mu}{}^{M}) = (\xi^{\mathcal{M}}, \Sigma_{\mu}{}^{M}) \ .$$

extended gauge transformation

$$(\hat{\mathcal{L}}_{\Xi} V)^{\mathcal{M}} := (\hat{L}_{\xi} V)^{\mathcal{M}} + Y^{\mathcal{M}}{}_{\mathcal{N}}{}^{\mathcal{P}}{}_{\mathcal{Q}} \partial_{\mathcal{P}} \xi^{\mathcal{Q}} V^{\mathcal{N}} + V^{\mu} \Sigma_{\mu}{}^{\mathcal{M}}$$

$\Sigma_{\mu}{}^{M}$ needed for closure of Diff. algebra

inherited from E

Frame formalism

$$\mathbb{E}_{\bar{\mathbb{C}}}{}^{\mathbb{M}} \qquad\qquad {}^{\mathbb{M}} = ({}^{\mu}, {}^{M} \; ,_{\mu}{}^{M})$$

Dynamical fluxes

$$(\hat{\mathcal{L}}_{\mathbb{E}_{\bar{\mathbb{A}}}} \mathbb{E}_{\bar{\mathbb{B}}})^{\mathbb{M}} = \mathbb{F}_{\bar{\mathbb{A}}\bar{\mathbb{B}}}{}^{\bar{\mathbb{C}}} \mathbb{E}_{\bar{\mathbb{C}}}{}^{\mathbb{M}}$$

Extended metric

$$\mathcal{G}^{\mathbb{MN}} = \mathcal{G}^{\mathbb{MN}}(g, B, \phi, C)$$

Metric, 2-form, dilaton, RR fields

Reduction ansatz

$U(Y)$ twist $\qquad Y^M \qquad M = 1, \ldots 56$

$$\mathbb{E}_{\bar{\mathbb{B}}}{}^{\mathbb{M}} = \mathbb{E}_{\bar{\mathbb{B}}}{}^{\mathbb{I}}(x)\mathbb{U}_{\mathbb{I}}{}^{\mathbb{M}}(Y)$$

frame

$$\Xi^{\mathbb{M}}(x, Y) = (U(Y)^{-1})^{\mathbb{M}}{}_{A}\,\widehat{\Xi}^{A}(x)\ ,$$

gauge parameters

Frame decomposition

$$\mathbb{E}_{\bar{\mathbb{A}}}{}^{\mathbb{M}}(x,y) = \begin{pmatrix} e_{\bar{a}}{}^{\mu}(x) & -e_{\bar{a}}{}^{\mu}(x)A_{\alpha}{}^{N}(x)U_{N}{}^{M}(y) & e_{\bar{a}}{}^{\alpha}(x)\hat{B}_{\alpha\nu}{}^{N}(x)U_{N}{}^{M}(y) \\ 0 & \Phi_{\bar{A}}{}^{N}(x)U_{N}{}^{M}(y) & \Phi_{\bar{A}}{}^{P}(x)\hat{C}_{P\nu}{}^{N}(x)U_{N}{}^{M}(y) \\ 0 & 0 & (e^{-1}(x))_{\nu}{}^{\bar{a}}\Phi_{\bar{A}}{}^{N}(x)U_{N}{}^{M}(y) \end{pmatrix}$$

$$\mathbb{F}_{\bar{\mathbb{A}}\bar{\mathbb{B}}}{}^{\bar{\mathbb{C}}}(x,Y) \longrightarrow \mathbb{F}_{AB}{}^{C}(Y) \quad \text{gaugings} \quad \mathcal{X}_{ABC}(U(Y))$$

$$\downarrow$$

$$\mathbb{F}_{\bar{\mathbb{A}}\bar{\mathbb{B}}}{}^{\bar{\mathbb{C}}}(x)$$

- Only space-time indices: GR
- Internal: scalars
- Mixed: gauged fields, kinetic terms

G.A, D. Marqués, M. Graña, A. Rosabal, (: arXiv:1302.5419)

Internal space

A generalized covariant derivative can be defined

A generalized Ricci tensor can be defined

$$\mathcal{R} = H^{\bar{A}\bar{B}}\mathcal{R}_{\bar{A}\bar{B}}$$

$$\frac{1}{4}\mathcal{R} = \frac{1}{672}\left(H^{AD}H^{BE}H_{CF}X_{AB}{}^{C}X_{DE}{}^{F} + 7H^{AB}X_{AC}{}^{D}X_{BD}{}^{C}\right) = V_{\mathcal{N}=8}$$

Scalar potential of $\mathcal{N} = 8$ gauged sugra

X_{ABC} gaugings in $\mathbf{912}$

$$X_{ABC} = \Omega_{ABC} - \Omega_{(BC)A} + 12K_{BC}{}^{DE}\Omega_{DEA} + \frac{2}{3}\omega_{A(B}\vartheta_{C)} + 8K_{ABC}{}^{D}\vartheta_{D}$$

$$\Omega_{\bar{A}\bar{B}}{}^{\bar{C}} = E_{\bar{A}}{}^{M}\partial_M E_{\bar{B}}{}^{N}(E^{-1})_N{}^{\bar{C}}$$

Dynamical fluxes → space-time forms contributions

i.e. gauge field strength

$$\mathbb{F}_{\bar{a}\bar{b}}{}^{\bar{C}} = \mathcal{H}_{\bar{a}\bar{b}}{}^{\bar{C}} = -e_{\bar{a}}{}^{\mu}(x)e_{\bar{b}}{}^{\nu}(x)(E^{-1})_M{}^{\bar{C}}(x)\big(F_{\mu\nu}{}^M(x) + B_{\mu\nu}{}^{IJ}(x)F_{(IJ)}{}^M\big)$$

$$F_{\mu\nu}{}^M = 2\partial_{[\mu}A_{\nu]}{}^M(x) - F_{[IJ]}{}^M A_\mu{}^I(x)A_\nu{}^J(x)$$

3-form

$$\begin{aligned}\mathbb{F}_{\bar{a}\bar{b}\bar{c}}{}^{\bar{C}} \quad = \quad & 3e_{\bar{a}}{}^{\alpha}(x)e_{\bar{b}}{}^{\beta}(x)e_{\bar{c}}{}^{\gamma}(x)(\Phi^{-1}(x))_N{}^{\bar{C}}\big[\\ & \partial_{[\alpha}B_{\beta\gamma]}{}^{IJ}(x) - 2A_{[\alpha}{}^I(x)B_{\beta\gamma]}{}^{AB}(x)F_{AB}{}^J \\ & +2(A_{[\alpha}{}^I(x)\partial_\beta A_{\gamma]}{}^J(x) - A_{[\alpha}{}^A(x)A_\beta{}^B(x)A_{\gamma]}{}^I(x)F_{AB}{}^J)\big]F_{IJ}{}^N.\end{aligned}$$

Covariant derivative of scalars

$$\begin{aligned}\mathbb{F}_{\bar{A}\bar{b}}{}^{\bar{C}} \quad = \quad & e_{\bar{b}}{}^{\mu}(x)(\Phi^{-1}(x))_N{}^{\bar{C}}\big(\partial_\mu\Phi(x)_{\bar{A}}{}^N - F_{IJ}{}^N A_\mu{}^I(x)\Phi(x)_{\bar{A}}{}^J\big) \\ & = e_{\bar{b}}{}^{\mu}(x)(\Phi^{-1}(x))_N{}^{\bar{C}} D_\mu\Phi(x)_{\bar{A}}{}^N.\end{aligned}$$

Final Comments

DFT:
Promotes stringy T-duality to a symmetry of field theory.

It defines an action for objects defined on a doubled space.

Consistency conditions for DFT, with twisted fields and gauge parameters, relax strong constraint on internal space. This suggests the existence of truly doubled backgrounds for which DFT is consistent.

Reduction fits electric (bosonic) sector of $\mathcal{N} = 4$ gauged sugra with explicit $O(6,6)$ symmetry.

Gaugings associated to non-geometric backgrounds in string theory, are geometric from the perspective of doubled geometry

EFT:
Promotes stringy U-duality to a symmetry of field theory.

Reduction fits (bosonic) sector of gauged $\mathcal{N} = 8$ sugra with explicit $E_{(7,7)}$ symmetry.

A geometry for the generalized(extended) space?

- Generalized covariant derivative
- Generalized torsion
- Generalized Ricii tensor
- Generalized Ricci scalar

(GR) Gravity = Geometry

(Stringy) gauged Supergravity= DFT (EFT) geometry

DFT (EFT) Purely geometrical description of strings ?

Outlook

EFT full action

Geometry

Connection with E_{11} ?

Relation between DFT(EFT) and string theory beyond tori

Full consistency, extra modes

fermions

Extensions, some references:

Heterotic formulation: Andriot, Hohm, Kwak

Type II: Hohm, Kwak, Zwiebach; Coimbra, Strickland-Constable, Waldram; Hohm, Kwak

Riemann Tensor: Hohm, Zwiebach
Hohm, Kwak; Jeon, Lee, Park
D. S. Berman, C. D. A. Blair, E. Malek,M. J. Perry,
Geisbuller,Marques, Nuñez, Penas

Generalized geometry and Non-geometry: Coimbra, Strickland-Constable, Waldram; Andriot, Larfors, Lust, Patalong

Dimensional reductions: Aldazabal, Baron, Marques, Nuñez; D. Geissbhuler,Strickland-Constable, Waldram; Graña,Marqués,Rosabal.

Double Sigma Models: Hull; Berman, Copland, Thompson

Noncommutativity and nonassociativity: Lust, Blumenhagen, Deser, Plauschinn, Rennecke

Supersymmetric DFT: Hohm, Kwak; Jeon, Lee, Park

U-duality, M-theory: Berman, Copland, Godazgar, Perry, Thompson; West;
Coimbra,Aldazabal,Graña,Marqués,Rosabal.

Branes and solitons: de Boer,Shigemori;Bergshoeff,Riccioni;Albertsson,Daia,Kao,Lin;Jensen