Conductance Fluctuations in Graphene Systems:
The Relevance of Classical Dynamics

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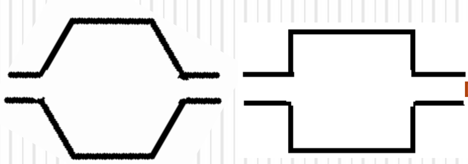
Conductance fluctuations in relativistic quantum dynamics

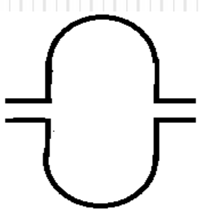
Purpose:

The nature of the corresponding classical dynamics can play a key role in the conductance-fluctuation pattern in RQD.



Integrable systems

Sharp conductance fluctuations



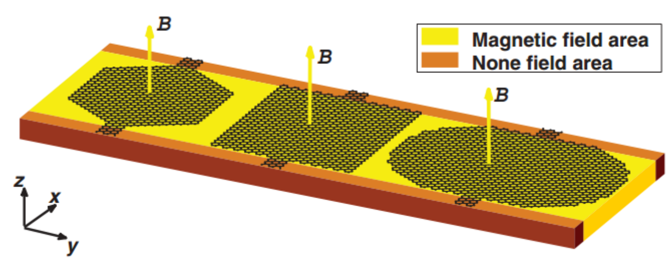
Chaotic system

Smooth conductance fluctuations

For a relativistic quantum system (graphene), we are interested in how the geometry influences the conductance fluctuations and how these fluctuations change when a magnetic field is applied.

Conductance fluctuations in graphene systems

OUTLINE



1. Model and calculation method

2. Periodic conductance oscillations and emergence of random conductance fluctuations

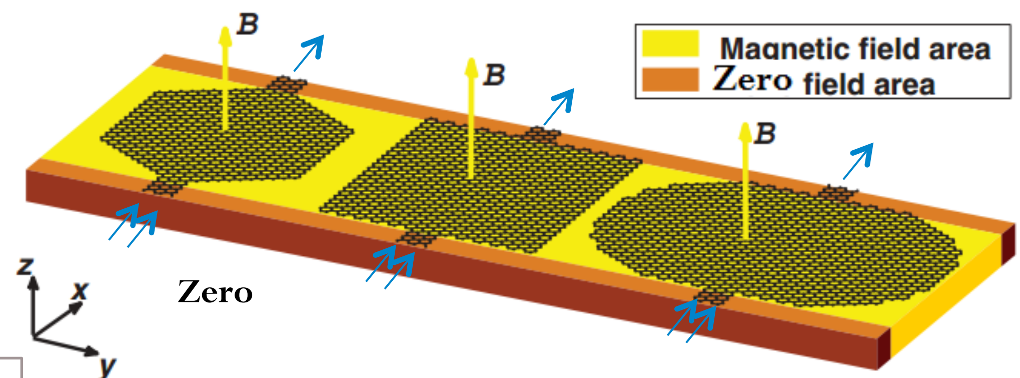
3.Semi-classical theory of regular conductance oscillations: scaling relationship

4.Universal transition to random conductance fluctuations

5. Summary

L.Ying, L. Huang, Y.-C. Lai, and C. Grebogi, "*Conductance fluctuations in graphene systems:The relevance of classical dynamics*" Phys. Rev. B 85 245448 (2012)

Model: vertical magnetic field applied in device region



$$H = \sum_{i,j} -t_{ij}(c_i^{\dagger} c_j + \mathrm{H.c.}),$$

$$t_{ij} = t_0 \exp(-i 2\pi \phi_{i,j})$$

$$\phi_{i,j} = (1/\phi_0) \int_j^i \mathbf{A} \cdot d\mathbf{l} \qquad \phi_0 = h/e$$

vector potential $\mathbf{A} = (-By, 0, 0)$

L. Ying, L. Huang, Y.-C. Lai, and C. Grebogi, Phys. Rev. B 85 245448 (2012)

Green's function: conductace and LDS calculations

$$\boxed{G(E) = (2e^2/h)T_G(E)}$$

$$T(E) = \mathrm{Tr}(\Gamma_L G_D \Gamma_R G_D^{\dagger})$$

$$G_D = (EI - H_D - \Sigma_L - \Sigma_R)^{-1}$$

$$\Gamma_{L,R} = i(\Sigma_{L,R} - \Sigma_{L,R}^{\dagger})$$

Local density of states (LDS):

$$\boxed{\rho = -\frac{1}{\pi}\mathrm{Im}[\mathrm{diag}(G_D)].}$$

Conductance fluctuations in graphene systems

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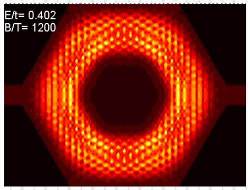

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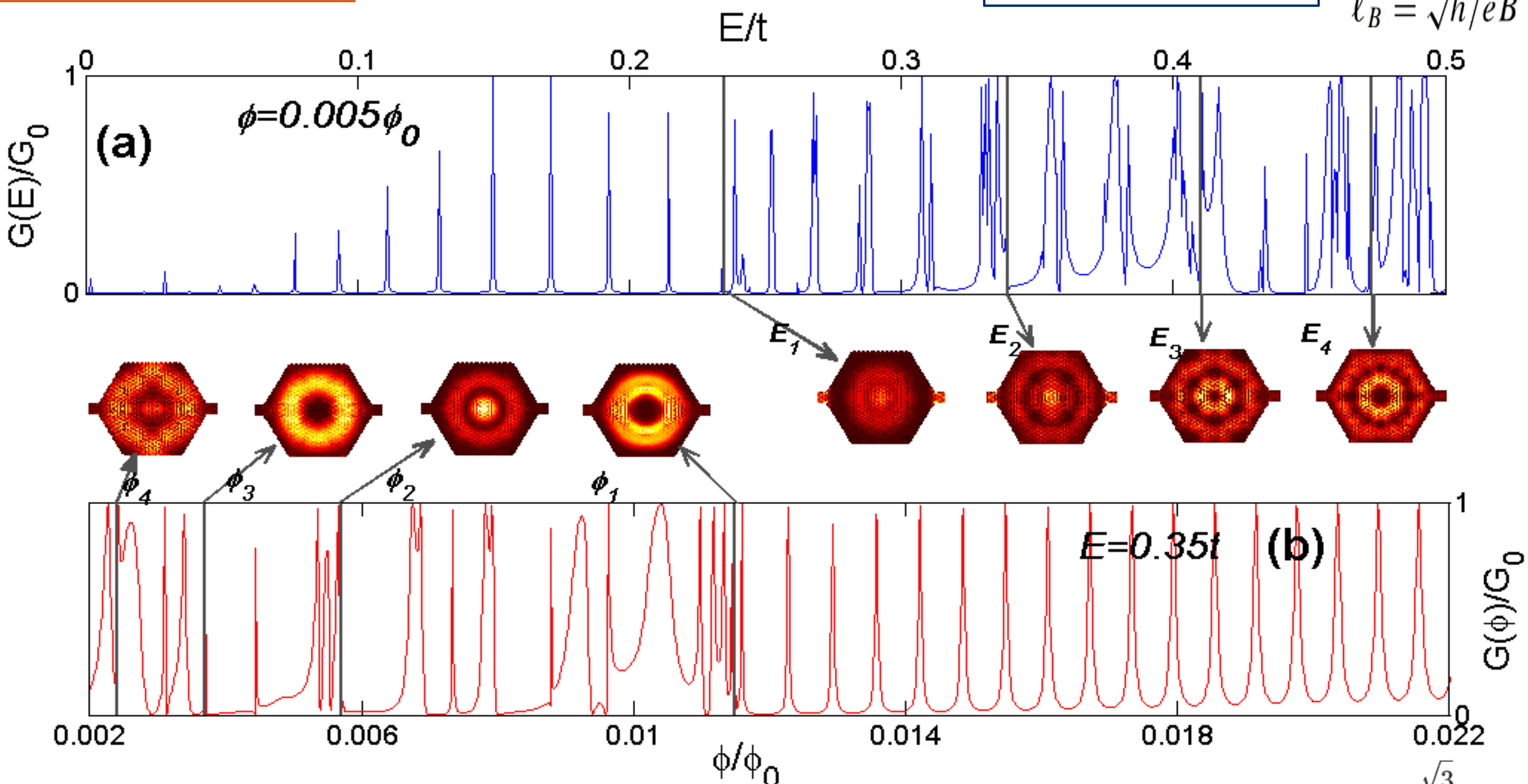
Landau levels: emergence of random conductance fluctuations

$$E(N) = \pm\omega_c\sqrt{N}$$

$$\omega_c = \sqrt{2}v_F/\ell_B$$

$$\ell_B = \sqrt{\hbar/eB}$$



$$\phi = BS_0 \;\; where \;\; S_0 = \frac{\sqrt{3}}{2}a_0^2$$

$$B(N) = \frac{\hbar E^2}{2ev_F^2}\frac{1}{N}$$

	1	2	3	4
E/t	0.2350	0.3395	0.4100	0.4730
Flux Phi	0.0115	0.0057	0.0037	0.0024

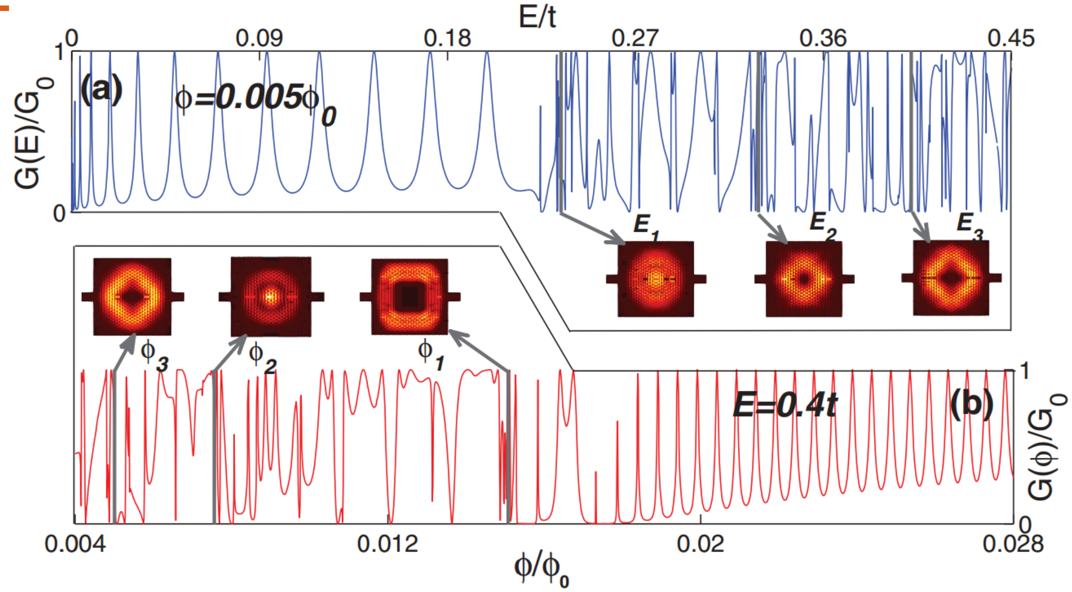
$E_1 : E_2 : E_3 : E_4$
$\approx 1 : \sqrt{2} : \sqrt{3} : 4$

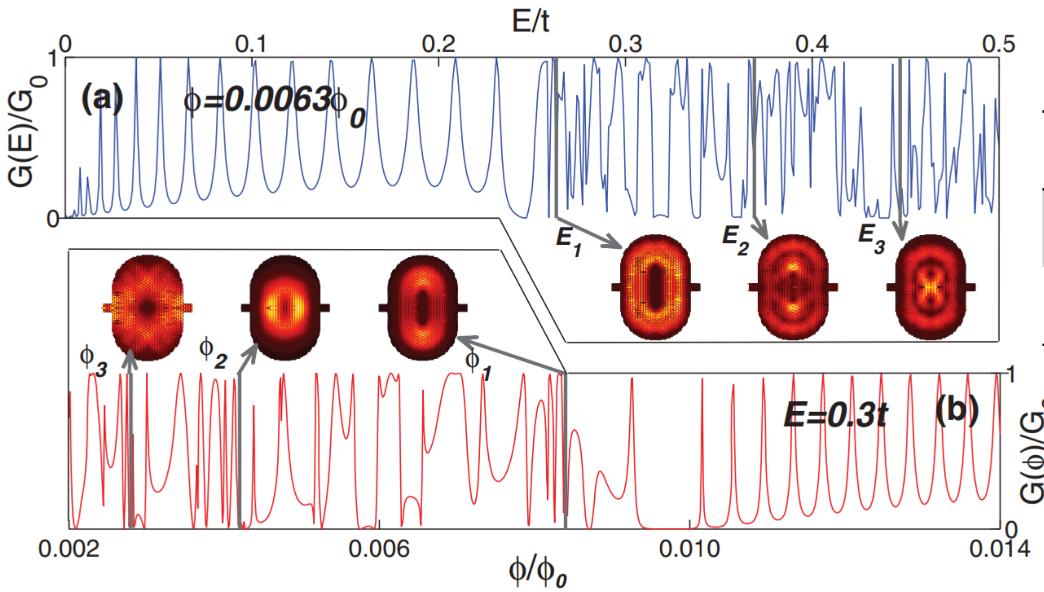
Ø1:Ø2:Ø3:Ø4≈1:1/2:1/3:1/4

Emergence of random conductance fluctuations: stadium and rectangle

	1	2	3
E/t	0.2344	0.3289	0.4021
Flux Phi	0.01508	0.00756	0.00501

$$E_1 : E_2 : E_3 : \approx 1 : \sqrt{2} : \sqrt{3}$$
$$\phi_1 : \phi_2 : \phi_3 \approx 1 : 1/2 : 1/3$$





	1	2	3
E/t	0.263	0.369	0.4471
Flux Phi	0.0084	0.0042	0.0028

$$E_1 : E_2 : E_3 : \approx 1 : \sqrt{2} : \sqrt{3}$$
$$\phi_1 : \phi_2 : \phi_3 \approx 1 : 1/2 : 1/3$$

Conductance fluctuations in graphene systems

OUTLINE

$$S/L = \frac{2\hbar S_0}{\sqrt{3}eat_0}\frac{\Delta E}{\Delta \phi}.$$

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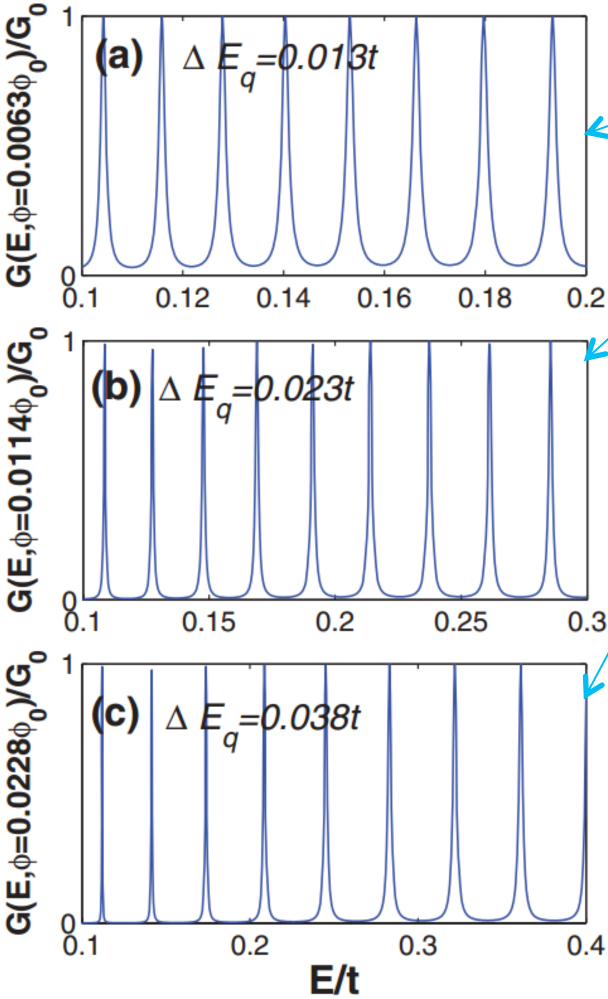
Semi-classical theory of regular conductance oscillations



N=12938

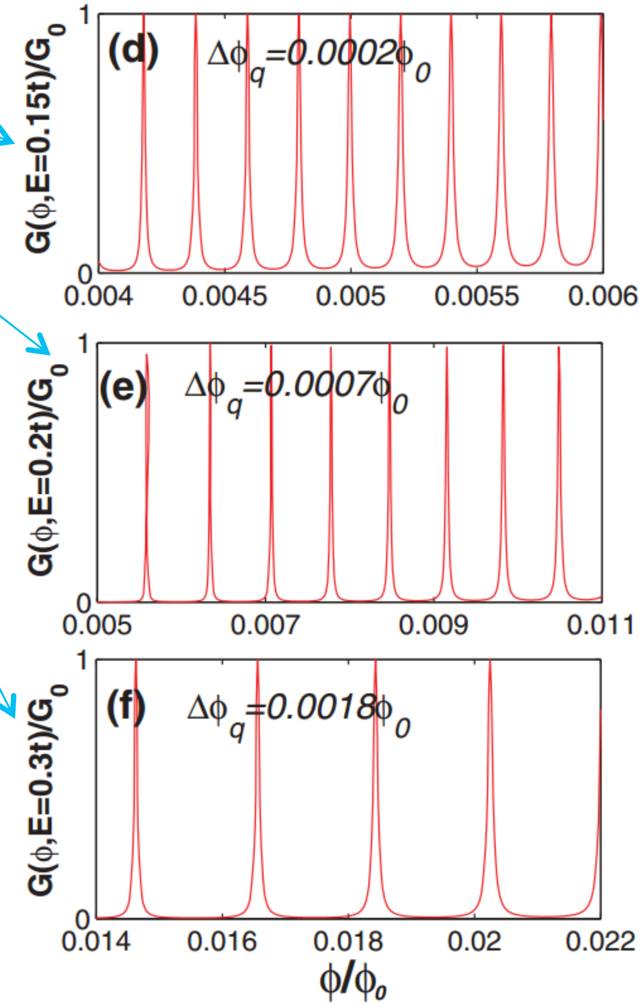
N=4158

N=1616

$$\Delta I = h$$

$$I = \oint \mathbf{p} \cdot d\mathbf{q}$$

$$\mathbf{p} = \hbar \mathbf{k} + e\mathbf{A}$$



$$\boxed{I = |\mathbf{p}|L = \hbar|\mathbf{k}|L + eBS}$$

Fixed magnetic flux Ø

$$E = \hbar v_F k$$

$$\Delta E_q = h v_F / L$$

L is the perimeter of periodic orbit

$\phi = BS_0 \;\; where \;\; S_0 = \frac{\sqrt{3}}{2} a_0^2$

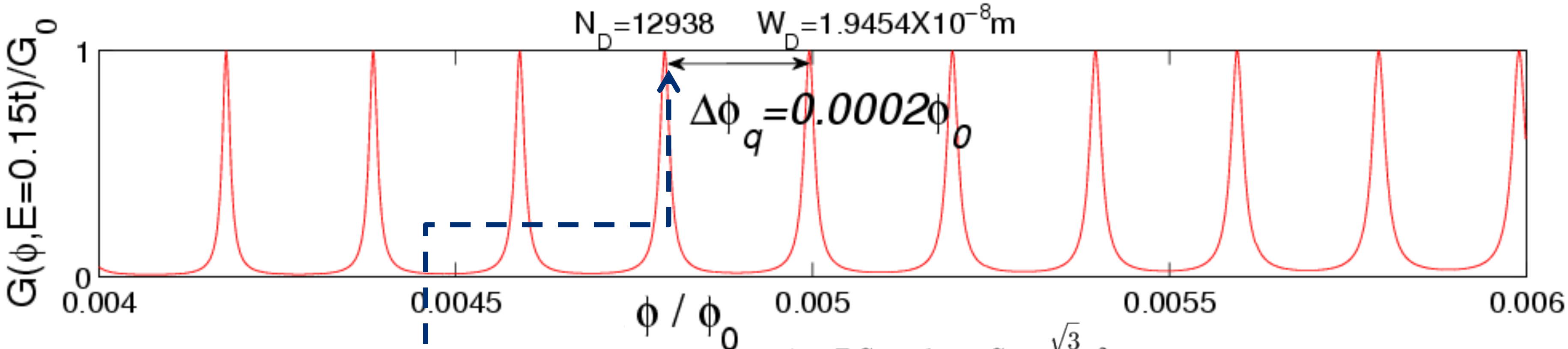
Fixed energy E

$$\Delta\phi = \Delta BS = \phi_0$$

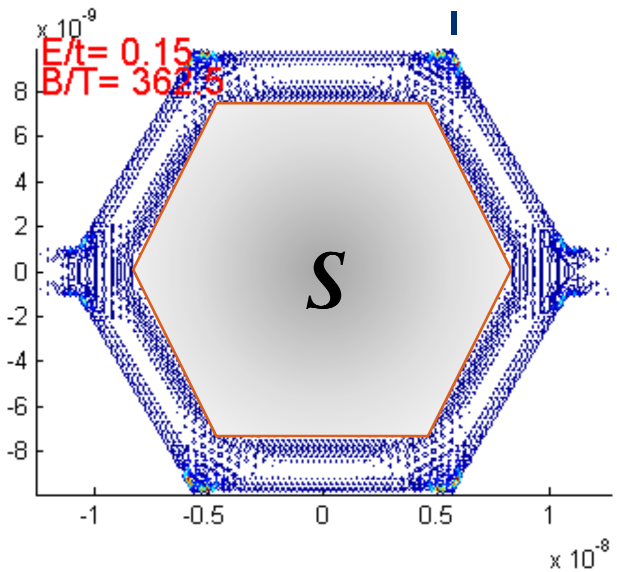
B-S relationship

$$\Delta\phi = \Delta B \cdot S = \phi_0 = h/e$$

$$= 4.12\times10^{-15}\,T\cdot m^2\ ??$$



$\phi = BS_0$ where $S_0 = \frac{\sqrt{3}}{2}a_0^2$



Inner ring:

$$S = \frac{\sqrt{3}}{2}W_d^2 \times \left(\frac{9}{9.8}\right)^2 = 2.60\times10^{-16}\,m^2$$

$$S' = \frac{\phi_0}{\Delta B} = \frac{\phi_0}{\Delta\phi / S_0} = 2.57\times10^{-16}\,m^2$$

$$S' \approx S$$

Device size scaling

$$S/L = k_F \ell_B^2$$

$$\begin{cases} \Delta E_q = h v_F / L \\ \Delta\phi = \Delta B S = \phi_0 \end{cases}$$

$$v_F = \sqrt{3} t_0 a / 2\hbar$$

$$\boxed{S/L = \frac{2\hbar S_0}{\sqrt{3} e a t_0} \frac{\Delta E}{\Delta\phi}}$$

For hexagonal geometry:

$S = \sqrt{3} D^2/2$ and $L = 2\sqrt{3} D$

$$\boxed{D_{\text{hex}} = \frac{12\hbar S_0}{\sqrt{3} e a t_0} \frac{\Delta E}{\Delta\phi}}$$

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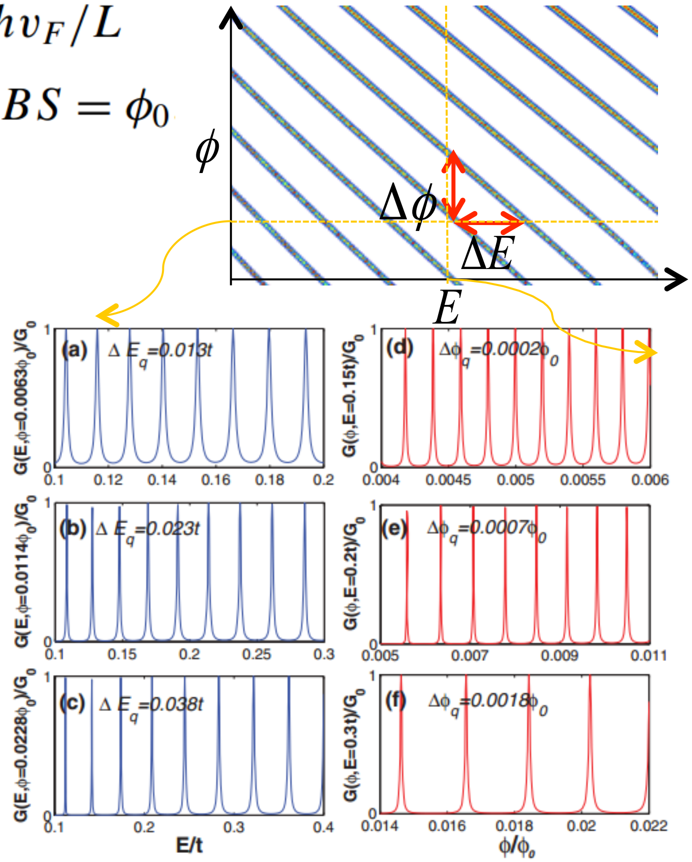

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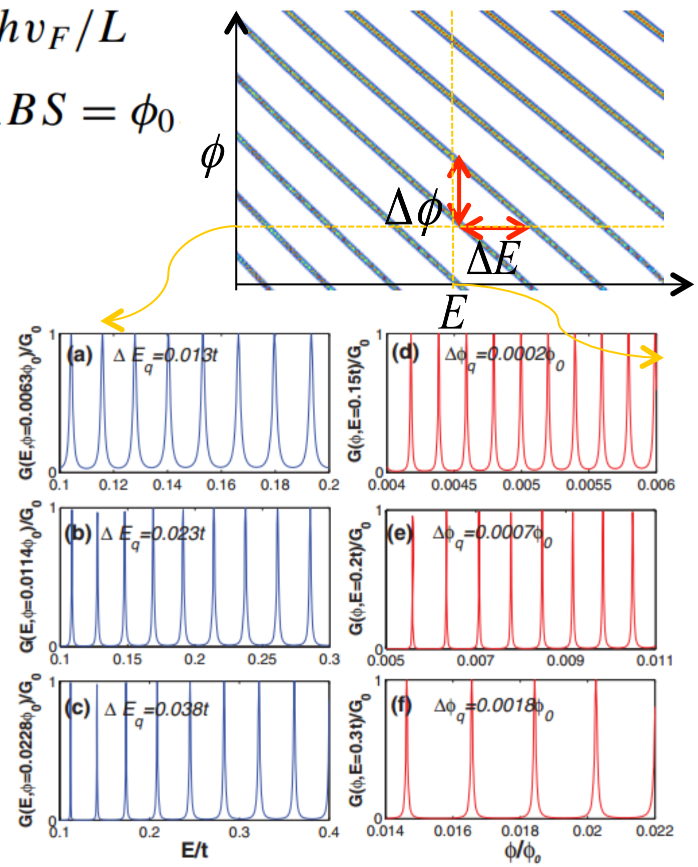

n(atoms N.)	Delta E_q/t	Delta Phi_0	D_hex (nm)	D_hex ratio
1(12938)	0.013	0.0002	13.926	2.94
2(4158)	0.023	0.0007	7.836	1.655
3(1616)	0.038	0.0018	4.734	1

Device size scaling

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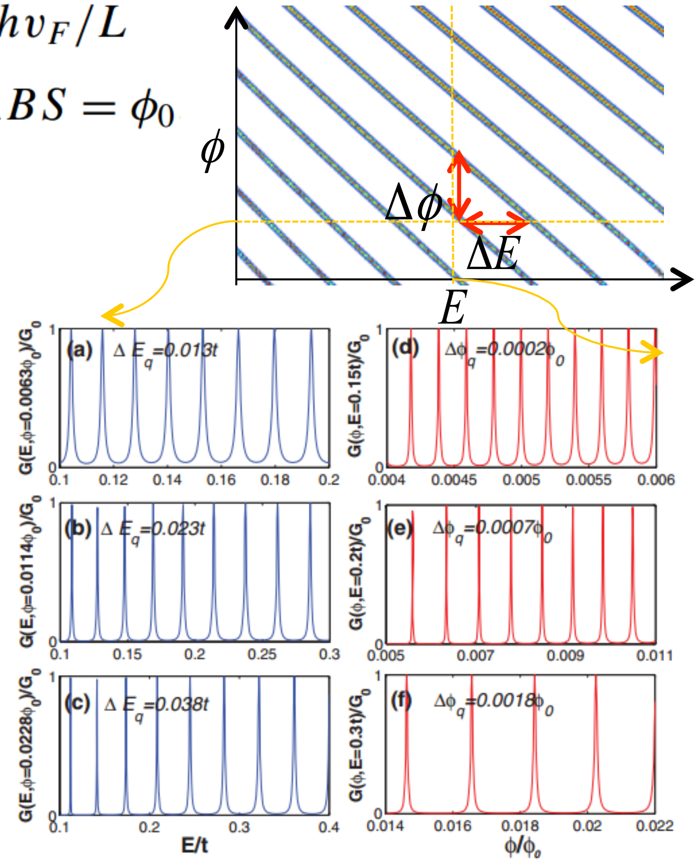
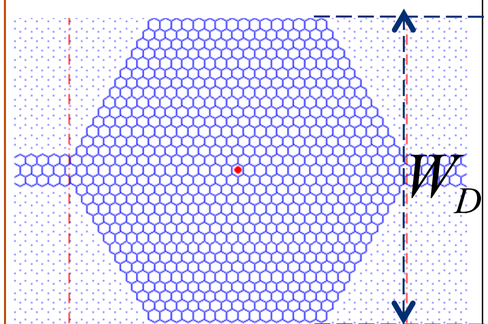


Size of device
Direct measurement



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≈

W_D	Size ratio
19.454	2.92
10.934	1.64
6.674	1

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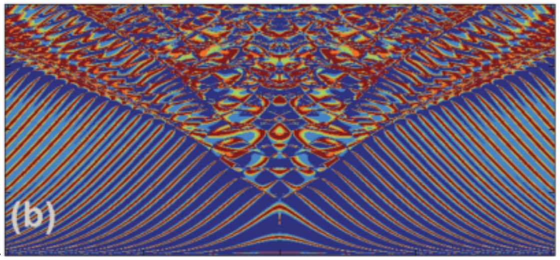
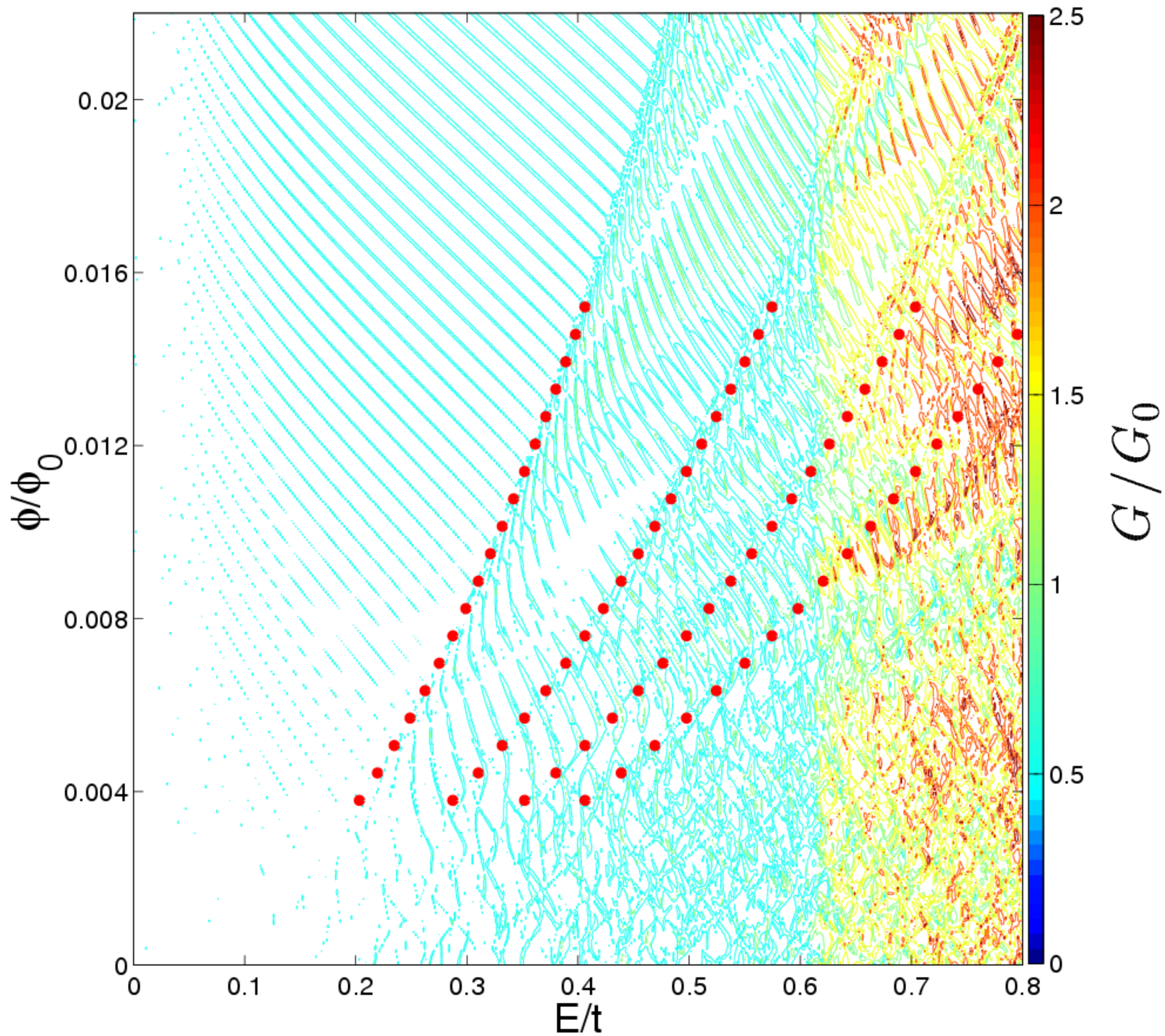
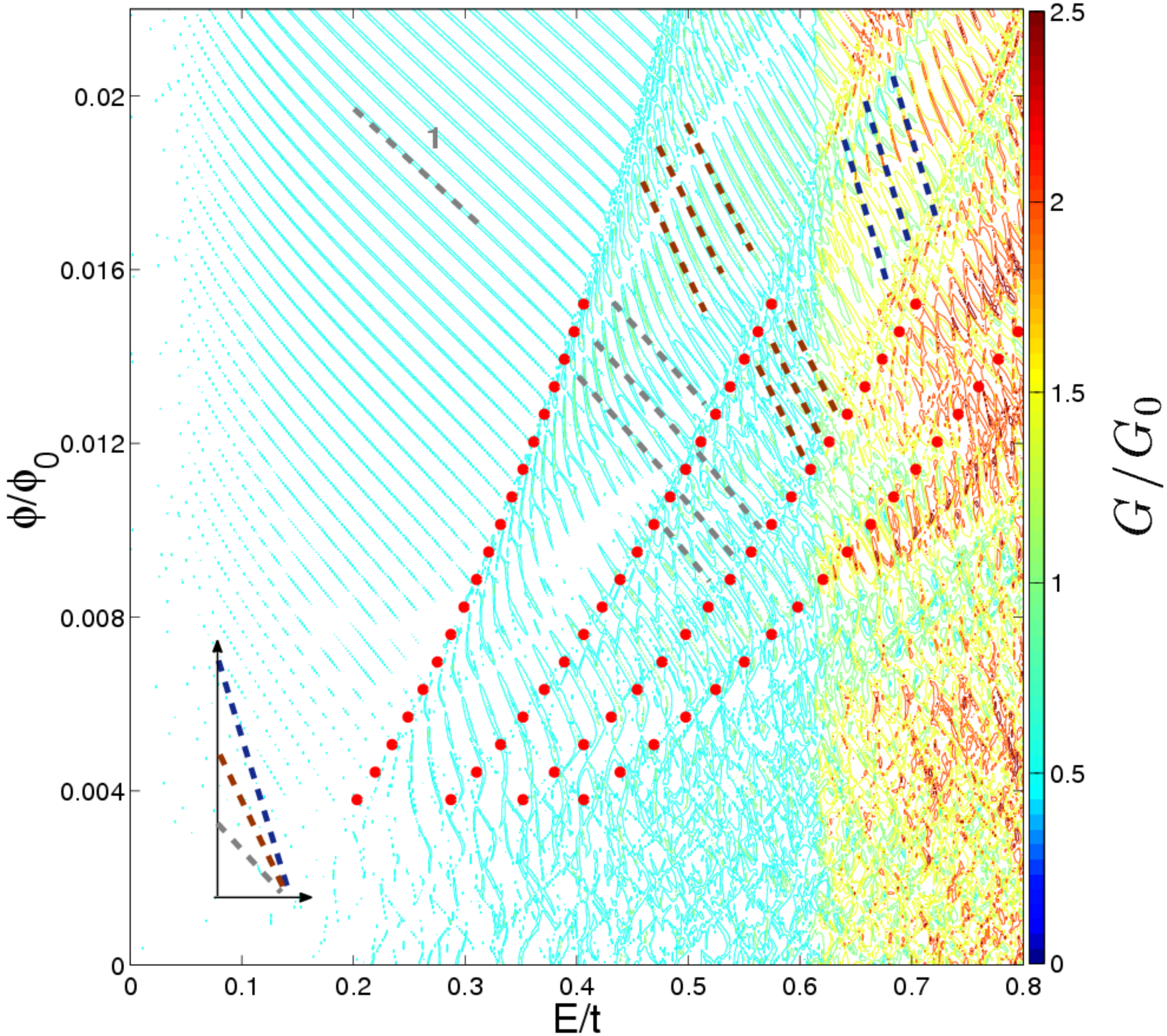


A global view of the conductance oscillations/fluctuations



Universal transition to random conductance fluctuations



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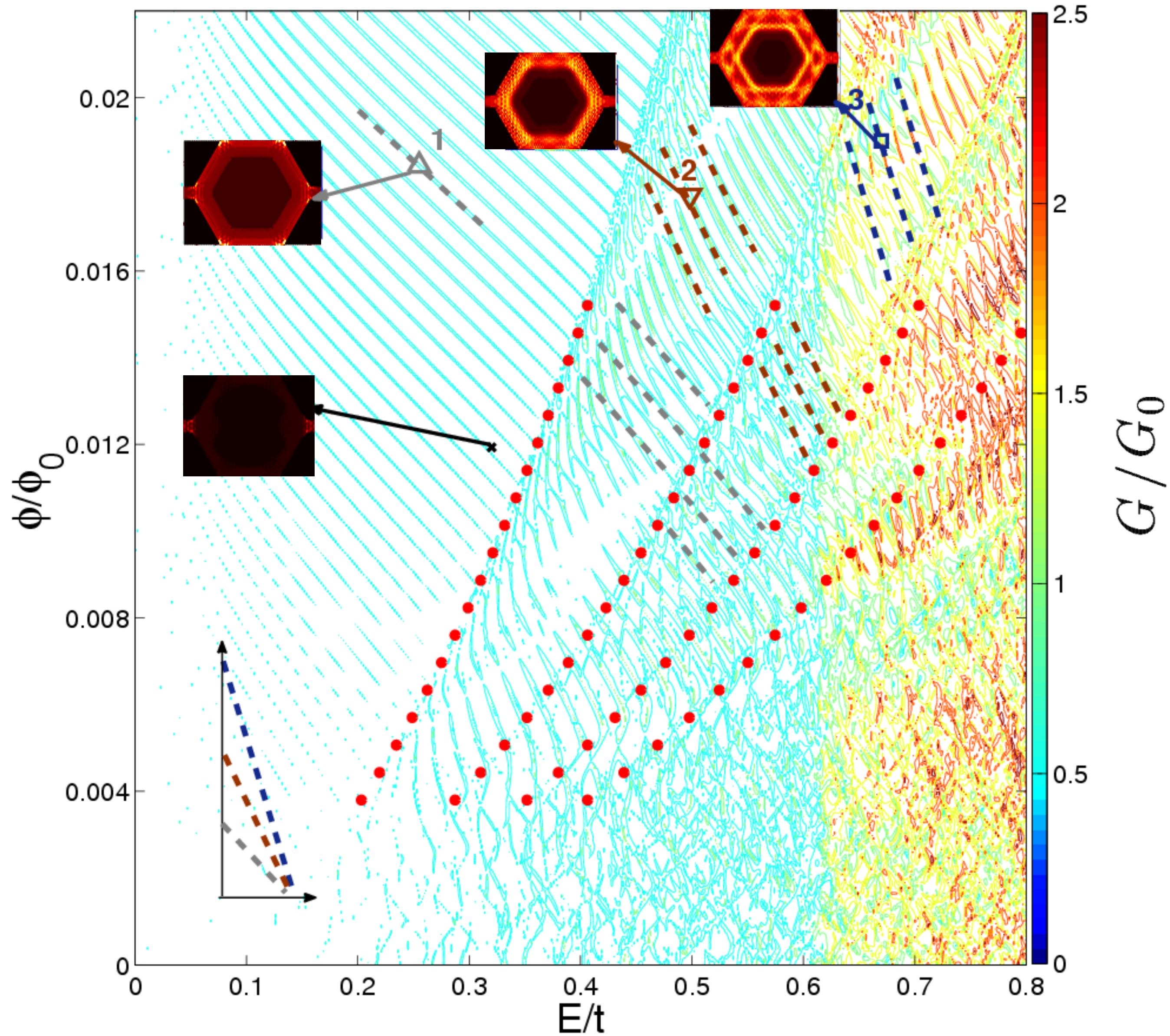
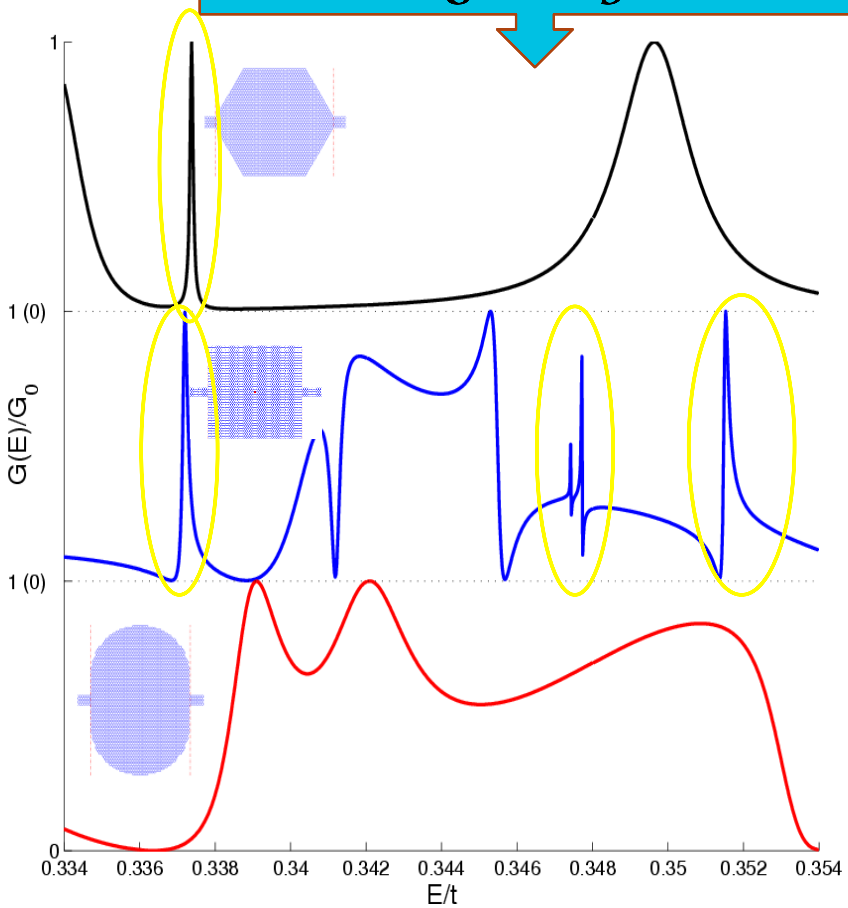


Universal transition to random conductance fluctuations

When magnetic field B=0



Universal transition to random conductance fluctuations

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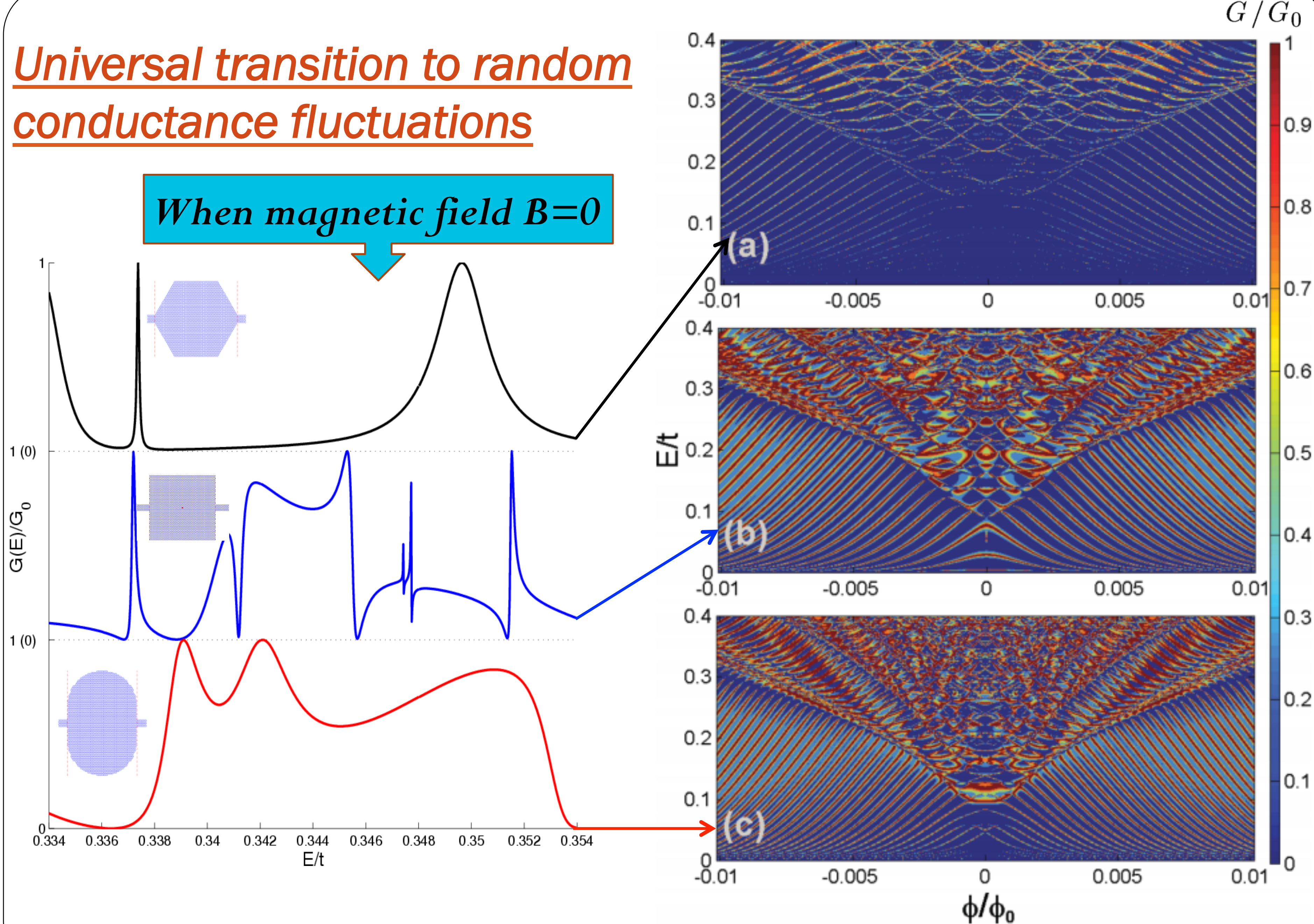

Conductance Fluctuations in Relativistic Quantum Dynamics

Summary

We use tight-binding framework and Green's function method to calculate the conductance for integrable and chaotic dot systems under a transversal magnetic field.

The results show Landau levels divide conductance curves into periodic-oscillation and random-fluctuation regions for hexagonal, rectangular and stadium shapes.

Bohr-Sommerfed Quantisation Condition elucidates the fine-scale structure.

From regular oscillation regions for different sizes, we verify the size scaling $S/L = \frac{2\hbar S_0}{\sqrt{3}eat_0}\frac{\Delta E}{\Delta \phi}$ relationship derived from the semi-classical theory.

L.Ying, L. Huang, Y.-C. Lai, and C. Grebogi, "*Conductance fluctuations in graphene systems:The relevance of classical dynamics*" Phys. Rev. B 85 245448 (2012)