

# Testing Dark Energy: a unified approach

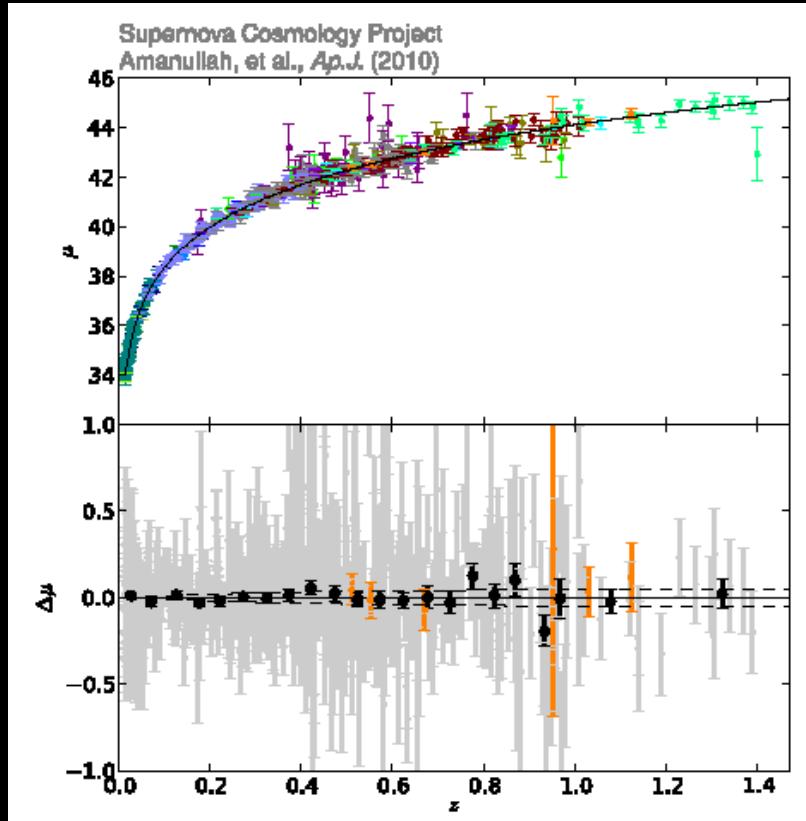
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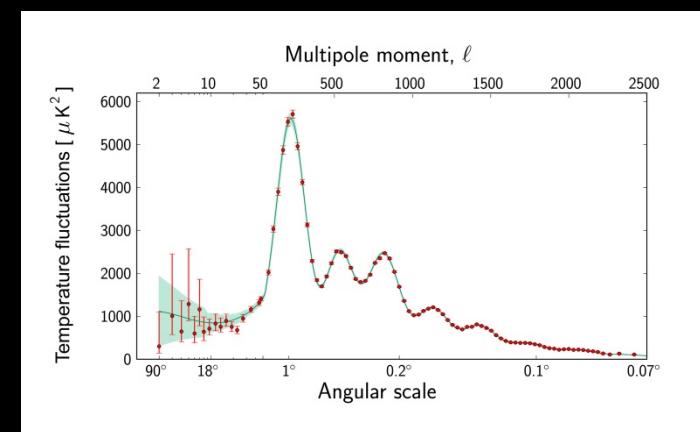
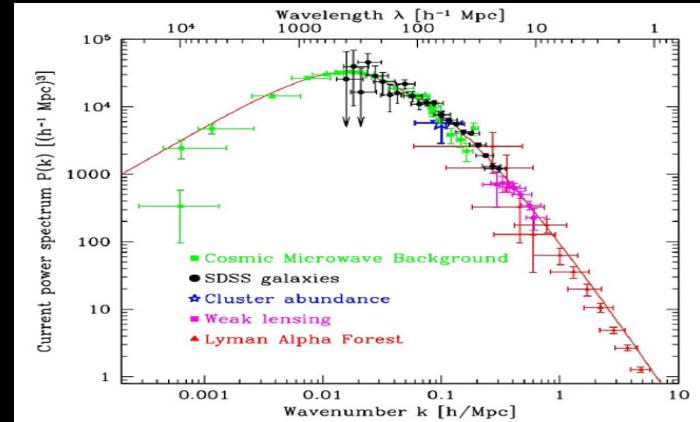
in collaboration with Frank Koennig, Martin Kunz, Mariele  
Motta, Ippocratis Saltas, Ignacy Sawicki, Alejandro Trilleras,  
Adrian Vollmer, Simone Fogli

Sao Paulo 2014

# Testing dark energy



expansion

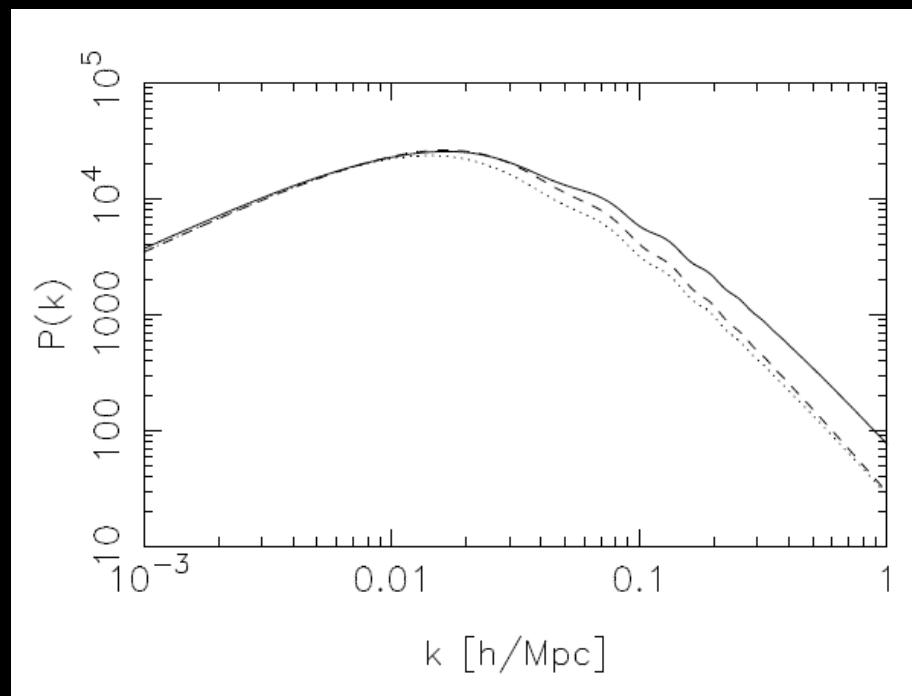


perturbations

# The two main problems of testing dark energy

## 1) Problem of initial conditions

For instance, how do we know if the shape of the power spectrum we observe is due to dark energy or to initial conditions?



# The two main problems of testing dark energy

## 1) Problem of initial conditions

e.g, how do we know if the shape of the power spectrum we observe is due to dark energy or to initial conditions?

## 2) Problem of design

If our model parameter space is sufficiently large, we can design a model to fit any observation

*Cosmological constant, Dark energy  $w=const.$ , Dark energy  $w=w(z)$ , Quintessence, scalar-tensor models, coupled quintessence, mass varying neutrinos,  $k$ -essence, Chaplygin gas, Cardassian, quartessence, quiescence, phantoms,  $f(R)$ , Gauss-Bonnet, anisotropic dark energy, brane dark energy, backreaction, degravitation, TeVeS, Galileons, KGB, multiple dark matter, ...*

# Prolegomena zu einer jeden künftigen Dark Energy physik

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Observations:

- Isotropy
- Large abundance
- Slow evolution
- Weak clustering

Theory:

- Scalar field?
- $\Omega_{\text{DE}} \approx \Omega_m$
- $W_{\text{eff}} \approx -1$
- $c_s \approx 1$

# The past ten years of DE research

$$\int dx^4 \sqrt{-g} \left[ R + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g} \left[ f(\phi)R + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g} \left[ f(\phi)R + K\left(\frac{1}{2} \phi_{,\mu} \phi^{,\mu}\right) + V(\phi) + L_{matter} \right]$$

$$\int dx^4 \sqrt{-g} \left[ f(\phi, \frac{1}{2} \phi_{,\mu} \phi^{,\mu})R + G_{\mu\nu} \phi^{,\nu} \phi^{,\mu} + K\left(\frac{1}{2} \phi_{,\mu} \phi^{,\mu}\right) + V(\phi) + L_{matter} \right]$$

Cosmological constant, Dark energy w=const, Dark energy w=w(z), quintessence, scalar-tensor model, coupled quintessence, k-essence, f(R), Gauss-Bonnet, Galileons, KGB,

# The Horndeski Lagrangian

The most general 4D scalar field theory with second order equation of motion

$$\int dx^4 \sqrt{-g} \left[ \sum_i L_i + L_{matter} \right]$$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\square\phi,$$

$$\mathcal{L}_4 = \underline{G_4(\phi, X)R + G_{4,X}[(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)]},$$

$$\mathcal{L}_5 = \underline{G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi)} - \frac{1}{6}\underline{G_{5,X}[(\square\phi)^3 - 3(\square\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)]}.$$

- First found by Horndeski in 1975
- rediscovered by Deffayet et al. in 2011
- no ghosts, no classical instabilities
- it modifies gravity!
- it includes f(R), Brans-Dicke, k-essence, Galileons, clustering DE etc etc
- Invariant under conformal and disformal transformations

# Massive gravity

Pauli-Fierz (1939) action: the only ghost-free quadratic action for a massive spin two field

$$\int d^4x \sqrt{g} R_g + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} (\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta})$$

The three capital sins of Pauli-Fierz theory:

- It does not reduce to massless gravity for  $m \rightarrow 0$  (vDVZ disc.)
  - It is linear
- It contains a ghost when extended to non-linear level (Boulware-Deser ghost)

# Massive gravity versus bigravity

- The first problem was partially solved by Vainshtein (1972): there exists a radius below which the linear theory cannot be applied;
- For the Sun, this radius is larger than the solar system!
- The second and third problems have been reconsidered very recently:

$$\begin{aligned} S = & -\frac{M_g^2}{2} \int d^4x \sqrt{-\det g} R(g) - \frac{M_f^2}{2} \int d^4x \sqrt{-\det f} R(f) \\ & + m^2 M_g^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^4 \beta_n e_n(\sqrt{g^{\alpha\beta} f_{\beta\gamma}}) + \int d^4x \sqrt{-\det g} L_m(g, \Phi) \end{aligned}$$

The only ghost-free local non-linear  
massive gravity theory is a second order theory!

deRham, Gabadadze, and coll., 2010  
Hassan & Rosen, 2011

# The next ten years of DE research

**Combine observations of background, linear  
and non-linear perturbations to reconstruct  
as much as possible the Horndeski & Massive Gravity model**

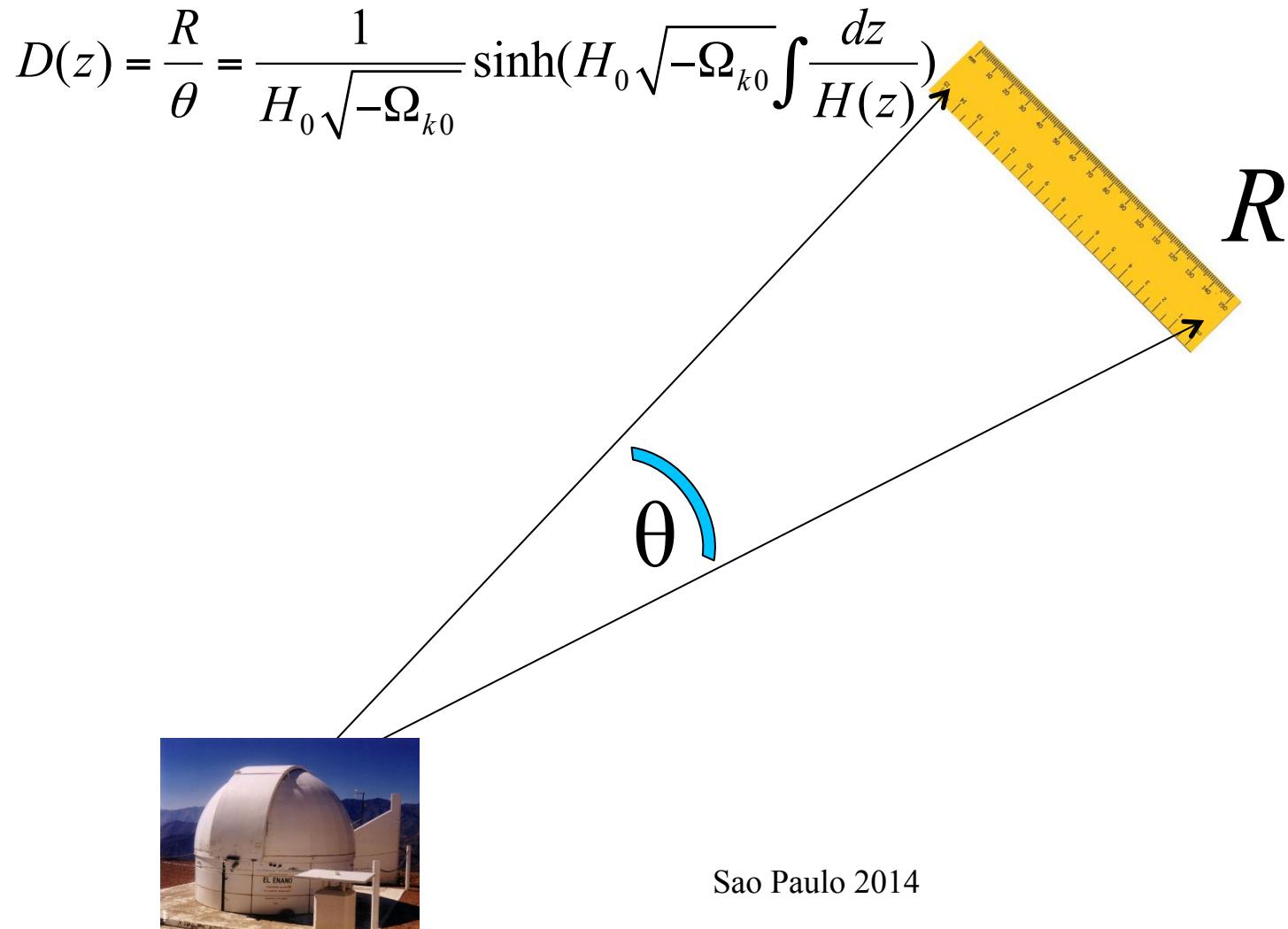
**... or to rule them out!**

# The Great Horndeski Hunt

**Let us assume we have only**

- 1) a perturbed FRW metric**
- 2) pressureless matter**
- 3) the Horndeski field**

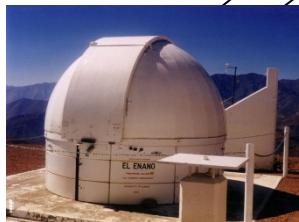
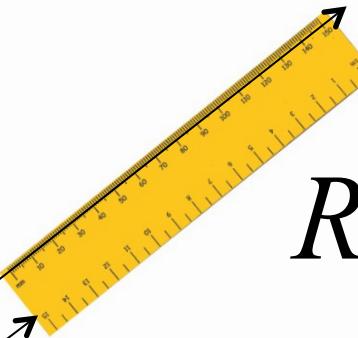
# Standard rulers



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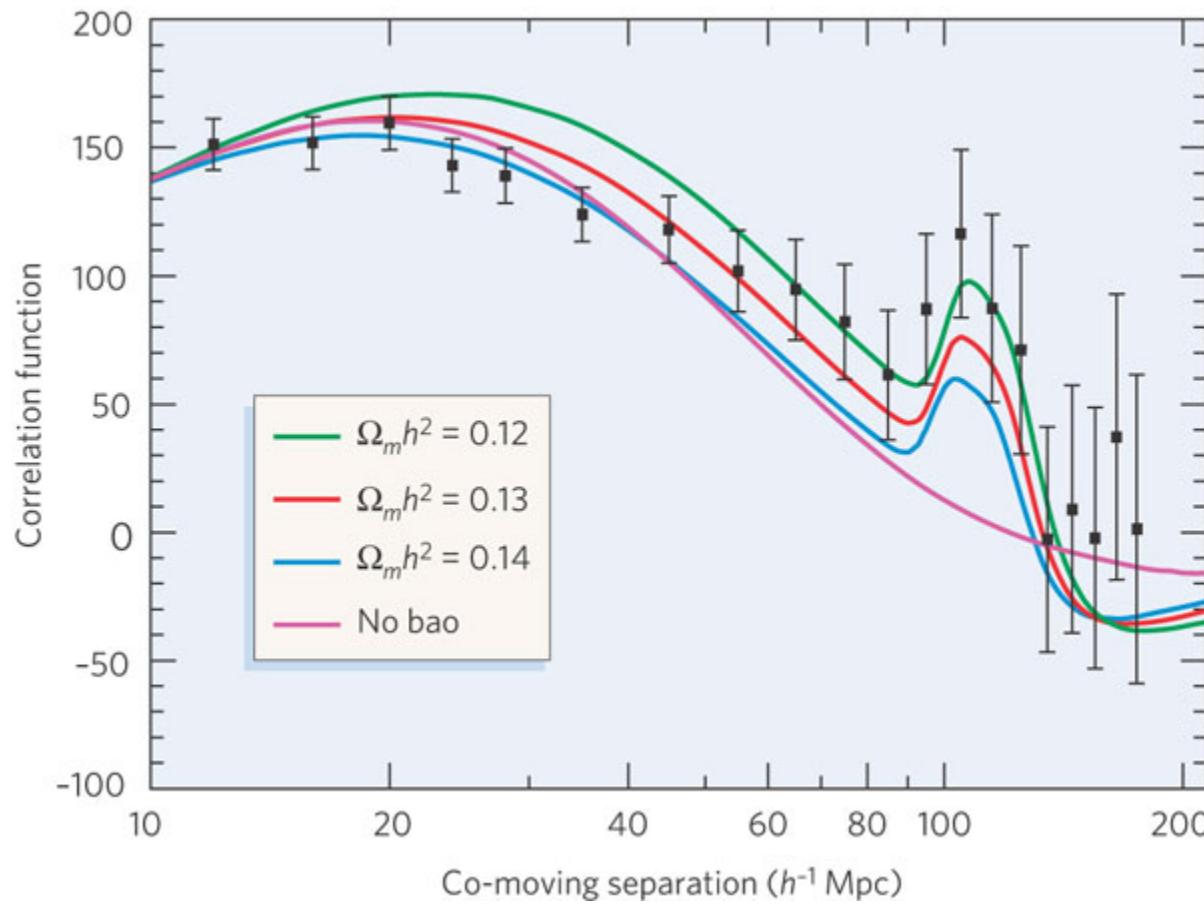
# Standard rulers

$$H(z) = \frac{dz}{R}$$



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# BAO ruler



Charles L. Bennett

Nature 440, 1126-1131(27 April 2006)

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# Background: SNIa, BAO, ...

Then we can measure  $H(z)$  and

$$D(z) = \frac{1}{H_0 \sqrt{-\Omega_{k0}}} \sinh(H_0 \sqrt{-\Omega_{k0}} \int \frac{dz}{H(z)})$$

and therefore we can reconstruct the  
full FRW metric

$$ds^2 = dt^2 - \frac{a(t)^2}{\left(1 - \frac{\Omega_{k0}}{4} r^2\right)^2} (dx^2 + dy^2 + dz^2)$$

# Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation

$$\nabla^2 \Psi = 4\pi G \rho_m \delta_m$$

- anisotropic stress

$$1 = -\frac{\Psi}{\Phi}$$

# Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation

$$\nabla^2 \Psi = 4\pi G Y(k, a) \rho_m \delta_m$$

- anisotropic stress

$$\eta(k, a) = -\frac{\Phi}{\Psi}$$

# Modified Gravity at the linear level

- standard gravity

$$Y(k, a) = 1$$

$$\eta(k, a) = 1$$


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- scalar-tensor models

$$Y(a) = \frac{G^*}{FG_{cav,0}} \frac{2(F + F'^2)}{2F + 3F'^2}$$

Boisseau et al. 2000  
Acquaviva et al. 2004  
Schimd et al. 2004  
L.A., Kunz & Sapone 2007

$$\eta(a) = 1 + \frac{F'^2}{F + F'^2}$$


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- f(R)

$$Y(a) = \frac{G^*}{FG_{cav,0}} \frac{1 + 4m \frac{k^2}{a^2 R}}{1 + 3m \frac{k^2}{a^2 R}}, \quad \eta(a) = 1 + \frac{m \frac{k^2}{a^2 R}}{1 + 2m \frac{k^2}{a^2 R}}$$

Bean et al. 2006  
Hu et al. 2006  
Tsujikawa 2007

- DGP

$$Y(a) = 1 - \frac{1}{3\beta}; \quad \beta = 1 + 2Hr_c w_{DE}$$

Lue et al. 2004;  
Koyama et al. 2006

$$\eta(a) = 1 + \frac{2}{3\beta - 1}$$


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- massive bi-gravity

$$Y(a) = \dots$$

$$\eta(a) = \dots$$

see F. Koennig and L. A. 2014

# Modified Gravity at the linear level

In the **quasi-static limit**, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k, a) = h_2 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right)$$

**k = wavenumber**

$$Y(k, a) = h_1 \left( \frac{1 + k^2 h_5}{1 + k^2 h_3} \right)$$

**$h_i$  = time-dependent functions**

De Felice et al. 2011; L.A. et al. PRD, arXiv:1210.0439, 2012

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# Modified Gravity at the linear level

$$\begin{aligned}
h_1 &\equiv \frac{w_4}{w_1^2} = \frac{c_T^2}{w_1}, \quad h_2 \equiv \frac{w_1}{w_4} = c_T^{-2}, \\
h_3 &\equiv \frac{H^2}{2X M^2} \frac{2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2 (\dot{w}_2 + \rho_m)}{2w_1^2}, \\
h_4 &\equiv \frac{H^2}{2X M^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 2w_1 \dot{w}_1 H + w_2 \dot{w}_1 - w_1 (\dot{w}_2 + \rho_m)}{w_1}, \\
h_5 &\equiv \frac{H^2}{2X M^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 4w_1 \dot{w}_1 H + 2\dot{w}_1^2 - w_4 (\dot{w}_2 + \rho_m)}{w_4},
\end{aligned}$$

$$\begin{aligned}
w_1 &\equiv 1 + 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X}), \\
w_2 &\equiv -2\dot{\phi}(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}) + \\
&\quad + 2H(w_1 - 4X(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X})) - \\
&\quad - 2\dot{\phi}XH^2(3G_{5,X} + 2XG_{5,XX}), \\
w_3 &\equiv 3X(K_X + 2XK_{XX} - 2G_{3,\phi} - 2XG_{3,\phi X}) + 18\dot{\phi}XH(2G_{3,X} + XG_{3,XX}) - \\
&\quad - 18\dot{\phi}H(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX}) - \\
&\quad - 18H^2(1 + G_4 - 7XG_{4,X} - 16X^2G_{4,XX} - 4X^3G_{4,XXX}) - \\
&\quad - 18XH^2(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX}) + \\
&\quad + 6\dot{\phi}XH^3(15G_{5,X} + 13XG_{5,XX} + 2X^2G_{5,XXX}), \\
w_4 &\equiv 1 + 2(G_4 - XG_{5,\phi} - XG_{5,X}\dot{\phi}).
\end{aligned}$$

De Felice et al. 2011; L.A. et al., PRD, arXiv:1210.0439, 2012  
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# Yukawa Potential

$$\eta(k, a) = h_2 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right)$$

Momentum space

$$Y(k, a) = h_1 \left( \frac{1 + k^2 h_5}{1 + k^2 h_3} \right)$$

$$\nabla^2 \Psi = 4\pi G Y(k, a) \rho_m \delta_m$$



$$\Psi = -\frac{GM}{r} h_2 \left( 1 + \frac{h_4 - h_5}{h_5} e^{-r/\sqrt{h_5}} \right) = -\frac{\bar{G}M}{r} (1 + Q e^{-mr}) \quad \text{Real space}$$

De Felice et al. 2011; L.A. et al. PRD, arXiv:1210.0439, 2012

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# Quasi-static approximation

$$c_s^2 k^2 \gg a^2 H^2$$

**From a wave equation:**

$$\begin{aligned} E_{\delta\phi} \equiv & D_1 \ddot{\Phi} + D_2 \ddot{\delta\phi} + D_3 \dot{\Phi} + D_4 \dot{\delta\phi} + D_5 \dot{\Psi} + D_6 \frac{k^2}{a^2} \dot{\chi} \\ & + \left( D_7 \frac{k^2}{a^2} + D_8 \right) \Phi + \left( D_9 \frac{k^2}{a^2} - M^2 \right) \delta\phi + \left( D_{10} \frac{k^2}{a^2} + D_{11} \right) \Psi + D_{12} \frac{k^2}{a^2} \chi = 0, \end{aligned}$$

**To a “Poisson” equation:**

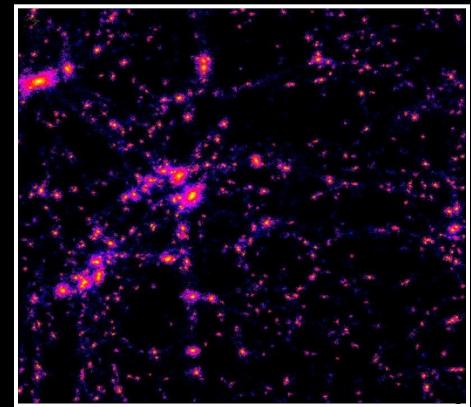
$$B_7 \frac{k^2}{a^2} \Phi + \left( D_9 \frac{k^2}{a^2} - M^2 \right) \delta\phi + A_6 \frac{k^2}{a^2} \Psi \simeq 0,$$

# Reconstruction of the metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

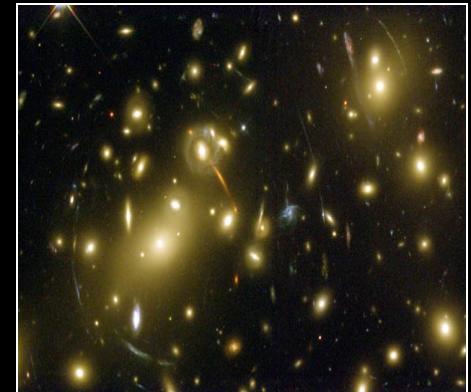
Non-relativistic particles respond to  $\Psi$

$$\delta_m'' + (1 + \frac{\mathcal{H}'}{\mathcal{H}})\delta_m' = -k^2\Psi$$



Relativistic particles respond to  $\Phi-\Psi$

$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$



# Deconstructing the galaxy power spectrum

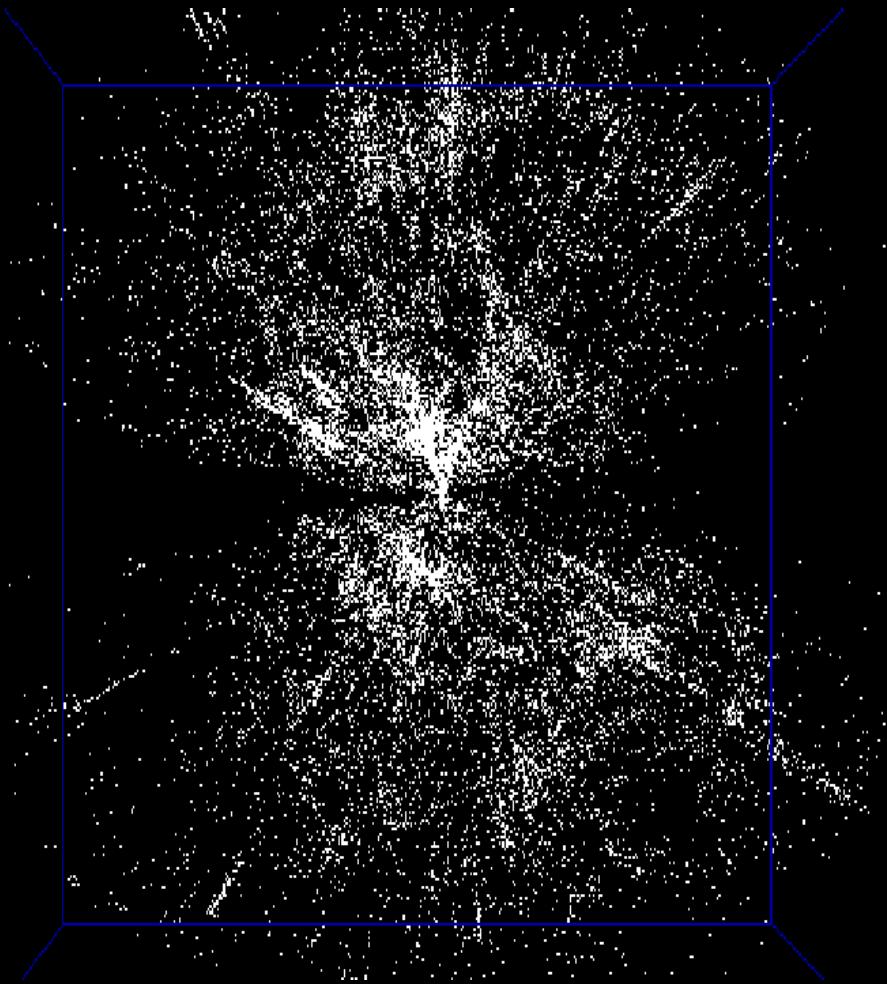
Galaxy clustering

$$\delta_{gal}(k, z, \mu) = Gb\sigma_8(1 + \frac{f}{b}\mu^2)\delta_{t,0}(k)$$

The equation shows the deconstruction of galaxy clustering into its components:

- Growth function ( $G$ )
- Redshift distortion
- Galaxy bias ( $b$ )
- Line of sight angle
- Present mass power spectrum ( $\delta_{t,0}$ )
- Normalization

Red arrows indicate the flow from the components to the final clustering equation. A red arrow also points from the "Line of sight angle" component to the term  $\frac{f}{b}\mu^2$ . A red bracket on the right side of the equation indicates the definition of the growth rate  $f = \frac{\delta'}{\delta}$ .



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# Three linear observables: A, R, L

$\mu=0$   
Amplitude  
**A**

clustering

$$\delta_{gal}(k, z, 0) = Gb\sigma_8 \delta_{m,0}(k) \equiv A$$

$\mu=1$   
Redshift distortion  
**R**

$$\delta_{gal}(k, z, 1) = G\sigma_8 f \delta_{m,0}(k) \equiv R$$

Lensing  
**L**

lensing

$$k^2 \Phi_{lens} = k^2 (\Psi - \Phi) = -\frac{3}{2} \Sigma G \Omega_m \sigma_8 \delta_{m,0}(k) \equiv L$$

$$\Sigma = Y(1 + \eta)$$

# The only model-independent ratios

**Redshift distortion/Amplitude**

$$P_1 = \frac{R}{A} = \frac{f}{b}$$

**Lensing/Redshift distortion**

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1 + \eta)}{f}$$

**Redshift distortion rate**

$$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

**Expansion rate**

$$E = \frac{H}{H_0}$$

**How to combine them to test the theory?**

# Summarizing....

Matter conservation equation  
independent of gravity theory

$$\delta_m'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right) \delta_m' = -k^2 \Psi :$$

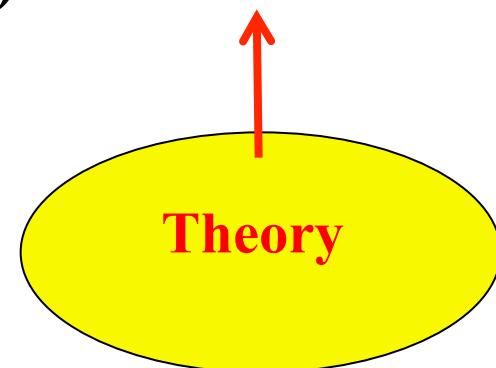
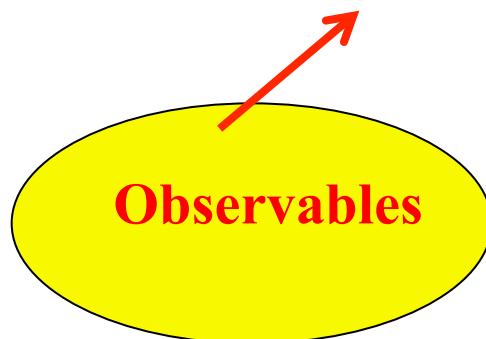
## Observables

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f} \quad P_3 = \frac{R'}{R} = \frac{f'}{f} + f \quad E = \frac{H}{H_0}$$

# The anisotropic stress is directly observable

A unique combination of model independent observables

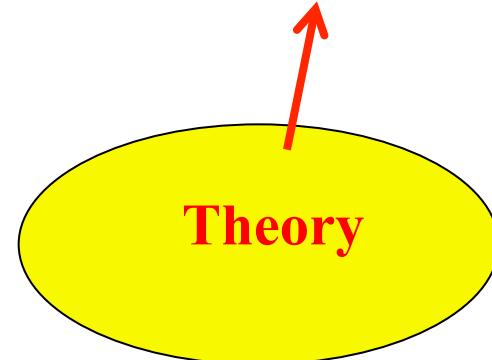
$$\frac{3P_2(1+z)^3}{2E^2(P_3 + 2 + \frac{E'}{E})} - 1 = \eta$$



# Testing the entire Horndeski Lagrangian

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2(P_3 + 2 + \frac{E'}{E})} - 1 = \eta = h_2 \left( \frac{1+k^2h_4}{1+k^2h_5} \right)$$



# Horndeski Lagrangian: not too big to fail

$$g(z, k) \equiv \frac{(REa^2)'}{LEa^2}$$

$$2g_{,k}g_{,kk} - 3(g_{,kk})^2 = 0$$

If this relation is falsified, the Horndeski theory is rejected\*

L.A., M. Motta, I. Sawicki,  
M. Kunz, I. Saltas, 1210.0439

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# Beyond the quasi-static condition

## General consistency relation

$$\begin{aligned} \eta\Gamma' + \eta'' + \Gamma(\eta\Gamma + 2\eta' + \bar{\alpha}_1\eta - \bar{\alpha}_2) + \\ + \bar{\alpha}_1\eta' + \bar{\alpha}_3\eta - \bar{\alpha}_5 + k^2(\bar{\alpha}_4\eta - \bar{\alpha}_6) = \bar{\alpha}_7\varpi. \end{aligned}$$

$$\Gamma \equiv \frac{\Psi'}{\Psi} = \frac{L'}{L} - \frac{\eta'}{1+\eta} - 1,$$

$$\varpi = \frac{1+\eta}{P_2}$$

M. Motta, L.A, I. Sawicki,  
M. Kunz, I. Saltas, 1305.0008  
2013

# Massive gravity versus bigravity

The only ghost-free local non-linear massive gravity theory

$$S = -\frac{M_g^2}{2} \int d^4x \sqrt{-\det g} R(g) - \frac{M_f^2}{2} \int d^4x \sqrt{-\det f} R(f)$$
$$+ m^2 M_g^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^4 \beta_n e_n(\sqrt{g^{\alpha\beta} f_{\beta\gamma}}) + \int d^4x \sqrt{-\det g} L_m(g, \Phi)$$

Two dynamical metrics!

Only one is coupled

$$g_{\mu\nu}, \quad f_{\mu\nu}$$

# Bigravity at the background level

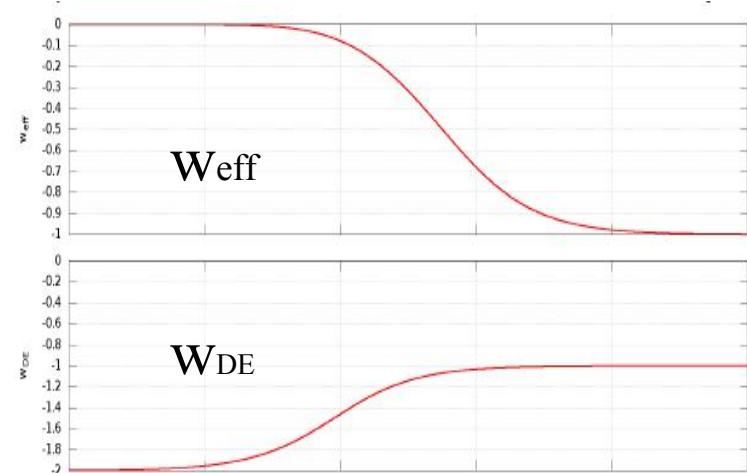
Let us drastically simplify the problem:

We assume two spatially flat FRW-like metrics,  
with two scale factors  $a(t)$ ,  $b(t)$

**At the background level, one finds that  
the problem is one-dimensional**

$$r' = \frac{3r\Omega_m(\beta_1 + 3\beta_2r + 3\beta_3r^2 + \beta_4r^3)}{\beta_1 - 3\beta_3r^2 + 2\beta_4r^3 + 3(\beta_1 + 2\beta_2r + \beta_3r^2)r^2}, \quad r \equiv \frac{b}{a}$$

**Final state always de Sitter**  
No problem in recovering an  
approximate LCDM behavior,  
fitting SN etc.



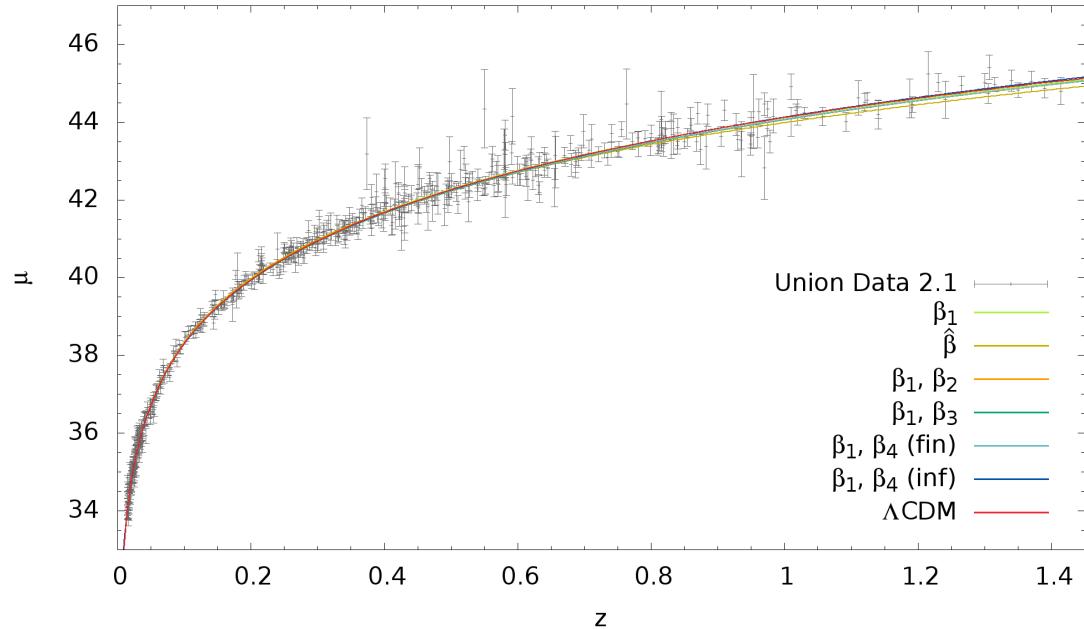
# Minimal Bimetric Model

$$\beta_{0,2,3,4} = 0,$$

$$\beta_1 \neq 0$$

$$r' = \frac{3r(1 - 3r^2)}{1 + 3r^2}, \quad r \equiv \frac{b}{a}$$

$$w_{MBM} = \frac{2}{\Omega_m - 2},$$



One parameter, analytical background behavior, as simple as LCDM, but fully distinguishable

$$\beta_1 = 1.38 \pm 0.03$$

$$w_{MBM} \sim -1.22 - 0.64 \frac{z}{1+z}$$

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# Bigravity at the linear level

The linear perturbations are much more complicated.  
6 d.o.f., instead of 2 in standard gravity, for scalar perturbations.

$$\Phi = -\frac{B_2 a^2 r}{2k^2 + 3B_2 a^2 r} k^2 \Delta E + \frac{3B_2 a^2 r}{2k^2 + 3B_2 a^2 r} \Phi_f + \frac{a^2 \rho_m}{2k^2 + 3B_2 a^2 r} \delta_m$$

$$\Psi = -\Phi - A_2 a^2 r \Delta E$$

$$\Phi_f = \frac{B_2 a^2}{3B_2 a^2 + 2k^2 r} k^2 \Delta E + \frac{B_2 a^2}{3B_2 a^2 + 2k^2 r} \Phi$$

$$\Psi_f = \frac{2A_2 a^2}{3B_2 a^2 + 2k^2(r+r')} k^2 \Delta E + \frac{6A_2 a^2}{3B_2 a^2 + 2k^2(r+r')} \Phi$$

$$+ \frac{3B_2 a^2}{3B_2 a^2 + 2k^2(r+r')} \Psi + \frac{6A_1 a^2 r - 2k^2(r+r')}{3B_2 a^2 + 2k^2(r+r')} \Phi_f$$

$$\Psi_f = -\Phi_f + \frac{A_2 a^2}{r+r'} \Delta E$$

Sub-horizon  
approximation

F. Koennig, M.Motta, L.A., in preparation

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# Bigravity at the linear level

In the quasi-static limit we obtain  
again the Horndeski form!

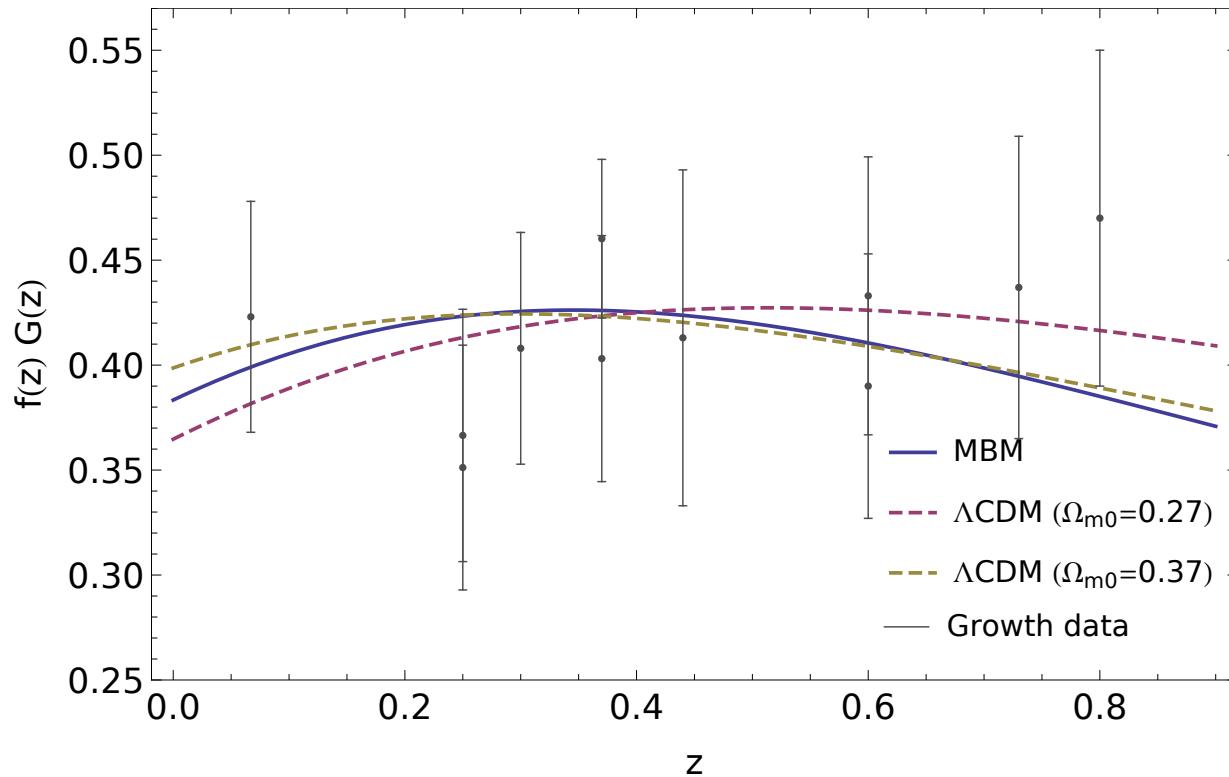
$$\eta(k, a) = H_2 \left( \frac{1 + k^2 H_4}{1 + k^2 H_5} \right)$$

$$Y(k, a) = H_1 \left( \frac{1 + k^2 H_5}{1 + k^2 H_3} \right)$$

F. Koennig, L.A. 1402.1988

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# Bigravity at the linear level



# Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

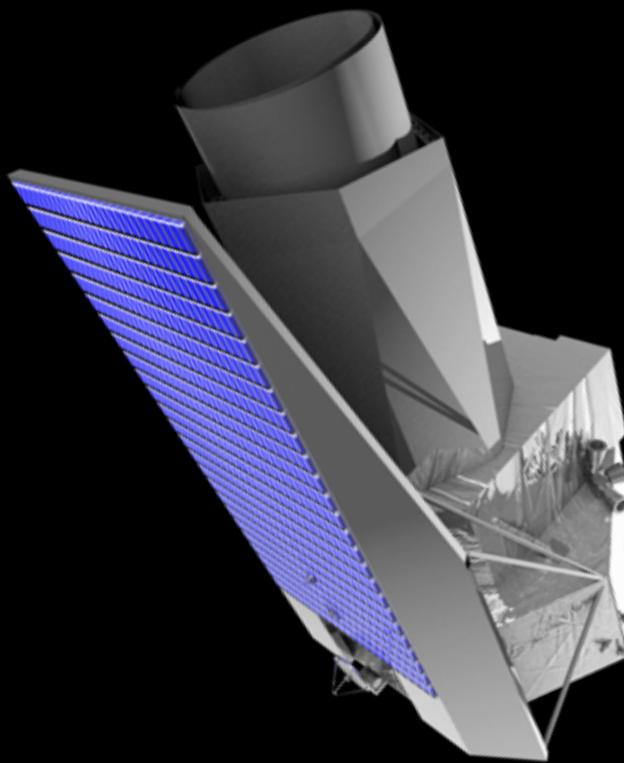
15,000 square degrees

70 million redshifts, 2 billion images

Median redshift  $z = 1$

PSF FWHM  $\sim 0.18''$

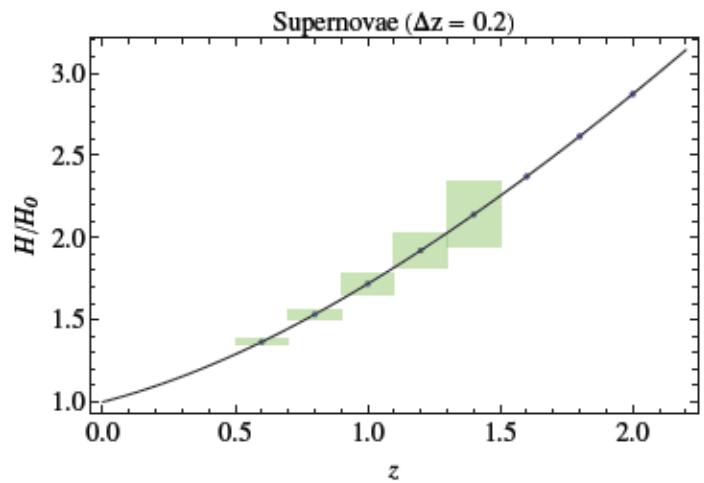
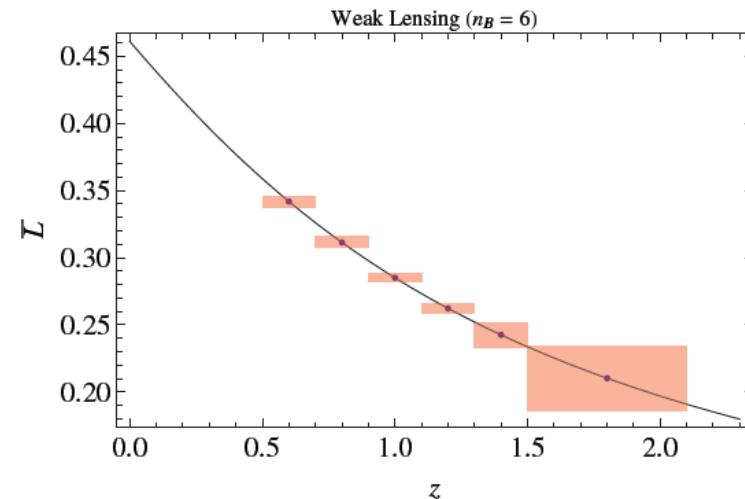
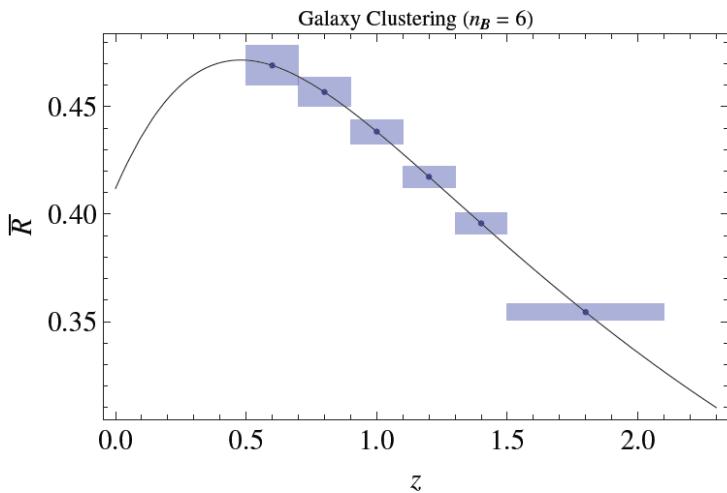
>1000 peoples, >10 countries



Euclid  
satellite

# Euclid forecasts...

Combining galaxy clustering, weak lensing and SN....



$$G\sigma_8 f \delta_{m,0}(k) \equiv R$$

100k SN (LSST)

$$-\frac{3}{2} \Sigma G \Omega_m \sigma_8 \delta_{m,0}(k) \equiv L$$

Sao Paulo 2014

# Results..

$$\eta(k,a) = H_2 \left( \frac{1+k^2 H_4}{1+k^2 H_5} \right)$$

Model 1:  $\eta$  constant for all z, k  
 Error on  $\eta$  around 1%

Model 2:  $\eta$  varies in z  
 Error on  $\eta$

| $\bar{z}$ | $P_1(\bar{z})$ | $\Delta P_1$ | $\Delta P_1(\%)$ | $P_2(\bar{z})$ | $\Delta P_2$ | $\Delta P_2(\%)$ | $P_3(\bar{z})$ | $\Delta P_3$ | $\Delta P_3(\%)$ | $(E'/E)(\bar{z})$ | $\Delta E'/E$ | $\Delta E'/E(\%)$ | $\bar{\eta}$ | $\Delta \bar{\eta}$ | $\Delta \bar{\eta}(\%)$ |
|-----------|----------------|--------------|------------------|----------------|--------------|------------------|----------------|--------------|------------------|-------------------|---------------|-------------------|--------------|---------------------|-------------------------|
| 0.6       | 0.766          | 0.012        | 1.6              | 0.729          | 0.011        | 1.6              | 0.134          | 0.13         | 99.              | -0.92             | 0.018         | 1.9               | 1            | 0.11                | 11.                     |
| 0.8       | 0.819          | 0.01         | 1.2              | 0.682          | 0.0088       | 1.3              | 0.317          | 0.12         | 38.              | -1.04             | 0.038         | 3.7               | 1            | 0.091               | 9.1                     |
| 1.        | 0.859          | 0.0093       | 1.1              | 0.65           | 0.0086       | 1.3              | 0.46           | 0.12         | 26.              | -1.13             | 0.085         | 7.5               | 1            | 0.089               | 8.9                     |
| 1.2       | 0.888          | 0.0092       | 1.               | 0.628          | 0.014        | 2.3              | 0.569          | 0.13         | 23.              | -1.21             | 0.11          | 9.5               | 1            | 0.097               | 9.7                     |
| 1.4       | 0.911          | 0.01         | 1.1              | 0.613          | 0.019        | 3.2              | 0.654          | 0.05         | 7.7              | -1.26             | 0.052         | 4.1               | 1            | 0.034               | 3.4                     |

# Results..

Model 3:  $\eta$  varies in z and k  
 Error on  $\eta$

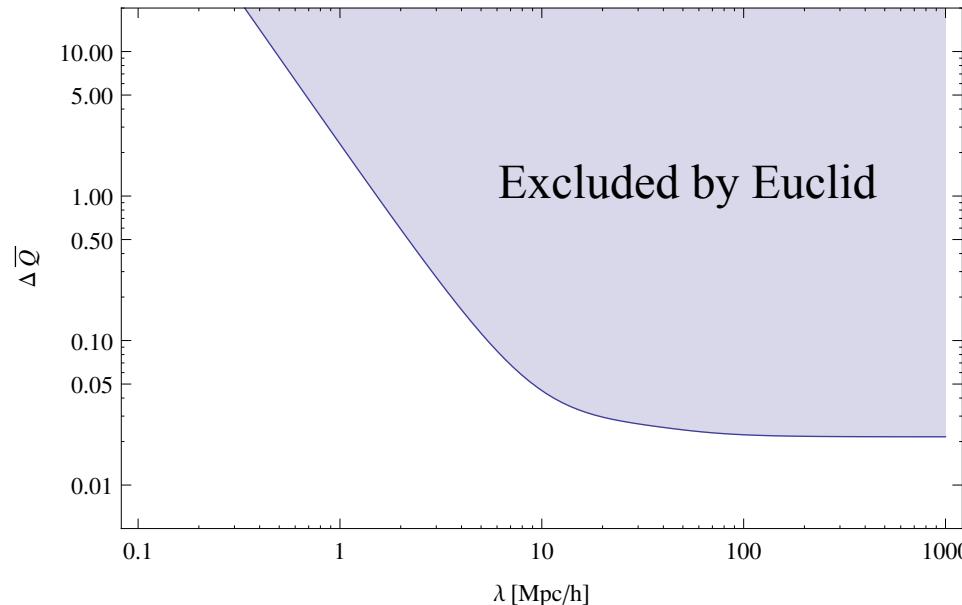
| $\bar{z}$ | $i$ | $P_1$ | $\Delta P_1$ | $\Delta P_1(\%)$ | $P_2$ | $\Delta P_2$ | $\Delta P_2(\%)$ | $P_3$ | $\Delta P_3$ | $\Delta P_3(\%)$ | $\bar{\eta}$ | $\Delta \bar{\eta}$ | $\Delta \bar{\eta}(\%)$ |
|-----------|-----|-------|--------------|------------------|-------|--------------|------------------|-------|--------------|------------------|--------------|---------------------|-------------------------|
| 0.6       | 1   | 0.14  | 18           |                  | 0.12  | 17           |                  | 1.4   | 1100         |                  | 1.1          | 120                 |                         |
|           | 2   | 0.766 | 0.032        | 4.1              | 0.729 | 0.030        | 4.1              | 0.134 | 0.33         | 240              | 1            | 0.26                | 26                      |
|           | 3   | 0.013 | 1.7          |                  | 0.015 | 2.0          |                  | 0.15  | 110          |                  | 0.12         | 12                  |                         |
| 0.8       | 1   | 0.11  | 13           |                  | 0.092 | 13           |                  | 1.2   | 380          |                  | 0.93         | 93                  |                         |
|           | 2   | 0.819 | 0.024        | 2.9              | 0.682 | 0.021        | 3.1              | 0.317 | 0.26         | 83               | 1            | 0.2                 | 20                      |
|           | 3   | 0.011 | 1.4          |                  | 0.013 | 1.9          |                  | 0.14  | 43           |                  | 0.1          | 10                  |                         |
| 1.0       | 1   | 0.093 | 11           |                  | 0.076 | 12           |                  | 1.1   | 240          |                  | 0.82         | 82                  |                         |
|           | 2   | 0.859 | 0.020        | 2.3              | 0.65  | 0.019        | 2.9              | 0.46  | 0.23         | 51               | 1            | 0.17                | 17                      |
|           | 3   | 0.011 | 1.2          |                  | 0.012 | 1.8          |                  | 0.14  | 31           |                  | 0.1          | 11                  |                         |
| 1.2       | 1   | 0.084 | 9.4          |                  | 0.074 | 12           |                  | 1.1   | 190          |                  | 0.78         | 78                  |                         |
|           | 2   | 0.888 | 0.017        | 2.0              | 0.628 | 0.021        | 3.3              | 0.569 | 0.23         | 40               | 1            | 0.16                | 16                      |
|           | 3   | 0.011 | 1.2          |                  | 0.017 | 2.7          |                  | 0.17  | 29           |                  | 0.12         | 12                  |                         |
| 1.4       | 1   | 0.079 | 8.7          |                  | 0.084 | 14           |                  | 0.79  | 120          |                  | 0.55         | 55                  |                         |
|           | 2   | 0.911 | 0.017        | 1.9              | 0.613 | 0.027        | 4.4              | 0.654 | 0.17         | 26               | 1            | 0.12                | 12                      |
|           | 3   | 0.013 | 1.4          |                  | 0.023 | 3.8          |                  | 0.14  | 21           |                  | 0.094        | 9.4                 |                         |

# A cosmological exclusion plot

Model 4:  $\eta$  has the Horndeski form  
Error on  $h_2, (h_4-h_5)$  in h/Mpc

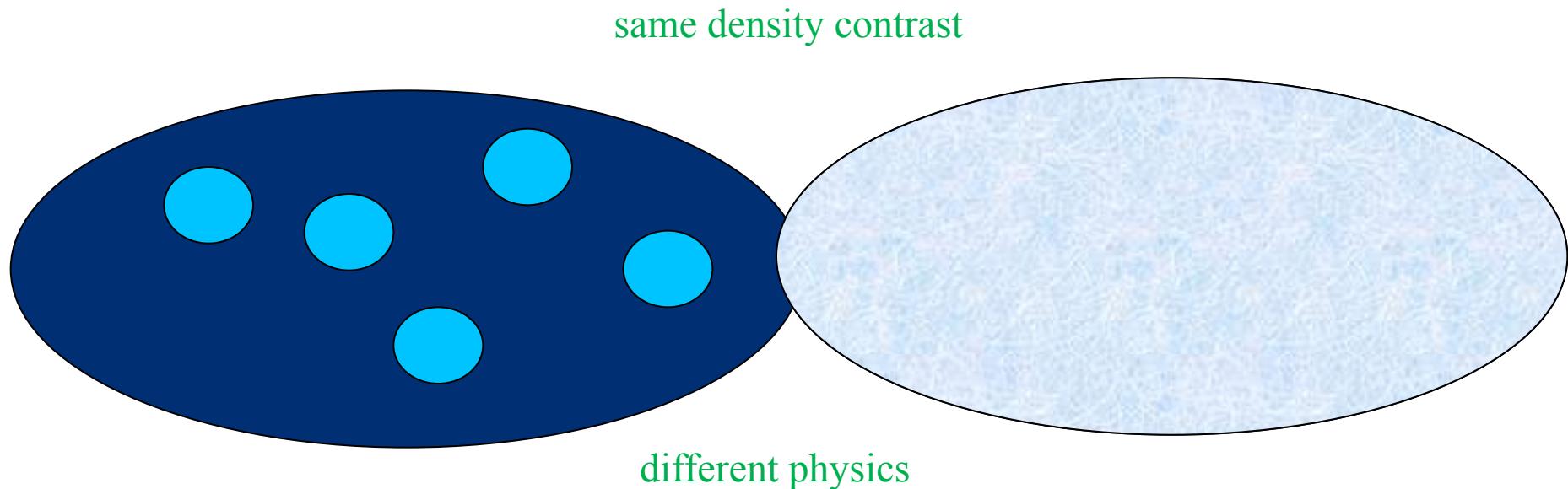
$$\eta = h_2 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right)$$

$$\eta = -\frac{\bar{G}\mu}{r}(1+Qe^{-r/\lambda})$$



# Under the carpet.

Problem of non-linearity:  
screening effects mix linear and non-linear scales



# Three Messages

1

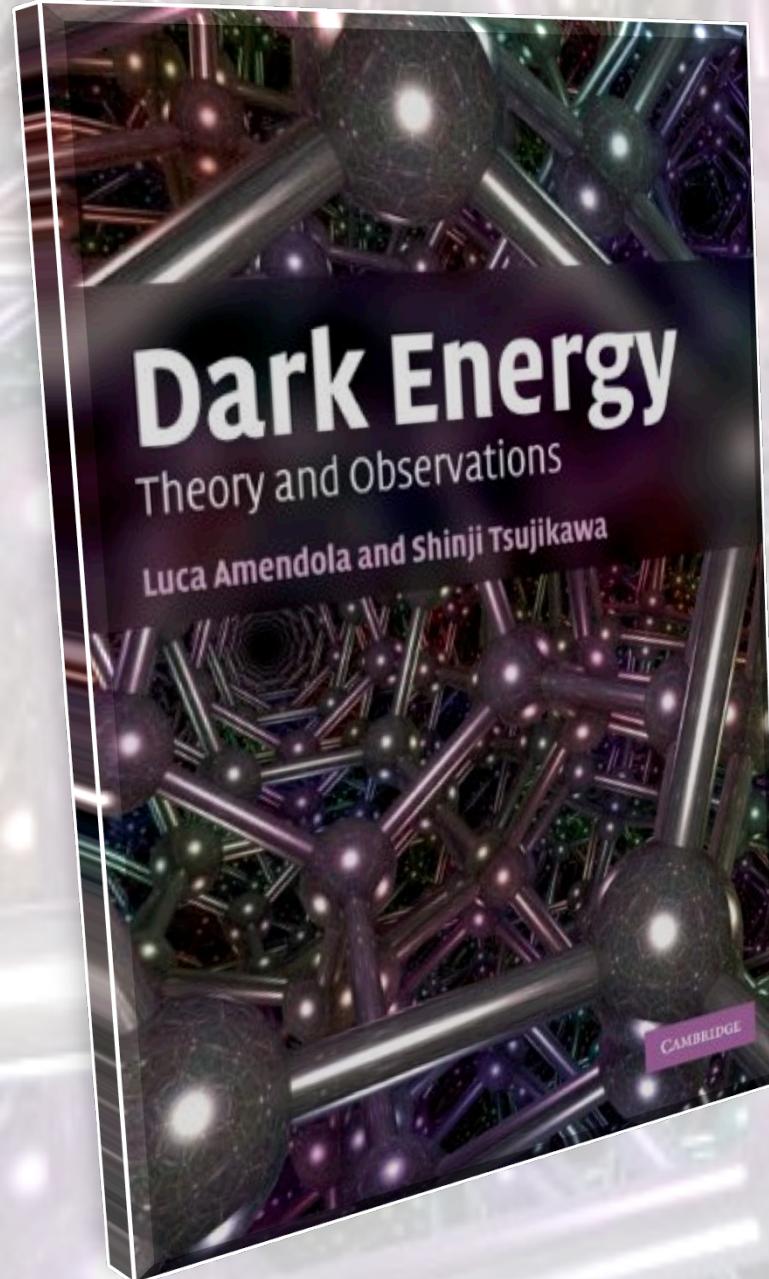
If DE is not a Horndeski field or massive gravity, then  
I don't know what could be

2

k-binned data are crucial!  
e.g. growth factor, redshift distortion parameter

3

Only by combining galaxy clustering and lensing  
can DE be constrained (or ruled out!) in a model-independent way



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