GLUON PROPAGATOR WITH DYNAMICAL QUARKS

Arlene Cristina Aguilar
UNICAMP, São Paulo, Brazil

Based on:
ACA, D. Binosi and J. Papavassiliou., Phys.Rev. D88 (2013) 074010; Phys.Rev. D86, 014032 (2012)

II Workshop on Perspectives in Nonperturbative QCD,
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Outline of the talk

- Motivation
- Main dynamical features of gluon propagator
- Inclusion of quark loops
- The effect of unquenching
- Comparison with lattice

Motivation

- In recent years, fruitful **synergy** between **lattice and SDEs.**

- **Most SDE studies** focus on Green's functions of **pure Yang-Mills**

  A.C. A., D. Binosi and J.Papavassiliou, Phys. Rev. D 78, 025010 (2008);
  P. Boucaud, J-P. Leroy, et al, JHEP 0806, 012 (2008).

- Majority of **lattice simulations** works in the **quenched limit** (no dynamical quarks)

  A. Cucchieri and T. Mendes, PoS LAT 2007 , 297 (2007); Phys. Rev. Lett. 100, 241601 (2008);
  I.L.Bogolubsky, et al, PoS LAT2007, 290 (2007).

- Must make the **transition to real-world QCD**

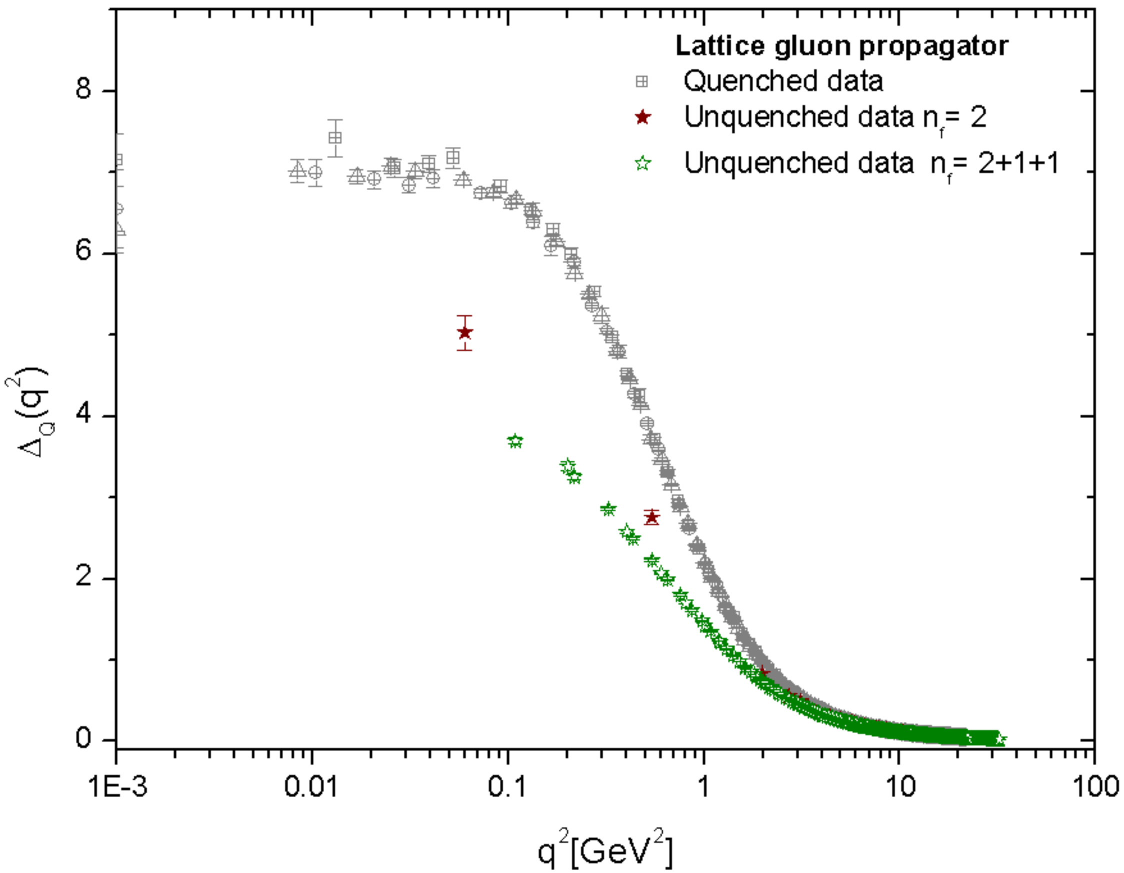
- **New unquenched lattice data** for gluon and ghost propagators

A. Ayala, et. al, Phys.Rev. D86 (2012) 074512

- **New SDE-based algorithm** for **estimating the quark-loop effects** on the gluon propagator

A.C. A., D. Binosi and J. Papavassliou,
Phys.Rev. D86, 014032 (2012); Phys.Rev. D88 (2013) 074010.

Lattice results



Quark	Current Mass
“up/down”	41.2 MeV
“strange”	95 MeV
“charm”	1.51 GeV

I.L.Bogolubsky, et al , **PoS LAT2007**, 290 (2007)
A. Ayala, et. al **Phys.Rev. D86 (2012) 074512**

- These lattice results suggest that in the presence of dynamical quarks:

1. Gluon propagator continues to saturate in the deep IR.
2. Overall suppression in the IR and intermediate regions.
3. Interpreting the saturation as a result of the gluon mass generation, i.e.

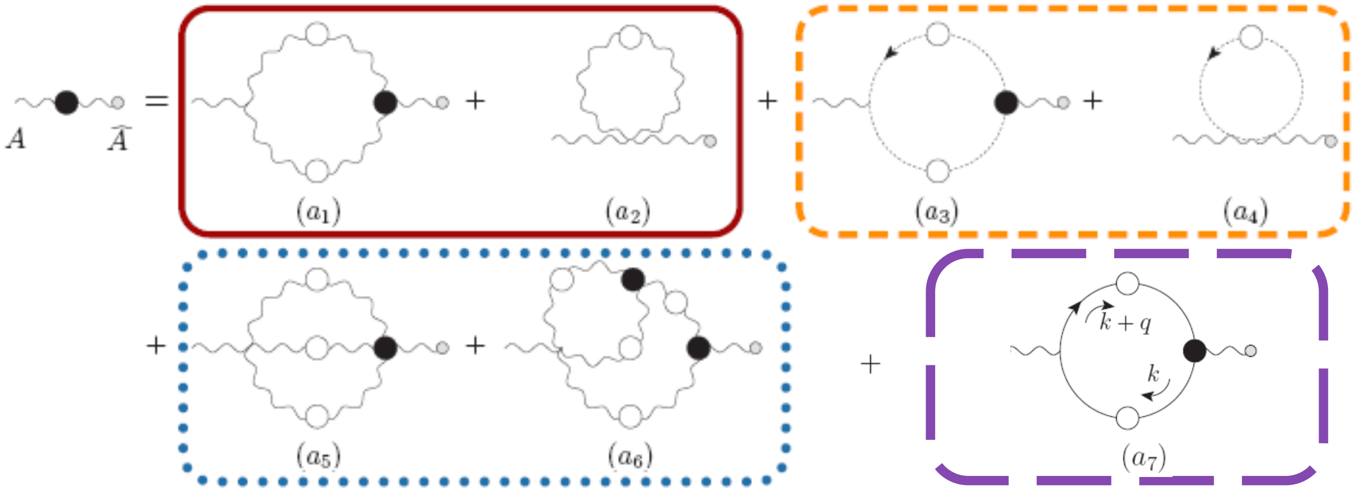
$$\Delta^{-1}(0) = m^2(0)$$

$$\Downarrow$$

Inclusion of quarks $\Rightarrow$ **Heavier gluon mass**

- We want to understand these features using the SDE

The gluon SDE (Landau gauge)



Quenched

$$\boxed{\widetilde{\Delta}^{-1}(q^2) = q^2 + \sum_{i=1}^{6} a_i}$$

Unquenched

$$\boxed{\widetilde{\Delta}^{-1}_{N_f}(q^2) = \widetilde{\Delta}^{-1}(q^2) + a_7}$$

$$\Delta(q^2) = [1 + G(q^2)]\widetilde{\Delta}(q^2);$$

Infrared finiteness $\Longleftrightarrow$ Gluon mass generation

- IR finiteness means:

$$\Delta^{-1}(q^2) = q^2 J(q^2) \quad \longrightarrow \quad \Delta_m^{-1}(q^2) = q^2 J_m(q^2) - m^2(q^2),$$

- **Coupled system of integral equations**

$$J_m(q^2) = 1 + \int_k \mathcal{K}_1(k, q, m^2, \Delta),$$
$$m^2(q^2) = \int_k \mathcal{K}_2(k, q, m^2, \Delta),$$

- In the limit $q^2 \rightarrow 0$

$$\mathcal{K}_2(q^2, m^2, \Delta_m) \neq 0$$

because of the inclusion of the massless poles.

Gluon mass generation in a nutshell

The **gauge invariant generation of a gluon mass** proceeds through the implementation of the **Schwinger mechanism**.

It **requires the existence of** a very **special** type of nonperturbative **vertices:**

1. they make possible that the SDE of the gluon propagator yields $\Delta^{-1}(0) \neq 0$;

2. they guarantee that **the WIs and STIs** of the theory **remain intact** - before and after mass generation;

3. they **decouple from on-shell amplitudes**.

R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973)
J. M. Cornwall and R. E. Norton, Phys. Rev. D 8 (1973) 3338
E. Eichten and F. Feinberg, Phys. Rev. D 10, 3254 (1974)

Special vertices

- Contain **massless poles**

- They are completely **longitudinally coupled**

$$P^{\alpha'\alpha}(q)P^{\mu'\mu}(r)P^{\nu'\nu}(p)\widetilde{V}_{\alpha'\mu'\nu'}(q,r,p) = 0.$$

- **Poles of nonperturbative origin** → (colored) tightly bound composite states bound states (vanishing mass) .

- These bound-state poles act as composite, longitudinally coupled **Nambu-Goldstone bosons**, maintaining gauge invariance (but, not associated with the spontaneous breaking of any continuous symmetry).

- Gauge invariance requires the simultaneous replacement

$$\Delta^{-1}(q^2) = q^2 J(q^2) \quad \longrightarrow \quad \Delta_m^{-1}(q^2) = q^2 J_m(q^2) - m^2(q^2),$$

$$\widetilde{\Gamma} \quad \longrightarrow \quad \widetilde{\Gamma}' = \widetilde{\Gamma}_m + \widetilde{V},$$

- The new vertex is given by

$$\widetilde{\Gamma}'_{\alpha\mu\nu}(q,r,p) = \left[\widetilde{\Gamma}_m(q,r,p) + \widetilde{V}(q,r,p)\right]_{\alpha\mu\nu}$$

- The gauge invariance requires that

$$q^\alpha \widetilde{V}_{\alpha\mu\nu}(q,r,p) = m^2(r^2)P_{\mu\nu}(r) - m^2(p^2)P_{\mu\nu}(p),$$

- **An explicit example:**

Schwinger mechanism "Turned off" ➡

$$q^\alpha \widetilde{\Gamma}_{\alpha\mu\nu}(q,r,p) = p^2 J(p^2)P_{\mu\nu}(p) - r^2 J(r^2)P_{\mu\nu}(r),$$

Schwinger mechanism "Turned on" ➡

$$\begin{aligned} q^\alpha \widetilde{\Gamma}'_{\alpha\mu\nu}(q,r,p) &= q^\alpha \left[\widetilde{\Gamma}_m(q,r,p) + \widetilde{V}(q,r,p)\right]_{\alpha\mu\nu} \\ &= [p^2 J_m(p^2) - m^2(p^2)]P_{\mu\nu}(p) - [r^2 J_m(r^2) - m^2(r^2)]P_{\mu\nu}(r) \\ &= \Delta_m^{-1}(p^2)P_{\mu\nu}(p) - \Delta_m^{-1}(r^2)P_{\mu\nu}(r)\,, \end{aligned}$$

The complete gluon mass equation

D. Binosi, D. Ibanez and J. Papavassiliou, Phys. Rev. D86, 085033 (2012)

$$m^2(q^2) = -g^2 C_A D(q^2) \int_k m^2(k^2) \Delta_\rho^\mu(k) \Delta^{\nu\rho}(k+q) \mathcal{K}_{\mu\nu}(k,q).$$

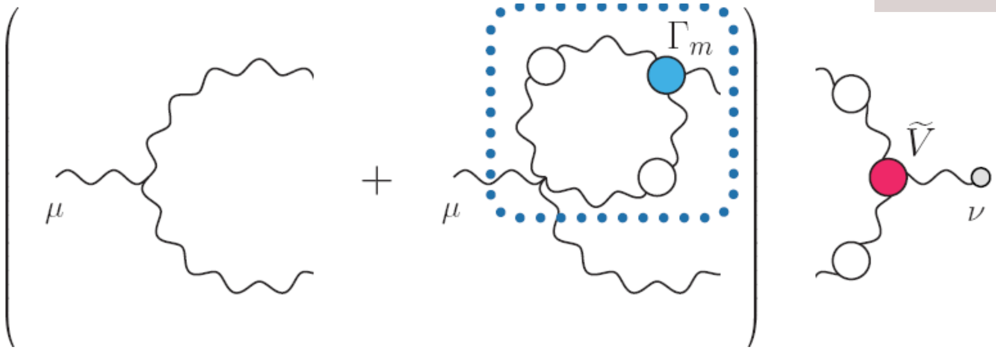
where $\mathcal{K}_{\mu\nu}(k,q) \; = \; [(k+q)^2 - k^2]\{1 - [Y(k+q) + Y(k)]\} g_{\mu\nu} + [Y(k+q) - Y(k)](q^2 g_{\mu\nu} - 2q_\mu q_\nu).$

$$1 + G(q^2) \approx F^{-1}(q^2)$$

$$D(q^2) = \frac{F(q^2)}{q^2}$$

- Diagramatically

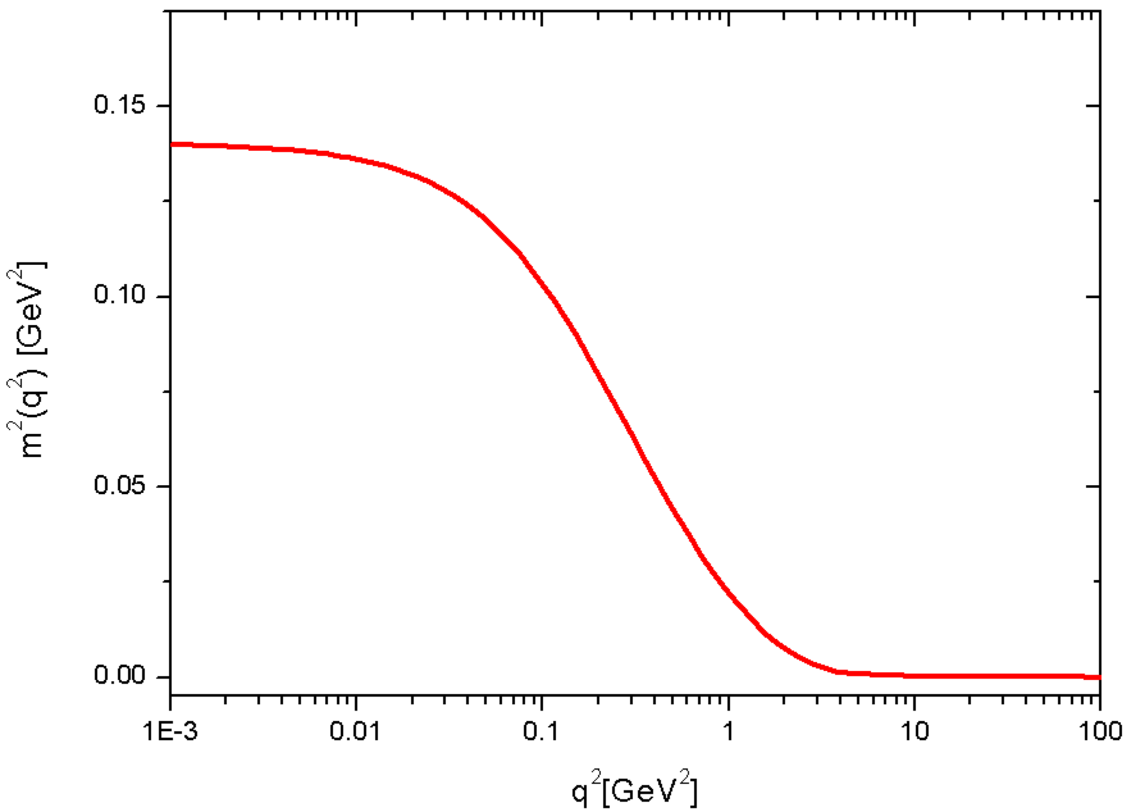
$$m^2(q^2) = D(q^2) \;\; q^\mu \;\; \times$$



$$\times q^\nu$$

- The solution depends on a subtle interplay between the shape of the full $\Delta(q^2)$ and the kernel $\mathcal{K}_{\mu\nu}(k,q)$.

Solution of the mass equation



- **Positive definite and monotonically decreasing gluon mass**

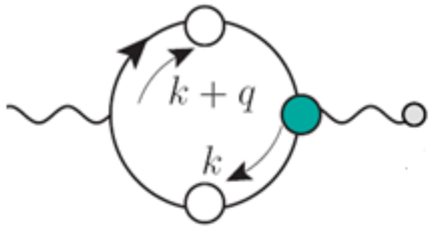
- **Solution normalized to coincide with lattice value $\Delta^{-1}(0) \approx 0.14\ GeV^{-2}$, namely $m = 375$ MeV.**

Unquenching the gluon propagator

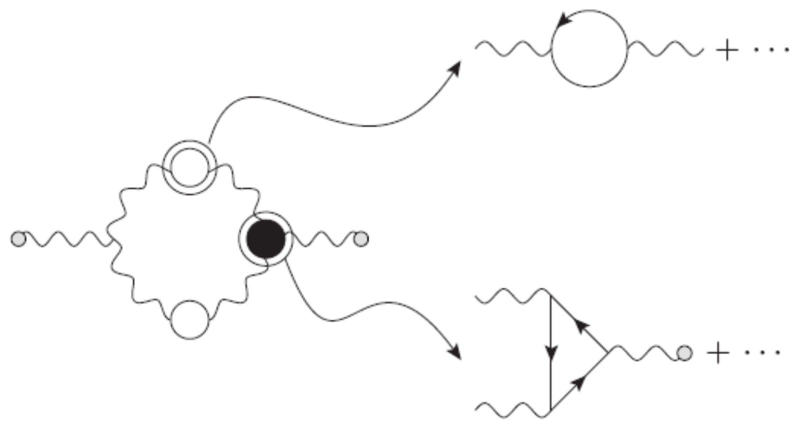
- We will assume that the main bulk of the quark contribution comes from the diagram a_7 (fully dressed quark loop), i.e.

$$\widetilde{\Delta}_{N_f}^{-1}(q^2) = \widetilde{\Delta}^{-1}(q^2) + a_7 + \text{"subleading corrections"}$$

$$a_7 = X(q^2) =$$



Subleading contributions

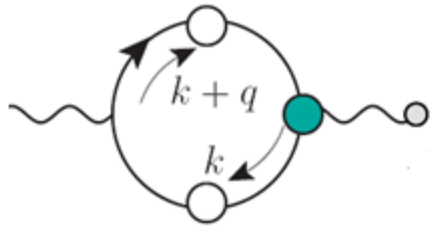


• There will be a nonlinear propagation of the changes induced due to X(q), which will also affect the original subset of purely Yang-Mills graphs (a_1 –a_6)
→ **Internal gluon propagator and the three-gluon vertex gets modified.**

• We assume that the inclusion of two light quark flavors (m = 300 MeV) may be considered as a **"perturbation"** to the quenched case.

• Our **operating assumption** is that these effects may be **relatively small** compared to those originating from graph a_7 (quark loop)

Leading effects: the quarks loop

$$X(q^2) =$$ 

$$X(q^2) = -\frac{g^2}{6}\int_k \mathrm{Tr}\left[\gamma^\mu S(k)\widehat{\Gamma}_\mu(k,-k-q,q)S(k+q)\right]$$

- The quark loop is transverse $\quad X_{\mu\nu}(q) = X(q^2)P_{\mu\nu}(q)$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2,$$

- Moreover, we have that

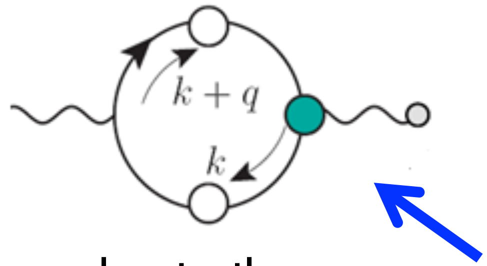
$$X(0) = 0$$

• **No direct** influence on the value of $\Delta(0)$;

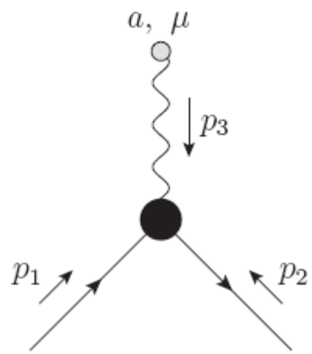
• However modifies it **indirectly,** due to the **change in the overall shape of $\Delta(q^2)$** throughout the entire range of momenta.

ACA and J.Papavassiliou, **Phys. Rev. D 81, 034003 (2010).**

The quark-gluon vertex



• In the PT-BFM scheme, the contribution to the gluon self-energy due to the quark loop has a special ingredient: **the fully-dressed quark-gluon vertex**



PT-BFM quark-gluon vertex

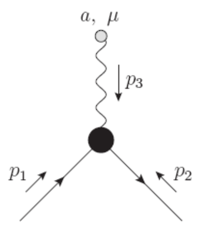
• The PT-BFM quark-gluon vertex satisfies the Ward identity

$$p_3^\mu \widehat{\Gamma}_\mu(p_1,p_2,p_3) = S^{-1}(-p_1) - \underbrace{S^{-1}(p_2)}_{\text{Quark propagator}}$$

• Instead the conventional Slavnov-Taylor identity

$$p_3^\mu \Gamma_\mu(p_1,p_2,p_3) = F(p_3)[S^{-1}(-p_1)\underbrace{H(p_1,p_2,p_3)}_{\text{Quark-ghost scattering kernel}} - \overline{H}(p_2,p_1,p_3)S^{-1}(p_2)]$$

Ansatz for the longitudinal part



• The most general tensorial structure for the longitudinal part is

$$\Gamma_\mu(p_1,p_2,p_3) = L_1\gamma_\mu + L_2(\not{p}_1 - \not{p}_2)(p_1-p_2)_\mu + L_3(p_1-p_2)_\mu + L_4\tilde{\sigma}_{\mu\nu}(p_1-p_2)^\nu$$

•Using the WI we find the form factors

$$L_1 = \frac{A(p_1)+A(p_2)}{2}; \quad L_2 = \frac{A(p_1)-A(p_2)}{2\,(p_1^2-p_2^2)}; \quad L_3 = -\frac{B(p_1)-B(p_2)}{p_1^2-p_2^2}; \quad L_4 = 0\,.$$

where the functions A(p) and B(p)

$$S^{-1}(k) = -i\,[A(k)\not{k} - B(k)] = -iA(k)\,[\not{k} - \mathcal{M}(k)]$$

and the dynamical mass is defined as the ratio

$$\mathcal{M}(k) = B(k)/A(k)$$

• The resulting vertex is known as **Ball Chiu (BC) vertex**

J.S. Ball and T.W. Chiu, Phys.Rev. D 22, 2542 (1980).
D. C. Curtis and M. R. Pennington, Phys. Rev. D 42, 4165 (1990

The unquenching formula

$\Delta_{Nf}(q^2)$ may be expressed as a **deviation** from $\Delta(q^2)$:

$$\Delta_{N_f}(q^2) = \frac{\Delta(q^2)}{1 + X(q^2)F^2(q^2) + \lambda^2(q^2)}$$

where

$$\lambda^2(q^2) = m^2_{N_f}(q^2) - m^2(q^2),$$

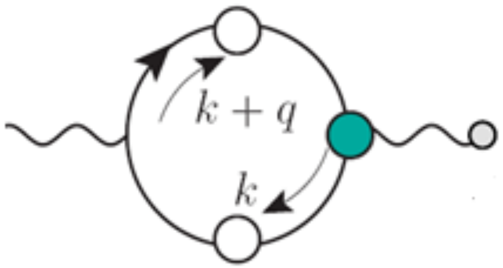
measures the **difference induced** to the gluon mass **due to the inclusion of quarks.**

ACA, D.Binosi and J.Papavassiliou, Phys.Rev. D86, 014032 (2012)

SDE-lattice synergy – Full system

$$\Delta^{-1}_{N_f}(q^2) = [1+G(q^2)]^{-2}\widehat{X}(q^2) + \overbrace{m^2_{N_f}(q^2) - m^2(q^2)}^{\lambda(q^2)} + \underbrace{\Delta^{-1}(q^2)}_{\text{lattice}}$$
(a)

$$m^2_{N_f}(q^2) = \int_k \mathcal{K}_2(k,q,\Delta_{N_f}),$$

$$\widehat{X}(q^2)P_{\mu\nu}(q) =$$


(b)

(c)

$$1 + G(q^2) \approx \underbrace{F^{-1}(q^2)}_{\text{lattice}}$$
(d)

Ingredients

$$\Delta_Q(q^2) = \frac{\Delta(q^2)}{1 + \left\{ \widehat{X}(q^2)\,[1 + G(q^2)]^{-2} + \lambda^2 \right\} \Delta(q^2)}$$

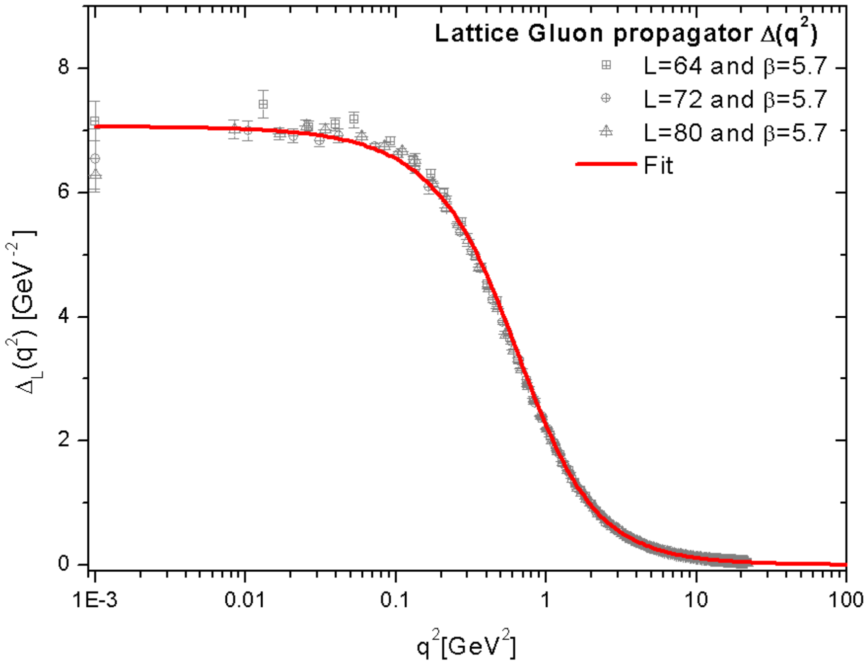
Use the relation

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$$

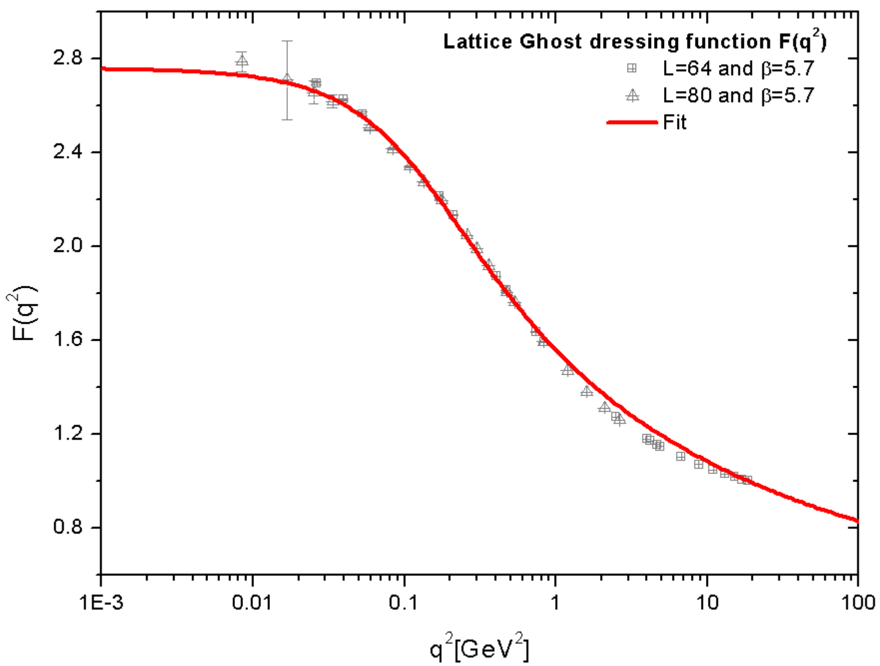
$$L(q^2) \ll G(q^2)$$

$$1 + G(q^2) \approx F^{-1}(q^2)$$

ACA, D.Binosi, J.Papavassiliou. and J.Rodriguez-Quintero, Phys.Rev. D 80 085018 (2009)



I.L.Bogolubsky, et al, PoS **LAT2007**, 290 (2007)

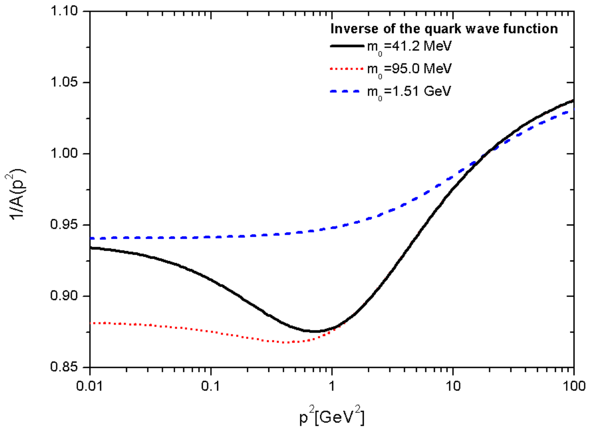


Gluon propagator and ghost dressing function renormalized at μ=4.3 GeV

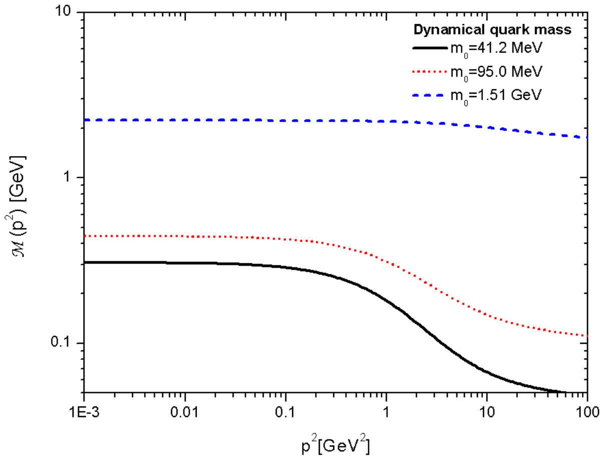
Calculating the quark loop - $X(q^2)$

$$S^{-1}(k) = -i\,[A(k)\not{k} - B(k)] = -iA(k)\,[\not{k} - \mathcal{M}(k)]$$

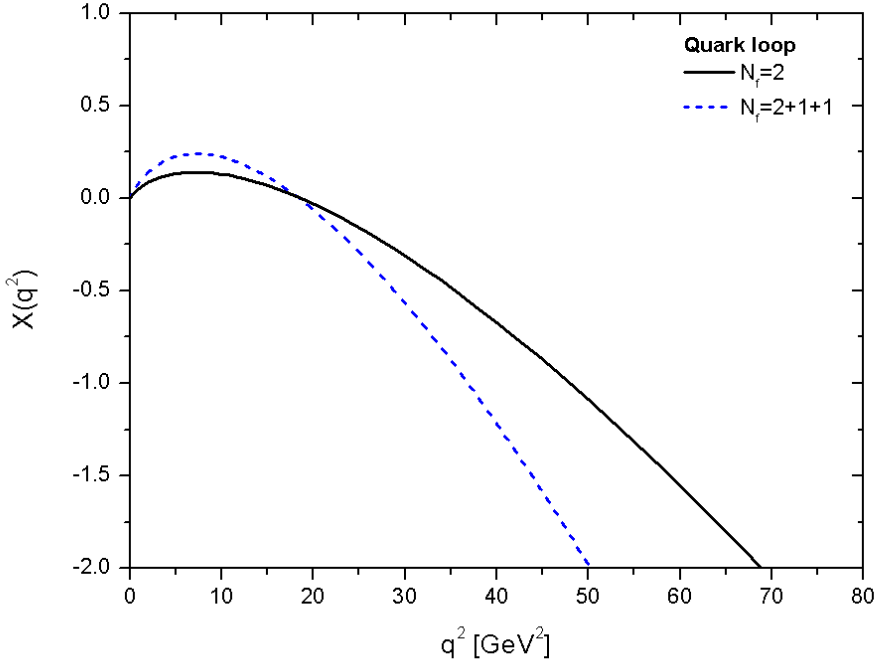
Quark wave function



Quark Masses



Quark loop

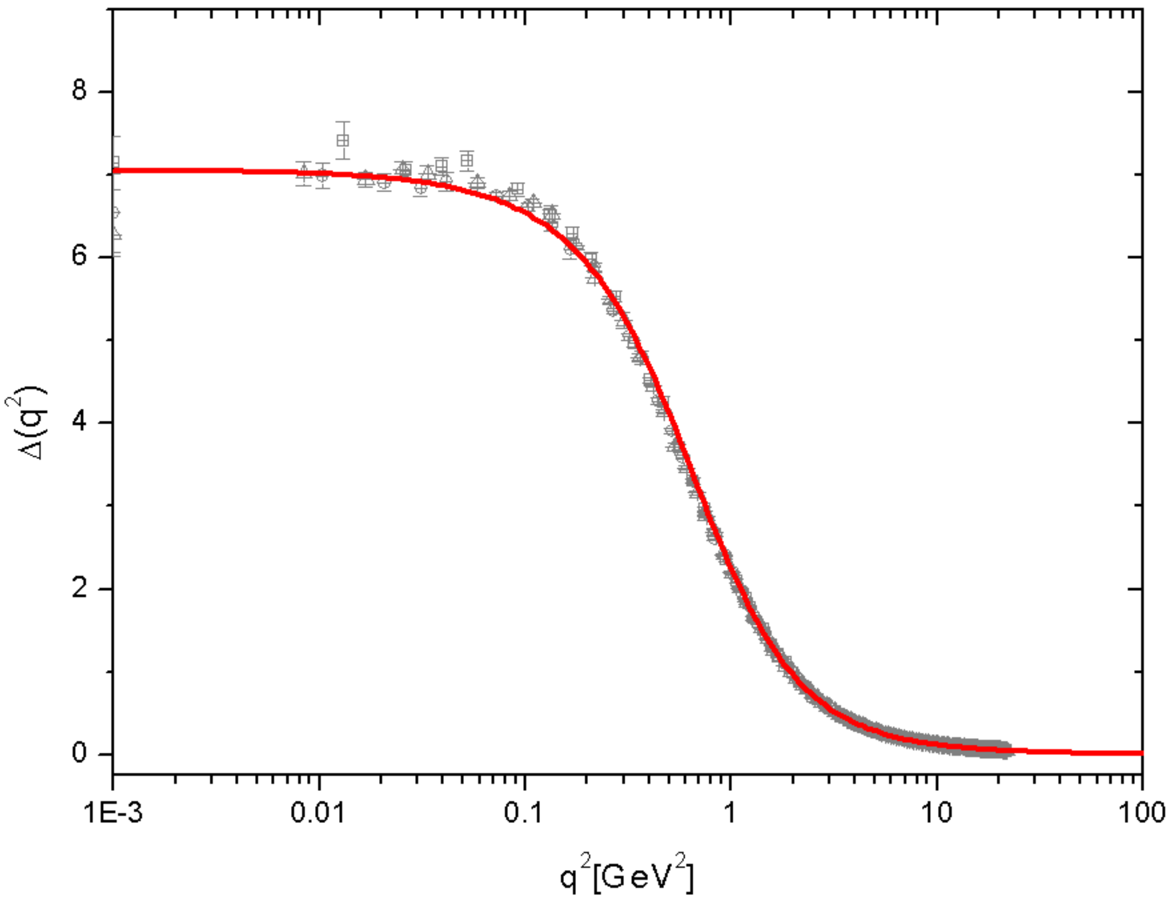


Quark	Current Mass	*Dynamical mass*
"up/down"	41.2 MeV	307 MeV
"strange"	95 MeV	445 MeV
"charm"	1.51 GeV	2.25 GeV

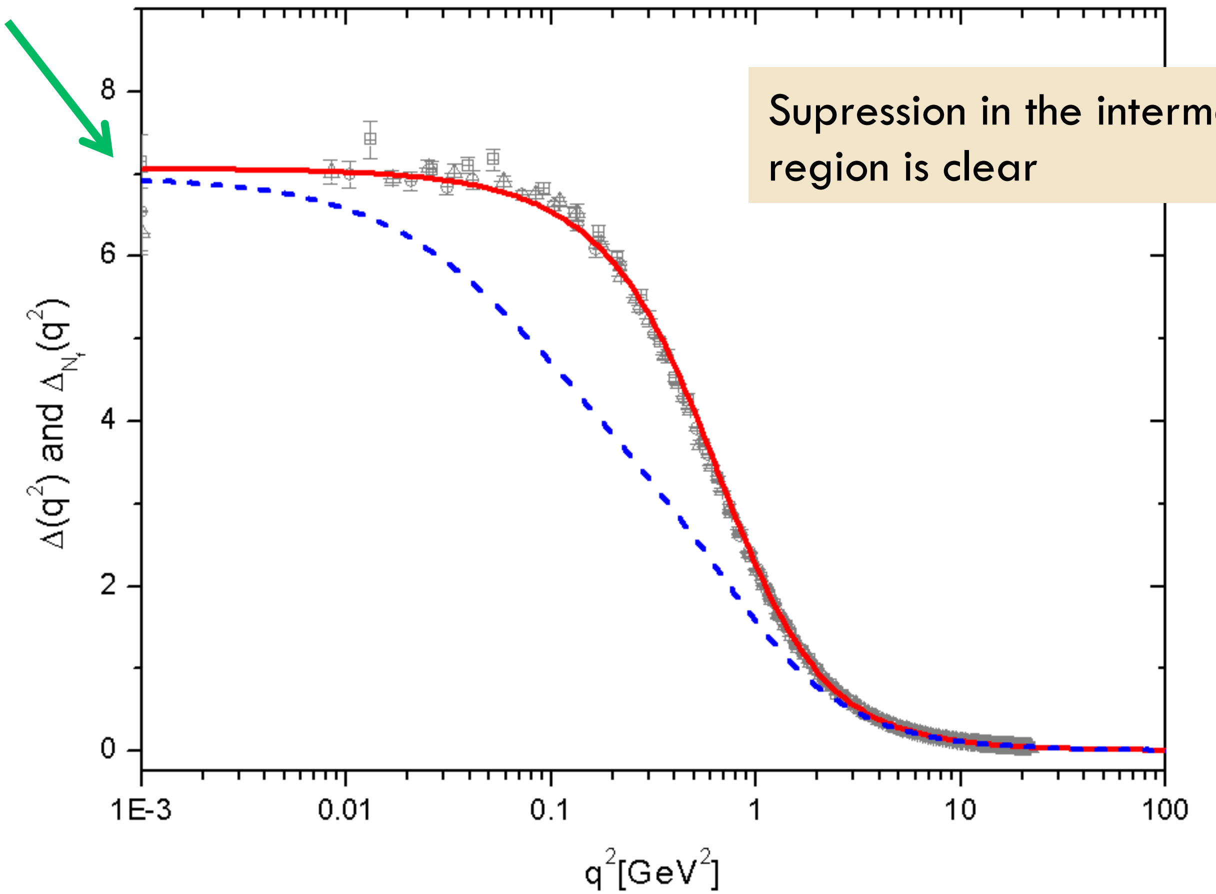
N_f	α_s (4.3 GeV)
0	0.222
2	0.285
2+1+1	0.331

- The case where $\lambda(q^2)= 0$ → **gluon mass equation turned off**

ACA, D. Binosi and J. Papavassiliou, Phys. Rev. D86, 014032 (2012)



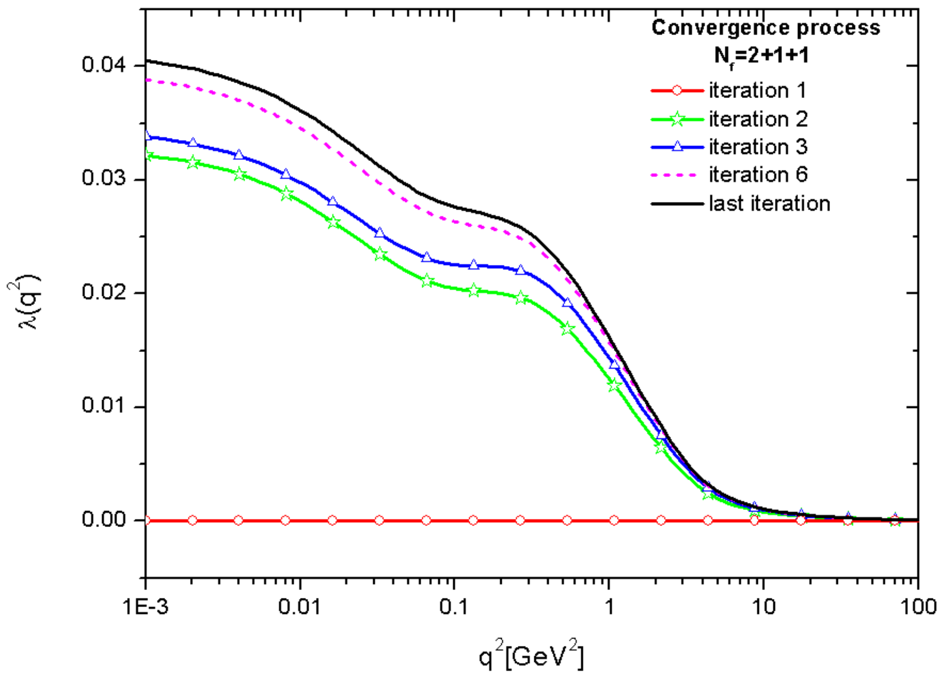
Since X(0)=0 → saturation point remains intact

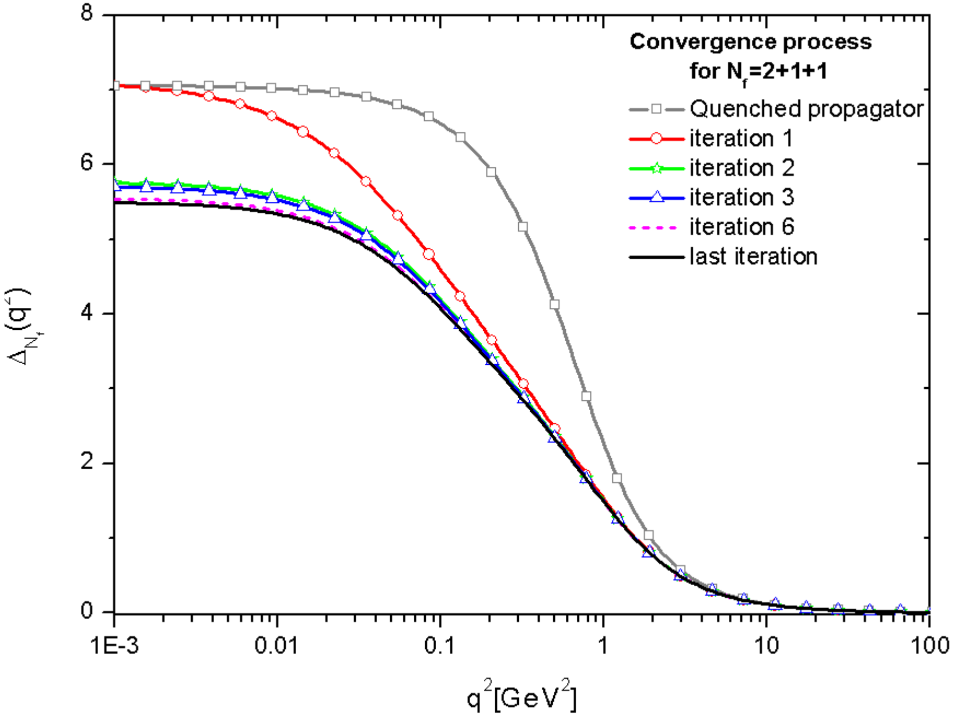


Supression in the intermediate region is clear

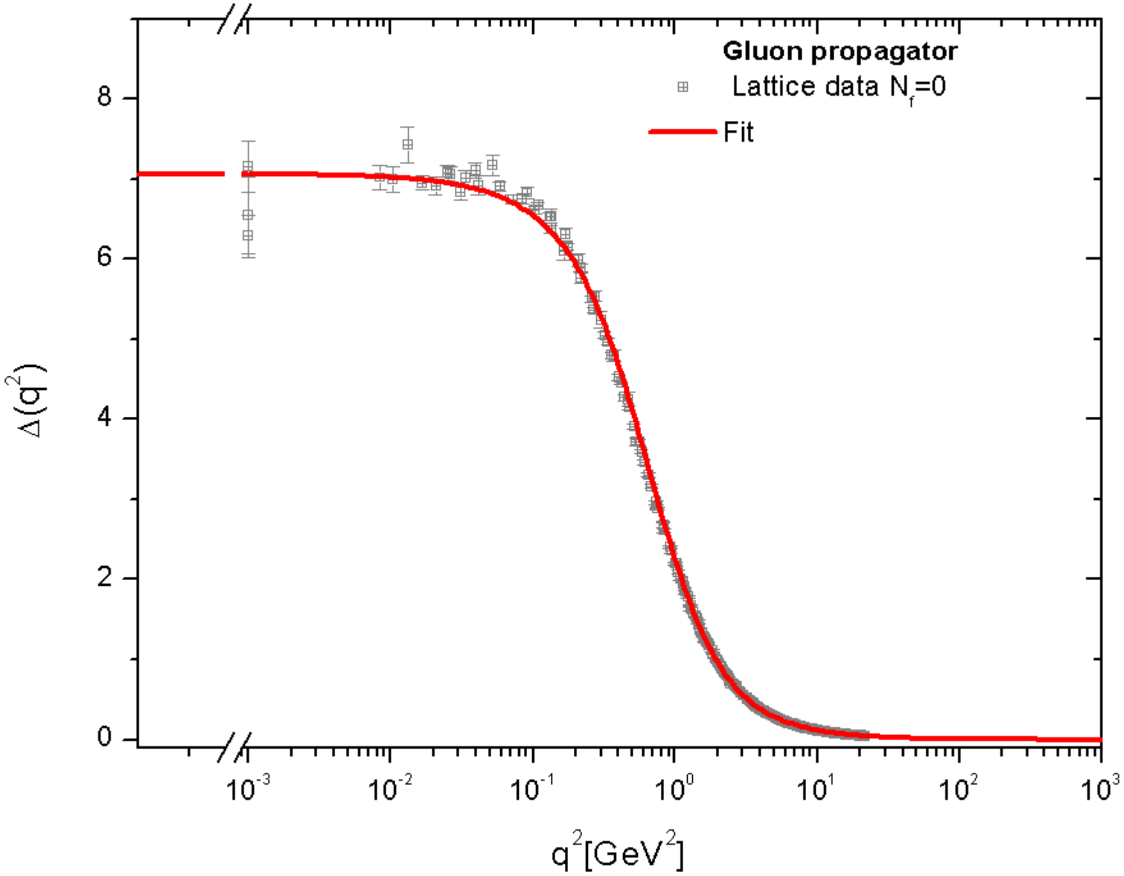
Full treatment: gluon mass equation turned on

- Convergence process for $\lambda(q^2) \neq 0$



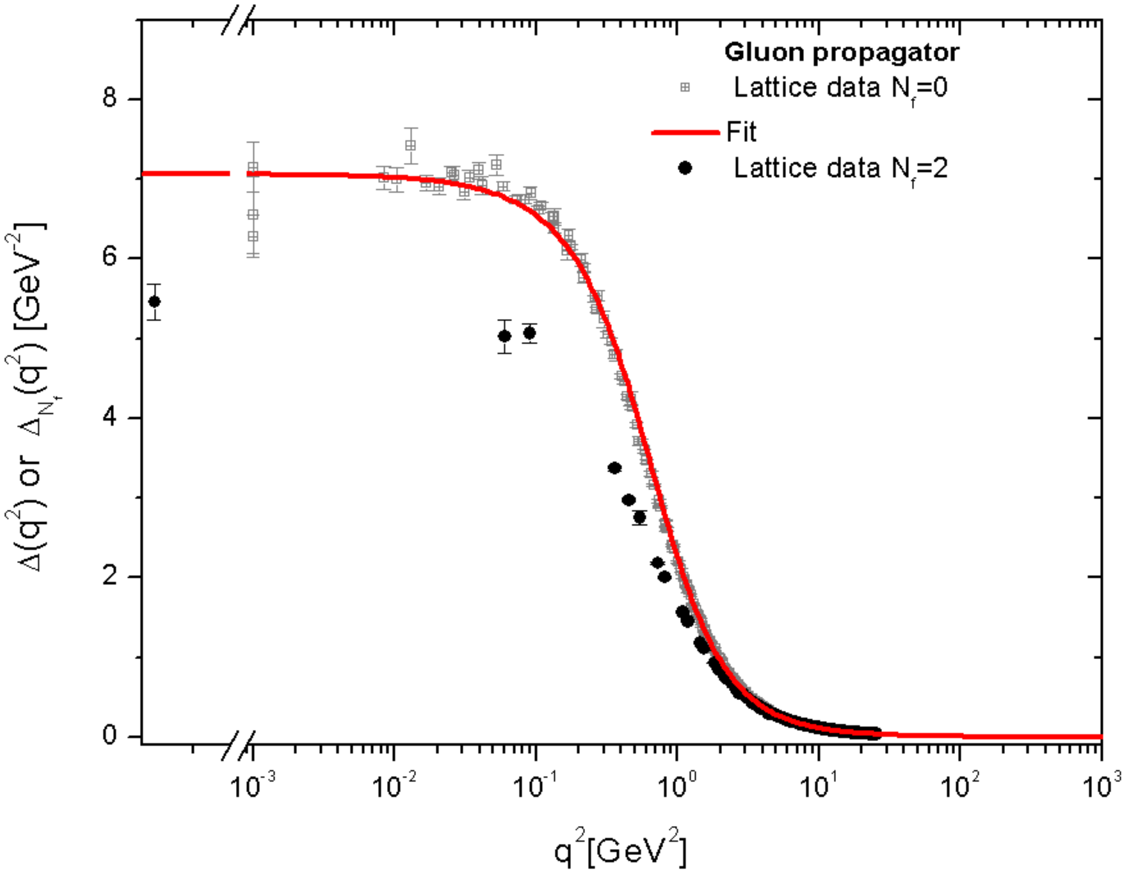


Comparison with the lattice



Quenched lattice propagator

The effect of two quarks

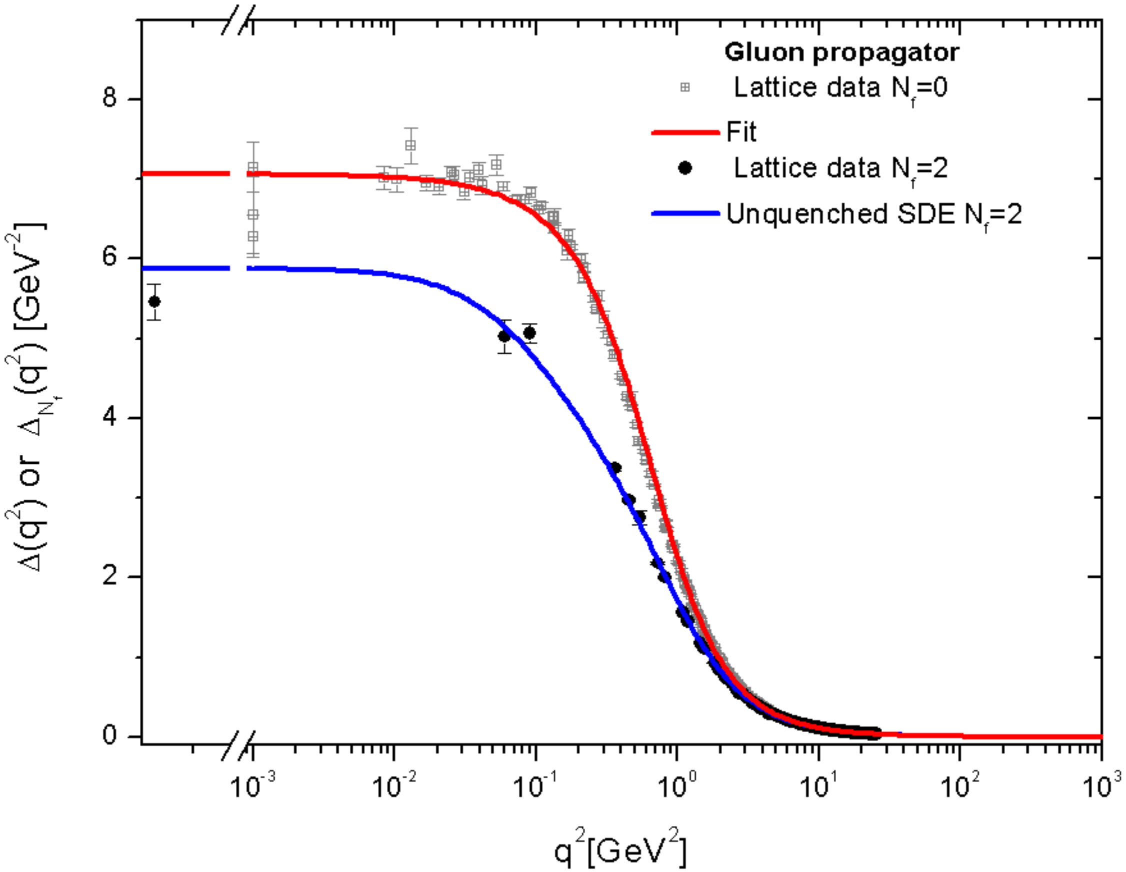


Unquenched lattice propagator ●

n_f = 2

Quark	Mass
up/down	41.2 MeV

The effect of two quarks

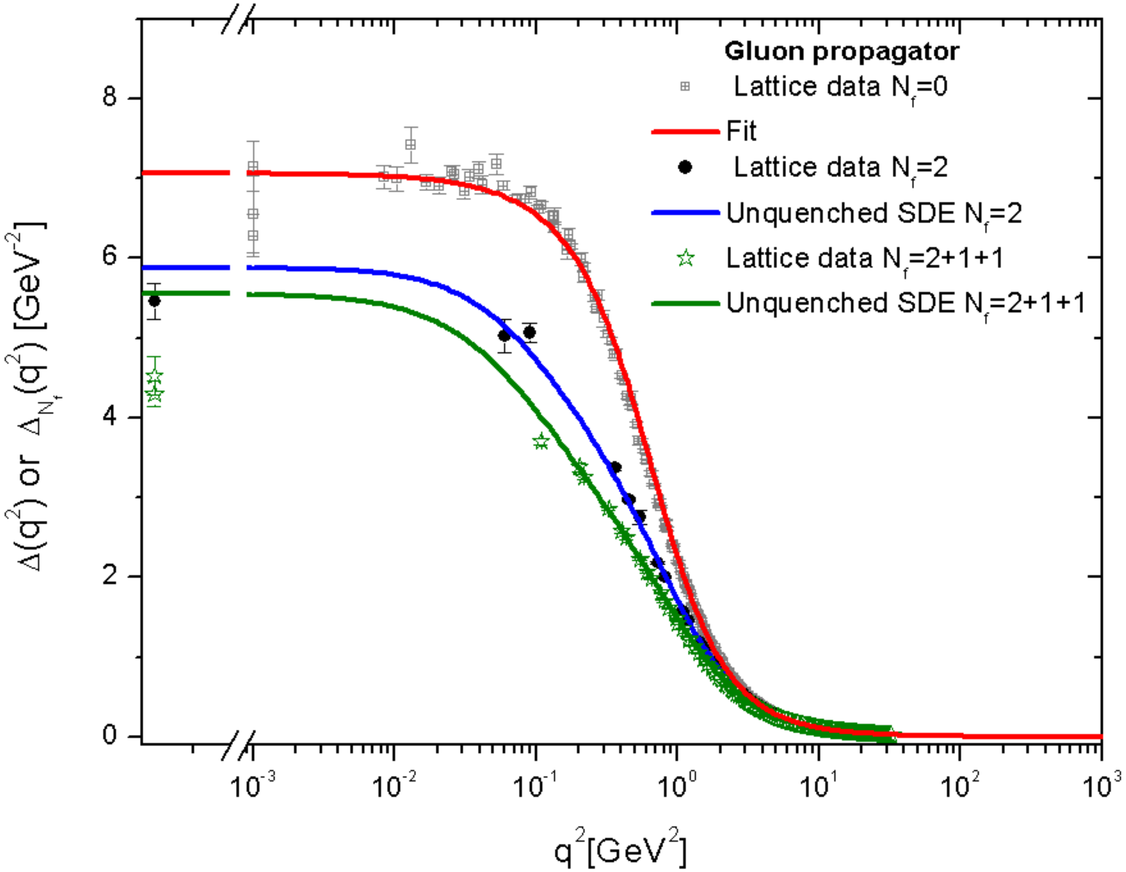


Unquenched SDE result

$n_f = 2$

Quark	Mass
up/down	41.2 MeV

The effect of adding heavier quarks

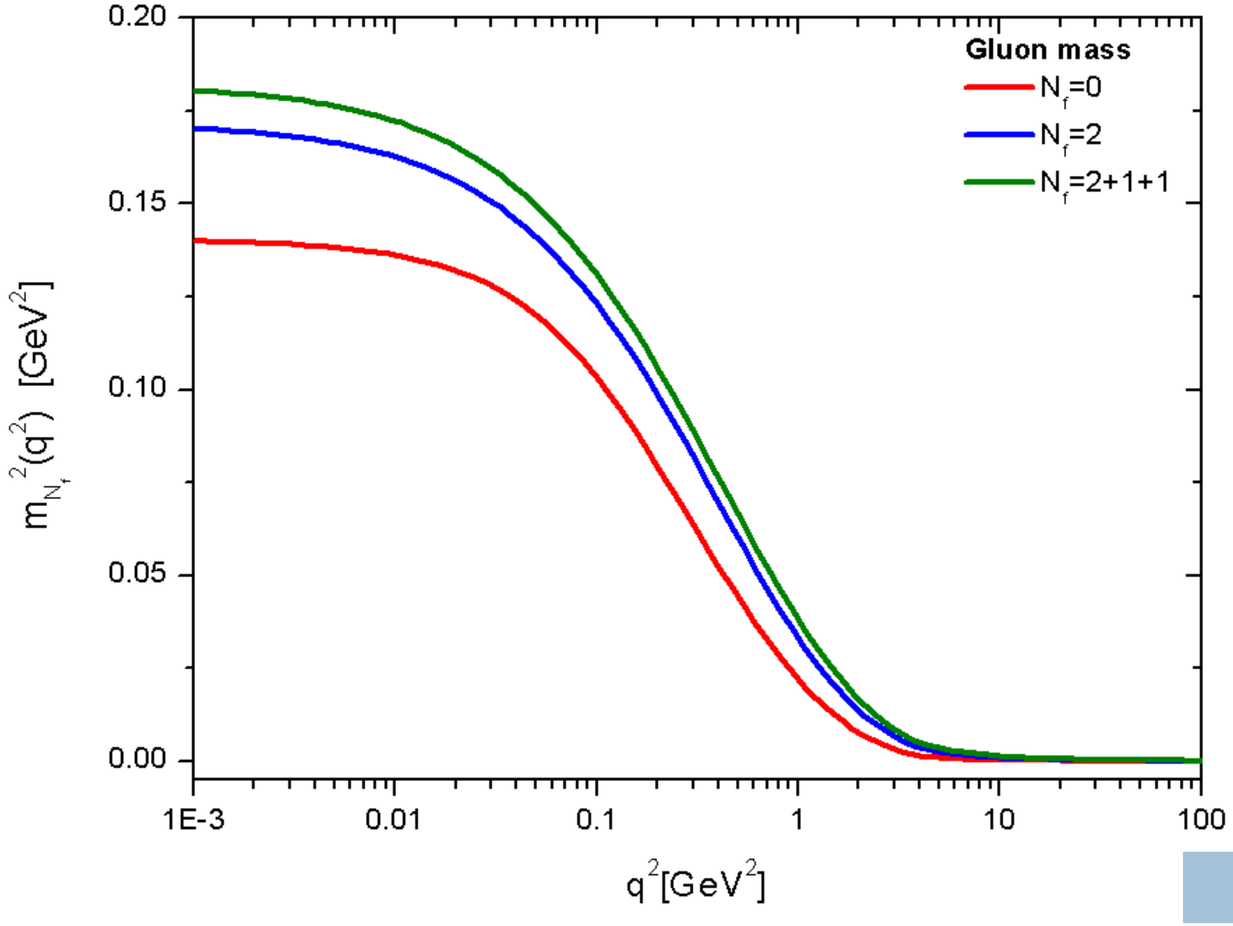


n_f = 2+1+1

Quark	Mass
up/down	41.2 MeV
strange	95 MeV
charm	1.51 GeV

Unquenched SDE result

The dynamical gluon mass

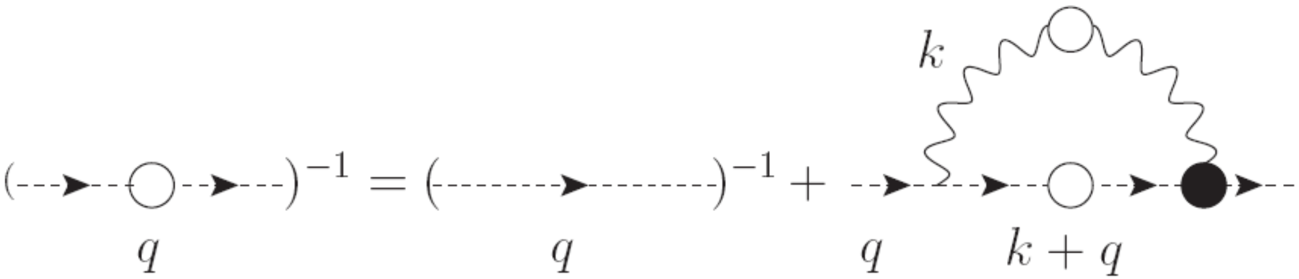


N_f	m(0)
0	375 MeV
2	413 MeV
2+1+1	425 MeV

ACA, D. Binosi and J. Papavassiliou Phys.Rev. D88 (2013) 074010

Unquenched effects on the ghost propagators

- The ghost SDE is given by

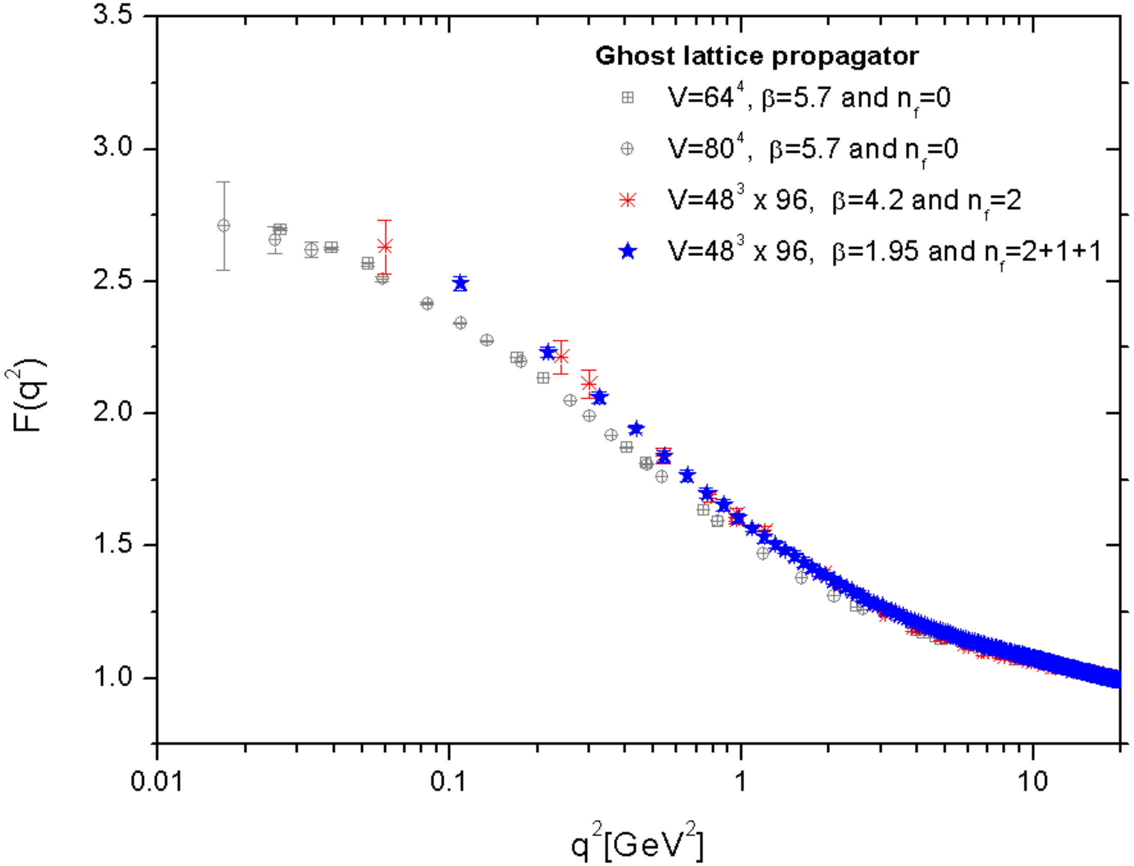


increases

$$iD^{-1}(q^2) = q^2 + ig^2 C_{\mathrm{A}} \int_k \Gamma^{\mu} \Delta_{\mu\nu}(k) \mathbf{\Gamma}^{\nu}(k,q) D(q+k).$$

Suppresses

N_f	α_s (4.3 GeV)
0	0.222
2	0.285
2+1+1	0.331



A. Ayala, et. al, Phys.Rev. D86 (2012) 074512

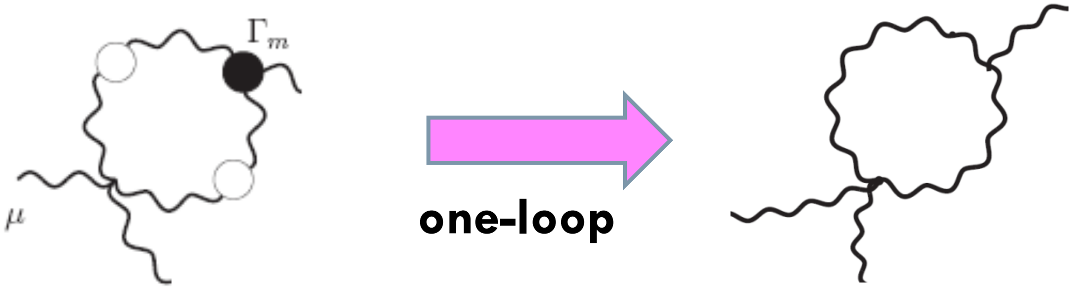
Conclusions

- **New SDE-based** method for **estimating quark effects**
- **Unquenched** gluon propagator computed as a **deviation from the quenched one.**
- **Quark loops suppress** intermediate **and infrared region.**
- **Gluon mass increases.**
- Good **agreement with lattice.**
- **Apply this method to TC models**, or QCD-like theories with fermions in higher representation.

Additional Slides

Simplifying the kernel

- **Perturbative one-loop expression for Y(k^2)**



$$Y_{\mathrm{R}}(k^2) = -\frac{1}{(4\pi)^2}\frac{5}{4}\log\frac{k^2}{\bar{\mu}^2}.$$

- **Rescaling**

$$\boxed{Y_R(k^2) \to C\, Y_R(k^2)}$$

- **C arbitrary parameter, models additional corrections.**

Contribution at zero momentum

• In the limit $q^2 \rightarrow 0$

$$\widehat{X}(0) = -\frac{2g^2}{d-1}\int_k \frac{1}{A^2(k^2-\mathcal{M}^2)^2}\left\{A\left[(2-d)k^2+d\mathcal{M}^2\right]+2A'k^2(k^2+\mathcal{M}^2)-4k^2B'\mathcal{M}\right\}.$$

by virtue of the identity

$$\int_k k^2 f'(k) + \frac{d}{2}\int_k f(k) = 0,$$

A. C. Aguilar and J.P., Phys. Rev. D 81, 034003 (2010).

setting

$$f(k) = \left[A(k)(k^2-\mathcal{M}^2(k))\right]^{-1}$$

$$\widehat{X}(0) = 0$$

$\Downarrow$

• No **direct** influence on the value of $\Delta(0)$;

• However modifies it **indirectly,** due to the **change** in **the overall shape of $\Delta(q^2)$** throughout the entire range of momenta.