The phase diagram in $T - \mu_B - N_c$ space

Based on PRL107:152301,2011 , Stefano Lottini , PRL111 012301 with Sascha Vogel,Bjoern Beauchle
Also , PRC82 (2010) 055202 ,with with Igor Mishustin , JHEP 1108 (2011) 097 with Piero Nicolini

G.Torrieri

UNICAMP

Synopsis

The basic problem with moderate μ_B

Large N_c : A short introduction

Surprises? Combining large N_c and moderate μ_B

An estimate from a percolation Ansatz

Towards a pheonomenolgy of quarkyonic matter in supernova and at FAIR/NICA/SPS/RHIC@low$\sqrt{s}$

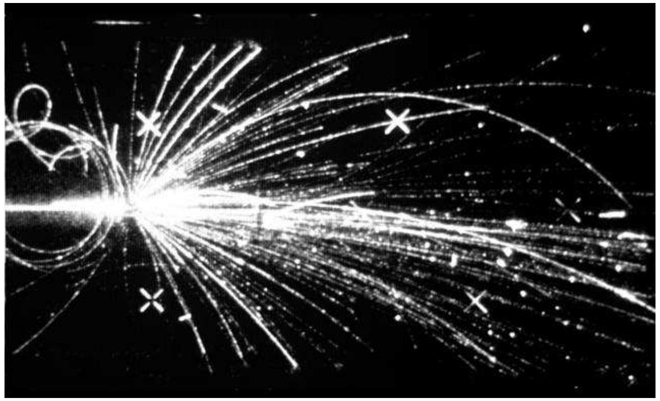
“The other” heavy ion program

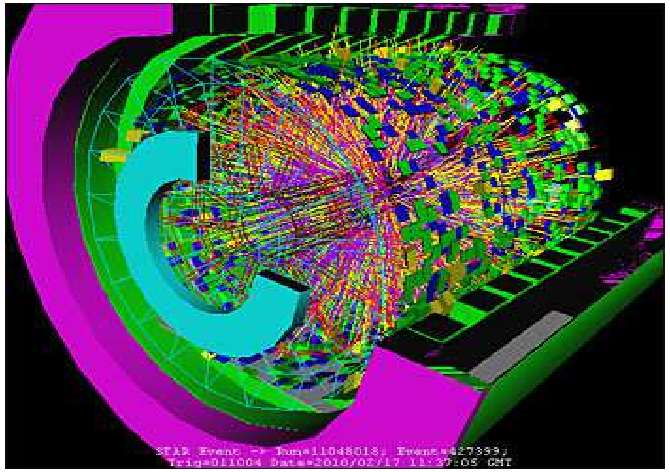
Recently you heard a lot about the LHC heavy ion program, but there is an equally exciting low energy program going on in parallel.

- RHIC low energy scan
- SPS experiment NA61 (CERN)
- FAIR (GIS,Darmstadt)
- NICA (Dubna,Russia)

Revisiting low $\sqrt{s}$

1988
(BEVALAC)

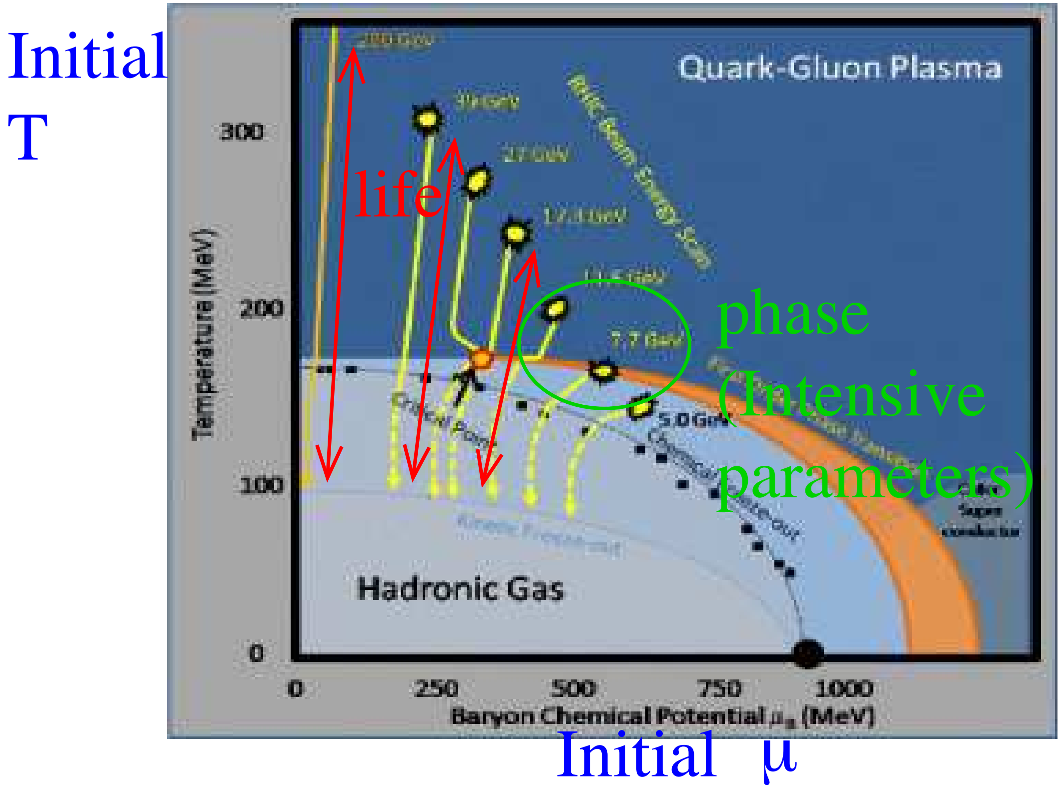




2012
(STAR)

Why low energy runs? Eliminating the $\sqrt{s}$-detector correlation!

To examine lower energies with modern dectectors and analysis. Luminosity/acceptance/triggering/analysis vastly progressed, allowing precision measurement of new observables at low $\sqrt{s}$. Since not all "interesting physics"@high $\sqrt{s}$

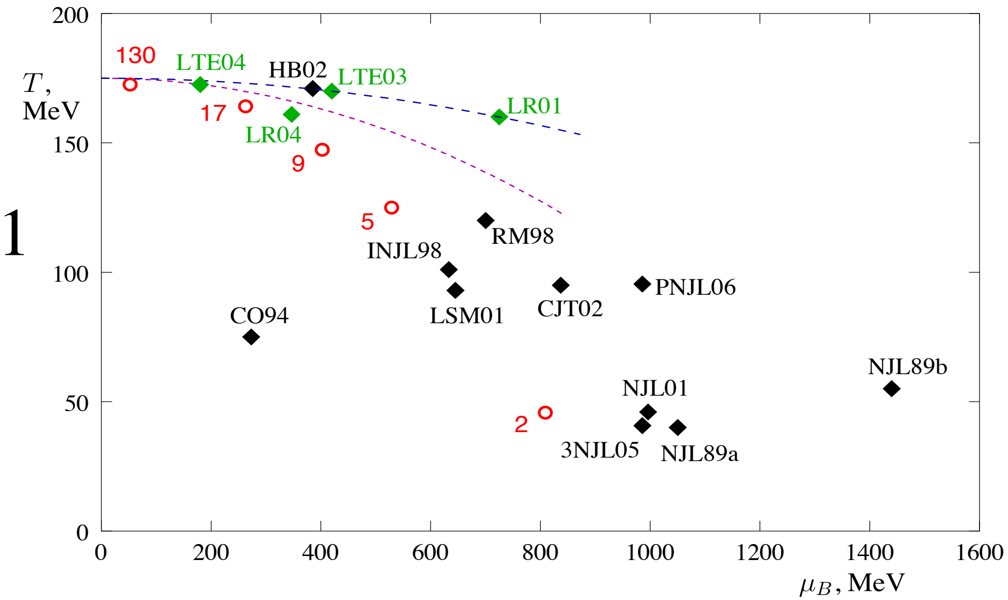


The basic idea: By scanning in $\sqrt{s}$, we generally decrease temperature <u>but</u> increase density! This way we can study denser phases of the system, perhaps relevant to <u>neutron stars</u>.

What can we discover? the critical point

"clear" signatures: divergence of fluctuations,higher cumulants, softening of the EoS (with "softest point" in 1st order phase).

M.Stephanov
hep–lat/0701002v1



But why are the points so spread out?? Plus, De Forcrand and Philipsen believe <u>no critical point</u>

The issue: QCD at $\mu_Q \geq \Lambda_{QCD}, T < T_c$ is really not understood

Hadronic or EFTs (σ,NJL,PNJL etc): based under the assumption that $p_i - p_j \ll \Lambda_{funamental}$
Only scale in QCD is $\Lambda_{fundamental} = \Lambda_{QCD}$, and $p_i - p_j \sim \mu_Q \sim \Lambda_{QCD}$

So EFT at $\mu_Q \simeq \Lambda_{QCD}$ means Taylor-expanding around 1!
For any operator $\hat{O}(x)$ (e.g. $q, P, ...$) Not quaranteed $\hat{O}^n \ll \hat{O}^{n-1}$ for any N

Lattice QCD has the sign problem, any expansion is good for $\mu_q \ll T$

AdS/CFT apart from the many unrealistic assumptions, classical Gauge dual depends on $N_c \to \infty$, on which more later

Any high density calculation is an essentially educated guess. Expect surprises

FAIR/NICA/RHICbes is a “shot in the dark”, requiring what if phenomenology (”If in FAIR regime X happens, we should see Y”)

And indeed there have been plenty of speculation of what we could find

- Coexistance between Confinement+pQCD (Mclerran,Pisarski,2007)
- Confinement+Chiral restoration (Fukushima,McLerran, 2008)
- Chiral spiral inhomogeneities (Kojo,Pisarski,Tsvelik, 2009)
- Generic chirally inhomogeneus regions (Buballa et al)
- Deconfinement+Chiral breaking (Fukushima,Csernai, 2009)...

The only hyerarchy that seems to be roughly correct is the large N_c limit
't Hooft, over 20 years ago, showed that provided a continuus limit exists where $N_c \to \infty, g_{YM} \to 0, g_{YM}^2 N_c \to \lambda$,

Not solution to all problems: g_{YM} weak, but λ has approximately same running as QCD, hence $\Lambda_{QCD} \sim N_c^0$
Theory still strongly coupled and confining below Λ_{QCD}

but in this limit drastic semplifications are possible, as some observables $\sim N_c^2$, some $\sim N_c^0$ etc. Plugging in $N_c = 3 \to \mathcal{O}(10)$ hierarchy

N_c scaling results...

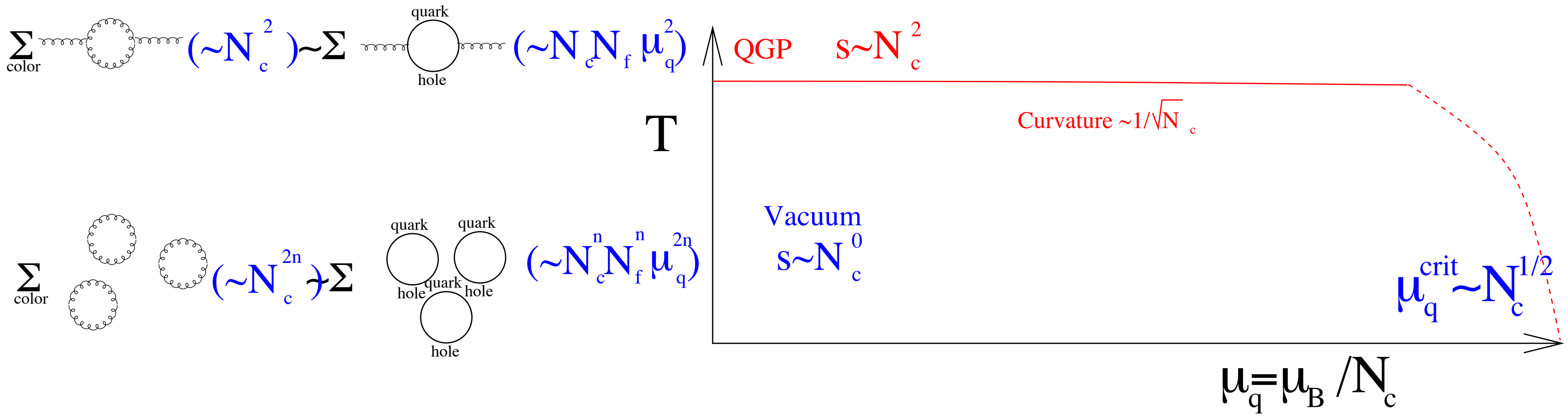
- Planar diagrams dominate, $\Rightarrow$ Strong force $\leftrightarrow$ strings
  Tension $\sim \lambda$,breaking probability $\sim N_c^{-1}$
  AdS/CFT ultimately comes from this analogy!

- Mesons $\rightarrow$ weakly interacting quasiparticles
  Confinement "survives" in $\sim N_c^{-1}$ coupling constant

- Baryons $\rightarrow$ strongly interacting semi-classical states
  Hyerarchy between light fast quantum quarks and baryons

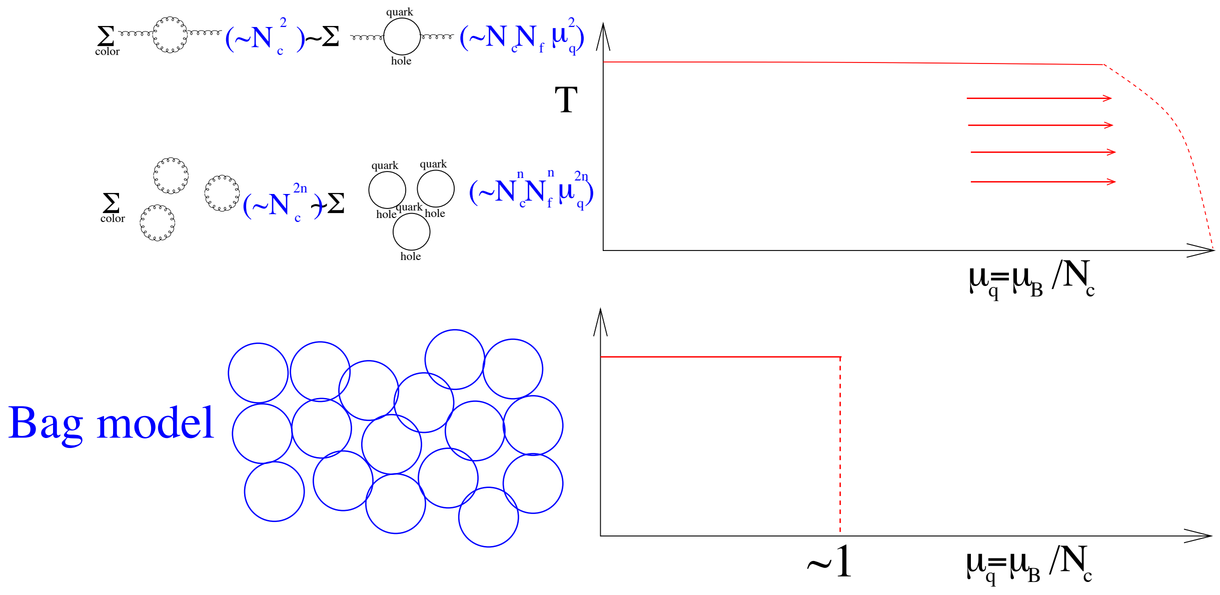
- The phase diagram...

If deconfinement ⇔ quark-hole loops "beat" gluon antiscreening...



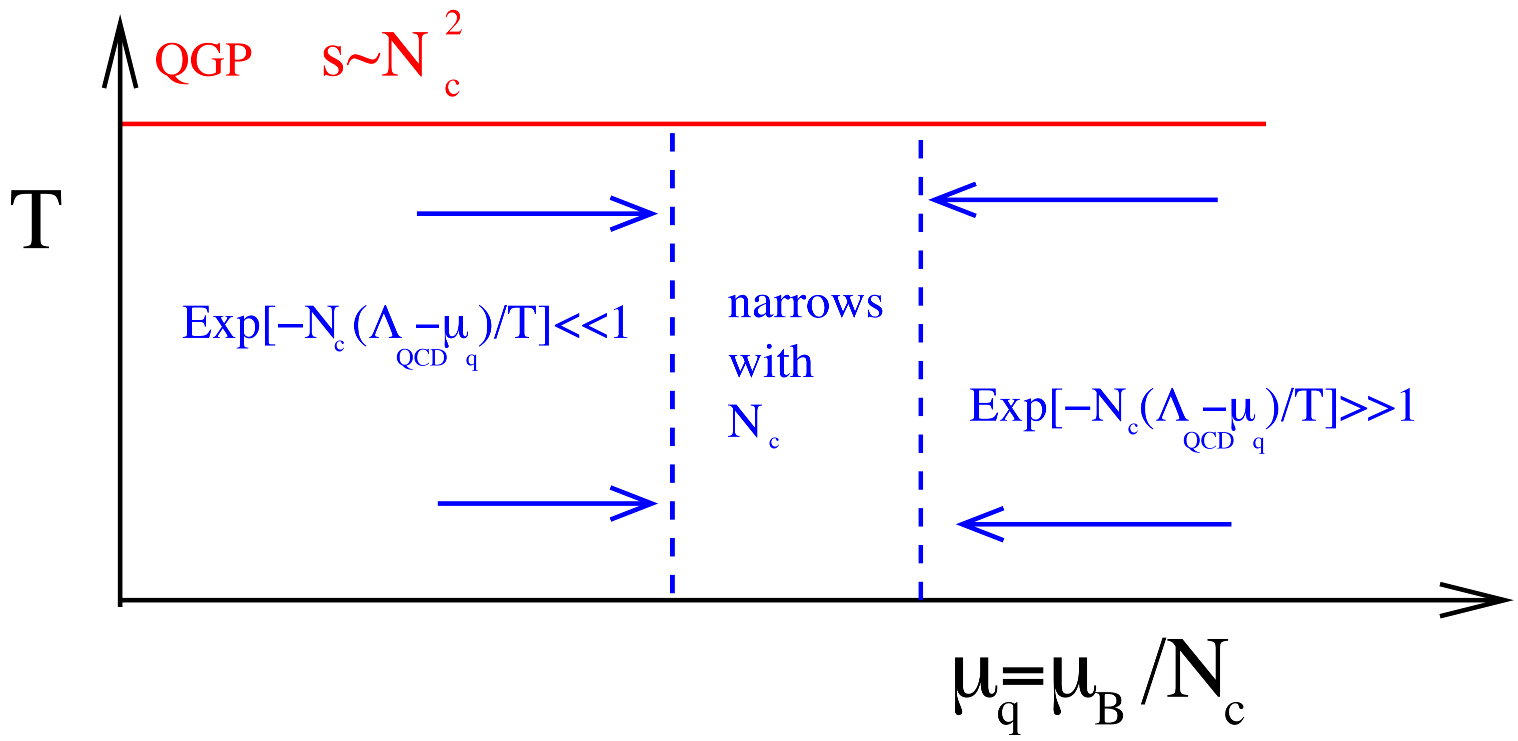
Deconfinement line flattens,for deconfinement $\mu_Q \sim N_c^{1/2} N_f^{-1/2} \Lambda_{QCD}$

NB: higher n order hyerarchy $\sim (N_c/N_f)^{n(n-1)}$, does not help!

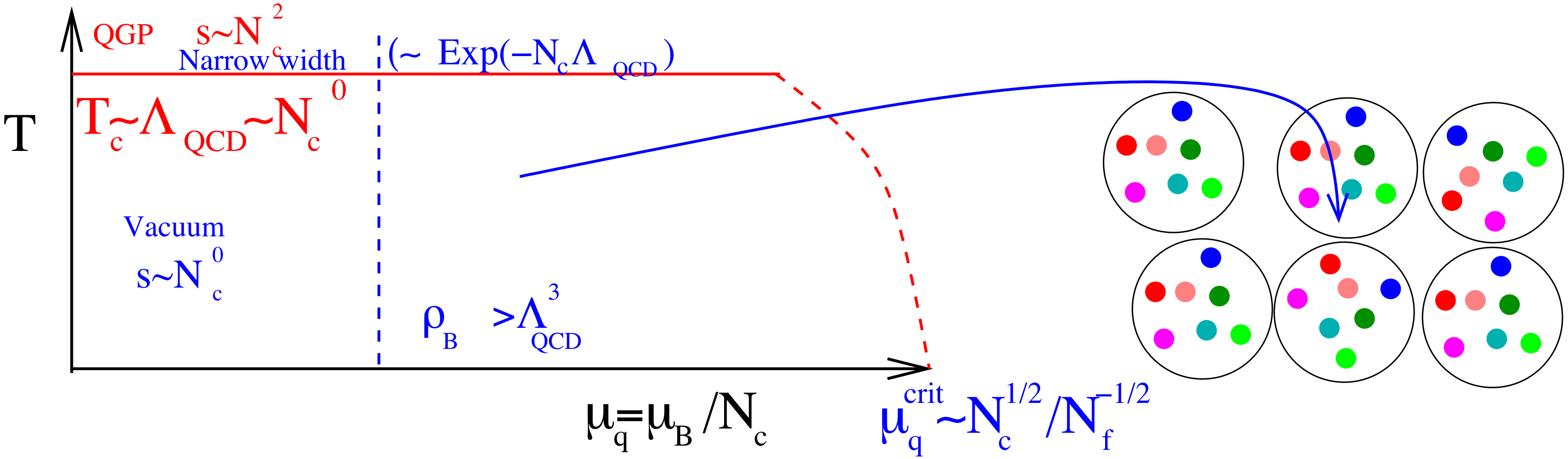


Note: Above is a big if

Above reasoning contradicts, for example, bag model intuition, where $\mu_Q^{crit} \sim T_c \sim \Lambda_{QCD} \sim N_c^0$. The "trick is" it assumes non-perturbative contributions to β-function/confinement order parameters don't have a different N_c dependence, which could dominate at $N_c = 3$. Lets continue to assume this , but its unproven! either alternative is insterstring



line separating "vacuum" from "dense nuclear matter" narrows , since baryon abundance in vacuum phase $\sim \exp(-N_c \Lambda_{QCD}/T) \to 0$

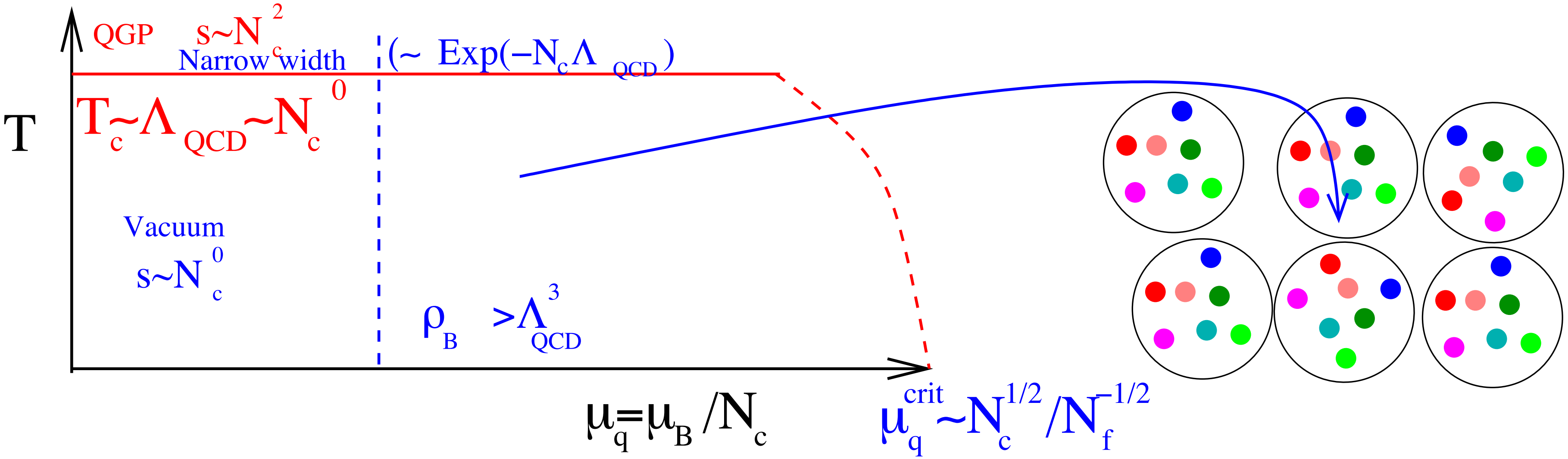


McLerran+Pisarski, arXiv:0706.2191: line at

$$\Lambda_{QCD} \leq \mu_Q \leq \sqrt{N_f/N_c}\Lambda_{QCD}$$

defines new "quarkyonic" phase!

NB: AGS,SIS $\mu_B \simeq 800$ MeV $< m_N$,so it might still be out there!

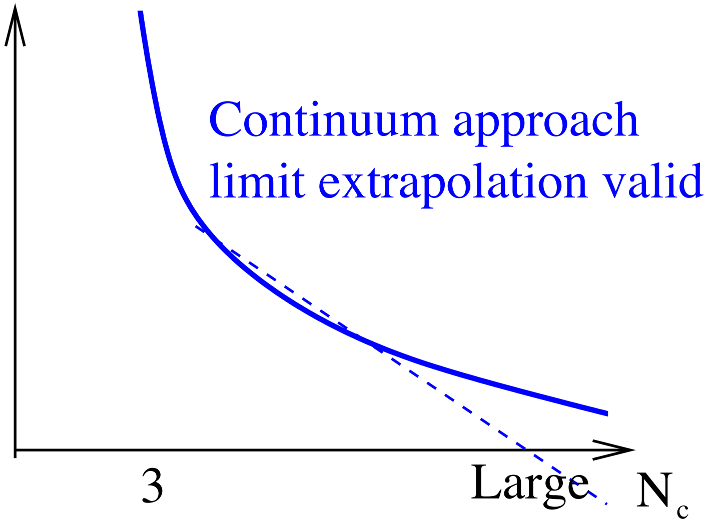


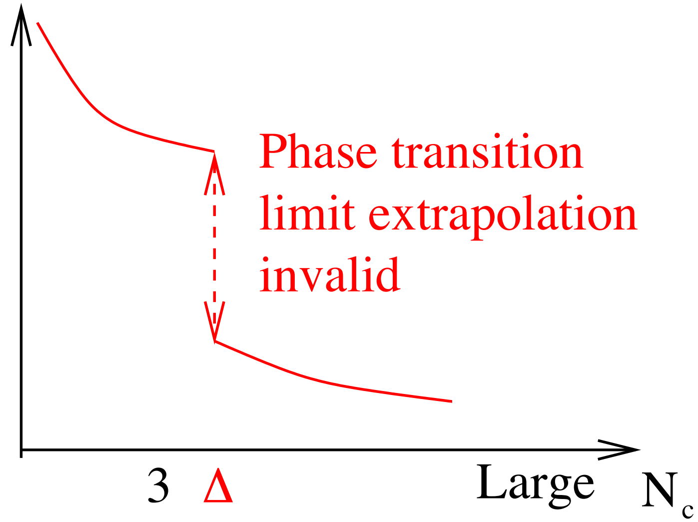
Inter-quark distance in this phase $\sim N_c^{-1/3} \to 0$, asymptotic freedom in configuration space! . Confined but quasi-free quarks below fermi surface and $P \sim N_c$ (quark-hole?)

NB: If color can propagate at inter-baryonic distances, "quarkyonic matter" $\equiv$ QGP, "bag model intuition" correct). otherwise , A new phase to look for at low energy, high density (Neutron stars, FAIR, NICA, etc.), In alternative to critical point,but...

Even if we assume our large N_c limits are under control....

Can we exclude phase transitions in N_f/N_c?





When you are expanding around the right vacuum, a $\sim 30\%$ correction is OK. When you are expanding around the wrong vacuum, any correction is catastrophic. Sometimes its easy to see this (tachyons!), sometimes not (confinement?)

$N_c \gg 1$ nucleons understood by Witten (NPB **160**, 57 (1979)), $\neq N_c = 3$

Quantity	$N_c \to \infty$ scaling	$N_c = 3$	QCD
$E^{binding}_{Nucleus}$	$N_c \Lambda_{QCD}$	1 GeV	10 MeV $\ll \Lambda_{QCD}, m_\pi$
$\Delta E_{spin-flip}$	$\sim \Lambda_{QCD}/N_c$	50MeV	$200 MeV \sim \Lambda_{QCD}$
Ground state	Crystal	Crystal	Liquid

Note $E^{nucleus}_{binding} \sim$ 10 MeV is a hyerarchy problem! It is much smaller than even the "massless" π scale. $N_c \gg 1$ is $N_c \Lambda_{QCD} \sim m_{nucleon} \sim \mathcal{O}(1000)$ times bigger. Needs explanation (especially as EMC effect shows quark wavefunction modified!!). Fit with Walecka model not an explanation

In fact, phase transitions in N_c are certain to happen !

Confined $SU(N_c)_{N_f=0}$ invariant under symmetry Z_N , spontaneously broken by deconfinement at high T.
These symmetry principles dictate that deconfinement is a phase transition, at $N_f = 0$

At $N_f/N_c \sim 1$, according to the lattice, deconfinement is a cross-over.

So, unless something weird is going on (GW point?) , there is a critical point in N_c for confinement.
"finding" a dual gravity description of this critical point,and measuring its critical exponents, an important test for Gauge/Gravity duality
(M.Sprenger,P.Nicolini,M.Kaminski,GT, work in progress)

In fact, phase transitions in N_c are certain to happen II

At $N_c \to \infty, \mu_B/N_c \sim \Lambda_{QCD}$, the ground state of nuclear matter is widely understood to be a Skyrme crystal I.Klebanov, Nucl.Phys.B262:133,1985

From that paper... *Of course , this treatment ignores the kinetic energy of skyrmions. It can be roughly estimated to be* $1/Mca^2 \sim 100$ *MeV. Energy of this order is enough to unbind the crystal at* $N_c = 3$
Roughly speaking... baryon mass $\sim N_c$, baryon Fermi motion energy $\sim N_c^0$ so baryon Fermi motion momentum $\sim N_c^{1/2}$, inter-baryon binding energy $\sim N_c$. As we go down in N_c, crystal melts into a fluid; This must be a phase transition, as symmetries change!

OK, but why should nuclear binding energy be sensitive to this?

The Landau algorithm:

a) Formulate simple picture of the problem
b) Solve it

The best physicist in the USSR is Yakov Frenkel, who uses in his papers only quadratic equations.
I am slightly worse, I sometimes use differential equations.

L.D.Landau, quoted in
BULLETIN OF THE American Mathematical Society
Volume 43, Number 4, October 2006, Pages 563–565

The Feynman algorithm
a) Write down the problem
b) Think REALLY hard
c) Write down solution

The rest of this talk: Toy models which hopefully reproduce the issues discussed until now!

Nuclei and their interactions at large N_c use the Van Der Waals EoS

$$\left(\rho^{-1} - b\right)\left(P + a\rho^2 - g\rho^3\right) = T$$

Only parameter is Λ_{QCD}, so all parameters will be in terms of it

b Is the excluded volume, $\sim \alpha \Lambda_{QCD}^{-3}$

a,g are the interaction, $\sim \beta, \gamma \Lambda_{QCD}^{3-5}$. For <u>any</u> radial interaction $V(r)$,they came out as terms in the expansion of $\prod_{ij} \int dx_{ij} e^{-\frac{V\left(x_{ij}\right)}{T}}$

Solvable analytically, universal, connected to black holes (A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, PRD 60, 064018 (1999))

How does α depend on N_c?

- α can't go below unity (deconfinement).
- In the large N_c limit, the only scale is Λ_{QCD} . It is therefore natural that

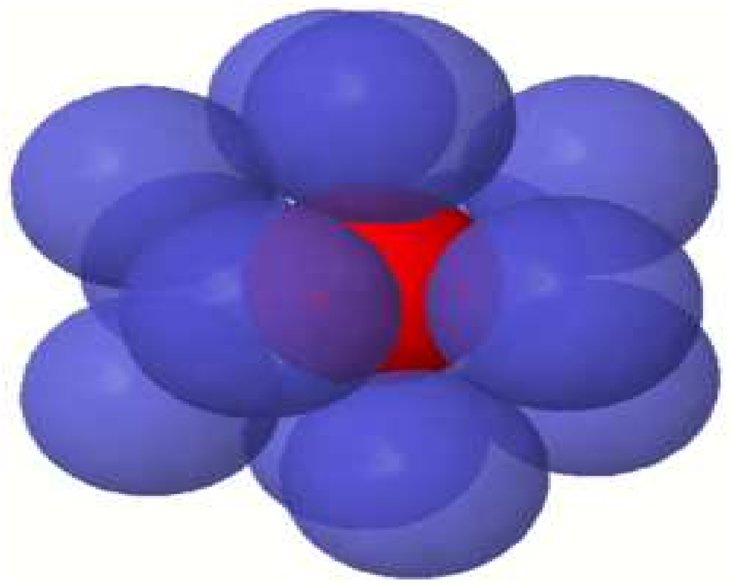
$$\lim_{N_c \to \infty} \alpha = \Lambda_{QCD}^{-3}$$

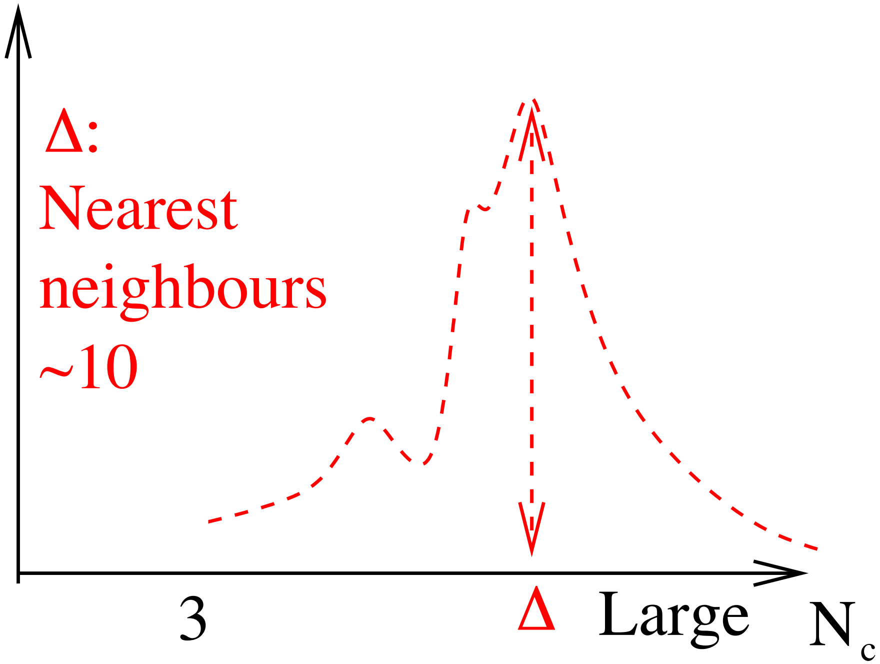
It can not have an $N_c^{a>1}$ leading term, since Baryon size does not diverge. But in our world, $\alpha \gg \Lambda_{QCD}^{3}$

$$\alpha \sim 1 + \frac{A}{N_c}$$

and the A term dominates!

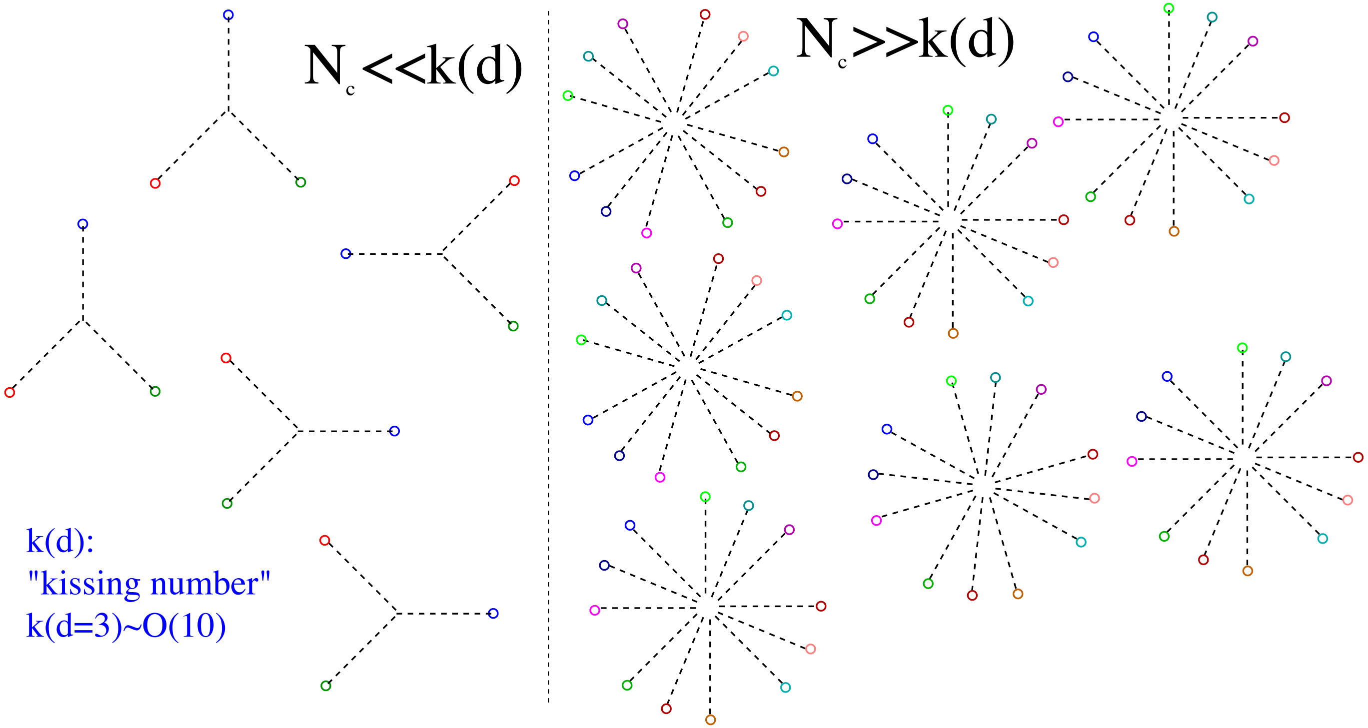
My guess is, we dont live in a large N_c world!





The other scale of the problem is the the number of neighbours in tightly packed system!
"kissing number", exact dependence on d unknown
$k(d) \sim 2^{\zeta d}, k(1,2,3,4) = 2,6,10,24$, of course $\sim N_c^0$, $k(d=3) \gg 3$
$2D$ (lightcone) world closer to large $N_c \Rightarrow$ implications for EMC effect?

Can we say anything more about a critical N_c?

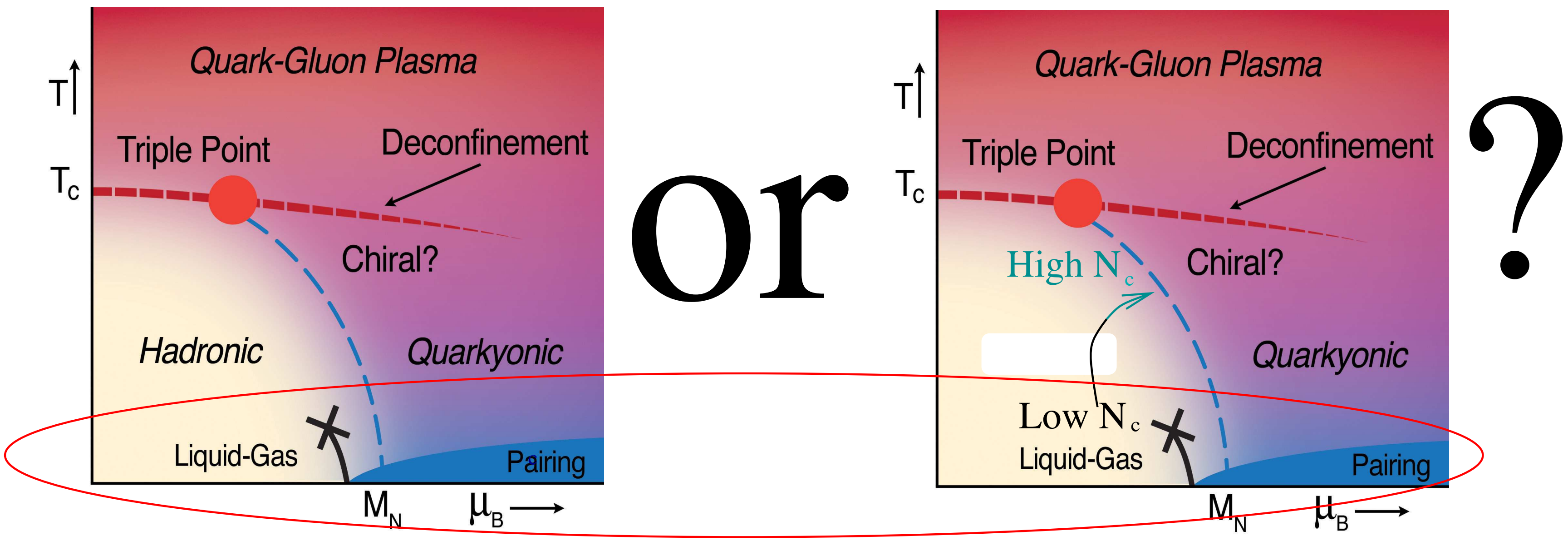


@$N_c \to \infty$ baryons <u>classical</u>. In-medium ($\rho_B \sim \Lambda_{QCD}^3$), $N_c \to \infty$ is when Pauli principle satisfied by color rotations :
$N_c \geq N_{neighbors} \sim k(d=3) \sim \mathcal{O}(10)$.

$$\alpha \sim 1 + \frac{N_N}{N_c} \sim 1 + \frac{k(d)}{N_c} \sim 1 + \frac{10}{N_c}\Big|_{3d}$$

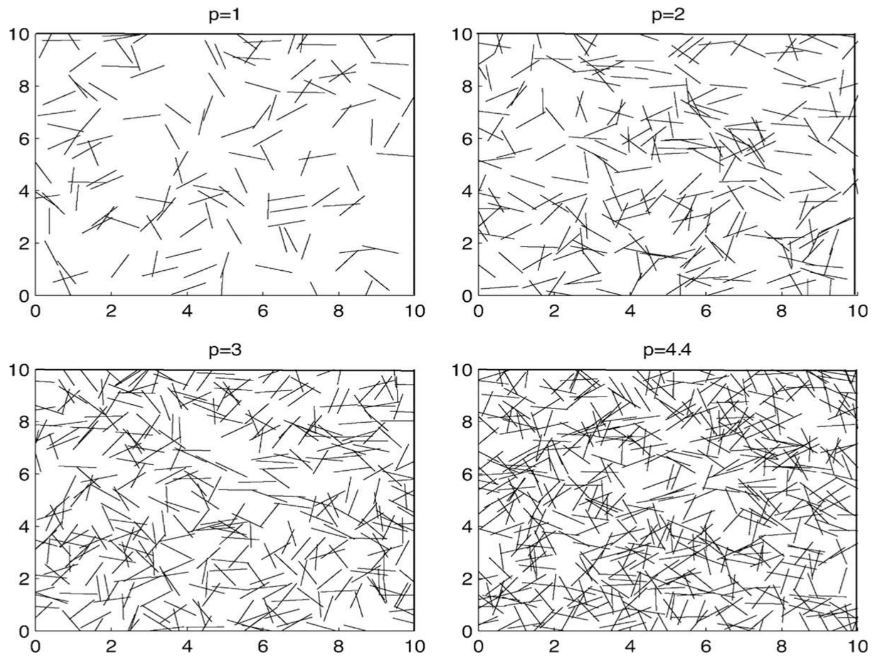
- Fits nuclear VdW at $N_c = 3$
- Compatible with strongly coupled nuclear matter at $N_c \gg 3$
- Understandable by Pauli exclusion principle
  Spin, flavor complicates things. But in our world $\Delta E|_{spin flip} \sim \Lambda_{QCD}$,
  flipping flavor suppressed

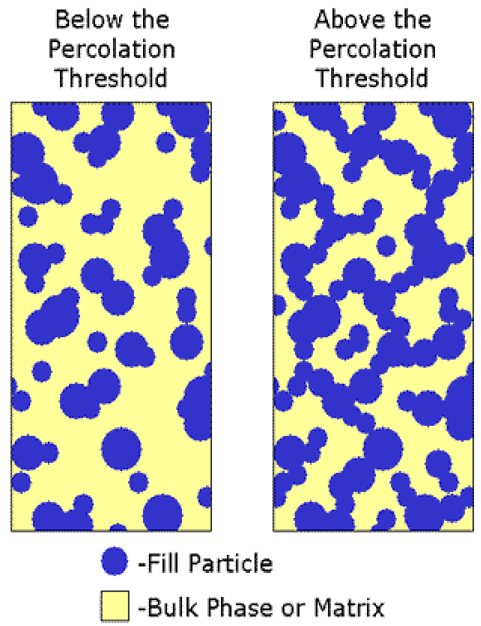
GT,I.Mishustin, PRC82 055202 such a quantum-to-classical transition might drive $E^{NN}_{binding} \sim \mathcal{O}\left(10\right) GeV \ll m_{\pi}, \Lambda_{QCD}$.



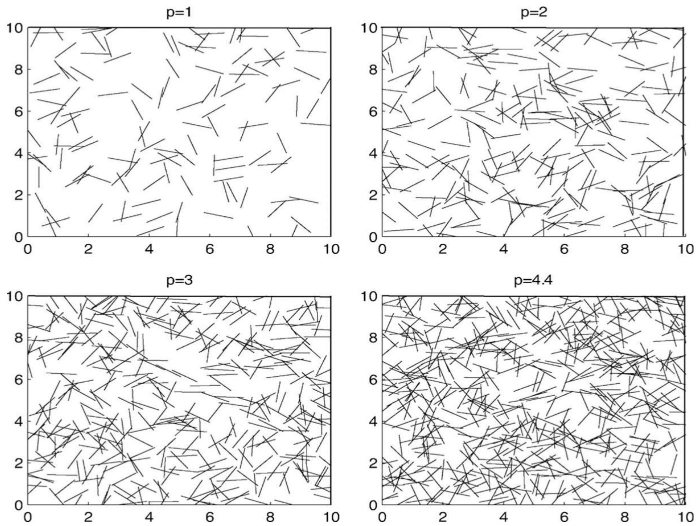
GT,I.Mishustin, PRC82 055202 "quarkyonic matter" might be nuclear matter at $N_c \gg N_{neighbours}$. Or not as depedence on flavor, density not so clear. But $N_{neighbors}$ scaling motivates percolation.

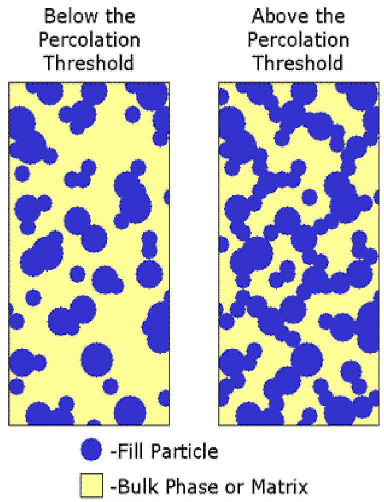
Percolation: the archetypal 2nd order transition





Basic idea: You have a (regular or irregular) lattice of sites, which can be "on" and "off" (links "switched on", particles "in sites", etc), with probability p . Count adjacent sites $\langle N_{sites} \rangle$. When $p \simeq p_c$, $\langle N_{sites} \rangle \to \infty$





- second order transition ($\langle N_{sites} \rangle \equiv$ correlation), with critical behavior.

- $p_c(1D) = 1, p_c(2D) \sim \mathcal{O}(0.5), p_c(3D) \sim \mathcal{O}(0.2)$ (depends on $N_{neighbors}$). So "small" $\sim N_c^{-1}$ correction could trigger it.

Some people have tried to describe deconfinement by percolation of strings/bags, but order of phase transition missed.

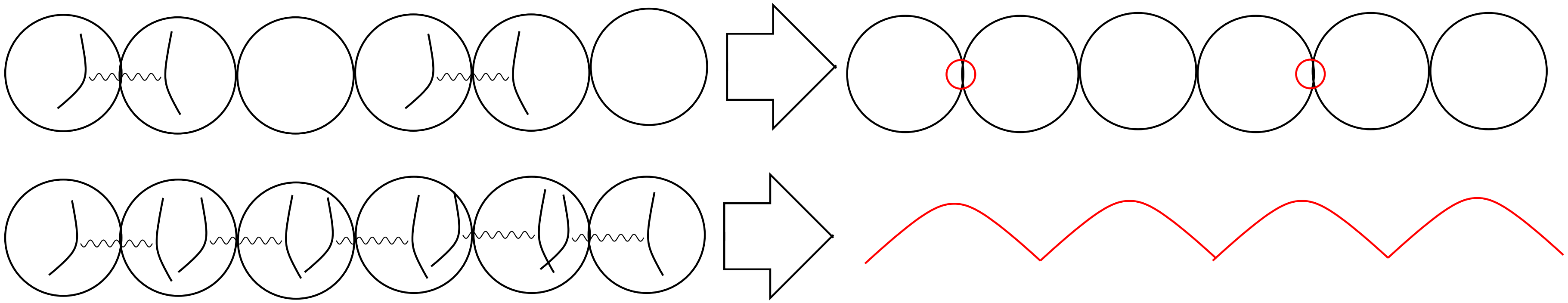
an EFT of $\mu_Q \sim \Lambda_{QCD}, N_c \gg 1$ matter

Baryons are heavy and immobile "background"

Quarks are <u>delocalized</u>, since $\rho_{baryon}^{-1/3} \leq R_{baryon}$ Such delocalization compatible with confinement

An immediate physical analogy: conductor in QED, with baryons playing the role of <u>atoms</u>.

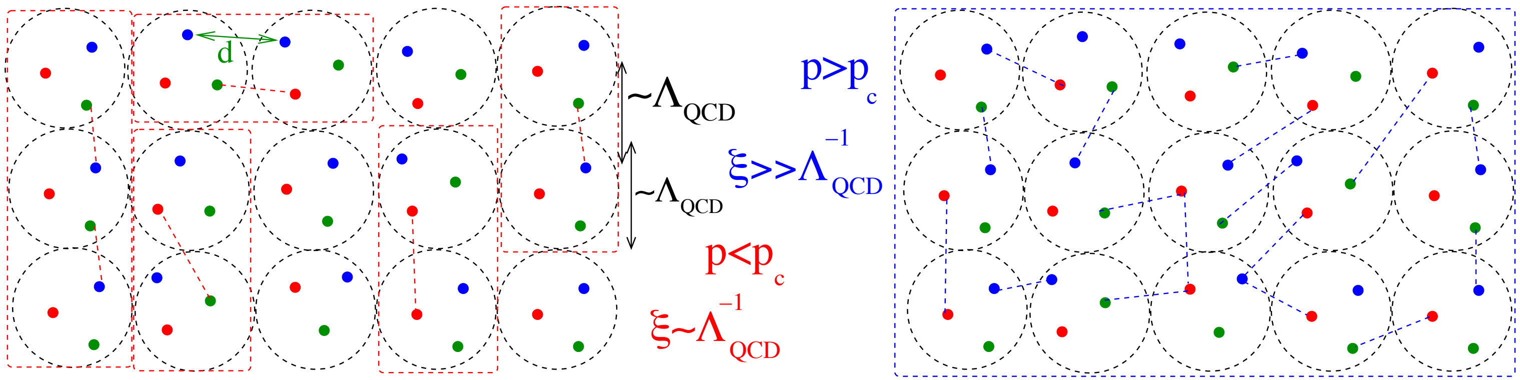
Such a “conducting phase”, not predicted by any EFT, could be the “surprise” we were looking for



But remember, conductor insulator phase transition is governed by number of electrons in the “conducting band”.

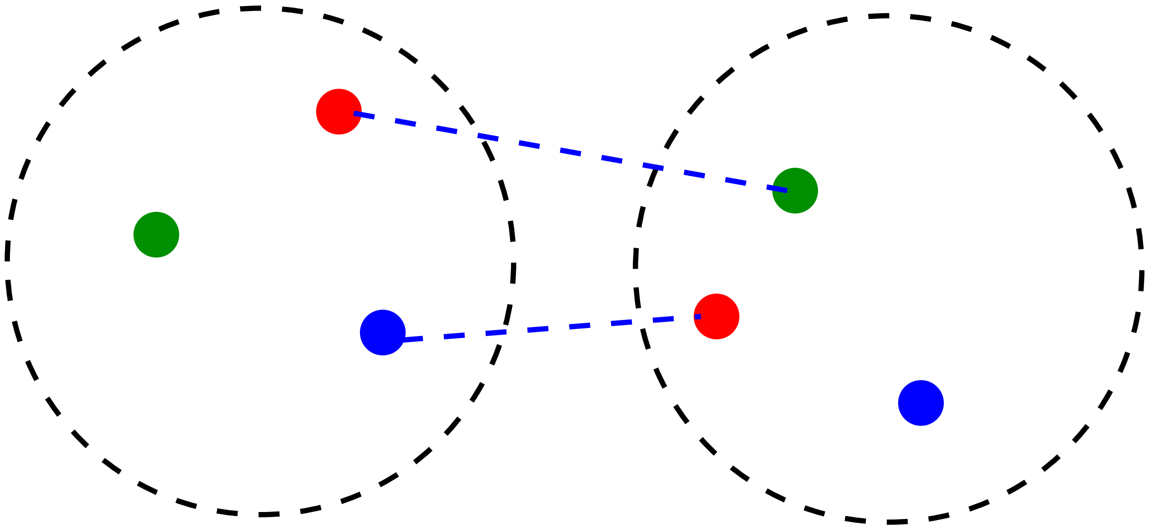
However , since Quark/baryon $\sim N_c$, conductor/insulator transition in full $T-\mu_Q-N_c$ space!

N_c scaling and Percolation at $\mu_Q = \Lambda_{QCD}$



Intuitively, relevance of percolation <u>clear</u>. With N_c colors, ways two baryons can interact with one another grows <u>fast</u> with N_c . Correlation length <u>diverges</u> at percolation, so existence of transition <u>independent</u> of microscopic details (within reason)

Calculating percolation probability at $\mu_Q = \Lambda_{QCD}$

$p \sim$ 

In large N_c limit, assume "perturbative" ($\sim \lambda N_c^{-1}$) interactions between "confining" quarks. Picture insensitive to further details

NB: all dependence on N_c only, the N_c vs $N_{neighbors}$ requirement for classical baryons <u>also</u> depends on N_f This transition <u>different</u> from VdW, as only scales with N_c!

An ansatz with confinement and correct N_c scaling

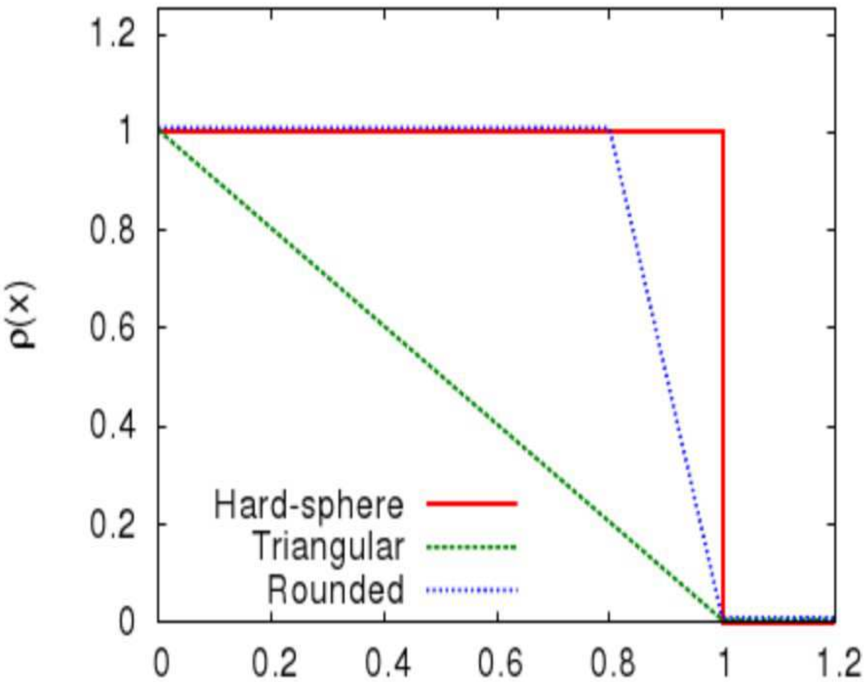
$$p = 1 - \left(q_{(1),ij}\right)^{(N_c)^{\alpha}} \quad , \quad q_{(1),ij} = \int f_A(x_i)\mathrm{d}x_i \int f_B(x_j)\mathrm{d}x_j \left(1 - F(|x_i - x_j|)\right)$$

Mathematically very similar to Glauber model, dont need to get σ exactly right to get N_{part} dependence. In same way, we put in sample propagators to get N_c dependence.

We assume a density distribution with a range of ρ s of the form

$$f_{A,B}(x) = \rho\left(\Lambda_{QCD}^{-1} - \left|x - x_{A,B}^{\text{center}}\right|\right)$$

A range of ρ considered



...and a range of probability amplitudes for the exchange $i \leftrightarrow j$ which respect

- Confinement (rapid fall-off at distances Λ_{QCD}^{-1})

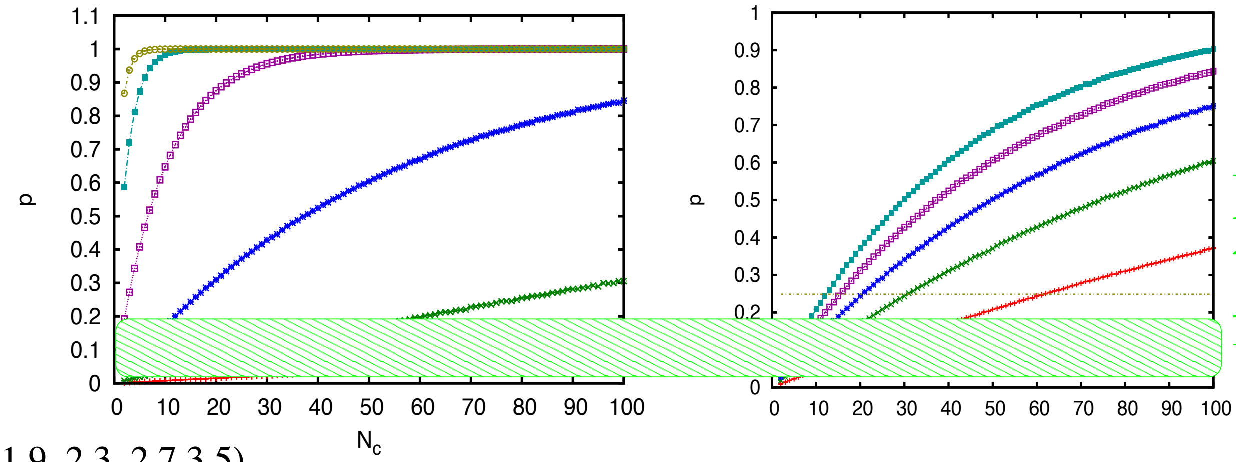
- N_c scaling ($\sim \lambda / N_c$)

$$F(y) = \frac{\lambda}{N_c} \mathcal{N} \begin{cases} \theta(1 - \frac{y}{r_T}) \\ \exp\left(- \frac{3}{4} \frac{y^2}{r_T^2} \right) \\ \frac{2r_T^2}{\pi y^2} \sin^2 \left(\frac{y}{r_T} \right) \end{cases}$$

(Θ-function and Gribov-Zwanziger propagators)

Varying r_T/Λ_{QCD}

(=0.7,1.0,1.5,1.9, 2.3, 2.7,3.5)



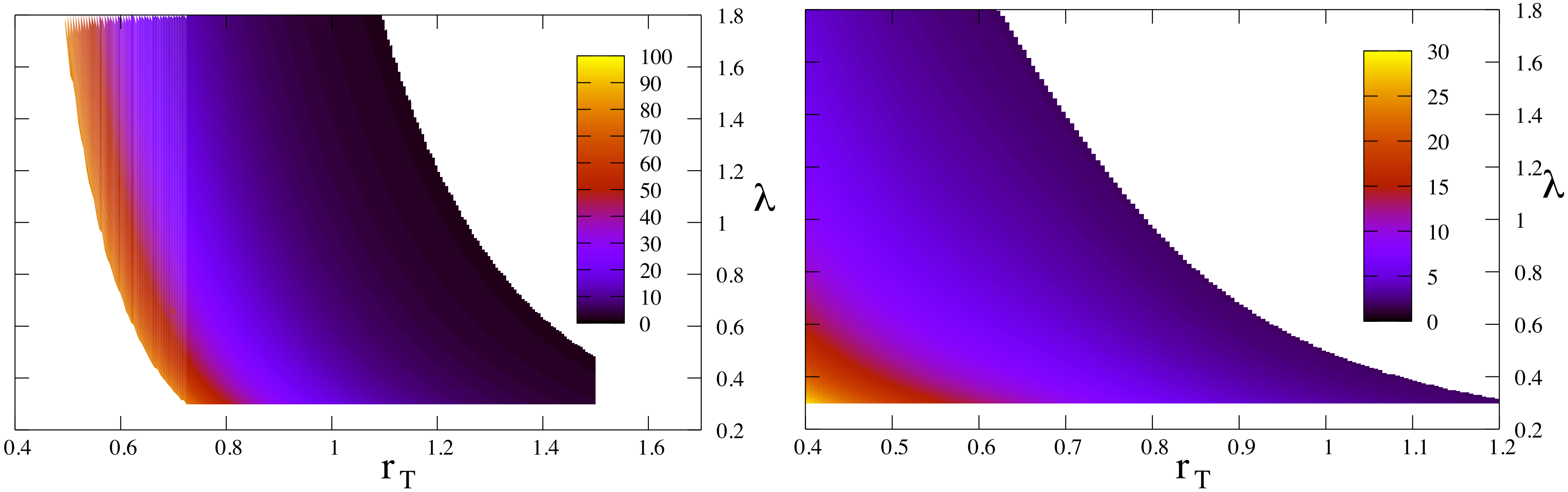
Varying λ

(=0.2,0.4,0.6,0.8,1.0)

Percolation thresholds for different packings

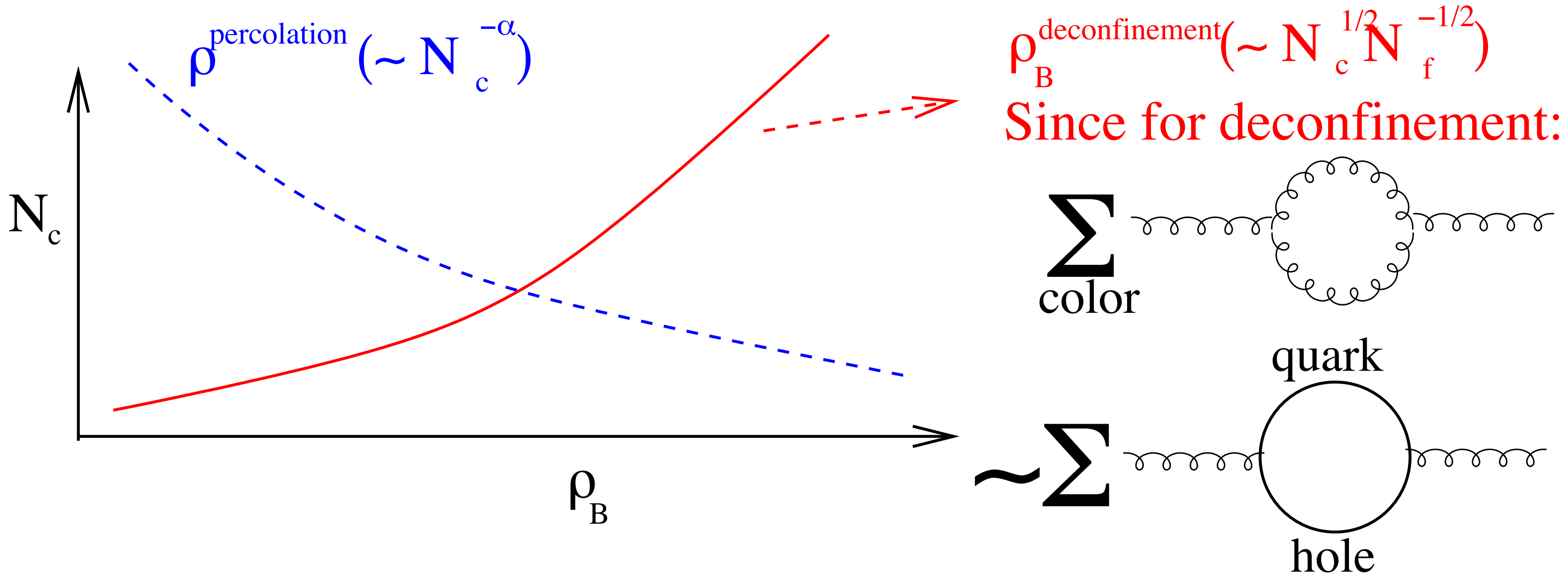
Rapid growth with N_c at $p = p_c$ independently of details of propagator. Transition seems universal at $N_c \sim \mathcal{O}(10)$

Critical N_c for Θ-function $P_{i\leftrightarrow j}$ in position and momentum



"typical" Parameters of order unity give a critical number of colors for percolation well above 3. These are lower limits, since we assume hexagonal lattice (Skyrme cubic and disordered p_c higher). So $N_c^{crit} = 3$ disfavored but<u>not</u> excluded at $\mu_Q = \Lambda_{QCD}, T = 0$.

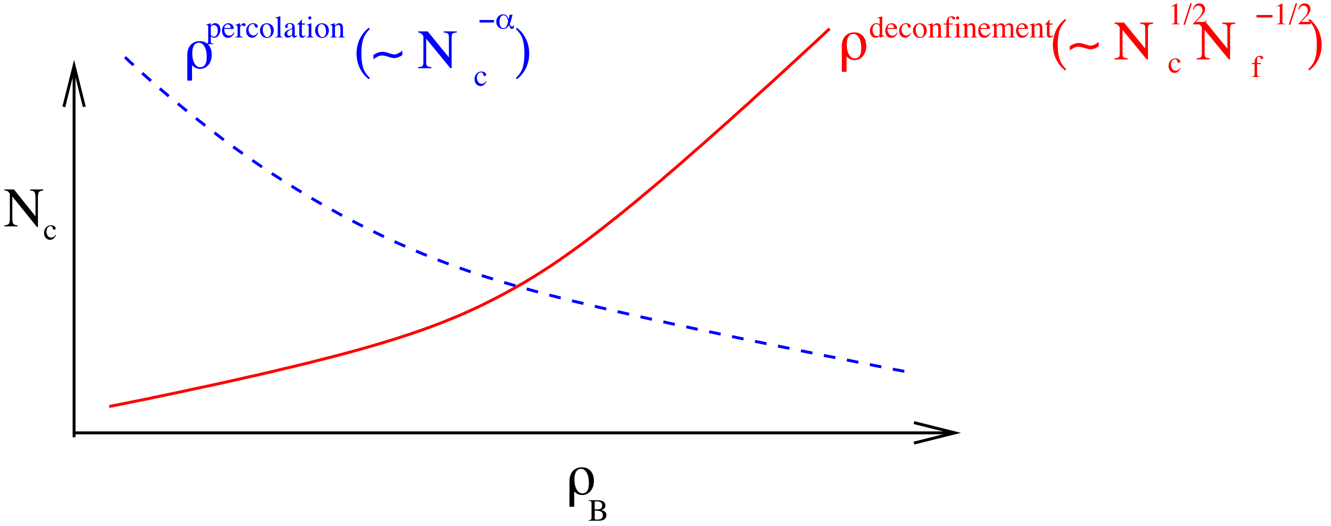
But lets vary μ_Q:Percolation and deconfinement



Percolation: $\rho - N_c$ anti correlated.

Deconfinement: $\rho - N_c$ correlated $\mu_B^{dec} \sim N_c^{1/2} N_f^{-1/2} m_B \sim N_c^{3/2} N_f^{-1/2} \mu_q$

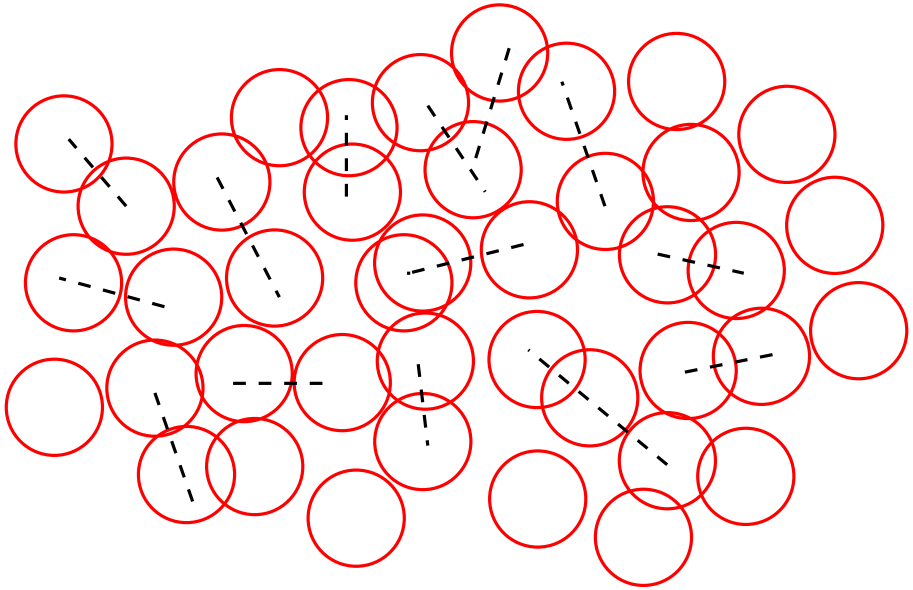
Remember 1 percolating quark negligible for wavefunction of hadron . Need $\mathcal{O}\left(N_c^{1/2} N_f^{-1/2} \right)$ or higher quarks to break hadron apart. But $N_c = 3$!!!



$N_c \leq N_c^{crit}$ Deconfinement happens below percolation, ie percolation transition does not exist separately from deconfinement

$N_c \geq N_c^{crit}$ Percolation, deconfinement separate (Quarkyonic phase?)

What is this critical N_c? Percolation in a "glass": Conceptually similar, technically more involved

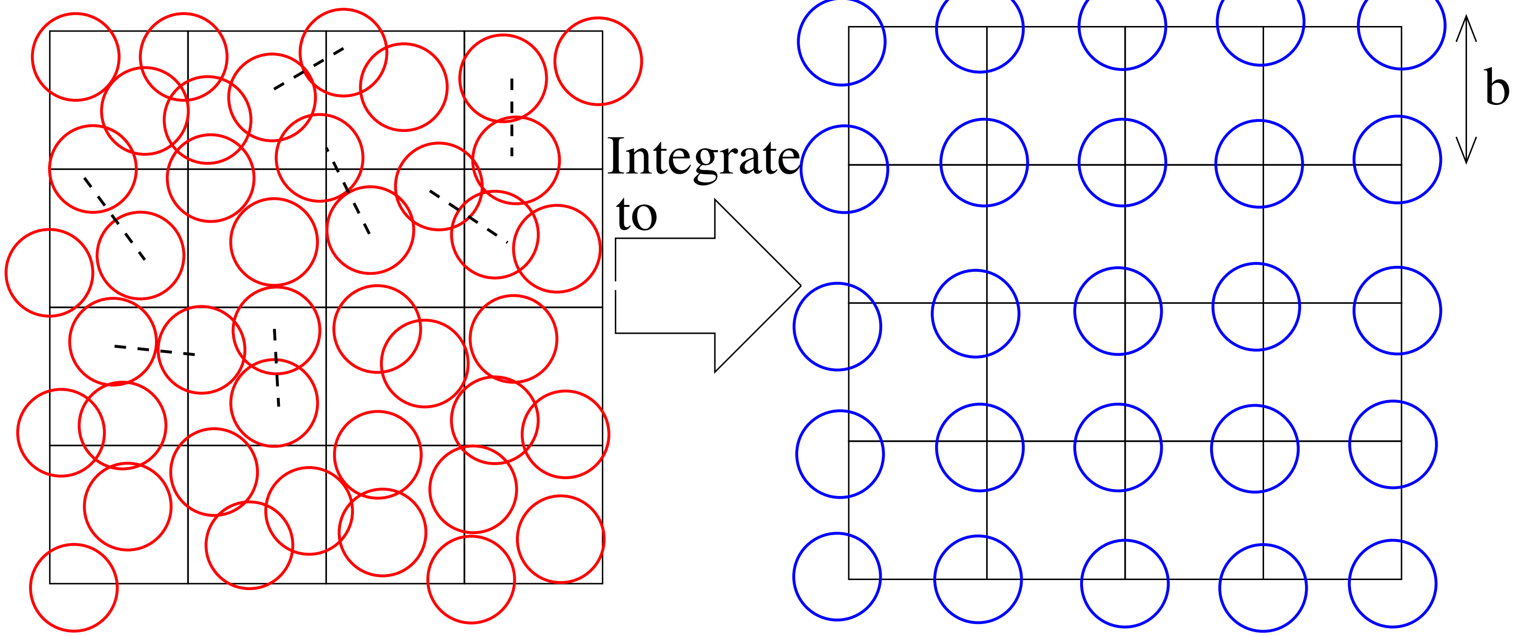


- "Nearest neighbor" <u>not</u> uniquely defined: Baryons <u>overlap</u>

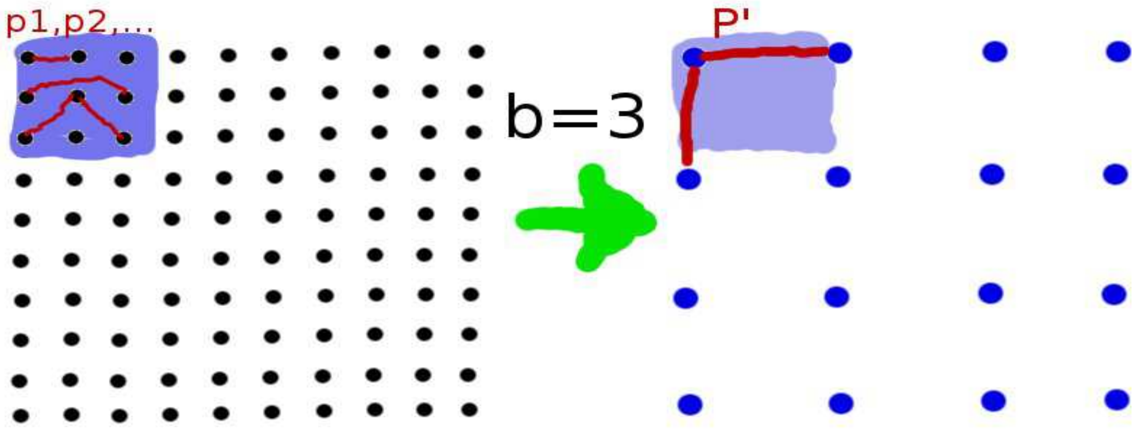
- Interactions to arbitrary distance $\rightarrow$ percolation for arbitrarily low tresholds?

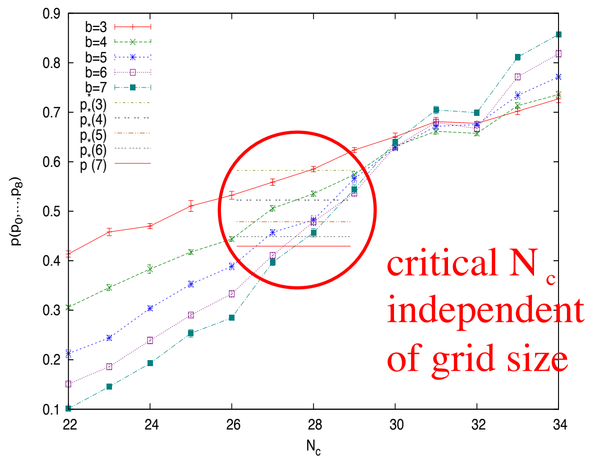
Solution:MC renormalization

Decimate glass to a cubic grid, over many "glass events". Do percolation over cubic grid



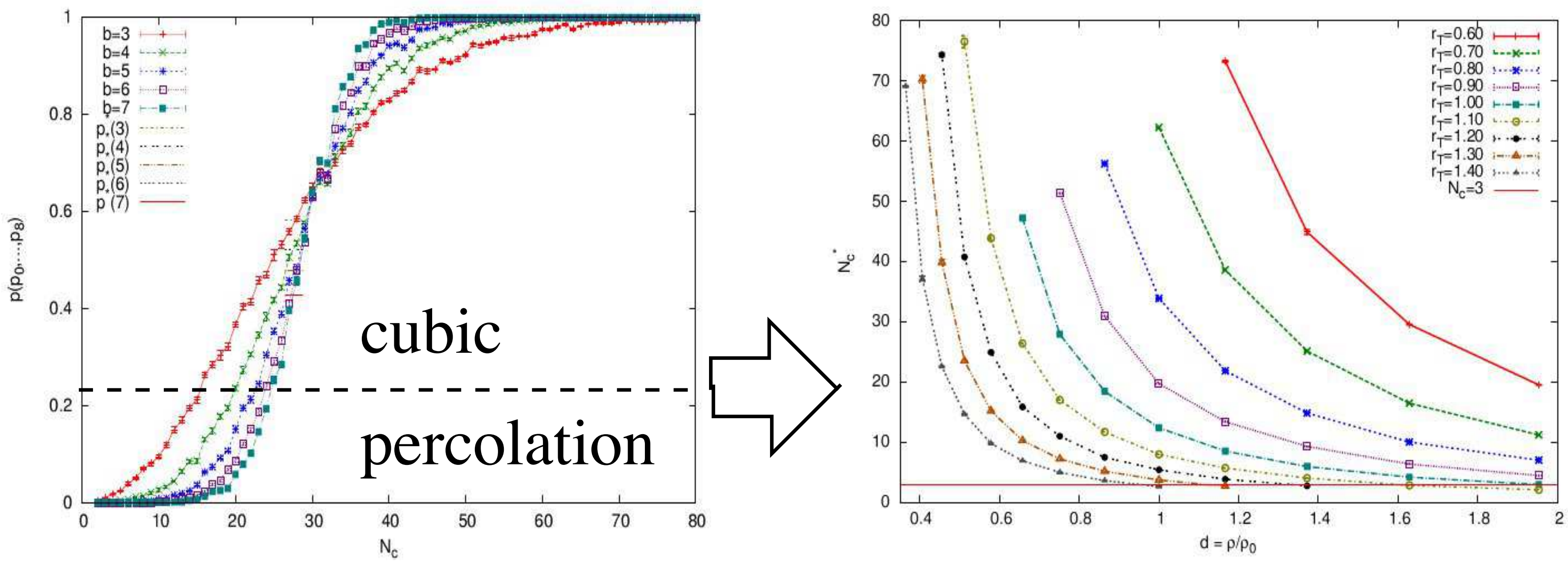
Since percolation at critical point, critical probability should be fixed point of renormalization step, independent of b





Gimel,Nicolai,Durand, J Phys A Math Gen 32 L515 (1999)

$$p^{*}\left(b, \Theta(x_T, \lambda, N_c)\right) = \Pi_{physical}\left(\Theta(x_T, \lambda, N_c)\right) + \beta b^{-y} \quad , \quad y = 0.81$$



Density and N_c tightly correlated. Percolation at $N_c = 3$ excluded at $\rho_B \sim \Lambda_{QCD}^3$. But could there be percolating region at $\Lambda_{QCD}^3 < \rho_B < \rho_B^{deconfinement}$?

Equations for confinement: Ideal gas of non-relativistic baryons,mesons

$$\frac{n^{conf}}{\Lambda_{\rm QCD}^3} = \mathcal{G}\sum_{n=1}^{\infty}(-1)^n\frac{n\gamma^2}{\beta}\sinh\left((\sqrt{N_c}\beta)^n\right)K_2\left(n\gamma\beta\right)$$

$$\frac{e^{conf}}{\Lambda_{\rm QCD}^3} = \mathcal{G}\sum_{n=1}^{\infty}3(-1)^n\frac{n\gamma^3}{\beta}\cosh\left((\sqrt{N_c}\beta)^n\right)\left(\frac{3}{\gamma\beta}K_2(n\gamma\beta)+K_1(n\gamma\beta)\right)$$

Where $\mathcal{G} = \frac{4\pi g_f g_s(N_c)}{(2\pi)^3\sqrt{N_f}}N_c^{5/2(T-T_c)^*}$ and

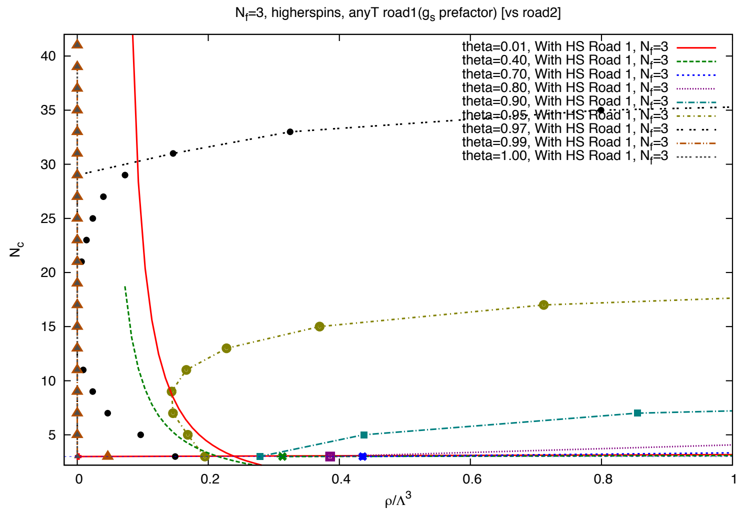
$$\frac{T}{\mu_B} = \frac{1}{\beta N_c^{1/2}} \quad , \quad \frac{m}{\mu_B} = \frac{\gamma}{N_c^{1/2}} \quad , \quad \frac{p}{\mu_B} = \frac{\alpha}{N_c^{1/2}} = 1\Big|_{deconfinement}$$

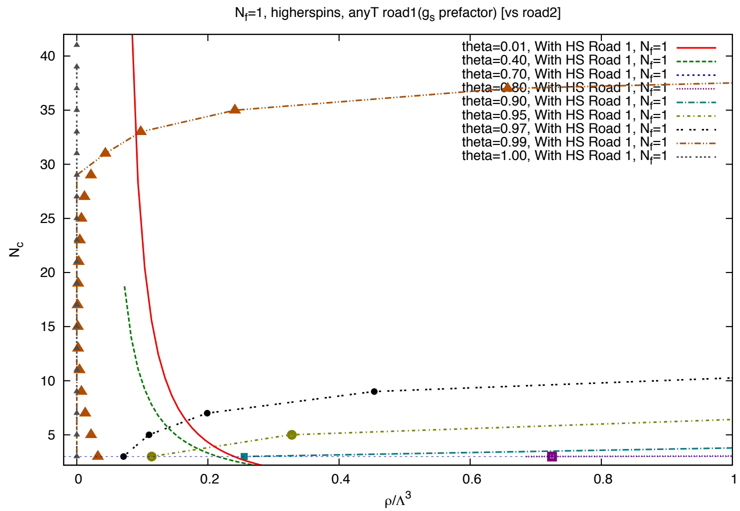
* $T \simeq 0$: All energy carried by baryons. $T \simeq T_c$: deconfinement happens at all μ_B : Parametrize confinement line by $T^2 + N_c^2\mu_q^2 = \mathcal{O}\left(1\right)\Lambda_{QCD}^2$

Quarkyonic phase <u>might</u> exist at $\Lambda_{QCD} \leq \mu_Q \leq N_c N_f^{-1} \Lambda_{QCD}$

In PRL we neglected Density-N_c curvature and fixed density to $\mu_B \sim \Lambda_{QCD}$

.





A sliver of $n - \rho - N_c = 3$ space which is percolating but confined seems to be there. Width depends a lot on whether $N_f = 2$ or $N_f = 3$.
"Systematic error too big . Need <u>phenomenology</u>!

Quarkyonic phenomenology on the lattice

Quenched lattice very close to N_c invariant (Panero et al), but need at least 1 flavor for the effects described here. One would need to vary $N_{f,c}$ at finite μ_Q , possibly $\mu_Q \sim \Lambda_{QCD}$

I can already see you making such a poster!

But hear me out!

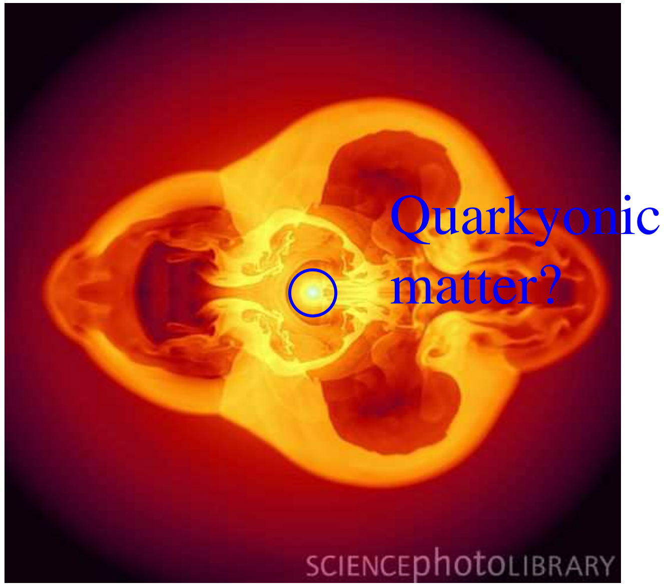
Strong coupling expansion Binding energy and EoS should drastically change with N_c, N_f (NB: Percolation sensitive to N_c, "kissing transition" to $N_c N_f$ so different)
Strong coupling expansion has no sign problem and relatively cheap!

"Baryon molecules" $T = 0$ wavefunction should drastically change shape with N_c

Hopping approximation and Reweighting found jump in baryon density at $N_c = 3, \mu_Q \simeq \Lambda_{QCD}$.
But this is "trivial" , due to high baryon mass!
Need to check pressure behavior with N_c . difficult but possible!

Astrophysical implications

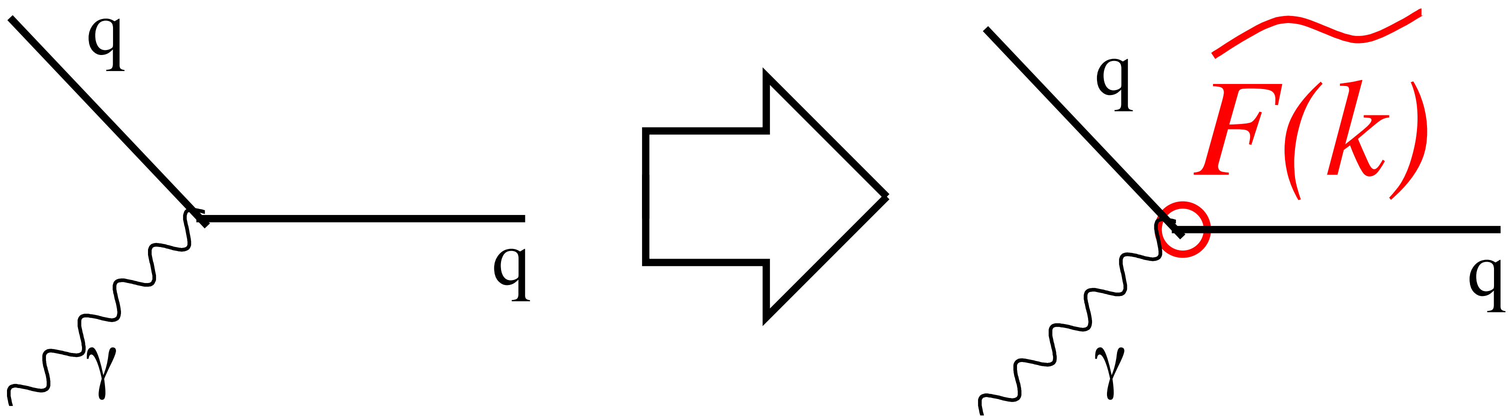
If quarkyonic phase realized in proto-neutron star , pressure, entropy $\sim \mathcal{O}(3)$ corresponding nuclear matter. EoS similar to pQCD (stiffer than nuclear matter), but no mixed phase/latent heat: Stiffness gradually turns on!.



Such an EoS might make it easier for supernovae to explode?

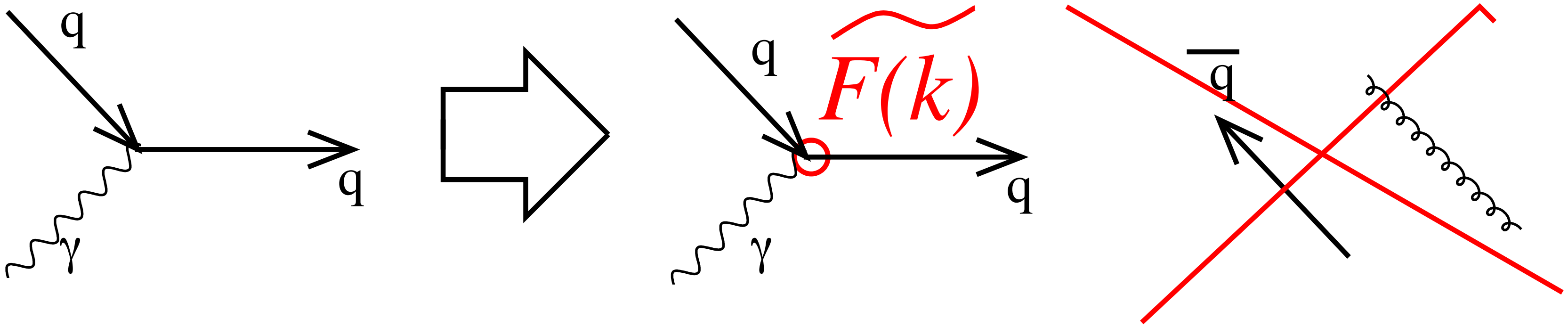
pQCD but not quite: the role of baryons

Unlike pQCD, quarkyonic matter's "vacuum" is a classical dense baryon state. Treating baryons as mean fields will give a momentum-dependent form factor



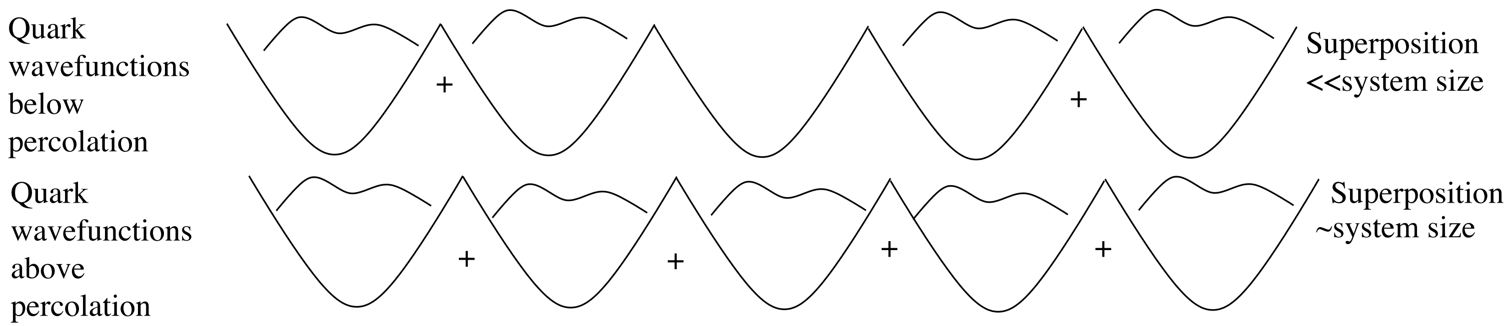
$F(k)$ gives the F.T. of the baryonic gluon content. For the equation of state, it should just be a $\mathcal{O}(1)$ normalization factor, but for scattering processes it is a qualitative difference from naive QCD. Spin-color-flavor separation can ensure color neutrality with quark-like degrees of freedom. Baryons motion doesent influence quarks up to N_c^{-1} corrections

NB: Quarks delocalized by tunneling, not confinement



Gluons, antiquarks still confined, only processes with outgoing quarks allowed!

From EoS to dynamics: An EFT of percolating matter

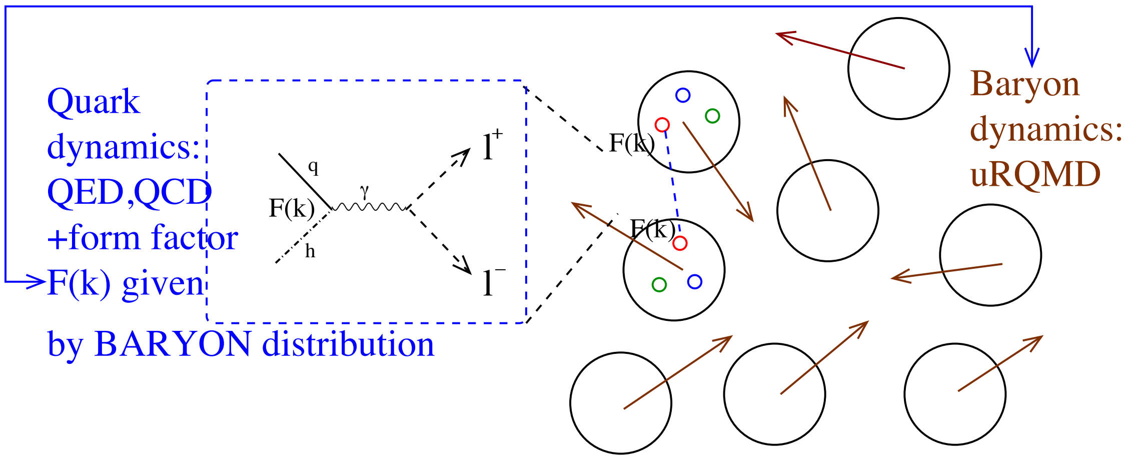


In percolation regime, asymptotically free quark wavefunctions of different baryons can superimpose across large distances.

Thus, even if $E_{state} \sim 1/L_{baryon} \sim N_c^0 \ll N_c^{1/2}\Lambda_{QCD}\Big|_{deconfinement}$ degrees of freedom quark-like, so $P \sim N_c, s \sim N_c$ (In the same way electrons in a metal have a much lower energy than ionization).

Periodic wavefunctions $\Rightarrow$ leading component <u>always</u> $p \geq \Lambda_{QCD}^{-1}$

Modeling quarkyonic matter for RHIC/NICA/FAIR

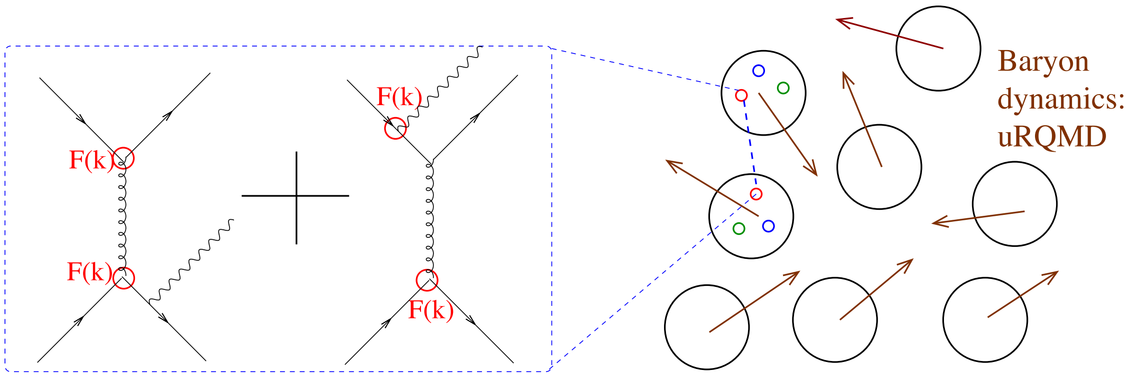


$R_{qq\to X} = \Psi(k)\Psi^*(k')M^2_{qq\to X}$ Where $M_{qq\to X}$ is the pQCD matrix element

$$\Psi(k) \sim \exp \sum_i \left[ikx_{0i}\right] F(k) \sim \exp\left[ikx_{0i} - \frac{k^2}{\Lambda_{QCD}}\right]$$

$F(k)$ is the quark function inside a "classical" proton potential well ($\sim$ Gaussian) and x_{oi} are the baryon locations. The latter is given by uRQMD.

Photon production in this approach



As antiquarks, gluons suppressed leading channel is quark Brehmsstrahlung.

$$\mathcal{M}^2 = L^2(k_1, k_2 \rightarrow k_3, k_4, p) + L^2(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4)$$

$$L^2 = -\frac{1}{4} e^2 \lambda^2 N_c^{-2} (k_2 - k_4)^{-4} Tr \left[\not{k}_4 \gamma^\sigma \not{k}_2 \gamma_\rho \right] Tr \left[\not{k}_3 Z_\sigma^\mu \not{k}_1 Z_\mu^\rho \right]$$

$$Z_\alpha^\beta = \gamma_\alpha (k_1 - p)^{-1} \gamma^\beta + \gamma^\beta (k_3 + p)^{-1} \gamma_\alpha$$

$$\frac{dN_\gamma}{d^3p} = \int \frac{d^4k_1}{k_1^0}\frac{d^4k_2}{k_2^0}\frac{d^4k_3}{k_3^0}\frac{d^4k_4}{k_4^0}\left(\mathcal{M}\left(k_1, k_2 \rightarrow k_3, k_4, p\right)\Psi(k_1)\Psi(k_2)\right)^2$$

- Quarkyonic quark wavefunctions
$\Psi(k) \sim \exp\sum_i \left[ikx_{0i}\right] F(k) \sim \exp\left[ikx_{0i} - \frac{k^2}{\Lambda_{QCD}}\right], uRQMD \Rightarrow x_{0i}$

- Can we go beyond $N_c \rightarrow \infty$ and incorporate baryon flow?
"Boosted quarkyonic" : Same wavefunction as above boosted to flow of a "random" baryon: An upper limit to N_c^{-1} backreaction (effect of baryon flow on quark wavefunction)

Calculate

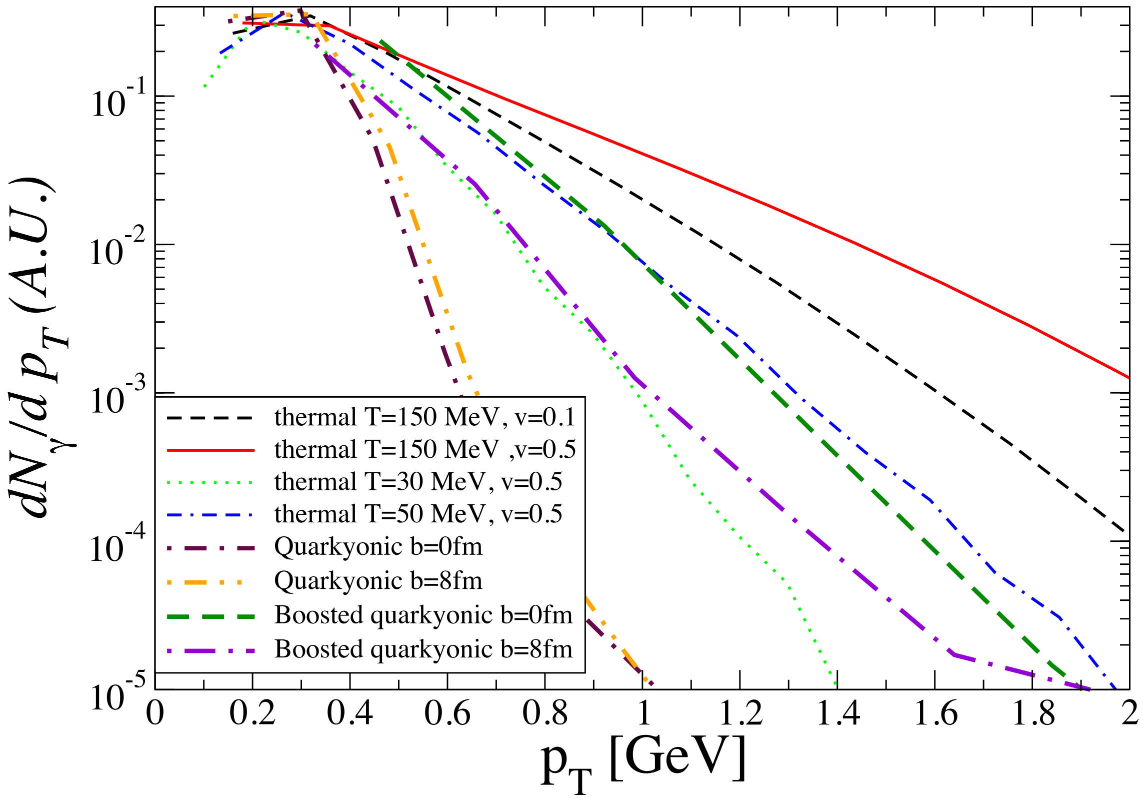
$$\frac{dN}{d^3p} = \frac{dN}{dp_T dy}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos\left(n\left(\phi - \Psi_{reaction}\right)\right)\right]$$

for

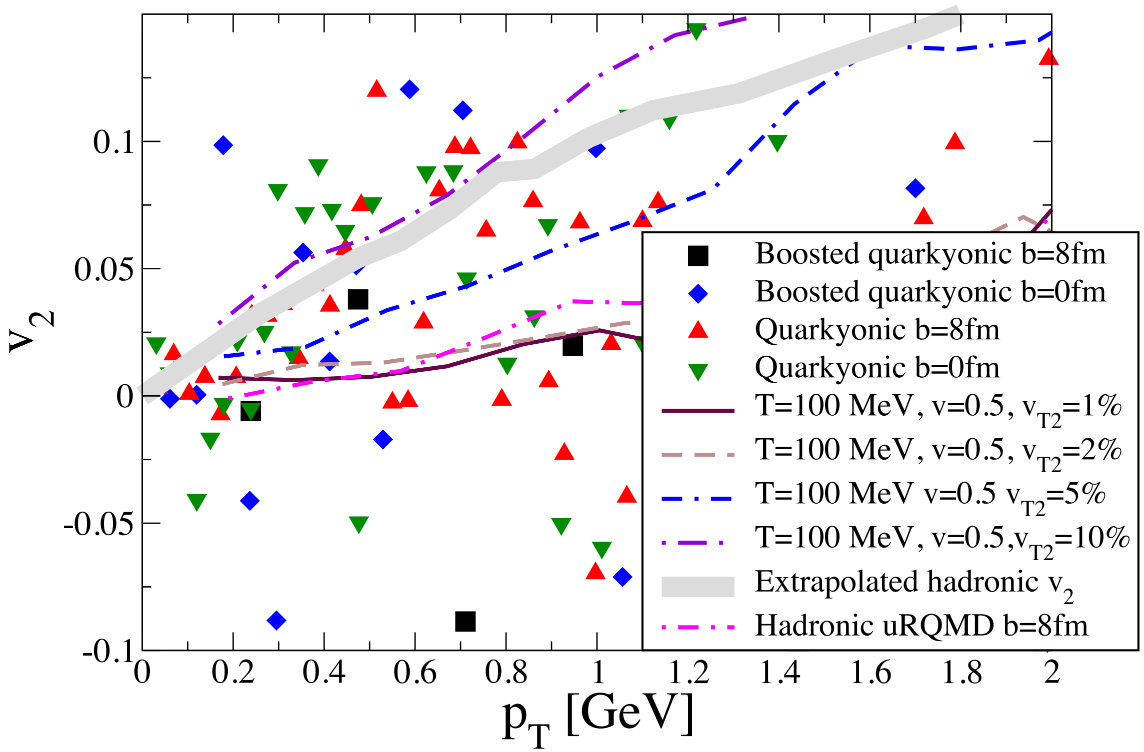
Quarkyonic and Boosted quarkyonic matter described above

thermalized QGP cross-sections described above and quark wavefunctions $\Psi(k)\Psi(k') = \delta(k'-k)\exp\left[-k_\mu u^\mu/T\right]$

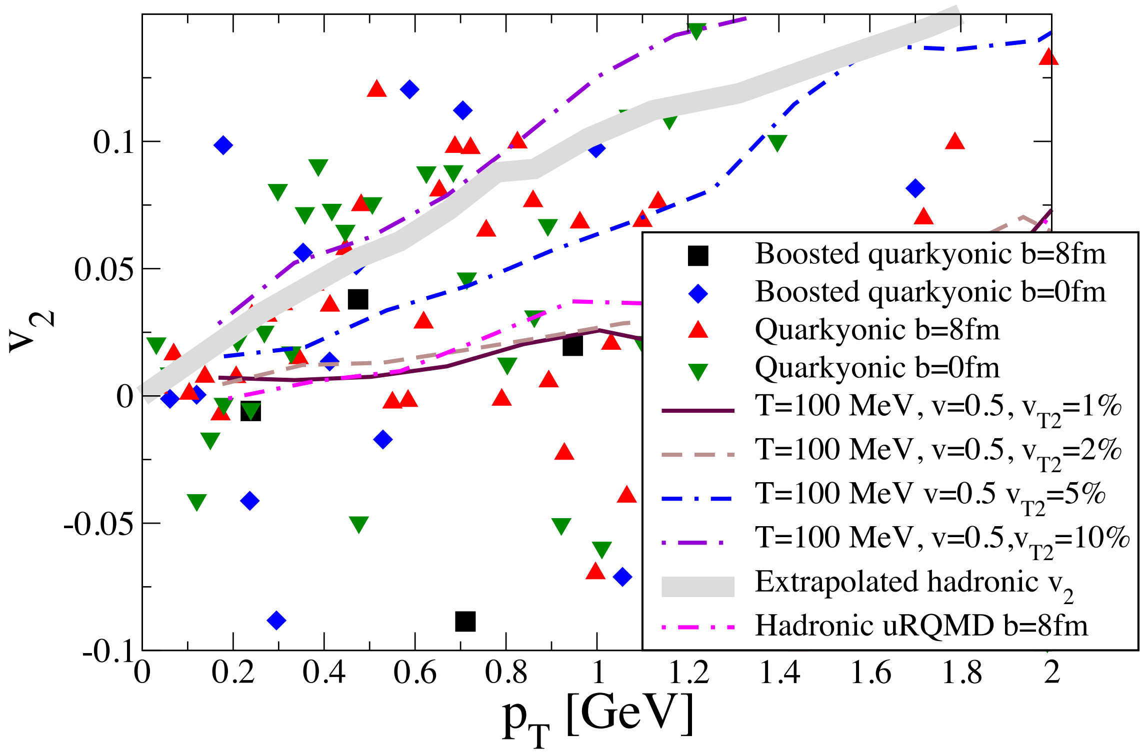
Hadron gas calculated with uRQMD molecular dynamics model (same as the one used for quarkyonic wavefunctions!)



Quarkyonic wavefunction similar to <u>cold</u> quark gluon plasma, unrealistic temperatures. NB: "boosted quarkyonic" increases flow, but still cold!



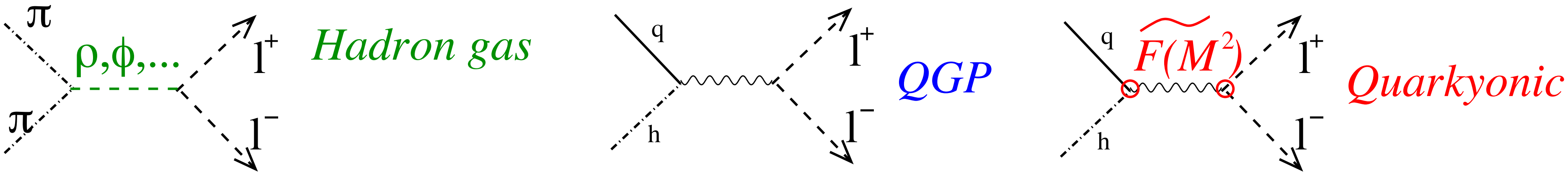
Random distribution of quark wavefunctions quenches total v_2 but produces big fluctuation in event and p_T : oscillation frequency $\sim p_T \rho_B^{-1/3}$



"pure" quarkyonic effect, it is due to sensitivity of quark wavefunctions to baryon location. signature?

dileptons potentially more direct probe but more complicated

Both quarks and holes needed Sensitivity to equilibration

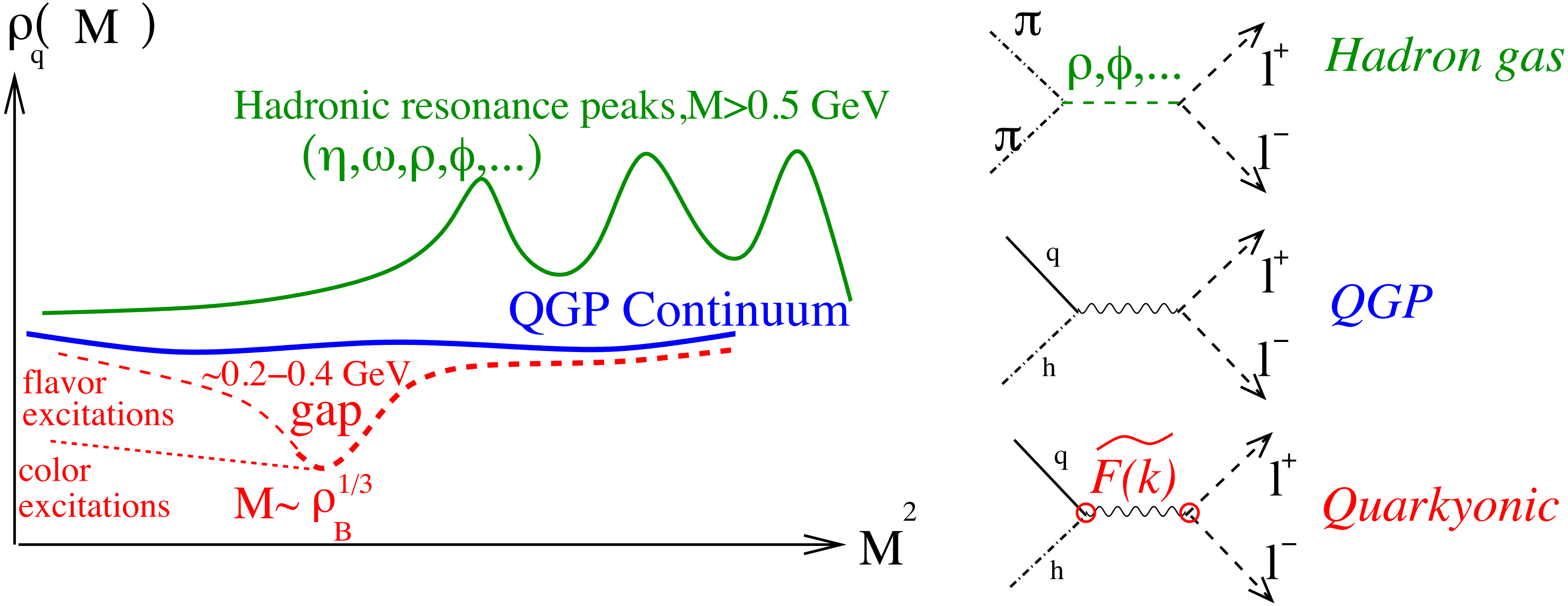


$\tilde{F}(M^2)$ connects baryon distribution to M^2 dilepton spectrum

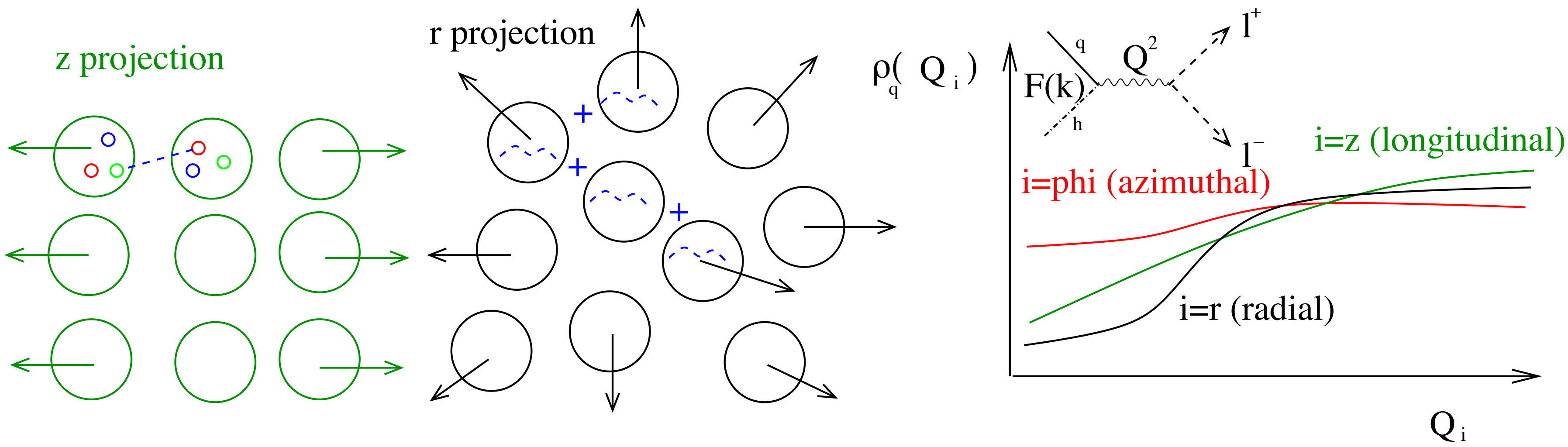
$$\langle \hat{\Psi} \rangle = \mathrm{Tr} \left\{ \{\exp \left[\frac{\hat{H} - \mu_q \hat{N}}{T} \right] \left[\frac{1}{3N} \left(\sum_{i,j,k}^{N} \hat{a}_i(k_i) \hat{a}_j(k_j) \hat{a}_k(k_k) \right) \right] \right\}$$

where a_i solutuions of <u>confining</u> potential wells centered around baryons,

$\hat{H} = \sum \hat{k}_i^2 + \sum_i^{baryons} V \left(\hat{x}_i^{baryon} - v_i^{baryon} t \right)$



If baryons were regular (pasta phase?) one could observe bloch waves! ("upside down resonance"?)



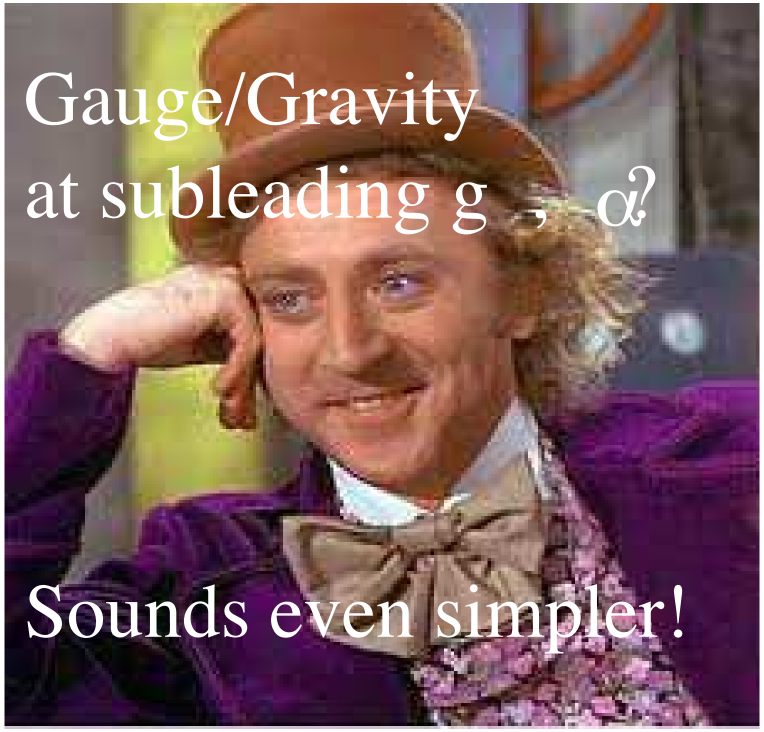
Event by event fireball structure not regular, but Collective structures exist in events flow profile (radial, longitudinal flow) and baryons have repulsive potential, soo structures in 3D dilepton spectral function $Q_{z,r,\phi}$ bound to exist!

Is there a Gauge/Gravity angle to all this?

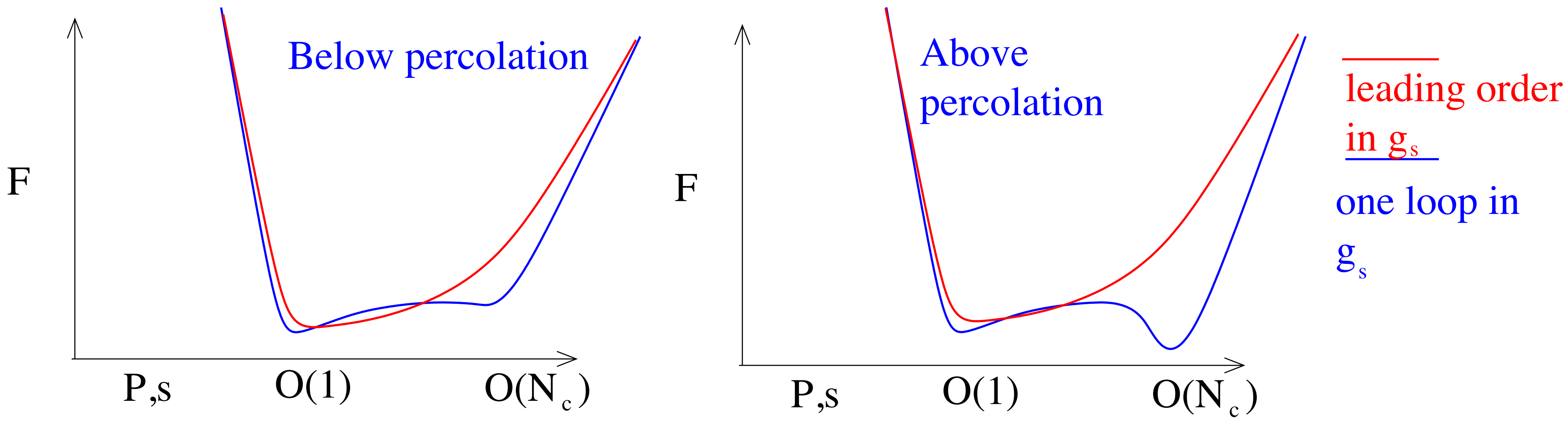
- Since phase transition happens at critical N_c , it can only be realized at subleading g_s . Asymptotic freedom limit for quark-quark interactions at large N_c also requires α' corrections!

- In string world flavor $\leftrightarrow$ D7,8 branes. So $N_c \sim N_f$ means so many overlapping branes string loops among them can not be neglected.

- This might explain why, despite compelling argument for $s \sim N_c @ \mu_q \geq \Lambda_{QCD}$, all AdS/CFT setups so far have $s \sim N_c^0$ in that regime.
  $P \sim s \sim N_c$ argument explicitly based on asymptotic freedom. Not implementable in supergravity.



I cannot see a sure road into percolation, but some qualitative insights could be obtained back at $\mu_Q \to 0$. remember the order of confinement!



Here is how to make arguments in previous slides compatible with AdS/CFT
Above leading order in g_s . Leading order misses auxiliary minimum where $s \sim N_c$ so only minimum at $s \sim N_c^0$. Van Der Waals example shows correction can be small (but not infinitesimal) for this to happen!

Confinement and black holes

In normal space, black hole decays and has a negative heat capacity $\rightarrow$ Thermodynamically unstable state!
Let's put the black hole in a reflecting box (One "physical way" of doing it: A negative cosmological constant,AdS!

Box large wrt black hole system (hole+gas) heat capacity still negative, black hole decays

Box small wrt black hole Hole and photons in box in thermal equilibrium, heat capacity positive, black hole stays

The two regimes connected by Hawking-Page phase transition (1st order).
According to Witten, confinement in d-flat or spherical space is dual to the Hawking-Page phase transition of a black hole in d+1 AdS space

The phase transition in N_c and gravity

In Gauge world , confinement critical point is understood in terms of broken symmetries (Z_N).

In Gravity world , Hawking-Page is most likely a transition because of naked singularity conjecture. You either have a black hole, with a singularity, or you don't!
(This is why I don't believe "bottom-up" models where confinement is a cross-over!)

Hence, making confinement into a cross-over is equivalent to smoothening black hole singularity

Non-commutative geometry-inspired Schwarzschild ansatz
P. Nicolini, A. Smailagic, E. Spallucci, Phys.Lett.B632:547-551,2006
The basic idea: Maintain "gravity" part classical but smear out energy momentum tensor. Black hole problem reduces to solving Einstein's equations for infinitely rigid Gaussian energy distribution

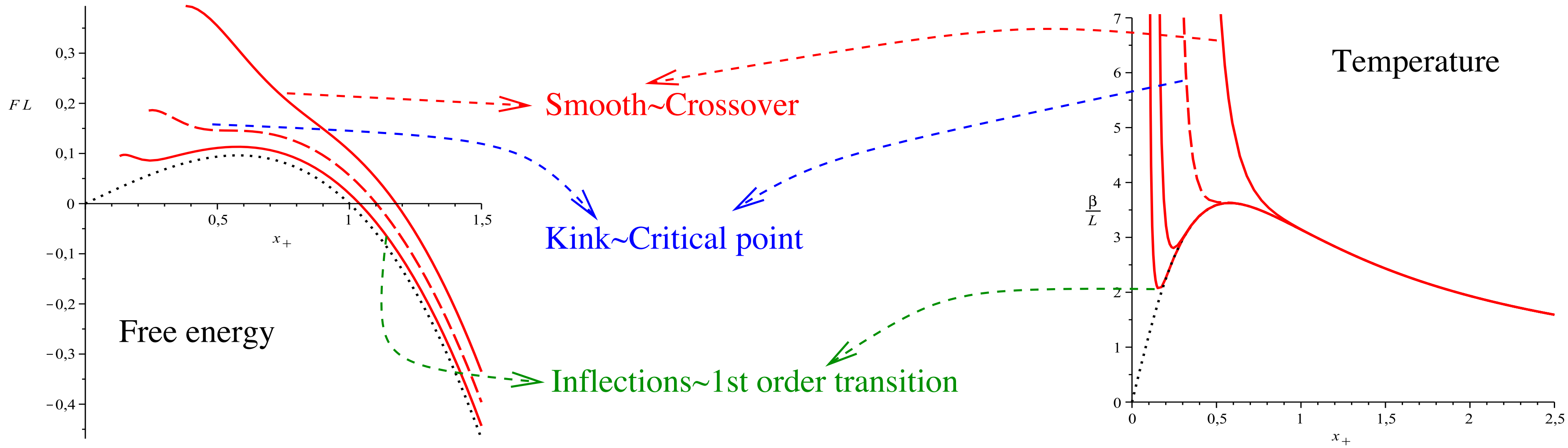
$$T_0^0 = \frac{1}{(2\pi l_p)^{3/2}} \exp\left[-\frac{x^2}{2l_p^2}\right] \underbrace{\Rightarrow}_{l_p \to 0} \delta(x)$$

Einsteins equations, spherical symmetry and $T^{\mu\nu}_{;\mu} = 0$ specify the problem uniquely.
Ansatz can be shown to be well-behaved (does not break unitarity and locality at distances long wrt l_p), Critical behaviour $\leftrightarrow$ universality! Insensitive to microscopic details of our model

Hawking entropy calculated the usual way. But...

Flat space Black hole heat capacity becomes positive after critical radius
$x_+^{planck} \sim l_p \rightarrow$ Ansatz used to study remnants

AdS space Van Der Waals-type phase diagram
If box small enough that $x_+^{planck} \sim L_{AdS}$, we reach critical point

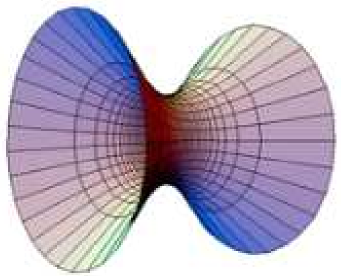


At critical $q = l_p \Lambda_{AdS}$ Hawking-Page transition becomes a <u>cross-over</u>, similar to Van der Waals gas. Critical $q^* = 0.18243 \simeq 1/6$ If $\Leftrightarrow \mathcal{O}(1)\, N_f/N_c$ surprisingly close, for 1 flavor, to $N_c = 6 = N_N^{d=2+1}$

Work in progress... a model of this type in AdS/CFT

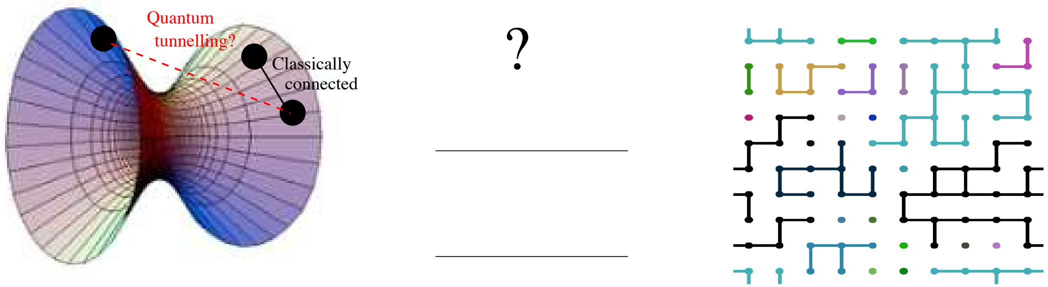
Does the Hawking page transition become a cross-over in Witten's original set-up, a Black hole on a sphere? ($AdS \times S_n$)?

Witten (hep–th/9803131v2)

Hawking–Page in  $\times$ 

$AdS_3 \neq AdS_1 \times S_n$ but is obvious that a similar critical point will happen in <u>all</u> setups with a Hawking-Page transition, although of course T_c and l_{pc} will change!

Is this the same as percolation? Not sure, but I think so!
Critical point behaviour identical to second order phase transition, and percolation is a 2nd order phase transition



Hawking-Page transition coincides with transition of a gas of black holes in AdS collapsing into a large black hole. It happens because of the interplay of black hole distance and the horizon. Non-commutativity fuzzes this over , so black holes can interact over super-horizon distances via quantum tunnelling. Very similar to percolation! Connection between Polyakov loops and percolation <u>not</u> trivial in Gauge picture, but understandable in gravity.

Can we make this ansatz testable?

The main effect of correction is to introduce a critical point of the Z_2 type (Shouldn't exist in a top-down system, and indeed doesent seem to!).

d	2	3	4	$Gravity$	$Gauge$
α	0	$0.110(1)$	0	R	$\langle L \rangle$
β	$1/8$	$0.3265(3)$	$1/2$	TdS/dT	C_V

In QCD can ,ideally, be read from the lattice, either in $T - N_f/N_c$ plane (hard) or $T - m$ plane (doable) In gravity,we can have a black hole in a Box or a brane setup. Universality can mean details of the theory secondary... critical exponents. And both sides are in Z_2 class!

If exponents match and remain critical, it would be very non-trivial: Stat Mech 101 says critical exponents set by universality class and number of dimensions. Holography is a counter-example! , as number of dimensions changes. In this setup we can measure critical exponents on both sides

Conclusions

- “naive” hadronic EFT unreliable for regime at $\mu_Q \simeq \Lambda_{QCD}$
- Large N_c expansion tells us quark degrees of freedom could appear even at confinement!
- On the other hand, not at all clear $\simeq \infty$
- Phenomenology of quarkyonic matter needed.

The best physicist in the USSR is Yakov Frenkel, who uses in his papers only quadratic equations.
I am slightly worse, I sometimes use differential equations.

L.D.Landau, quoted in
BULLETIN OF THE American Mathematical Society
Volume 43, Number 4, October 2006, Pages 563–565