Some Chapters from the Do-It-Yourself Hadron Theory Manual

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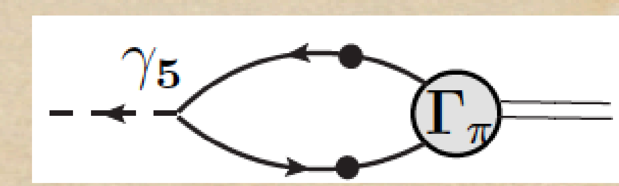
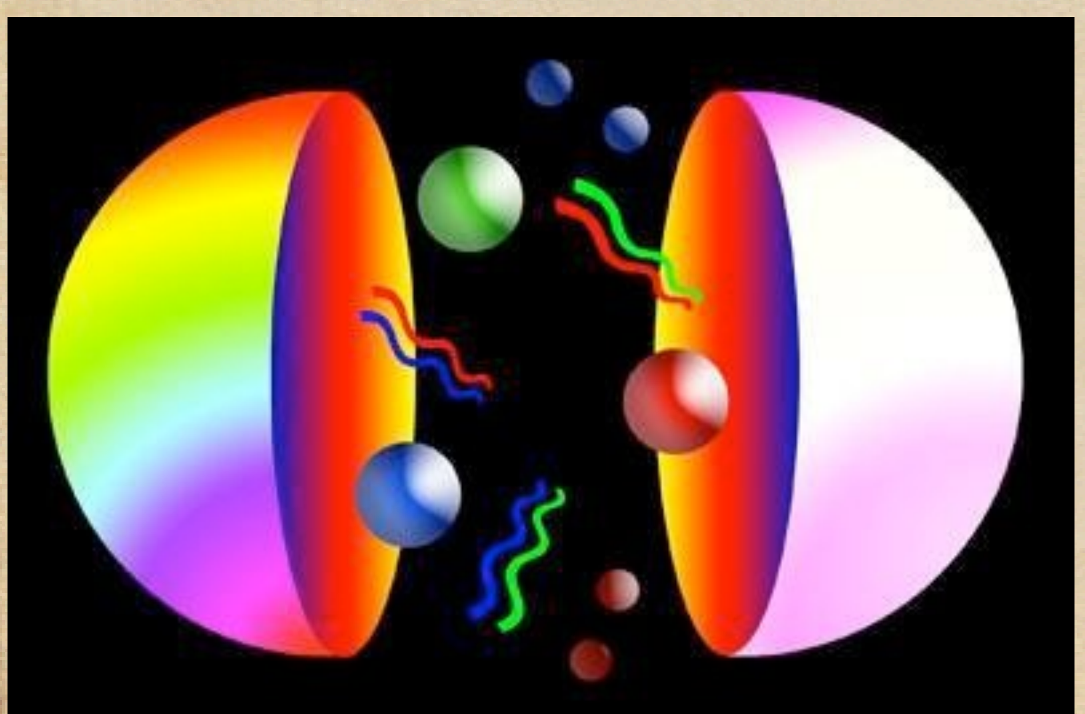
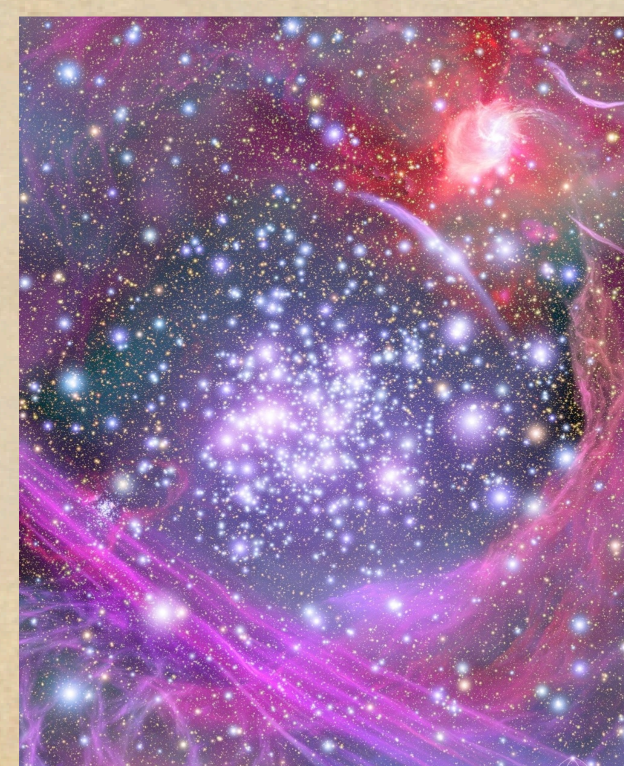
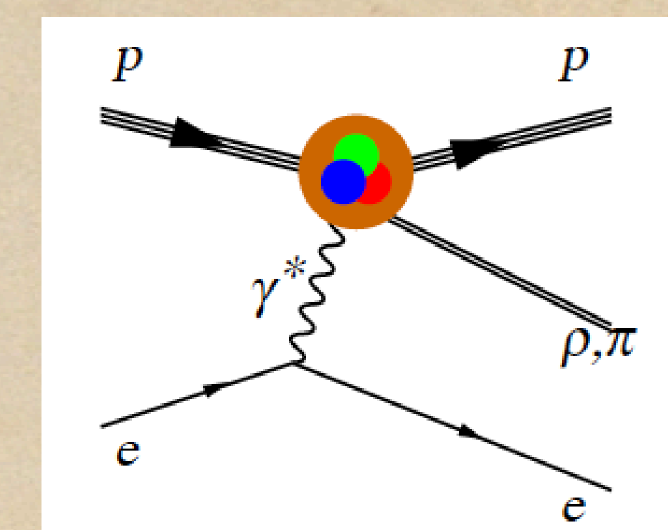








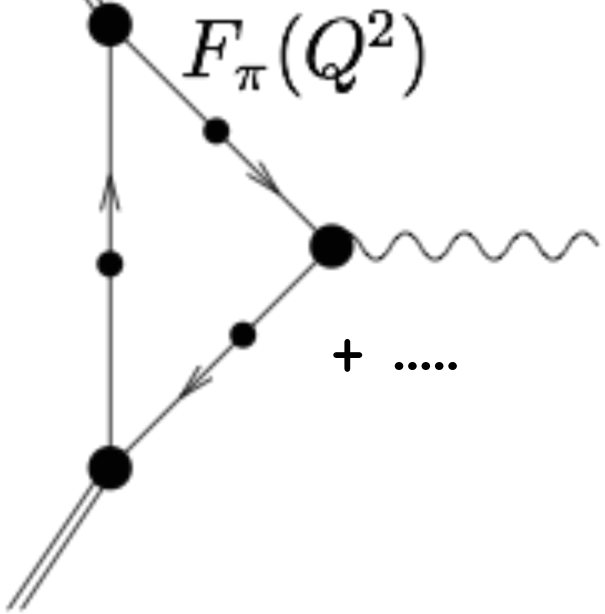
Topics:

- Some remarks on how use DSE-QCD modeling methods yourself
- Scale for soft to hard transition in $\mathbf{F}_\pi(\mathbf{Q}^2)$?
- Leads to Pion Distribution Amplitude
- Asymptotic/conformal QCD is not UV-QCD. Implications for JLAb 12 GeV for form factors typified by $\mathbf{F}_\pi(\mathbf{Q}^2)$
- Pion transition FF
- Pion Parton Distribution Fns and pion loop contributions

- Hadron DAs are amplitudes involved in hard exclusive processes, eg $\mathbf{F}_\pi(\mathbf{Q}^2)$
- Hadron PDFs are probabilities involved in hard inclusive processes, eg $\mathbf{q}_\pi(\mathbf{x})$

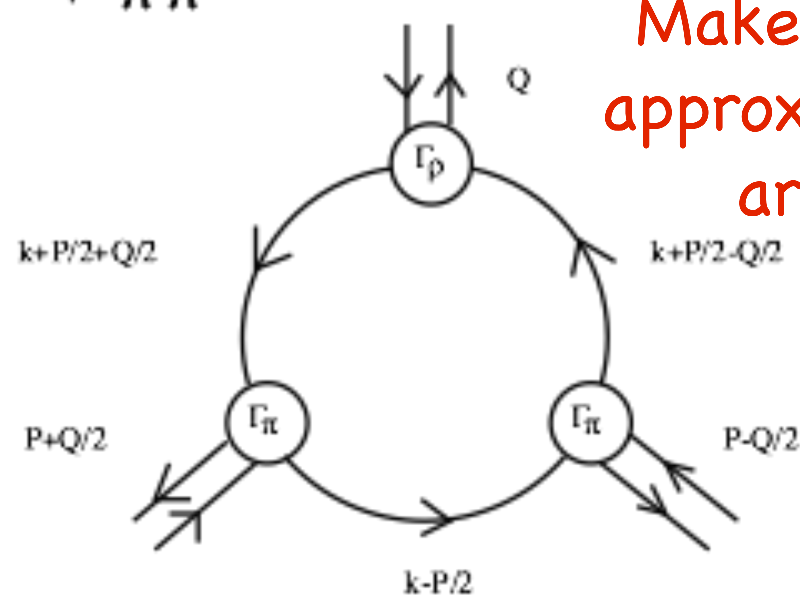
To Calculate Meson Observables

Pion form factor $F_\pi(Q^2)$



decay constant f_π

Strong decay $\rho \rightarrow \pi\pi$

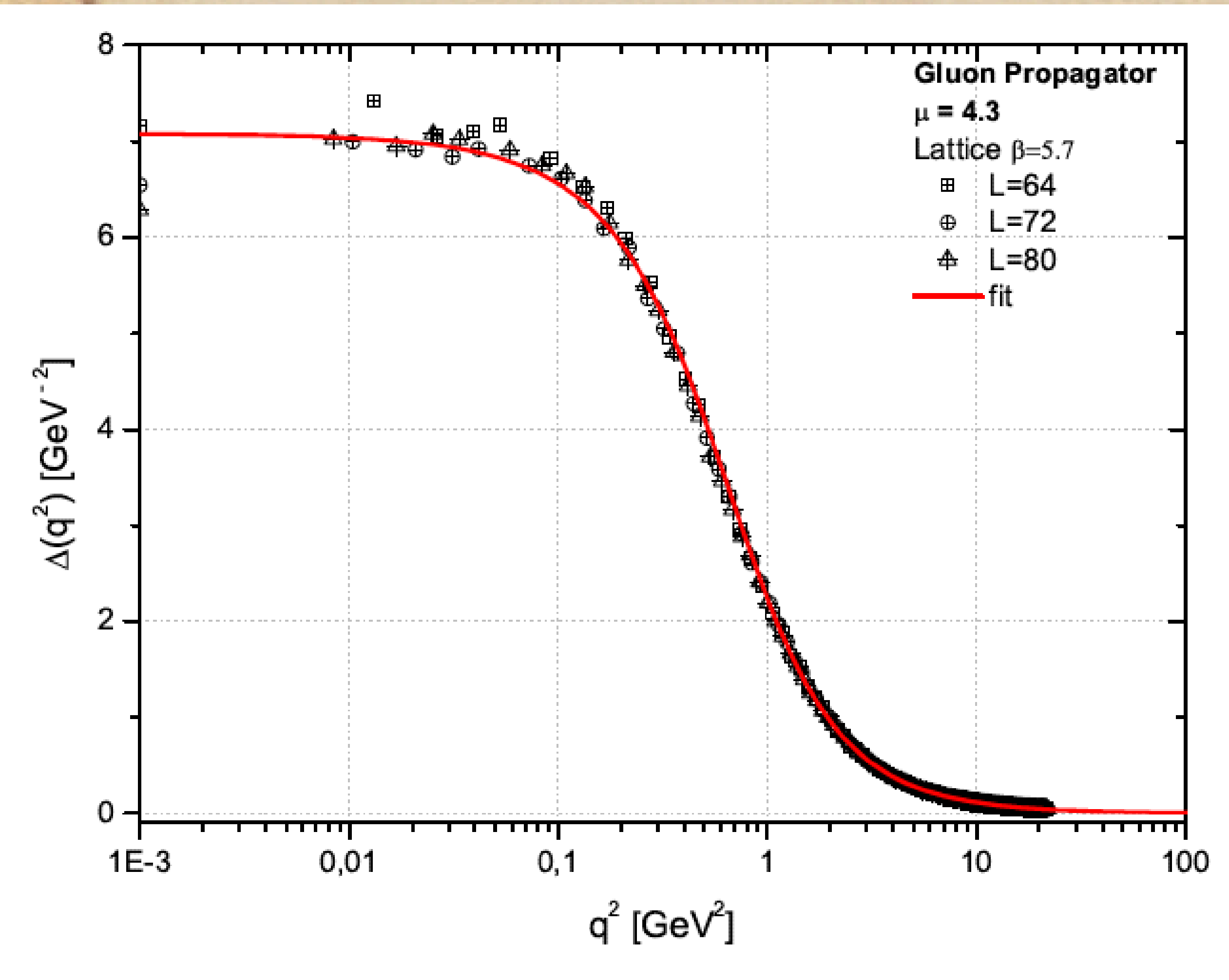


Not likely to need an exact QCD result for the dressed objects required here. Make an educated guess for a good approximation, seek forgiveness if you are really wrong---don't seek permission first.

DSE-based Modeling of QCD for Hadron Physics

- Most common: Rainbow-ladder truncation of QCD's eqns of motion.
- Naturally implements DCSB, conserved vector current, PCAC...
- RL truncation only good for vector & pseudoscalar mesons, q-qq descriptions of baryons with AV and S diquarks. More general BSE kernel now available.
- At the very least: DSE continuum QCD modeling suited for surveying the landscape quickly from large to small scales; finding out which underlying mechanisms are dominant.
- Unifying DSE treatment of light front quantities (PDFs, GPDs, DA) with other aspects of hadron structure: masses, decays, charge form factors, transition form factors.....

Modern Context for Rainbow-Ladder Kernel



Landau gauge, lattice – QCD gluon propagator, I.L.Bogolubisky *et al.*, PosLAT2007, 290 (2007)

DSE studies with lattice props Aguilar & Papavassiliou, arXiv:1010.5815

Identified enough stength for physical DCSB

$$\Rightarrow m_G(k^2) \qquad \mathbf{m_G(0) \sim 0.5 - 0.7\ GeV}$$

$$\mathbf{K^{RL}_{BSE}} = \frac{4\pi\hat{\alpha}_{\rm eff}(\mathbf{q^2})}{\mathbf{m_G^2(q^2) + q^2}} \Rightarrow \frac{\hat{\alpha}_{\rm eff}(\mathbf{0.1})}{\pi} \approx \mathbf{3 - 4}$$

A more modern RL kernel: S. Qin, L. Chang, C.D. Roberts, D.J. Wilson, PRC84, 042202 (2011).

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	($0.241^\dagger$)^3
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ s^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6\ s^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

Propagator Inflection Point = Confinement

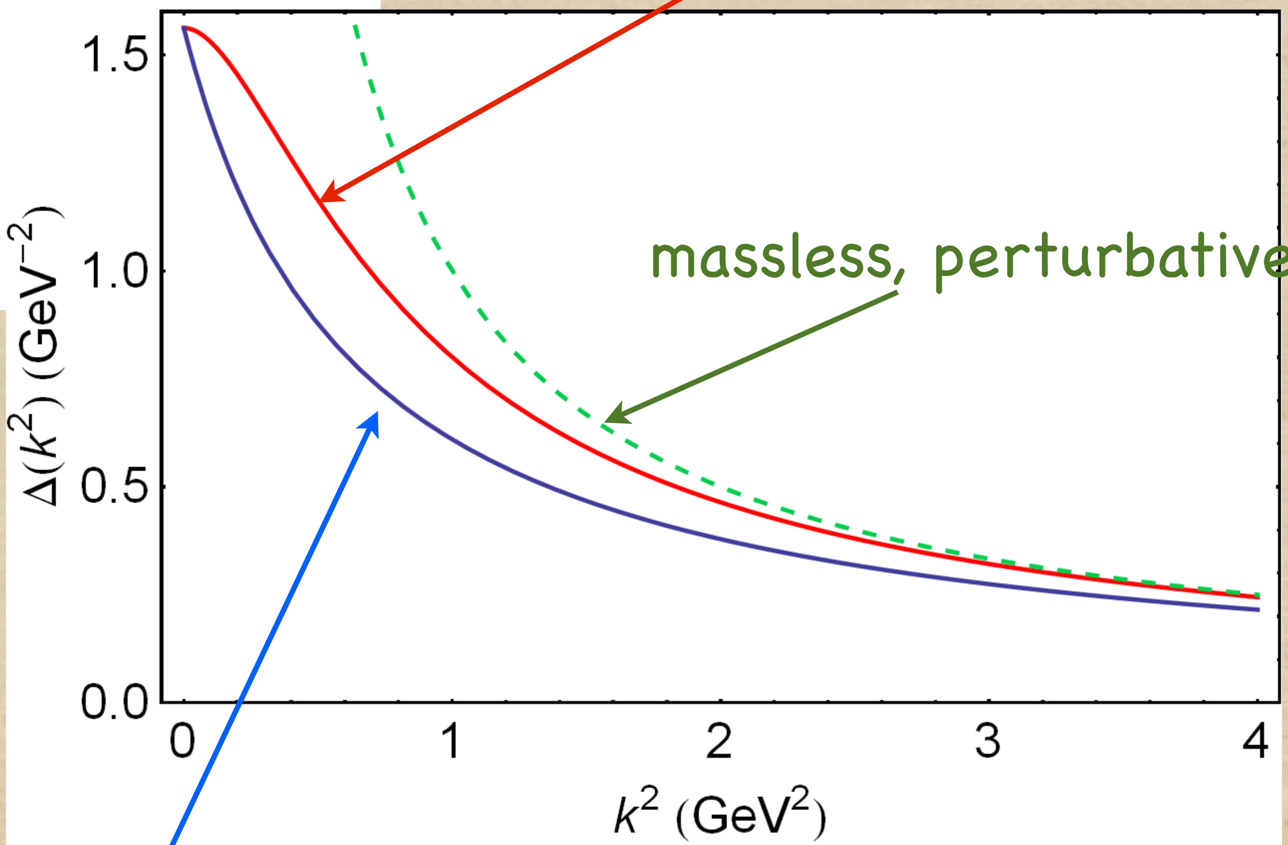
$$\Delta(\mathrm{p}^2) = \int_0^\infty d\sigma \, \frac{\rho(\sigma)}{\mathrm{p}^2 + \sigma}$$ has an inflexion pt at p_I^2

$$\Delta'(\mathrm{p}_\mathrm{I}^2) = -\int_0^\infty d\sigma \, \frac{\rho(\sigma)}{(\mathrm{p}_\mathrm{I}^2 + \sigma)^2}$$ has a minimum

$$\Delta''(\mathrm{p}_\mathrm{I}^2) = 2\int_0^\infty d\sigma \, \frac{\rho(\sigma)}{(\mathrm{p}_\mathrm{I}^2 + \sigma)^3} = 0$$

$\mathrm{p}_\mathrm{I}^2 > 0 \quad \Rightarrow \rho(\sigma)$ is not $+$ve definite

gluon mass runs. inflection pt



massless, perturbative gluon

constant mass, unconfined gluon

Propagator Inflection Point = Confinement

$$\mathrm{EG}: \quad \Delta(p^2) = \frac{1}{M_g^2(p^2) + p^2} \Rightarrow$$

$$\mathrm{A}): \quad M_g^2(p^2) = \mathrm{const} \quad \Rightarrow \Delta''(p^2) > 0$$

$$\mathrm{B}): \quad M_g^2(p^2) = \frac{M^4}{p^2 + M^2} \quad \Rightarrow -2 < \Delta''(p^2) < +\infty$$

Hence for some $p_I^2 > 0 \quad \Delta''(p_I^2) = 0$ ie confinement

Confining Representations of Propagators & Bethe-Salpeter Equations

$$\Delta_S(T) = \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} \sigma_S(p^2),$$

$$\Delta_S^{\text{free}}(T) = \frac{1}{\pi}\int_0^\infty d\varepsilon \cos(\varepsilon T)\frac{\mu}{\varepsilon^2+\mu^2} = \frac{1}{2}e^{-\mu T}$$

$$\sigma_S(p^2) = \frac{\mu}{2}\left[\frac{1}{p^2+\mu^2-i\rho^2} + \frac{1}{p^2+\mu^2+i\rho^2}\right],$$

a function with poles at $p^2+\sigma^2\exp(\pm i\theta)=0$, where

$$\sigma^4 = \mu^4+\rho^4, \quad \tan\theta = \rho^2/\mu^2,$$

then

$$\Delta_S(T) = \frac{\mu}{2\sigma} e^{-\sigma T\cos\frac{\theta}{2}} \cos\left(\sigma T \sin\frac{\theta}{2} + \frac{\theta}{2}\right).$$

PHYSICAL REVIEW C 68, 015203 (2003)

Analysis of a quenched lattice-QCD dressed-quark propagator

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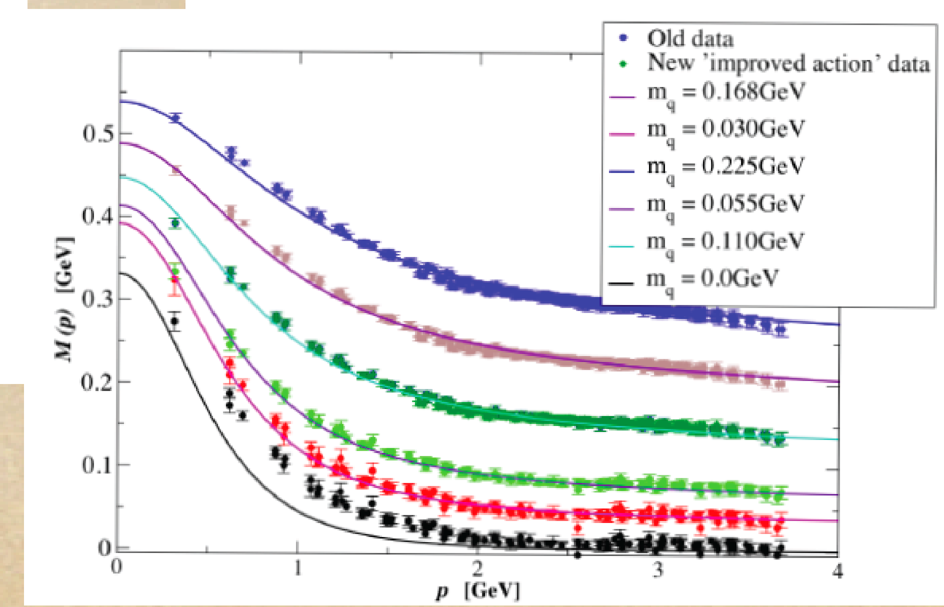
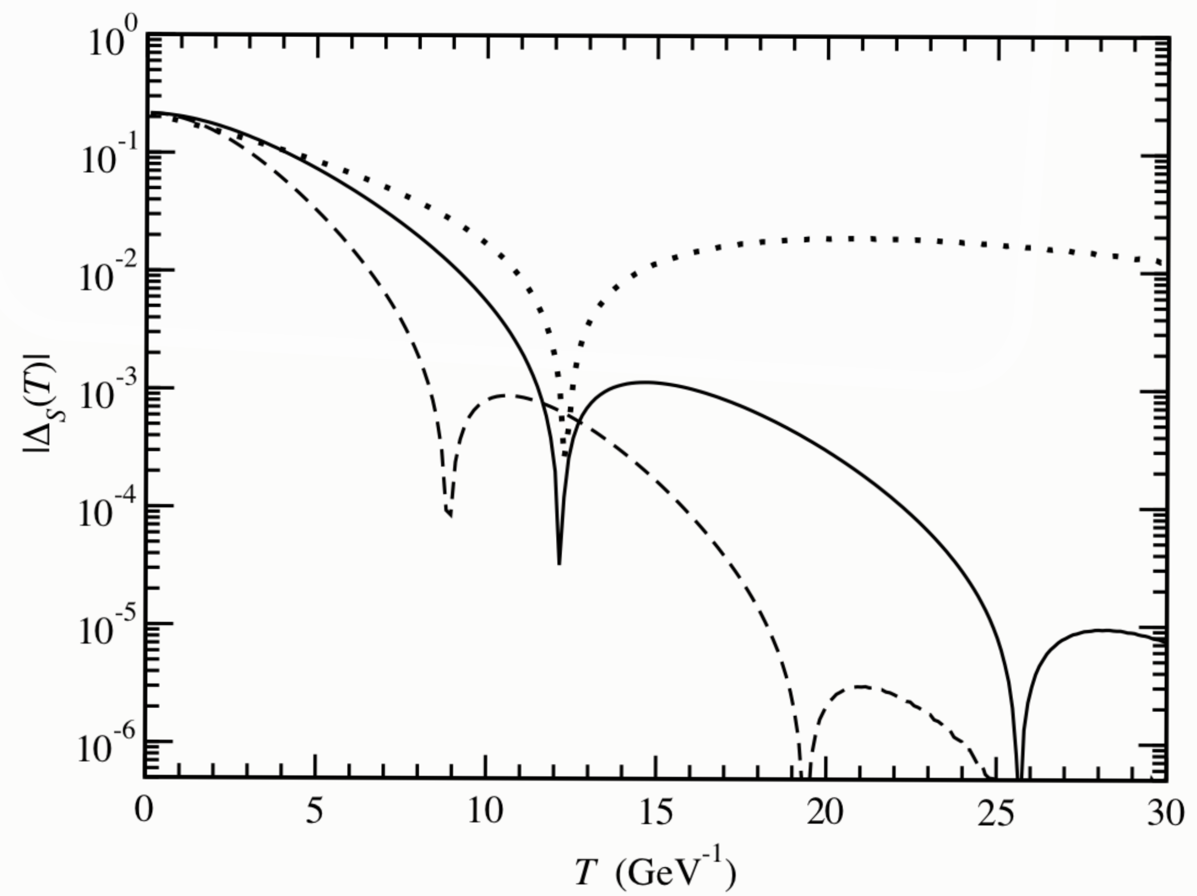


Quark Confinement—positivity violation

- Confinement/positivity analysis (Osterwalder-Schrader axiom No. 3)
- Fourier transf $\sigma_S(p_4, \vec{p} = 0)$ to Eucl time T

$$\Delta_S(T) = \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} \sigma_S(p^2),$$

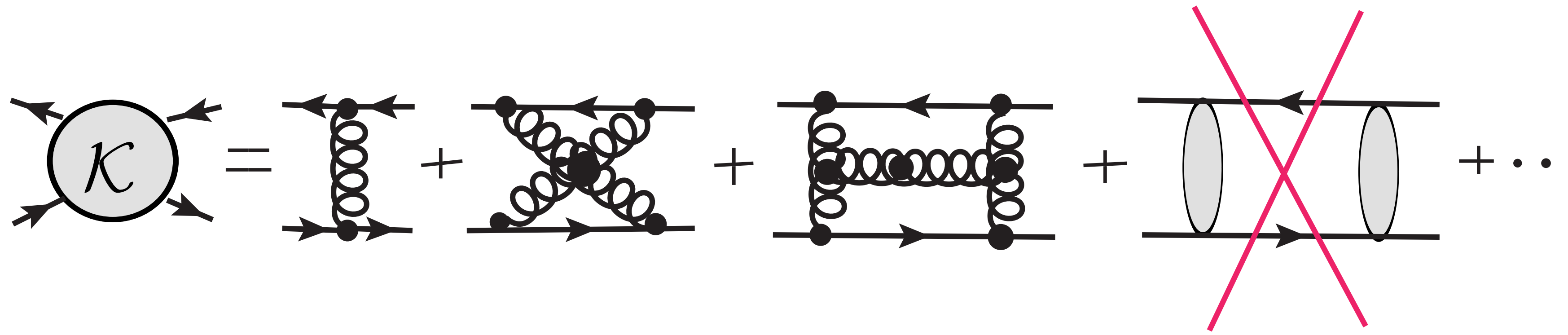
$$\sigma_S(p^2) = \frac{\mu}{2}\left[\frac{1}{p^2+\mu^2-i\rho^2} + \frac{1}{p^2+\mu^2+i\rho^2}\right],$$



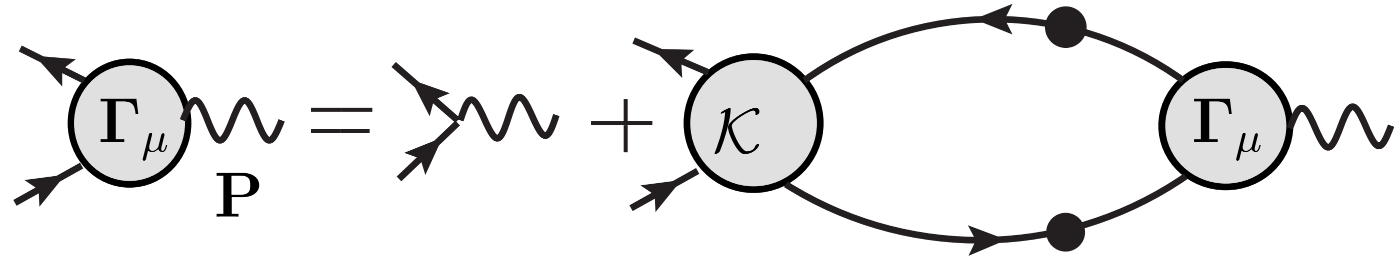
solid = lattice prop, dashed = MT DSE, dotted = cc pole eg

Bethe-Salpeter Eqns of Field Theory

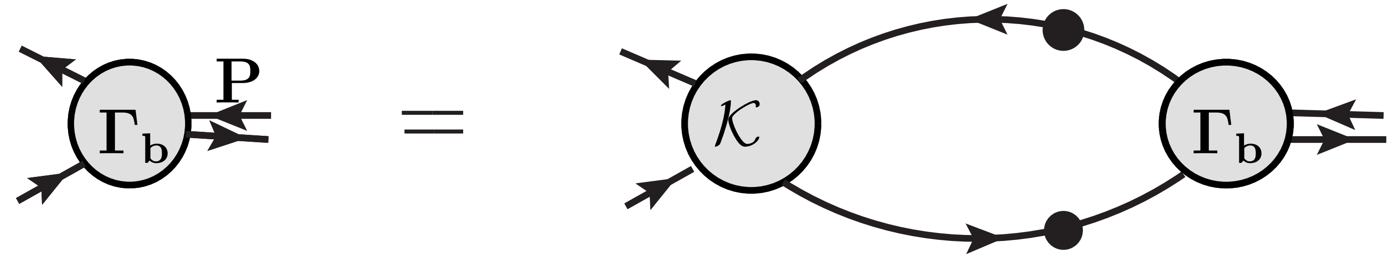
Kernel:



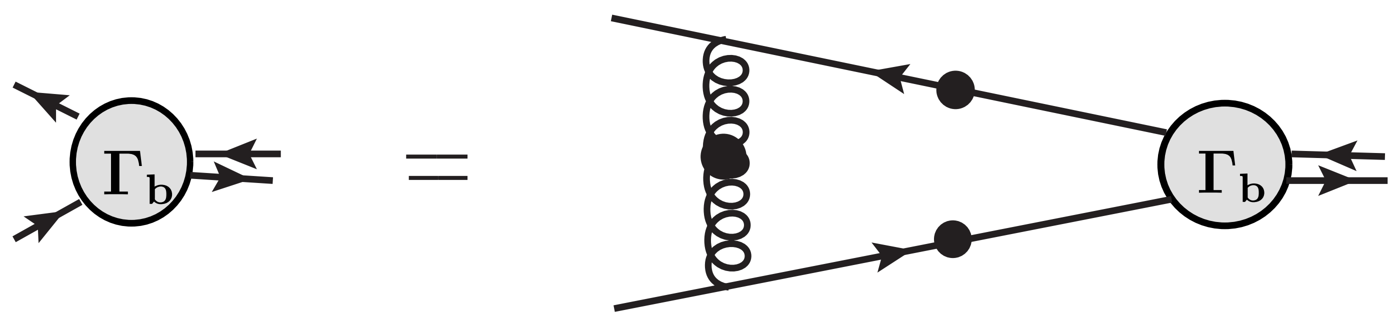
amputated $\langle 0|\, \mathbf{T}\, \mathbf{A}_\mu(\mathbf{0})\, \mathbf{q}(\mathbf{x})\, \bar{\mathbf{q}}(\mathbf{y})\, |0\rangle$



amputated $\langle 0|\, \mathbf{T}\, \mathbf{q}(\mathbf{x})\, \bar{\mathbf{q}}(\mathbf{y})\, |\mathbf{b}(\mathbf{P})\rangle$



Ladder Approx



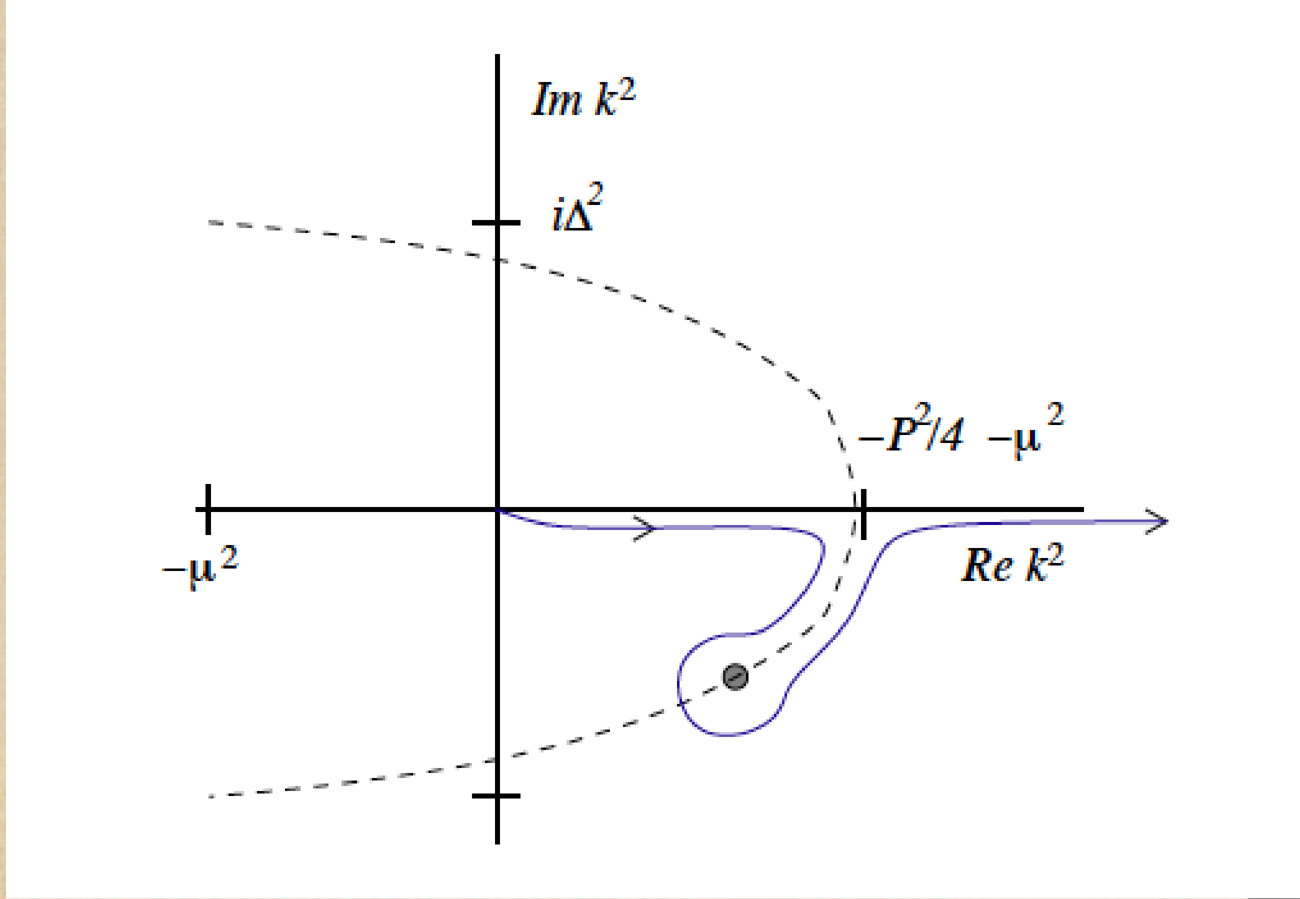
$$\text{In first eqn } \Gamma_\mu(\mathbf{q};\mathbf{P}) \;\to\; \frac{\Gamma_b(\mathbf{q},\mathbf{P})\,\mathbf{P}_\mu \text{ x const}}{\mathbf{P}^2+\mathbf{m}_b^2} \;\Rightarrow\; \text{second eqn for } \Gamma_b$$

Euclidean BSE: requires analytic continuation in external hadron P^2 after integration

$$F(Q^2, P^2 = -M^2) = \lim_{P^2 \to -M^2} \int d^4k \, I(k, P, Q)$$

$$\neq \int d^4k \lim_{P^2 \to -M^2} I(k, P, Q) \;, \quad \text{above"threshold"}$$

Complex conjugate quark mass poles in propagator give REAL meson masses—-sink = source



Improvement over Rainbow-Ladder Truncation: Dressed q-g Vertex -> Ansatz -> 4pt fn -> General BSE Kernel

$$\Gamma^{a}_{\mu}(p', p) = \frac{\lambda^{a}}{2}\{\Gamma\mu^{BC}(p', p) - \eta\, \sigma_{\mu\nu}\, q_{\nu}\, [\frac{M(p'^{2}) - M(p^{2})}{p'^{2} - p^{2}}] + \cdots\}$$

$M_{a_1} - M_{\rho} = 455\ \mathrm{MeV(expt)},\ 115\ \mathrm{MeV(DSE - RL)},\ 480\ \mathrm{MeV(DSE - DB\ Kernel)}$

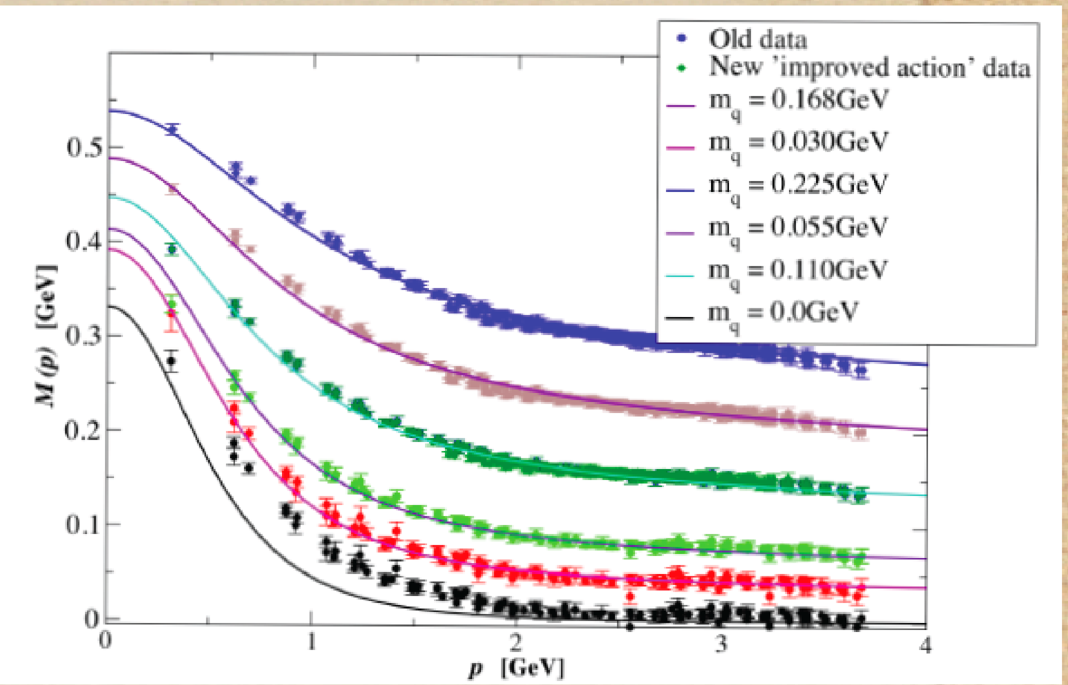
---L. Chang and C.D. Roberts, arXiv:1104.4821

K_{BSE} from this $\Gamma^{a}_{\mu}(p', p)$: L. Chang, C.D. Roberts, PRL **103** 081601 (2009)

Pion BSE Wavefn/PDA is Very Tightly Constrained

$$\mathbf{AV - WTI : m_q \to 0, P \to 0 \Rightarrow \Gamma_{\pi q\bar{q}}(k^2) = i\gamma_5 \frac{\frac{1}{4}trS_0^{-1}(k)}{f_\pi^0} + \mathcal{O}(P)}$$

- Its as good as the chiral quark propagator is
- Key to seeing: q condensate is "in-hadron"
- Constrained by pion el charge FF (hard & soft)
- ....by pion transition FF (CLEO,CELLO,BaBar,



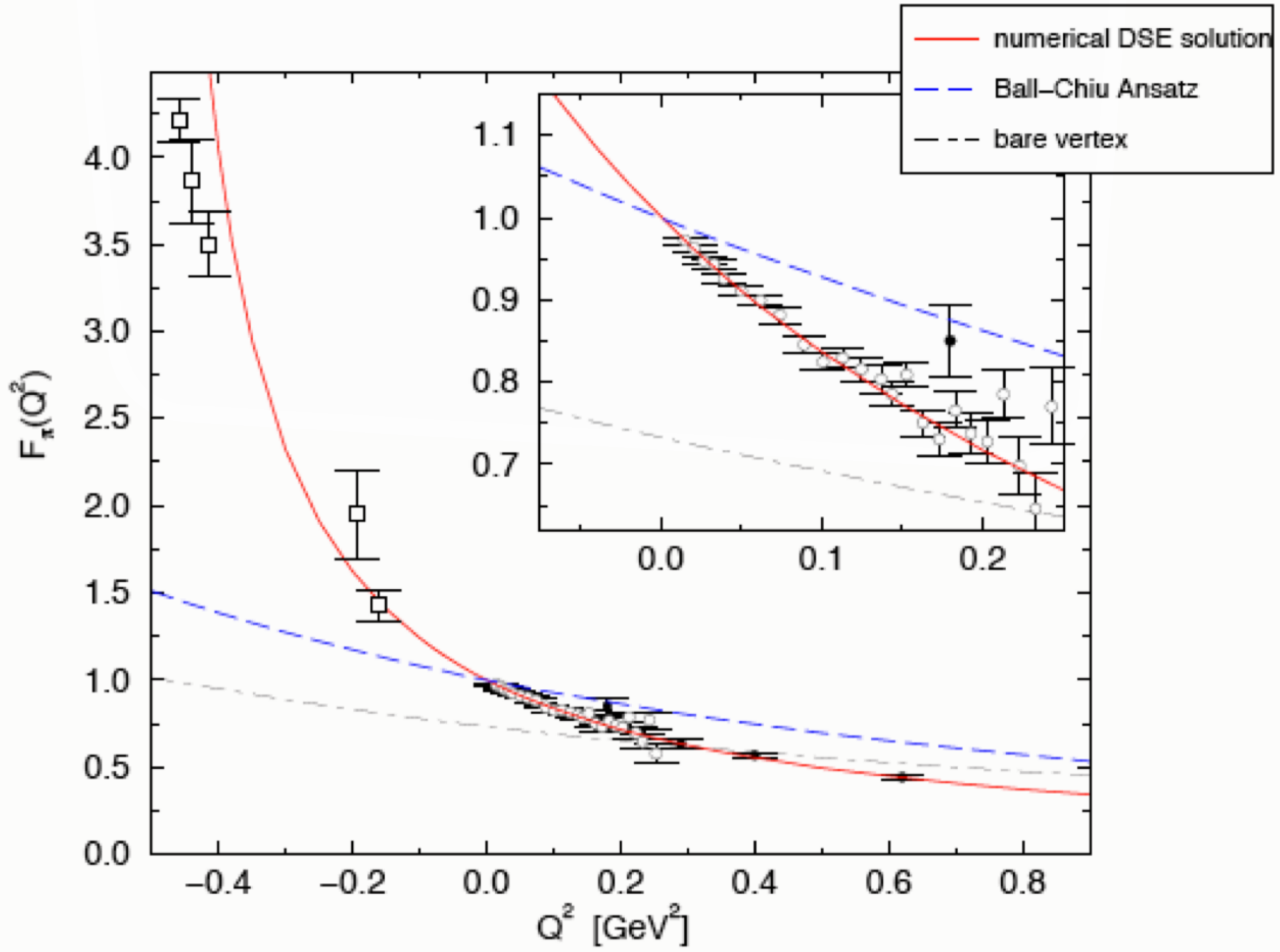
- ....by PDFs & DAs of pion in exclusive processes (hard & soft scales)
- ....by ew decay constant (IR+UV), chiral quark condensate (UV)...
- etc

Pion $F(Q^2)$: Low Q^2

(P Maris & PCT, PRC 61, 045202 (2000)

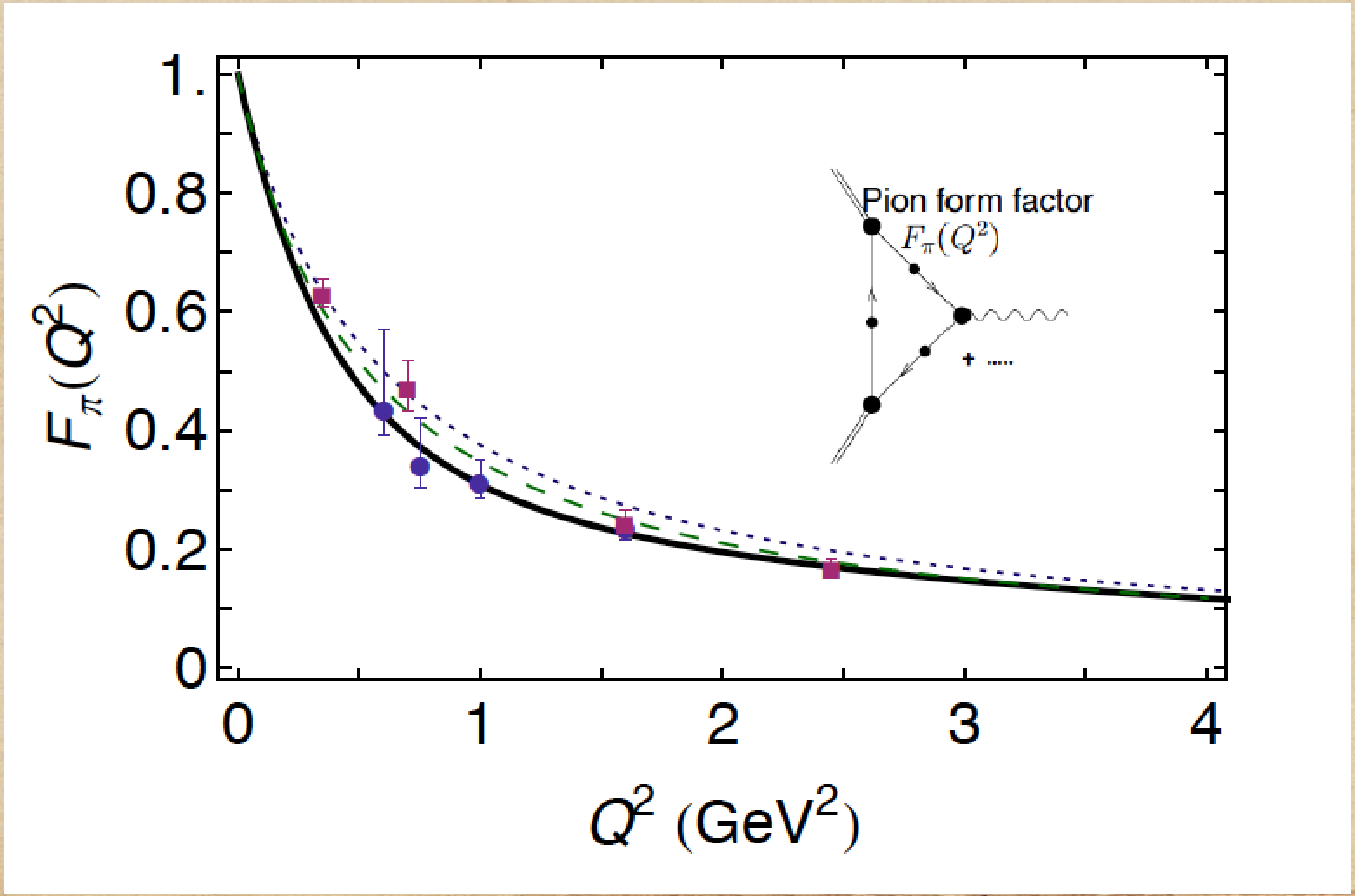
(P. Maris & PCT, PRC 62, 0555204 (2000)

$r_\pi^{\rm DSE} = 0.68$ fm $\qquad r_\pi^{\rm expt} = 0.663 \pm .006$ fm

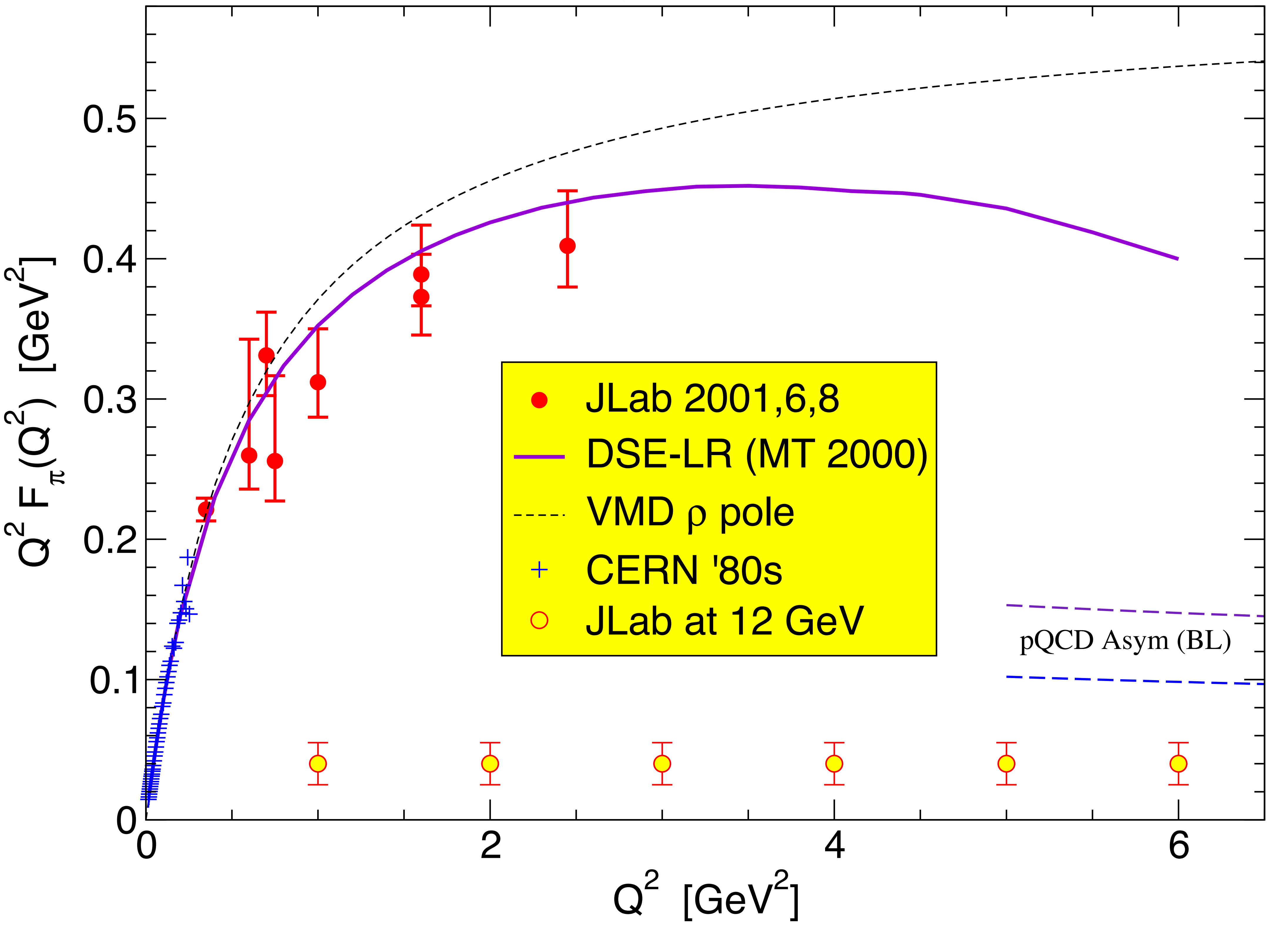


Previous DSE Limited Result 2000

P. Maris and P.C. Tandy, PRC62, 055204, (2000)

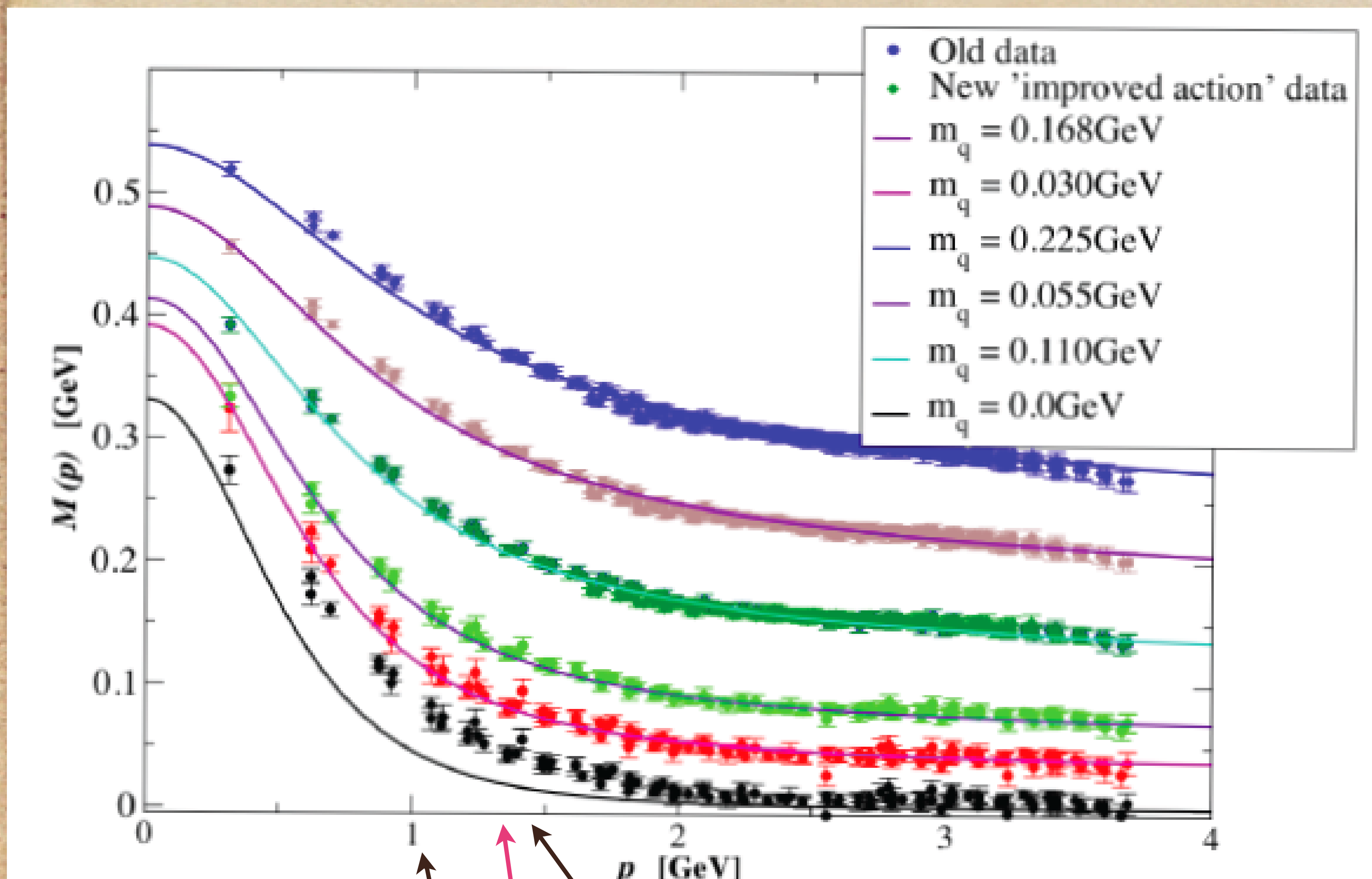


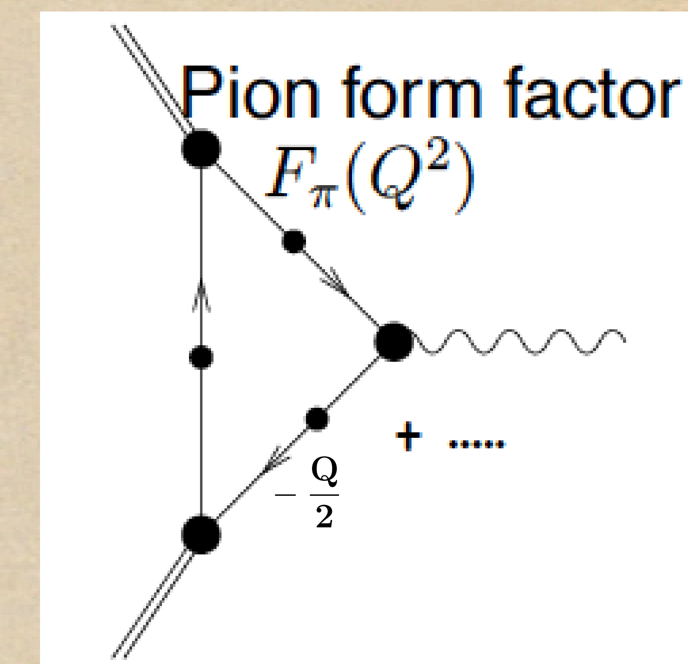
P. Maris and P.C. Tandy, PRC62, 055204, (2000)



Jab data: G. Huber et al., PRC78, 045203 (2008)

Transition from constituent to parton quark





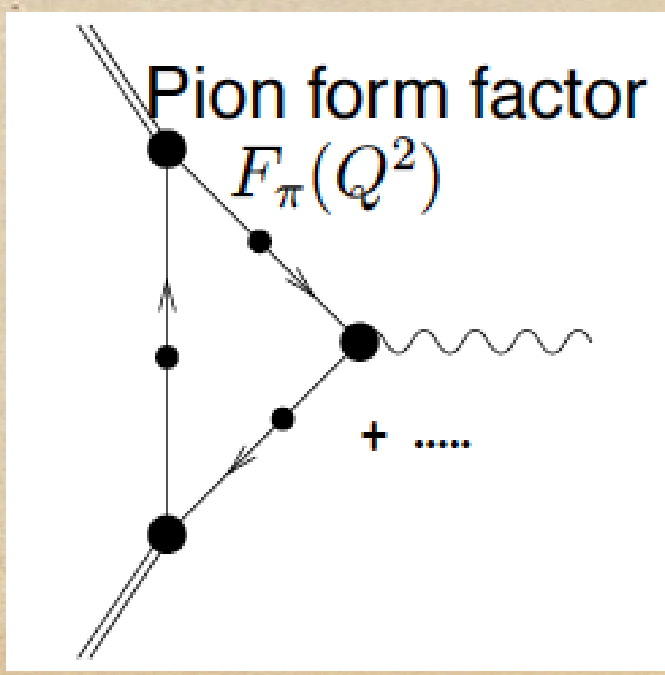
$$4\ \mathbf{GeV^2} = \mathbf{Q^2} \Leftarrow \frac{\mathbf{Q}}{\mathbf{2}} \qquad \frac{\mathbf{Q}}{\mathbf{2}} \Rightarrow \mathbf{Q^2} = 8\ \mathbf{GeV^2}$$

JLAb 12 GeV

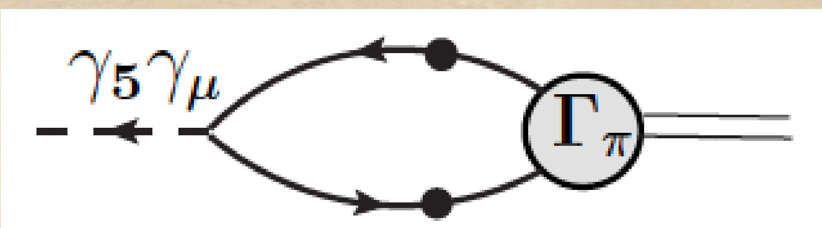
KENT STATE UNIVERSITY

Uncovering The UV Behavior of The Pion Charge Form Factor

L. Chang, I.C. Cloet, C.D.Roberts, S.M. Schmidt, P.C. Tandy, PRL, October (2013)



Pion distribution ampl



Better/Faster DSE Approach to Form Factors for Any Q^2 , and PDFs/GPDs, DAs as defined in light-cone momenta......eliminate an approximation

Must inform and learn from expt using all available tools: symmetries, analytic methods, numerical approaches, intuition, approximation, light-cone field theory, art of educated guessing; fists, knees, elbows......

Feynman Integral Method/Representation

- Need all momentum integral variables to appear in denominators that are powers of quadratic forms, with possible finite powers of momenta in numerator.
- 4-dim momentum integrals done symbolically/analytically
- Often called the Perturbation Integral Repn, via the Nakanishi Repn [PR 130, 1230 (1963)] of general BSE amplitudes and kernels.
- Represent DSE solutions for propagators as a sum of a few free propagators with complex conjugate masses [LQCD strongly favors this]
- Accommodates quark and gluon confinement via violation of spectral positivity
- Calc hadron observables while trusting in "Weinberg's Thm": QFT has no essential content other than analyticity, unitarity, physical mass thresholds, causality, cluster decomposition,.....etc.
- Some of this has been applied to solve the BSE and discuss Minkowski <--> Euclidean issues and LC issues, eg Karmanov et al, Mathiot et al,...

Fit Existing BSE Ampls, DSE solns for S(k) for Feyn Integral Method

$$\Gamma_\pi(\mathbf{q^2}, \mathbf{q}\cdot\mathbf{P}) = \gamma_5 \{\mathbf{E}_\pi(\mathbf{q^2}, \mathbf{q}\cdot\mathbf{P}) + \not{P}\, \mathbf{F}_\pi(..) + \not{q}\, \mathbf{q}\cdot\mathbf{P}\ \mathbf{G}_\pi(..) + \sigma : \mathbf{qP}\ \mathbf{H}_\pi(..)\}$$

Use Nakanishi Representation (1965) :- $\mathcal{F} = \mathbf{E},\ \mathbf{F},\ \mathbf{G},\ \text{or}\ \mathbf{H}$

$$\mathcal{F}(\mathbf{q^2}; \mathbf{q}\cdot\mathbf{P}) = \int_{-1}^{1} \mathbf{d}\alpha \int_0^\infty \mathbf{d\Lambda}\ \{\frac{\rho_{\mathrm{IR}}(\alpha; \mathbf{\Lambda})}{(\mathbf{q^2} + \alpha\mathbf{q}\cdot\mathbf{P} + \mathbf{\Lambda^2})^{\mathbf{m+n}}} + \frac{\rho_{\mathrm{UV}}(\alpha; \mathbf{\Lambda})}{(\mathbf{q^2} + \alpha\mathbf{q}\cdot\mathbf{P} + \mathbf{\Lambda^2})^{\mathbf{n}}}\}$$

$$\rho_{\mathrm{IR}}(\alpha; \mathbf{\Lambda}) \to \rho_{\mathbf{1}}(\alpha)\ \delta(\mathbf{\Lambda} - \mathbf{\Lambda}_{\mathrm{IR}_1})\ +\ \cdots \mathbf{3}$$

npQCD info is in the variables and constants that are not momenta---Wick rotation is trivial as in pert thy.

$$S(q) = \sum_{k=1}^{3} \left(\frac{z_k}{i\ \not{q} + m_k} + \frac{z_k^*}{i\ \not{q} + m_k^*} \right)$$

Works for u-, d-, s-, c-, b-quarks.
Also for lattice-QCD propagators.

N. Souchlas, PhD thesis KSU, (2009), J. Phys. G37, 115001 (2010)

Bruno, Eduardo, de Melo et al: lattice gluon prop = 1ccp repn, solve BSE for $\rho(\alpha_{\mathbf{i}}, \mathbf{\Lambda_j})$

Precise Details of Complex p^2 Behavior of S(p^2) Don't Matter

J. Phys. G: Nucl. Part. Phys. **37** (2010) 115001 N Souchlas

Table 3. Pseudoscalar and vector meson masses: experimental data and calculated masses using the gap or the 3ccp fit for the quark propagators. In the fourth column of the table we have the relative percentage differences between the gap and the experimental meson masses: $\Delta m/m^{\mathrm{exp}} = (m^{\mathrm{gap}} - m^{\mathrm{exp}})/m^{\mathrm{exp}}$, while in the last column we list the relative percentage differences between 3ccp and gap masses: $\Delta m/m^{\mathrm{exp}} = (m^{\mathrm{3ccp}} - m^{\mathrm{gap}})/m^{\mathrm{gap}}$. All masses are in GeV. Experimental data are from [30].

Meson	Exp.	Gap	$\Delta m/m^{\mathrm{exp}}$ (%)	3ccp	$\Delta m/m^{\mathrm{gap}}$ (%)
$\pi(u\bar{u})$	0.139	0.138	−0.7	0.044	−68.1
$\rho(u\bar{u})$	0.769	0.742	−3.5	0.750	+1.1
$K(us)$	0.494	0.497	+0.6	0.465	−6.4
$K^*(us)$	0.896	0.936	+4.5	1.00	+6.8
$(s\bar{s})$(fict.)	–	0.696	–	0.663	−4.7
$\phi(s\bar{s})$	1.019	1.072	+5.2	1.078	+0.6
$\eta_c(c\bar{c})$	2.980	3.035	+1.8	3.007	−0.9
$J/\psi(c\bar{c})$	3.097	3.235	+4.5	3.180	−1.7
$\eta_b(b\bar{b})$	9.300	9.585	+3.1	9.347	−2.5
$\Upsilon(b\bar{b})$	9.460	9.685	+2.4	9.440	−2.5

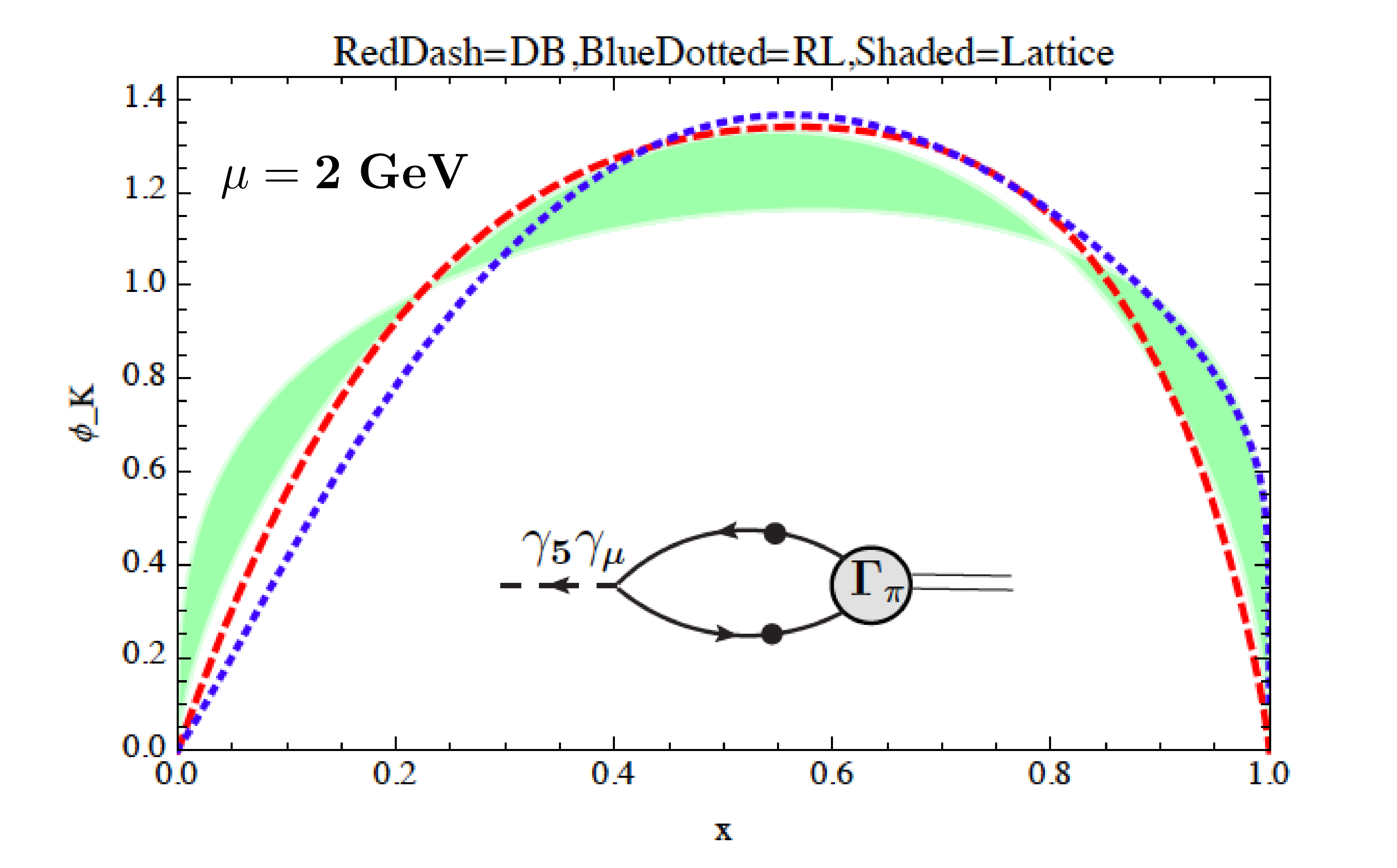
J. Phys. G: Nucl. Part. Phys. **37** (2010) 115001 N Souchlas

Table 5. Pseudoscalar and vector meson electroweak decay constants: experimental data and calculated constants using the gap or the 3ccp fit for the quark propagators. In the fourth column of the table are the relative percentage differences between the gap and the experimental values: $\Delta f/f^{\mathrm{exp}} = (f^{\mathrm{gap}} - f^{\mathrm{exp}})/f^{\mathrm{exp}}$ and in the last column we have the relative percentage differences between 3ccp and gap decay constants: $\Delta f/f^{\mathrm{exp}} = (f^{\mathrm{3ccp}} - f^{\mathrm{gap}})/f^{\mathrm{gap}}$. All decay constants are in GeV. Experimental data are from [30].

Meson	Exp.	Gap	$\Delta f/f^{\mathrm{exp}}$ (%)	3ccp	$\Delta f/f^{\mathrm{gap}}$ (%)
$\pi(u\bar{u})$	0.131	0.131	0.0	0.131	0.0
$\rho(u\bar{u})$	0.218	0.207	−5.0	0.240	+15.9
$K(us)$	0.159	0.155	−2.5	0.155	0.0
$K^*(us)$	0.225	0.241	+7.1	0.218	−9.5
$(s\bar{s})$	–	0.182	–	0.184	+1.1
$\phi(s\bar{s})$	0.228	0.259	+13.6	0.279	+7.7
$\eta_c(c\bar{c})$	0.340	0.387	+13.8	0.362	−6.5
$J/\psi(c\bar{c})$	0.416	0.415	−0.2	0.340	−18.1
$\eta_b(b\bar{b})$		0.692		0.547	−20.9
$\Upsilon(b\bar{b})$	0.700	0.682	−2.6	0.517	−24.1

Kaon Distribution Amplitude

Skewness from flavor symmetry breaking



DSE results; Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy to be publ (2014)

$\mu = 2$ GeV

Kaon Distribution Amplitude

Kaon BSA takes the form:

$$\Gamma_K(q;P) = \gamma_5 [iE_K(q;P) + \gamma\cdot P F_K(q;P) + \gamma\cdot q\, G_K(q;P) + \sigma_{\mu\nu}q_\mu P_\nu H_K(q;P)]\,, \quad (1)$$

the E_K, F_K... can be decomposed as

$$\mathcal{F}(q,P) = \mathcal{F}_{even}(q,P) + q\cdot P \mathcal{F}_{odd}(q,P) \quad (2)$$

for both $\mathcal{F}_{even}$ and $\mathcal{F}_{odd}$, we use the following form to fit the data

$$\begin{aligned}\mathcal{F}_\sigma(q,P) &= \frac{1}{N_{norm}}[\int_{-1}^{1} d\alpha\,\rho_0(\alpha)\frac{(U_0-U_1-U_2)\Lambda_\sigma^{2n_0}}{(q^2+\alpha\, q\cdot P+\Lambda_\sigma^2)^{n_0}} + \int_{-1}^{1} d\alpha\,\rho_1(\alpha)\frac{U_1\Lambda_\sigma^{2n_1}}{(q^2+\alpha\, q\cdot P+\Lambda_\sigma^2)^{n_1}} \\ &+ \int_{-1}^{1} d\alpha\,\rho_2(\alpha)\frac{U_2\Lambda_\sigma^{2n_2}}{(q^2+\alpha\, q\cdot P+\Lambda_\sigma^2)^{n_2}}] \\ \rho_i(\alpha) &= \frac{\Gamma(\nu_i+\frac{3}{2})}{\sqrt{\pi}\,\Gamma(\nu_i+1)}(1-\alpha^2)^{\nu_i} \quad (3)\end{aligned}$$

The u quark and s quark are fitted with the form

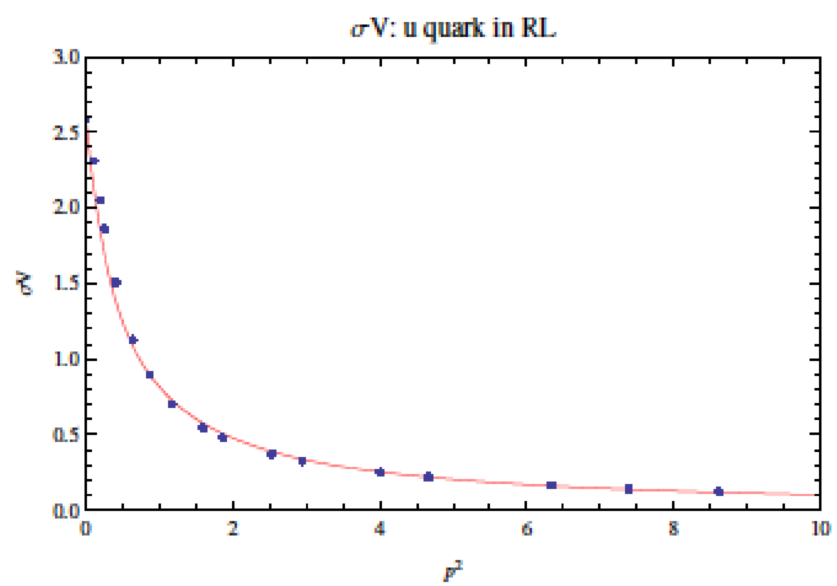
$$S(q) = \sum_{i=1}[\frac{Z_i}{i\gamma\cdot q+m_i} + \frac{Z_i^*}{i\gamma\cdot q+m_i^*}] \quad (4)$$

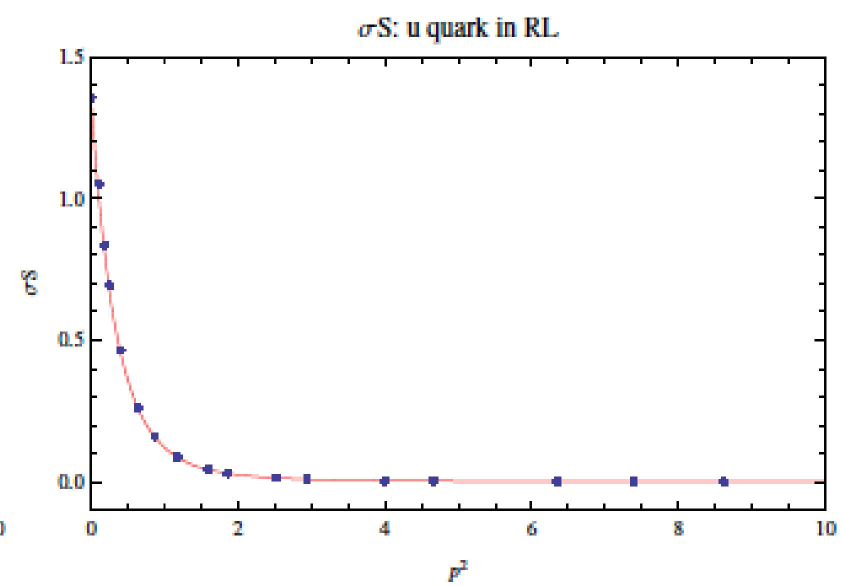
DSE results; Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy to be publ (2014)

$\mu = 2$ GeV Kaon Distribution Amplitude

	E_{even}	E_{odd}	F_{even}	F_{odd}	G_{even}	G_{odd}
ν_0	-0.7124	0.169	1.326	5.61636	1.0	-0.1
ν_1					-0.7	
ν_2	1.0	0.0	0.0	0.0	0.0	0.0
U_0	1.0	0.7	0.4176	0.2053	-8.19E-4	0.2839
U_1					0.25	
U_2	6.83E-3	3.598E-4	9.0E-4	5.6E-6	-1.0E-5	7.026E-4
n_0	5	8	5	8	10	6
n_1					12	
n_2	1	2	1	2	2	2
Λ_σ	1.795	1.97	1.49	1.61	2.1	1.463

Table 4: U1 is zero here so all the related parameters are ignored.





DSE results; Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy to be publ (2014)

KENT STATE UNIVERSITY

NSF

$\mu = \mathbf{2\ GeV}$

Kaon Distribution Amplitude

Skewness from flavor symmetry breaking

	pion	kaon
mass /GeV	0.1380	0.4944
decay constant /GeV	0.0927	0.1095

Table 1: Results Obtained

u	$\mathrm{Re}(Z_i)$	$\mathrm{Im}(Z_i)$	$\mathrm{Re}(m_i)$	$\mathrm{Im}(m_i)$
1	0.3768	0.7116	0.7112	0.2228
2	0.1381	0.0	-0.7788	0.7548

Table 2: u quark in RL case, fitting plot can be seen in Fig.1 and Fig.2

s	$\mathrm{Re}(Z_i)$	$\mathrm{Im}(Z_i)$	$\mathrm{Re}(m_i)$	$\mathrm{Im}(m_i)$
1	0.4467	0.15	0.7215	0.2922
2	0.1564	0.00514	-1.454	0.7396

Table 3: s quark in RL case, fitting plot can be seen in Fig.3 and Fig.4

DSE results; Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy to be publ (2014)

The Pion Charge Form Factor: Transition from npQCD to pQCD

$$\mathbf{F_\pi(Q^2 = uv) = \int_0^1 dx \int_0^1 dy\, \phi_\pi^\star(x;Q)\, [T_H(x,y;Q^2)]\, \phi_\pi(y;Q)}$$

---LFQCD, Brodsky, LePage PRD (1980)

$$\mathbf{Q^2 >> \Lambda^2_{QCD} :\ Q^2 F_\pi(Q^2) \to 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, \omega_\phi^2(Q^2) \ +\ \mathcal{O}(1/Q^2)}$$

$$\mathbf{\omega_\phi(Q^2) = \frac{1}{3}\int_0^1 dx\, \frac{\phi_\pi(x;Q)}{x} \quad \to 1\,, \quad Q^2 \to \infty}$$

$$\mathbf{16\,\pi\, f_\pi^2\, \alpha_s(Q^2) \ \approx 0.1 \text{ at } Q^2 \sim 3-4 \text{ GeV}^2\ . \quad (\text{JLab, theory} \Rightarrow \ \sim 0.45)}$$

$$\mathbf{\text{But, recent DSE theory} \Rightarrow \phi_\pi(x;\mu = 2 \text{ GeV}) \Rightarrow \omega_\phi^2 = 3.3}$$

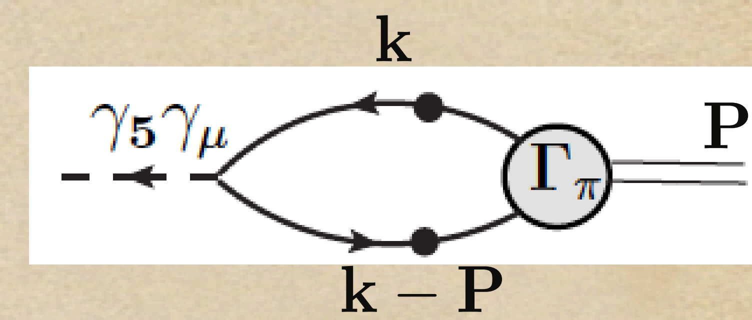
PRL **111,** 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I. C. Cloët,[2] C. D. Roberts,[2] S. M. Schmidt,[3] and P. C. Tandy[4]

Pion Distribution Amplitude

$$f_\pi\, \phi_\pi(x) = \int \frac{d\lambda}{2\pi}\, e^{-ixP.n\lambda}\, \langle 0|\, \bar{q}(0)\, \gamma_5 \not{n}\, q(\lambda n)\, |\pi(P)\rangle$$



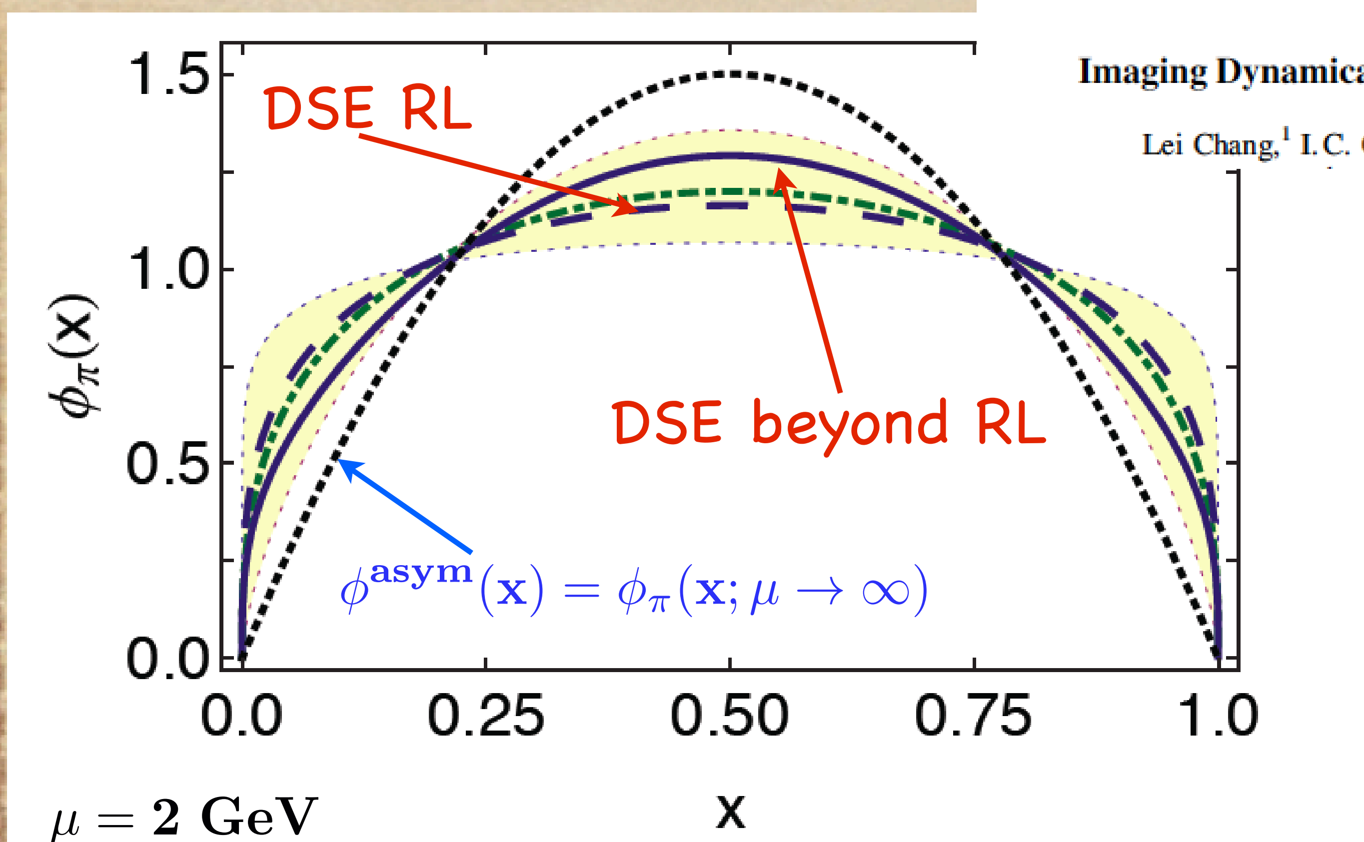
$$f_\pi \langle x^m\rangle_\phi = \frac{Z_2 N_c}{P\cdot n}\, tr \int_k \left(\frac{k\cdot n}{P\cdot n}\right)^m \gamma_5 \not{n} \left[S(k)\, \Gamma_\pi(k-\frac{P}{2};P)\, S(k-P)\right]$$

BS wavefn $\chi_\pi(k-\frac{P}{2})$

PRL 110, 132001 (2013) PHYSICAL REVIEW LETTERS week ending 29 MARCH 2013

Imaging Dynamical Chiral-Symmetry Breaking: Pion Wave Function on the Light Front

Lei Chang,[1] I. C. Cloët,[2,3] J. J. Cobos-Martinez,[4,5] C. D. Roberts,[3,6] S. M. Schmidt,[7] and P. C. Tandy[4]



Broadening of PDA is an expression of DCSB ---long sought after in LF QFT

Evolution to higher scales is EXTREMELY SLOW
Not much change up to LHC energy

Pion Distribution Amplitude

ERBL (~1980): $\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\,\{\,\mathbf{1 + \Sigma_{n=2,4\cdots}\;\; a_n}(\mu)\,\mathbf{C_n^{3/2}(2x-1)}\,\}$

Efficient representation
at low scales, eg DSE result:

$$\phi_\pi(\mathbf{x};\mu) = \mathbf{N}_\alpha\;\mathbf{x}^\alpha(\mathbf{1-x})^\alpha\,\{\,\mathbf{1 + \Sigma_{n=2}^\infty\;\; \tilde{a}_n}(\mu)\,\mathbf{C_n^{\alpha+1/2}(2x-1)}\,\}$$

Evolution to higher scales is
EXTREMELY SLOW
Not much change up to LHC energy

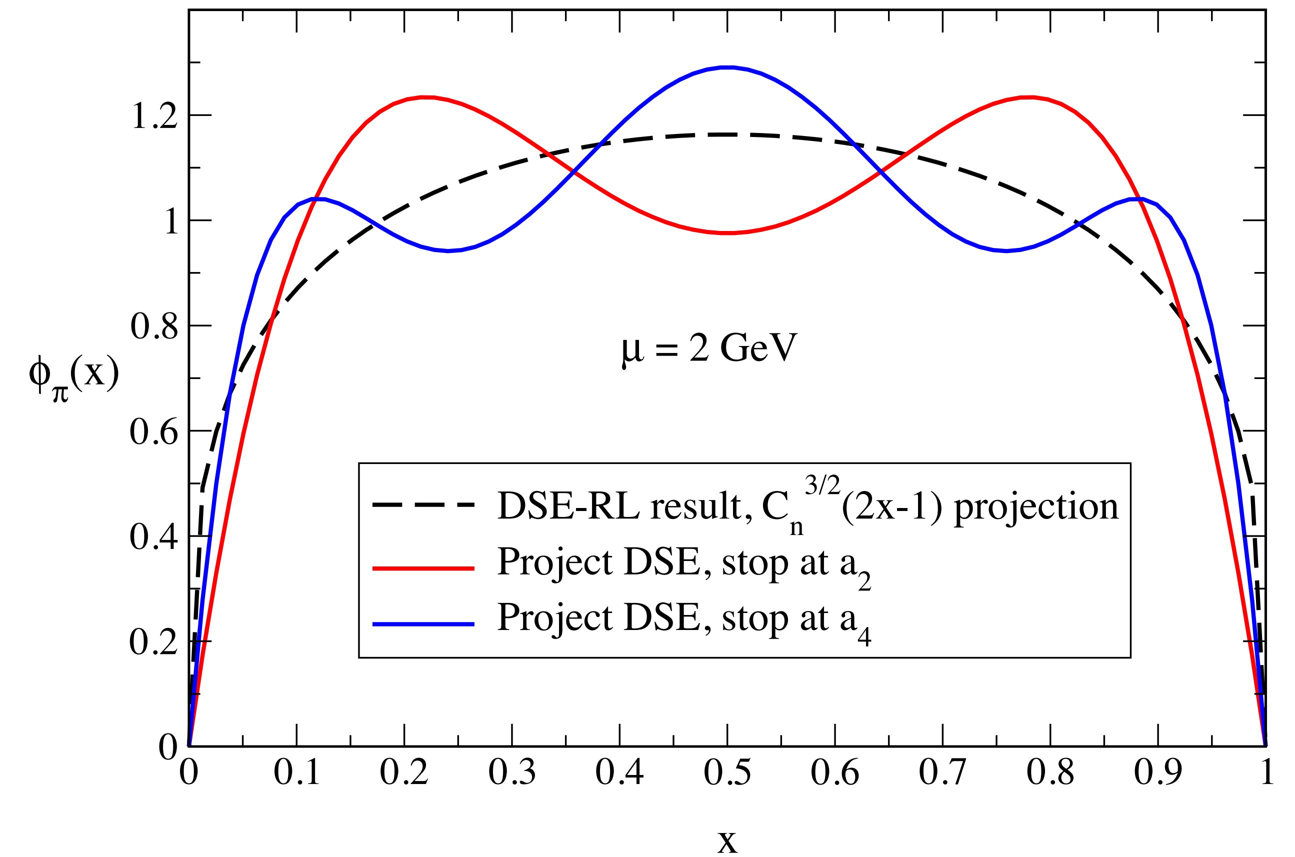
Low Truncation of ERBL Projection of DSE PDA

$$\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\ \{\ \mathbf{1} + \mathbf{\Sigma_{n=2,4\cdots}}\ \ \mathbf{a_n}(\mu)\ \mathbf{C_n^{3/2}}(\mathbf{2x-1})\ \}$$

DSE soln

{{0, 1.}, {2, 0.233104}, {4,0.112135},
{6, 0.0683202}, {8, 0.0469145},
{10, 0.0346469}, {12, 0.0268732},
{14, 0.0215933}, {16, 0.0178199},
{18, 0.0150159}, {20, 0.0128672},
{22, 0.0111788}, {24, 0.00982438},
{26, 0.00871886}, {28, 0.00780296},
{30, 0.00703438}, {32, 0.0063823},
{34, 0.00582279}, {36, 0.00534272},
{38, 0.00493277}, {40, 0.00447911}}

+.....



A double-humped PDA is ruled out by V. Braun, I. Filyanov, Z. Phys. C44, 157 (1989) →

$$\phi_\pi^{\mathbf{QCDSR}}(\mathbf{x}=\mathbf{1/2};\mu=\mathbf{2}) = \mathbf{1.2 \pm 0.3}$$

The Pion Charge Form Factor: Transition from npQCD to pQCD

$$\mathbf{F}_\pi(\mathbf{Q^2} = \mathbf{uv}) = \int_0^1 \mathbf{dx} \int_0^1 \mathbf{dy}\; \phi_\pi^\star(\mathbf{x};\mathbf{Q})\, [\mathbf{T}_\mathrm{H}(\mathbf{x},\mathbf{y};\mathbf{Q^2})]\, \phi_\pi(\mathbf{y};\mathbf{Q})$$

---LFQCD, Brodsky, LePage PRD (1980)

$$\mathbf{Q^2} >> \Lambda^2_{\mathrm{QCD}} :\; \mathbf{Q^2 F}_\pi(\mathbf{Q^2}) \to \mathbf{16}\,\pi\, \mathbf{f}_\pi^2\, \alpha_\mathbf{s}(\mathbf{Q^2})\, \omega_\phi^2(\mathbf{Q^2}) + \mathcal{O}(\mathbf{1}/\mathbf{Q^2})$$

$$\omega_\phi(\mathbf{Q^2}) = \frac{\mathbf{1}}{\mathbf{3}} \int_0^1 \mathbf{dx}\, \frac{\phi_\pi(\mathbf{x};\mathbf{Q})}{\mathbf{x}} \quad \to \mathbf{1}\,, \quad \mathbf{Q^2} \to \infty$$

$\mathbf{16}\,\pi\,\mathbf{f}_\pi^2\,\alpha_\mathbf{s}(\mathbf{Q^2}) \approx \mathbf{0.1}$ **at** $\mathbf{Q^2} \sim \mathbf{3 - 4}$ $\mathbf{GeV^2}$. (**JLab, theory** $\Rightarrow\; \sim \mathbf{0.45}$)

But, recent DSE theory $\Rightarrow \phi_\pi(\mathbf{x};\mu = \mathbf{2}\ \mathbf{GeV}) \Rightarrow \omega_\phi^2 = \mathbf{3.3}$

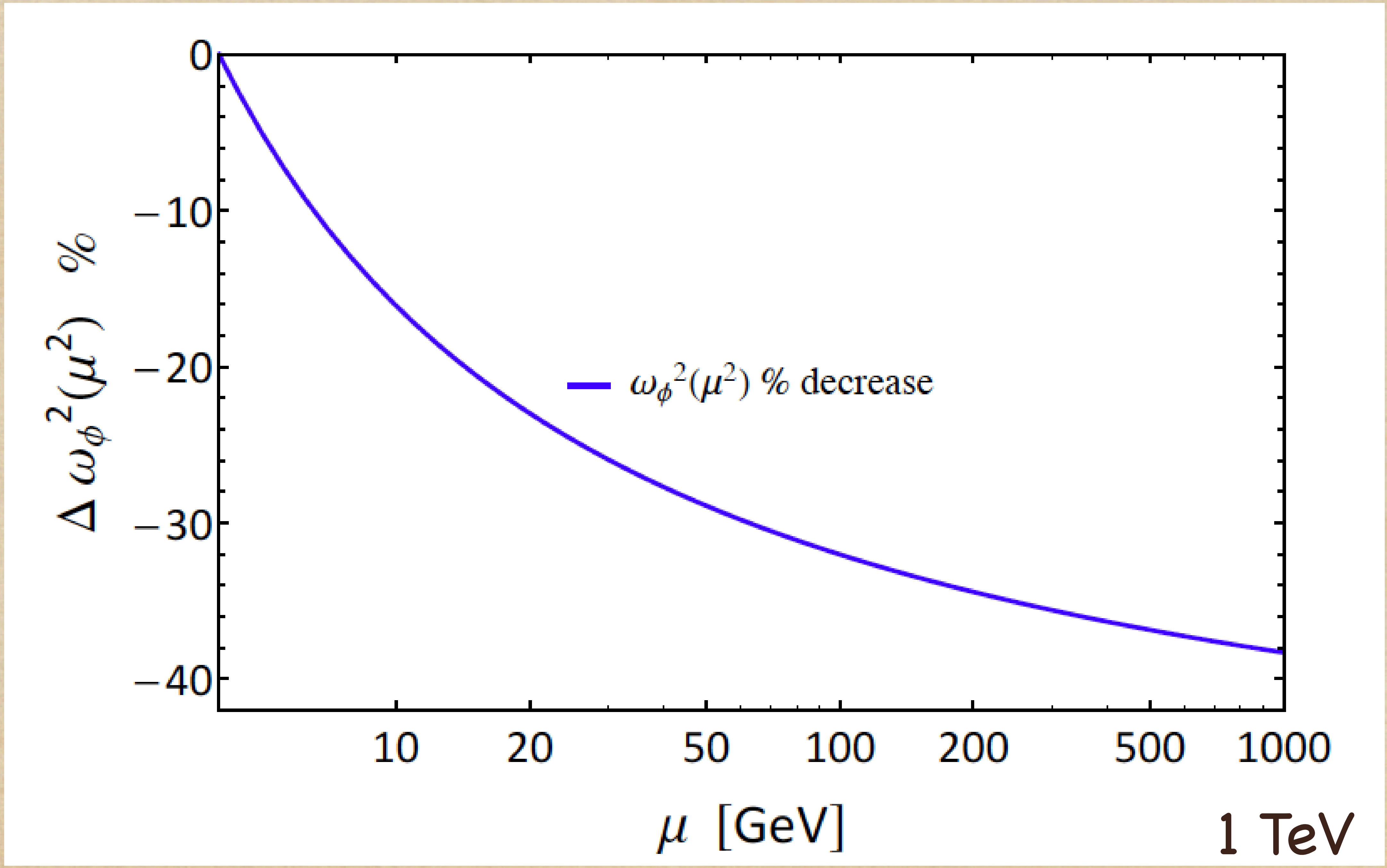
PRL **111,** 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I.C. Cloët,[2] C.D. Roberts,[2] S. M. Schmidt,[3] and P. C. Tandy[4]

UV-QCD is not Asymptotic QCD

$$\mathbf{Q^2} >> \mathbf{\Lambda^2_{\rm QCD}} : \ \mathbf{Q^2 F_\pi(Q^2)} \rightarrow \mathbf{16}\,\pi\,\mathbf{f_\pi^2}\,\alpha_\mathbf{s}(\mathbf{Q^2})\ \omega^\mathbf{2}_\phi(\mathbf{Q^2})\ +\ \mathcal{O}(\mathbf{1/Q^2})$$



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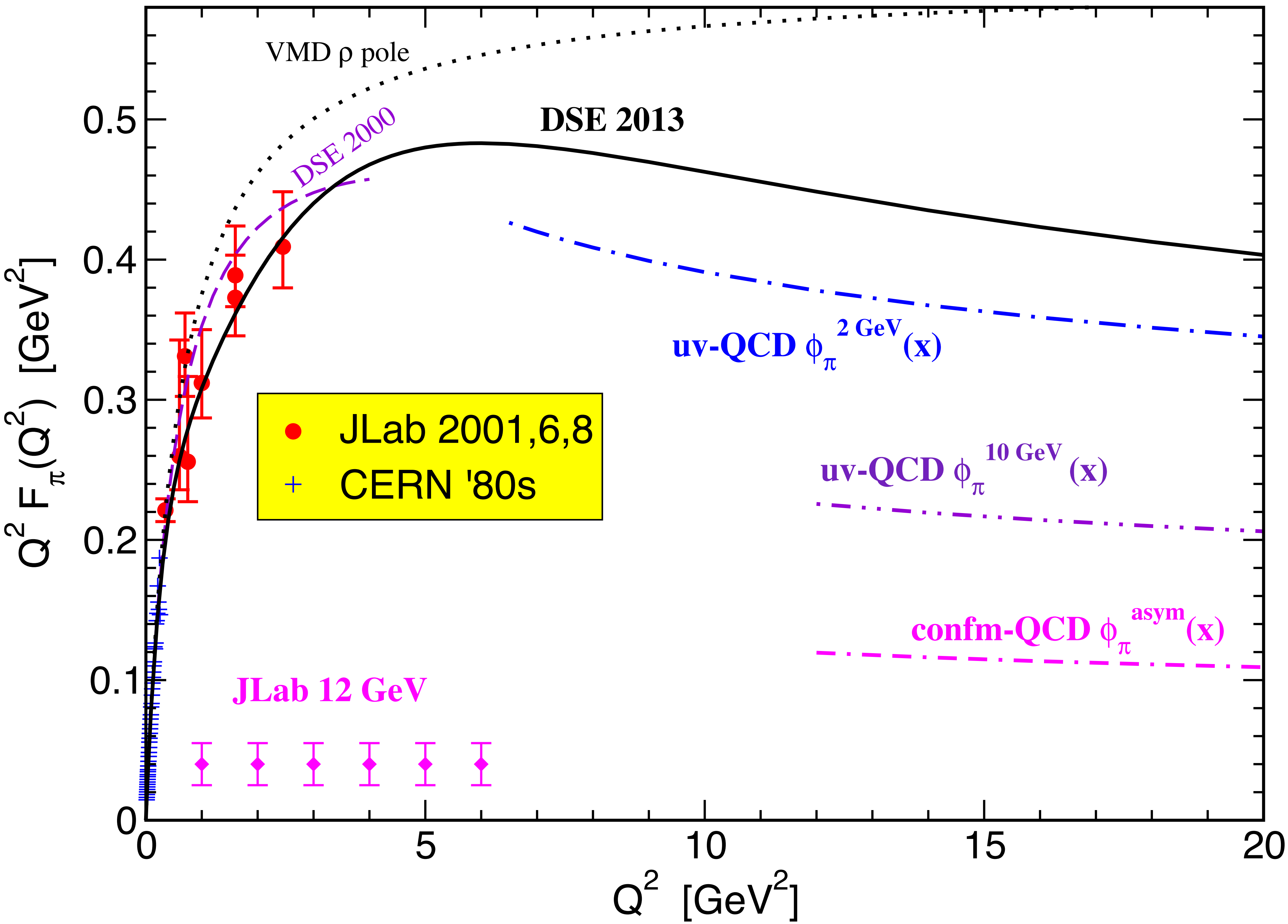
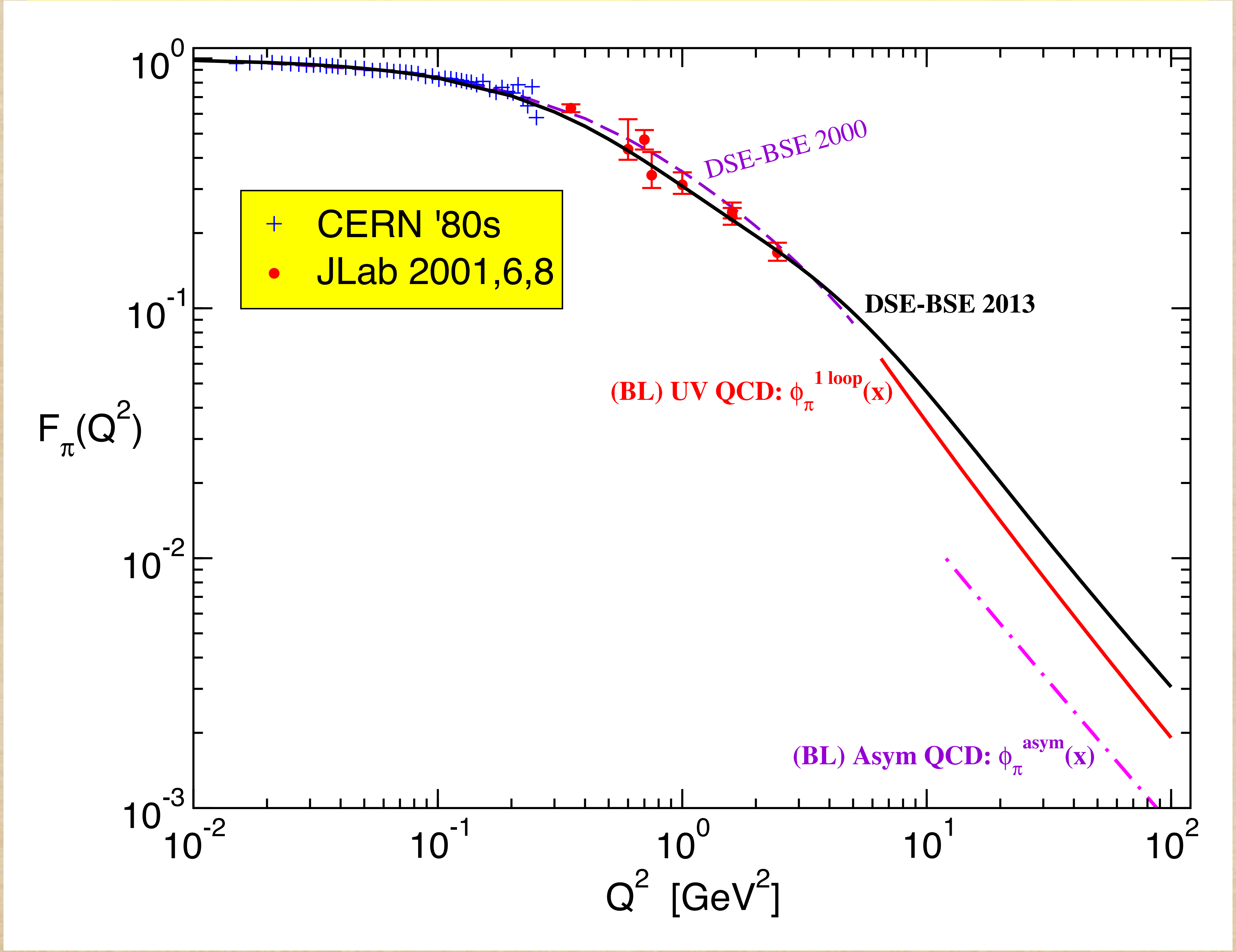
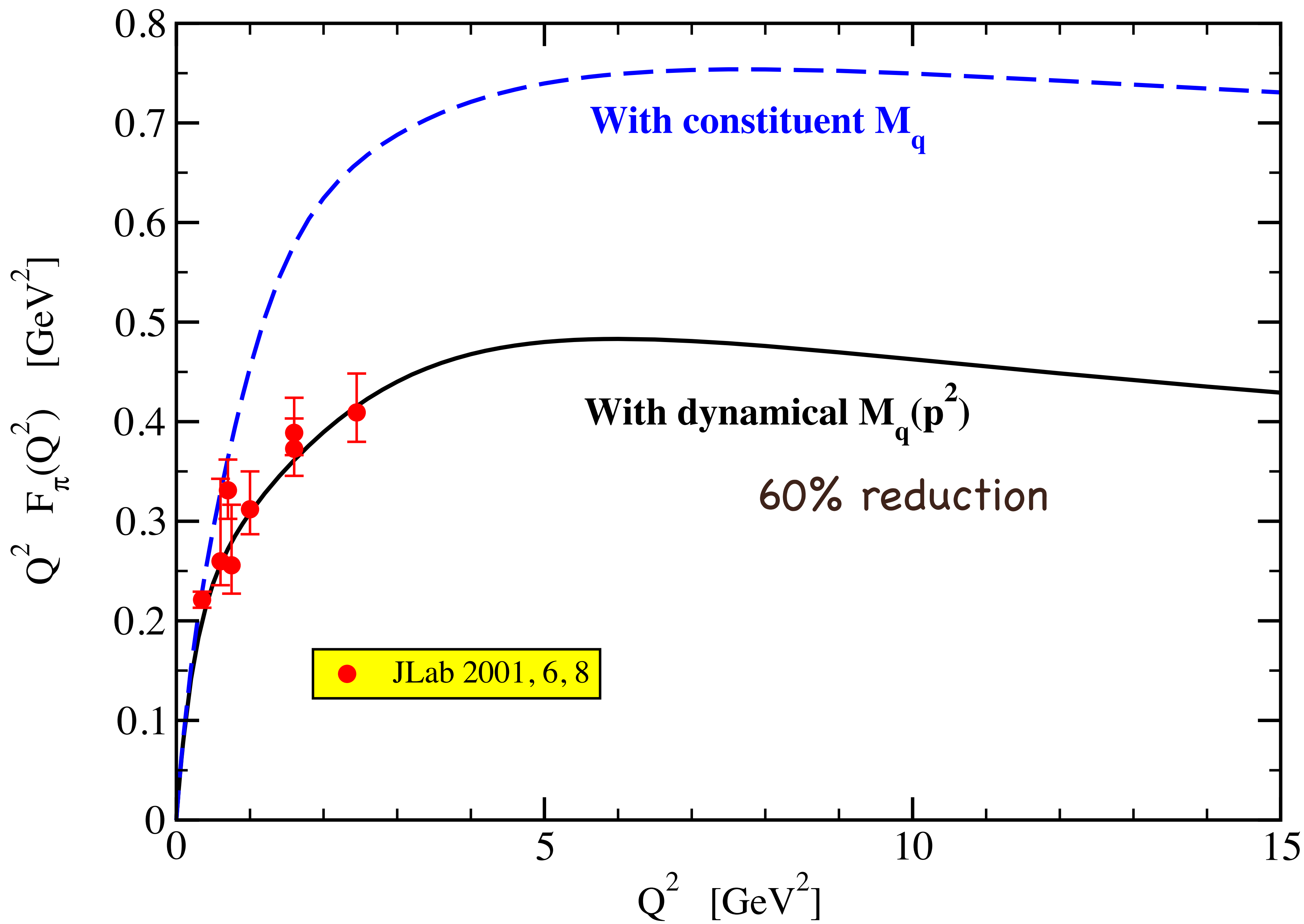


Jab data: G. Huber et al., PRC78, 045203 (2008)

Pion Form Factor: Broad Picture



Pion Form Factor: Running q Mass Fn Effect



Jab data: G. Huber et al., PRC78, 045203 (2008)

Summary

- DCSB: A large u/d quark constituent mass is generated from almost nothing for the same reason & and by the same mechanism that makes the pion almost massless!
- DCSB causes the shape of the pion DA to be significantly broader than the asymptotic-QCD DA at accessible scales for hadron physics, and a new analysis technique shows that lattice-QCD moments say the same thing. [DCSB identified in a LF-defined quantity.]
- The scale running of distribution amplitudes is exceedingly SLOW---even at LHC scales asymptotic-QCD for DAs and form factors they influence there are persistent sizeable npQCD effects and DCSB in the hadron states.
- The elastic form factor of the pion makes a transition from non-perturbative/constituent quark behavior to partonic perturbative behavior for Q^2 at 6-8 GeV^2 and the relevant extension of the Brodsky-LePage uv-QCD leading formula is just 15% below the recent DSE calculation there.
- The new DSE approach is applicable to form factors for all spacelike Q^2.
- DSE-QCD can now be applied to light-front-defined bound state properties as a fn of momentum fraction x. Meson DAs and PDFs work out well, nucleon PDFs and GPDs await...

Continuum QCD, Dyson-Schwinger Eqns and Hadron Physics

Collaborators:

- Craig Roberts, Argonne National Lab, USA
- Adnan Bashir, University of Michoacan, Morelia, Mexico
- Ian Cloet, Argonne National Lab, USA
- Yuxin Liu, Peking Univ, China
- Lei Chang, Peking U, Argonne/Julich/Univ Adelaide, Australia
- Chao Shi, Nanjing Univ, [visiting Kent State U]
- Konstantin Khitrin, PhD student, Kent State Univ, USA
- Javier Cobos-Martinez, Univ of Sonora, Mexico

The End

Thank you !