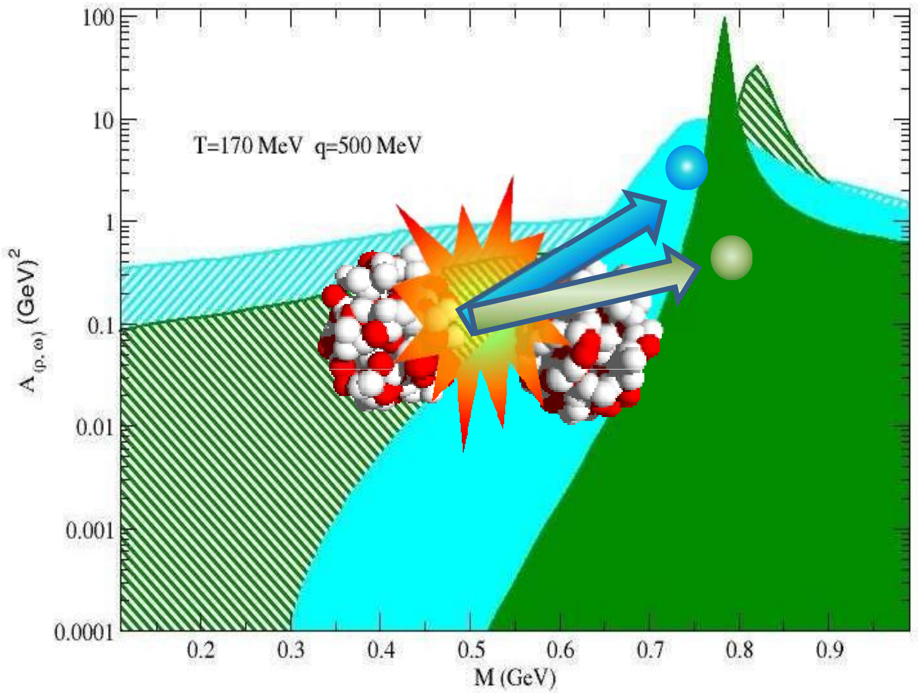
Vector Meson Spectral Functions in Medium

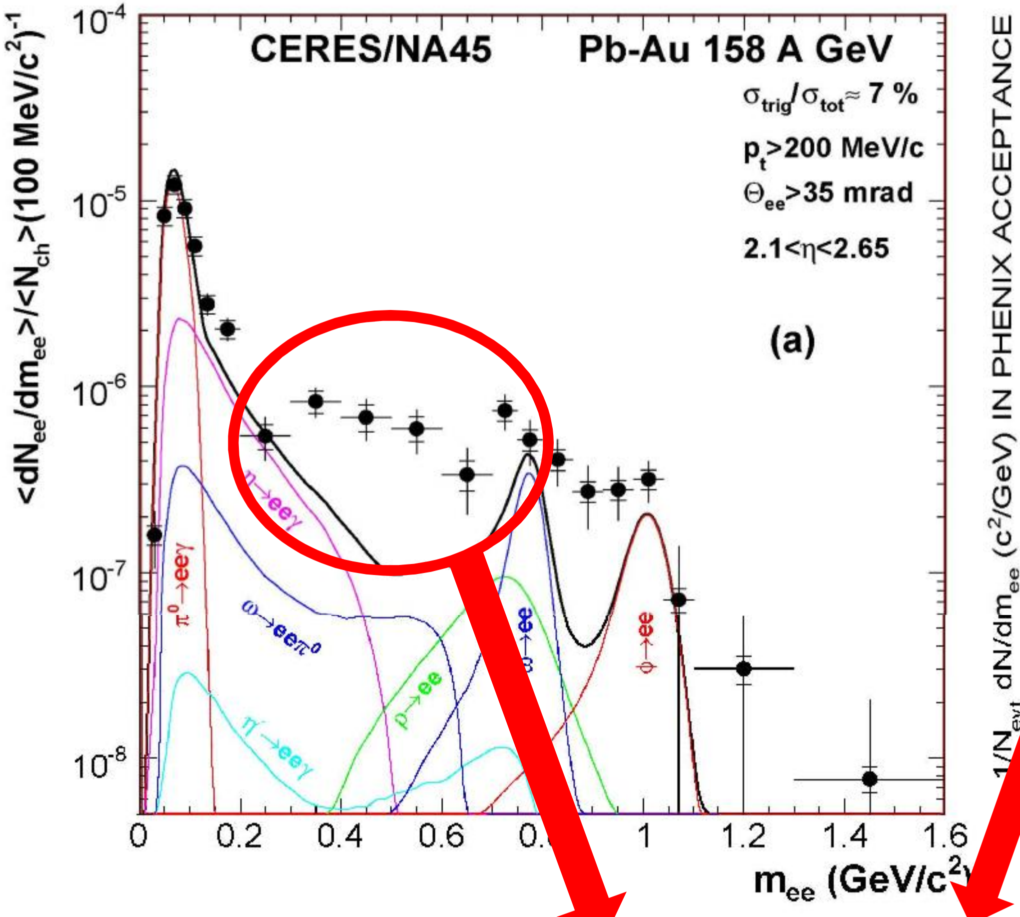


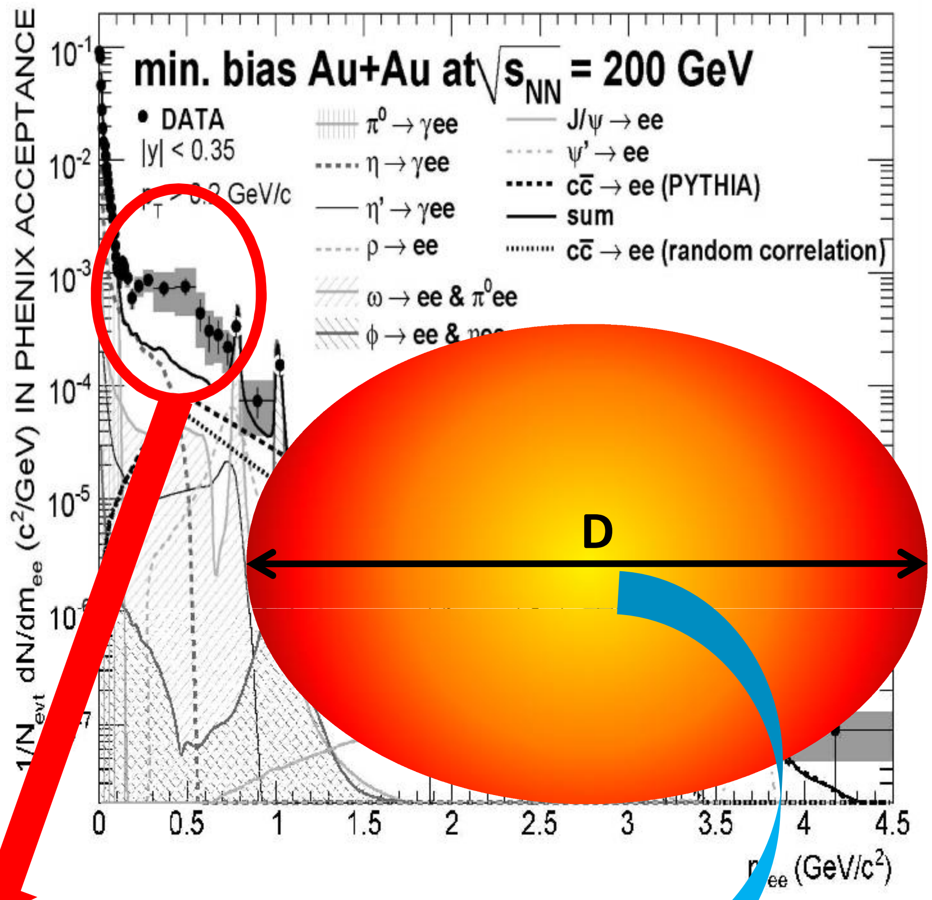
Outline of the talk ………………

- ✓ *Motivation*
- ✓ *Formalism*
- ✓ *Results*

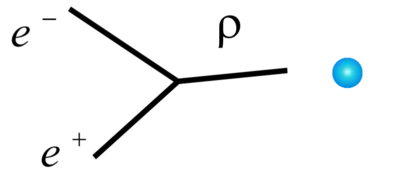
Sabyasachi Ghosh
(UNESP, IFT)

Experimental motivation :





Low mass enhancement
due to in-medium modification
of light vector mesons (specially ρ).



$$\text{Mean free path}: \lambda \propto \frac{1}{\sigma_{\text{e.m.}}} >> D$$

$$L_{\rm QCD} = \sum_{q=u,d} \bar{q}(i\gamma^{\mu} D_{\mu} - m_q) q + \text{kinetic part of gluon}$$

$$q = \frac{1}{2}(1+\gamma^5)q + \frac{1}{2}(1-\gamma^5)q$$
$$= \quad q_L \quad + \quad q_R$$

$$L_{\rm QCD} = \sum_{q=u,d} \{(q_L i\not{D} q_L + q_R i\not{D} q_R) - (q_L m_q q_R + q_L m_q q_R)\}$$

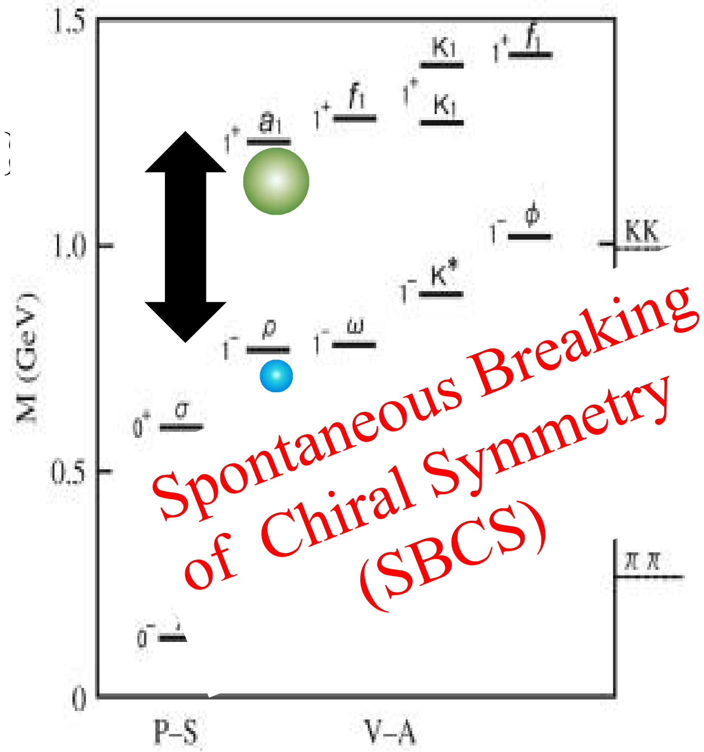
Mass term ⇨ 0

$$q_{L,R} \rightarrow U_{L,R}\, q_{L,R} \quad (U_{L,R} = \mathrm{e}^{i\vec{\alpha}_{L,R}\cdot\vec{\tau}/2})$$

remain invariant under global $SU(2)_L \times SU(2)_R$ symmetry & associated conserved Noether currents

$$J^{\mu a}_{L,R} = \bar{q}_{L,R}\gamma^{\mu}(\tau^a/2) q_{L,R}$$

Real world



Vector current

$$J^{\mu a}_V = \bar{q}\gamma^{\mu}(\tau^a/2)q = J^{\mu}_R + J^{\mu}_L \quad (1^-)$$

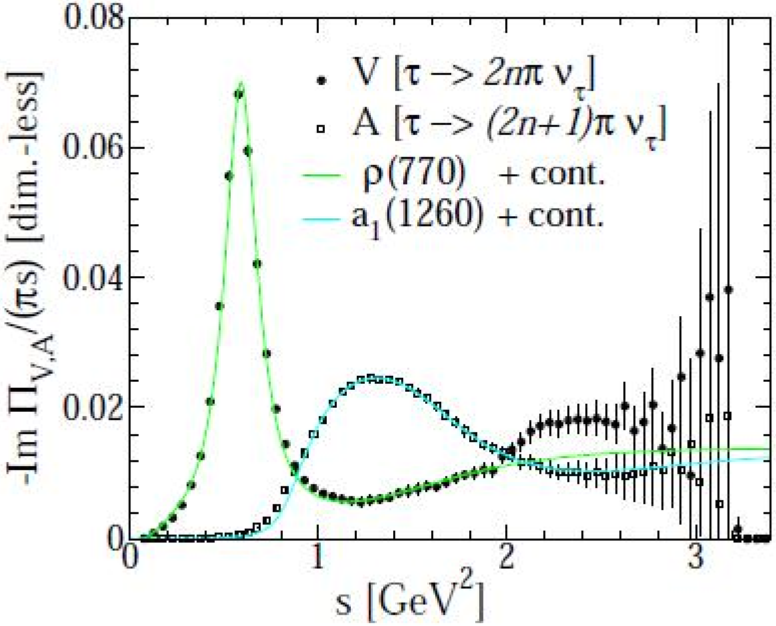
Axial vector current

$$J^{\mu a}_A = \bar{q}\gamma^{\mu}\gamma^5(\tau^a/2)q = J^{\mu}_R - J^{\mu}_L \quad (1^+)$$

Parity doublet

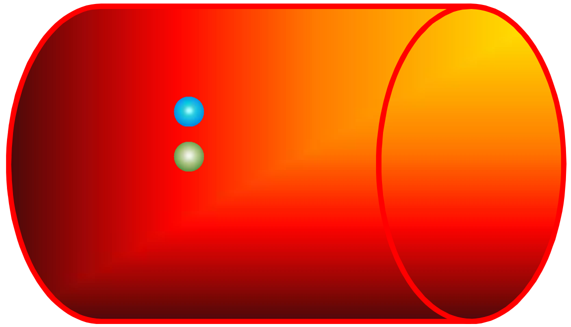
Broken Chiral Symmetry → Restored Chiral Symmetry

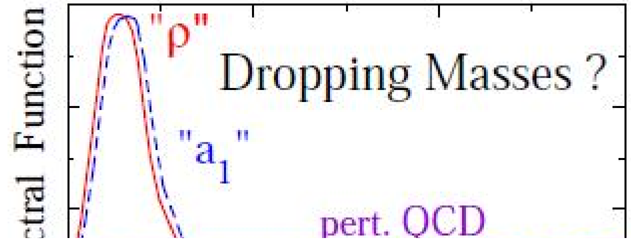
Vacuum



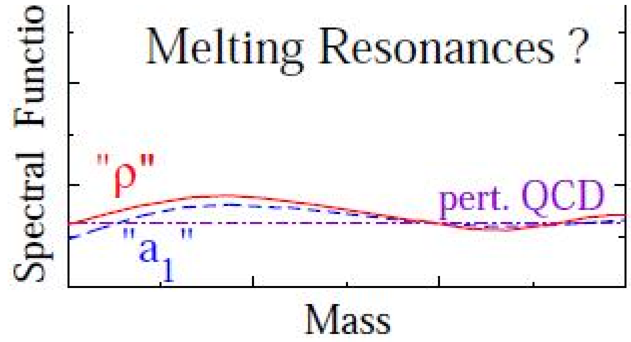
Spectral function of ρ and a_1 are measured via hadronic decays of **τ lepton** at **LEP** by **ALEPH** and **OPAL**

Strongly interacting matter





experimental data from heavy ion collision seems to prefer melting scenario



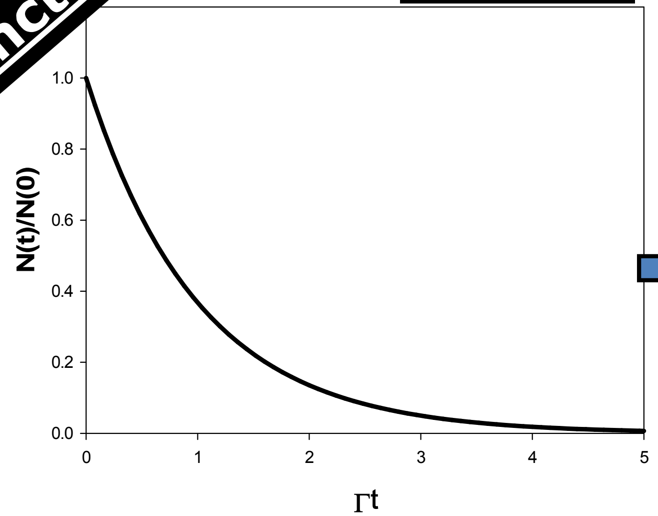
The **spectral properties** of vector mesons and axial Vector mesons **in strongly-interacting matter** may indicate about Chiral Symmetry Restoration (CSR).

Rapp et.al. arXiv:0901.3289

spectral function :

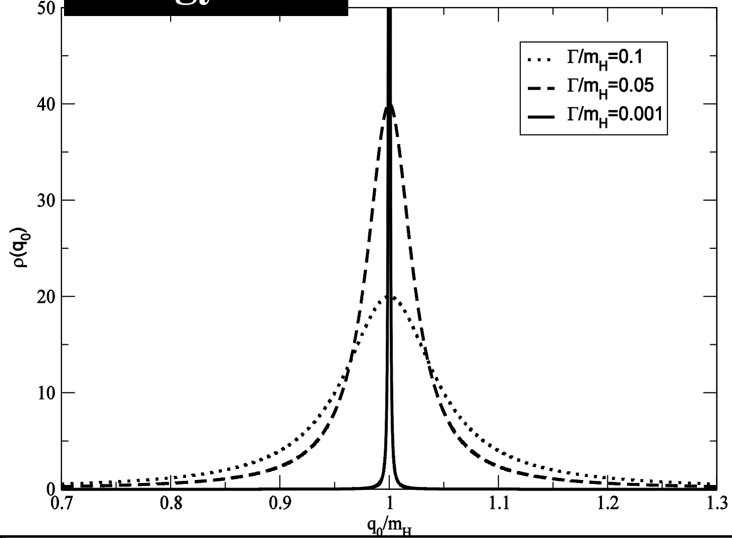
Probabilistic amplitude of Unstable particle

time axis



ırier nsform

energy axis



$$\frac{N(t)}{N(0)} = \left|\frac{\psi(t)}{\psi(0)}\right|^2 = \exp(-\Gamma t)$$

$$\psi(t) \sim \exp(i m_H t - \Gamma t/2)$$

$$\frac{N(q_0)}{N(m_H)} = \left|\frac{\tilde{\psi}(q_0)}{\tilde{\psi}(m_H)}\right|^2 = (\Gamma/2)\rho(q_0)$$

$$\tilde{\psi}(q_0) \sim \frac{1}{(q_0 - m_H) - i\Gamma/2}, \; \rho(q_0) = \mathrm{Im}\,\tilde{\psi}(q_0)$$

$$\lim_{\Gamma\to 0} \rho(q_0) \sim \delta(q_0 - m_H)$$

QFT definition of spectral function

$$x = (t, \vec{x}) \Rightarrow q = (q_0, \vec{q})$$

$$\lim_{\Gamma\to 0} \rho(q) \sim \delta(q^2 - m_H^2)$$

$$\sim \mathrm{Im}\,\frac{1}{q^2 - m_H^2 + i\eta}$$

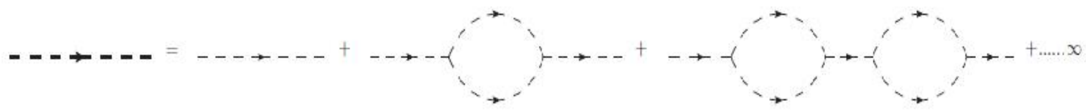
Propagator ⟸

$$<0|\psi(x_1)\psi(x_2)|0>$$

Vacuum free propagator

$$\lim_{\Gamma \to 0} \rho(q) \sim \delta(q^2 - m_H^2)$$

$$\sim \mathrm{Im}\,\frac{1}{q^2 - m_H^2 + i\eta}$$



Vacuum interacting propagator

$$D(\mathrm{q}^2) = \frac{1}{\mathrm{q}^2 - m^2 - \Pi(\mathrm{q}^2)}$$

Essence of RTF

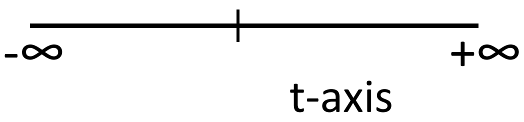
Time evolution operator

$$\exp(-i\,H\,t) = \exp(-\beta\,H)$$

$$t \to \tau = -i\,\beta$$

~ density matrix

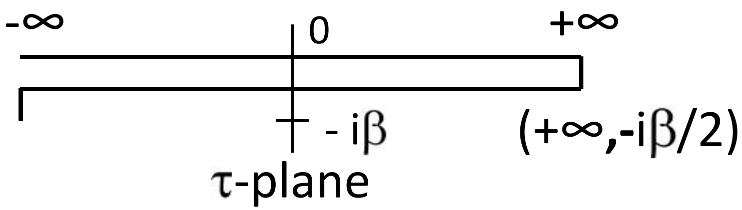
Field Theory of vacuum



$$\int_{-\infty}^{\infty} dt$$

$$\int_{\text{contour}} d\tau$$

Field Theory at finite temperature



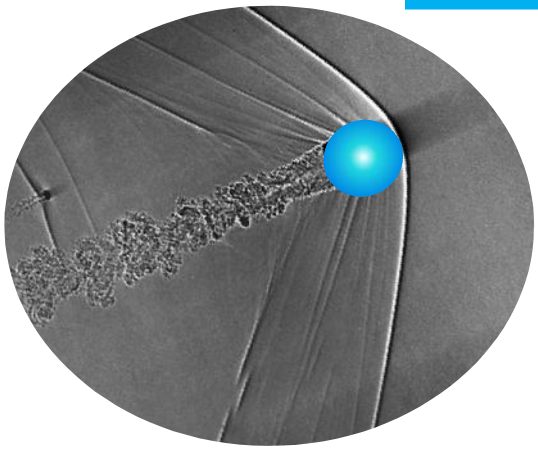
Diagonalization

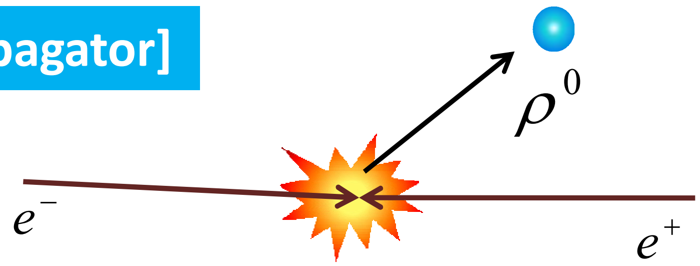
$$\overline{D}(\mathrm{q}^0, \vec{q}, T) = \frac{1}{\mathrm{q}^2 - m^2 - \overline{\Pi}(\mathrm{q}^0, \vec{q}, T)}$$

D^{ab} thermal propagator

Thermal self-energy Π^{ab}

Spectral function = Im [propagator]

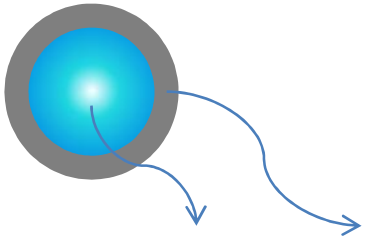




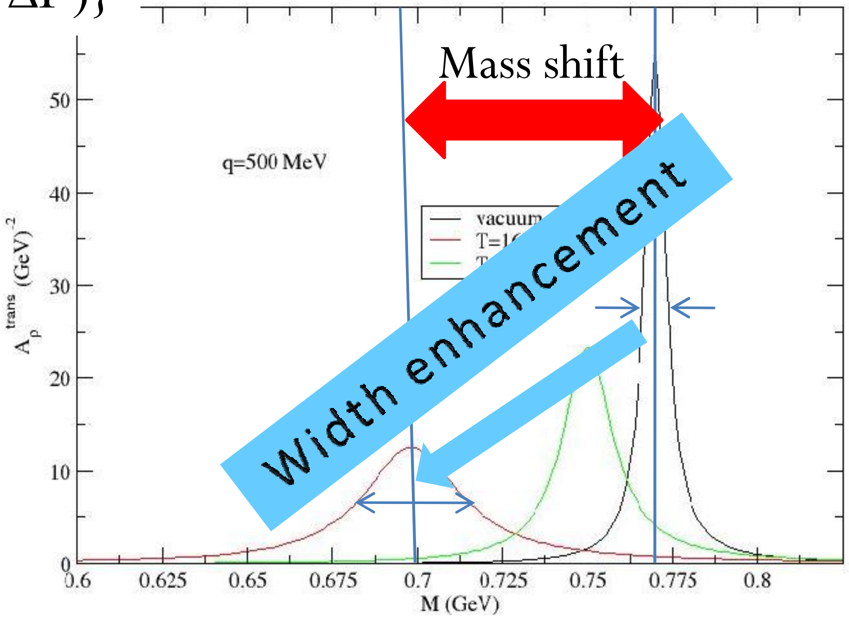
$$A_v(q) = \mathrm{Im}\left[\frac{1}{\{q^2 - (m_{phy}^2) - iq\Gamma_{decay}\}}\right]$$

$$A_{tot}(q) = \mathrm{Im}\left[\frac{1}{\{q^2 - (m_{phy} + \Delta m)^2 - iq(\Gamma_{decay} + \Delta\Gamma)\}}\right]$$

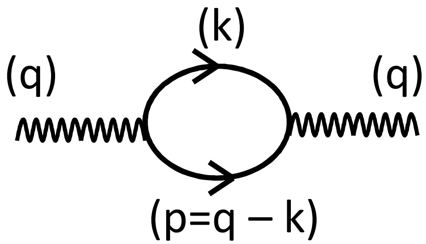
$$\overline{\Pi}_{total} = \overline{\Pi}_{vac} + \overline{\Pi}_{th}$$



$$m_{bare}^2 + \mathrm{Re}\overline{\Pi}_{vac} + \mathrm{Re}\overline{\Pi}_{th} = m_{phy}^2 + \mathrm{Re}\overline{\Pi}_{th}$$
$$= (m_{phy} + \Delta \mathrm{m})^2$$



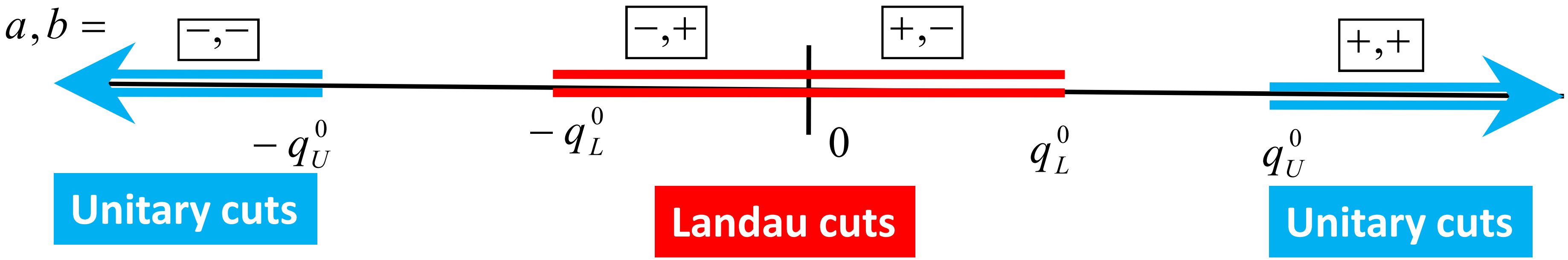
Self Energy of ρ for mesonic loops :



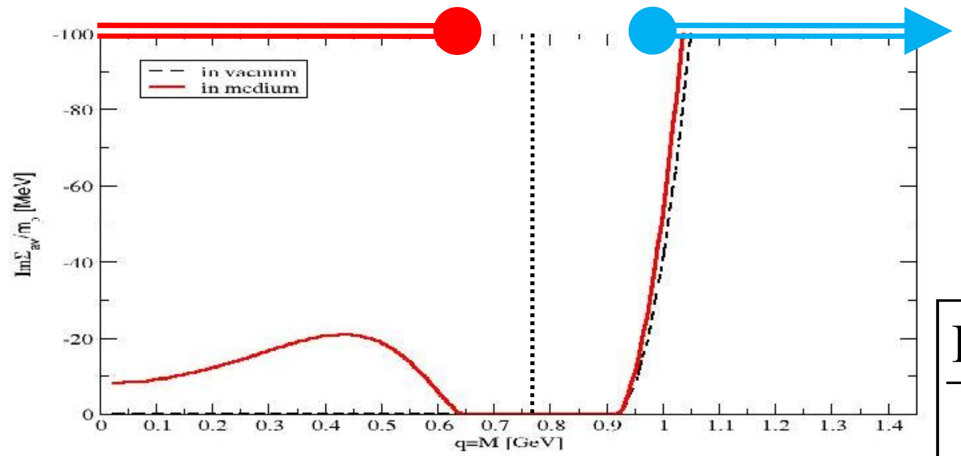
$$D_{11} = \text{vacuum} + \text{Thermal}$$

$$\Pi_{11}^{\mu\nu}(q,T) = \int \frac{d^4k}{(2\pi)^4} \mathrm{N}^{\mu\nu}(\mathrm{q},\mathrm{k})\ \mathrm{D}_{11}(\mathrm{k},\mathrm{T})\ \ \mathrm{D}_{11}(\mathrm{p}=\mathrm{q}-\mathrm{k},\mathrm{T})$$

$$\mathrm{Im}\,\Pi_{11}^{\mu\nu}(q,T) = \int d^3k \sum_{a,b} (..)\,\delta(q_0 - a\,\omega_k - b\,\omega_p)$$

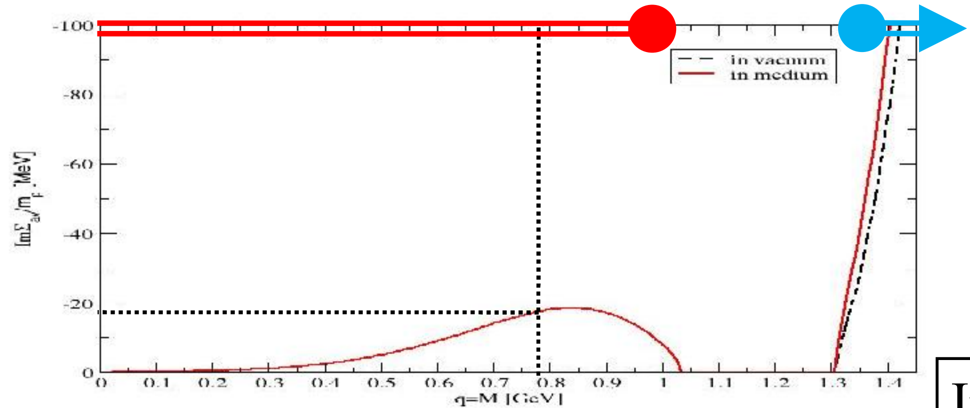


$$q_L^0 = \sqrt{\vec{q}^{\,2} + (m_p - m_k)^2} \qquad q_U^0 = \sqrt{\vec{q}^{\,2} + (m_p + m_k)^2}$$



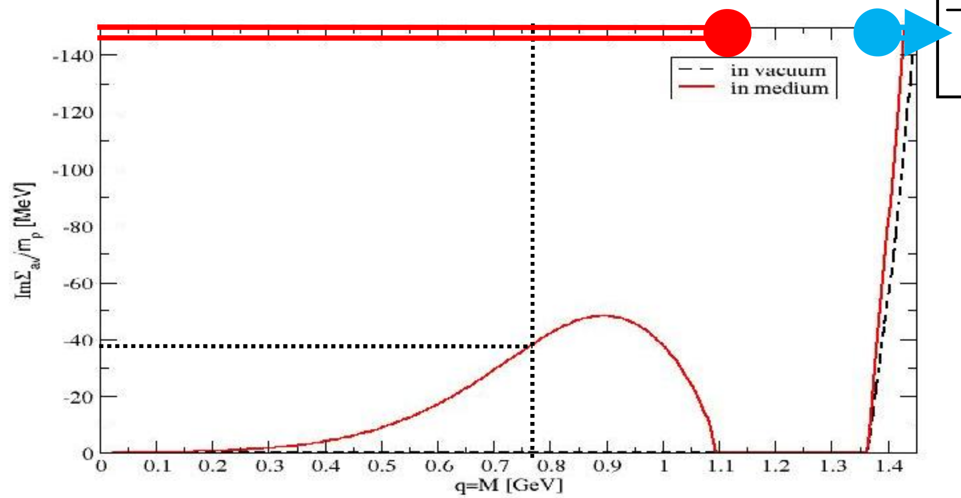
$$k,\, p = \pi\ \omega(782)$$
$$\pi\ h_1(1170)$$
$$\pi\ a_1(1260)$$

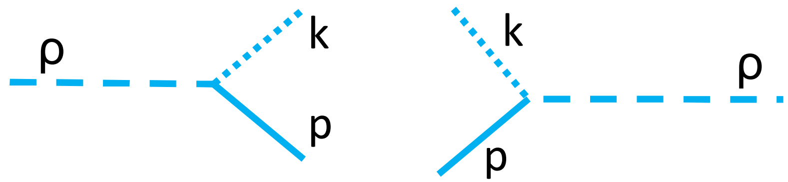
$$\frac{\mathrm{Im}\,\overline{\Pi}_L^M}{m_\rho} \sim \mathrm{vac} \otimes \{n_k(1+n_p) - (1+n_k)n_p\}$$



Mesonic collision rate (Γ_C^M)

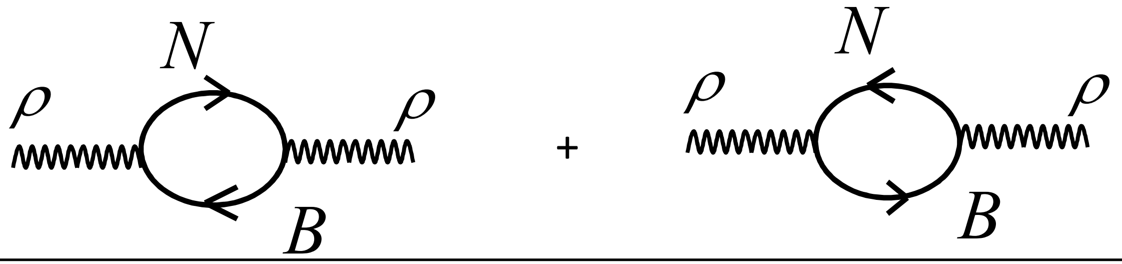
$$\frac{\mathrm{Im}\,\overline{\Pi}_U^M}{m_\rho} \sim \mathrm{vac} \otimes \{(1+n_k)(1+n_p) - n_k n_p\}$$



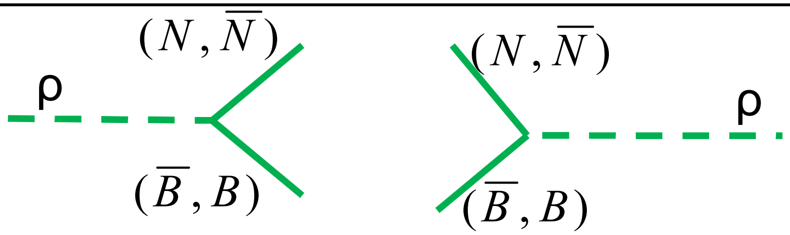


Bose enhancement of decay rate $(\Gamma_{B.e.}^M)$

Self Energy of ρ for baryonic loops :

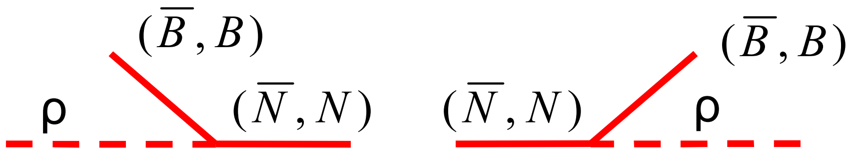


$$\frac{\mathrm{Im}\,\overline{\Pi}_U^B}{m_\rho} \sim \mathrm{vac} \otimes \left[\ \{(1-n_N^+)(1-n_B^-) - n_N^+ n_B^-\} \ + \ \{(1-n_N^-)(1-n_B^+) - n_N^- n_B^+\}\ \right]$$

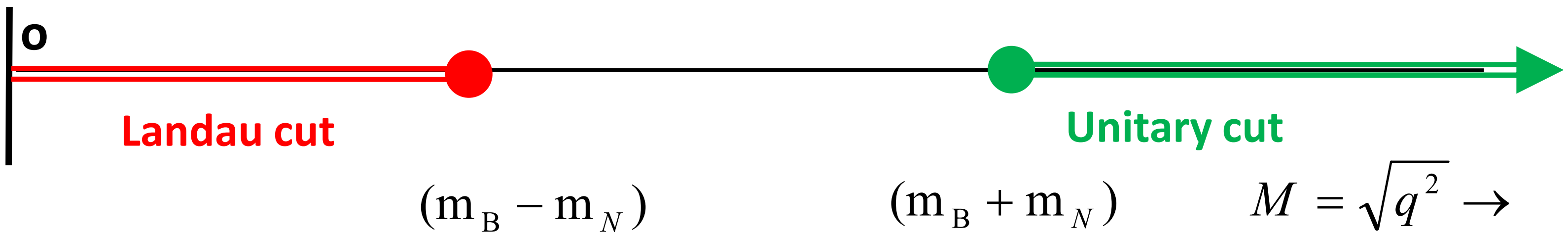


Pauli blocking of decay rate $(\Gamma_{P.b.}^B)$

$$\frac{\mathrm{Im}\,\overline{\Pi}_L^B}{m_\rho} \sim \mathrm{vac} \otimes \left[\ \{(1-n_N^-)n_B^- - n_N^-(1-n_B^-)\} \ + \ \{(1-n_N^+)n_B^+ - n_N^+(1-n_B^+)\}\ \right]$$



Bosonic collision rate (Γ_C^B)



Landau cut

$(m_B - m_N)$

=(0.94-0.94) GeV = 0 GeV [min]

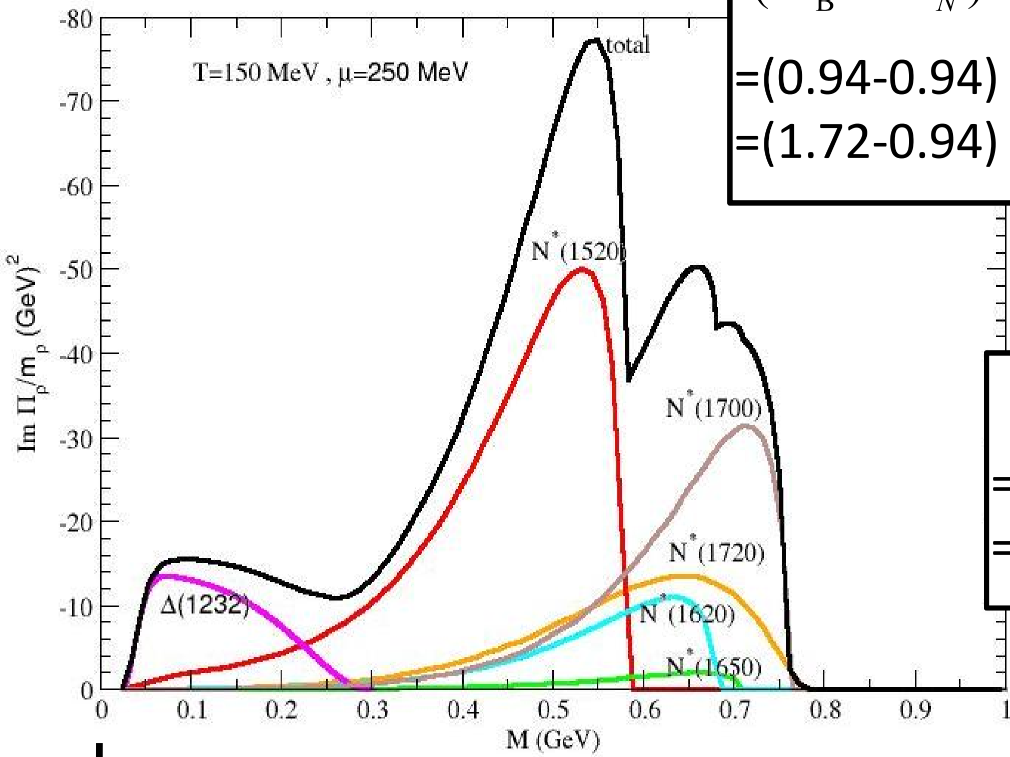
=(1.72-0.94) GeV = 0.78 GeV [max]

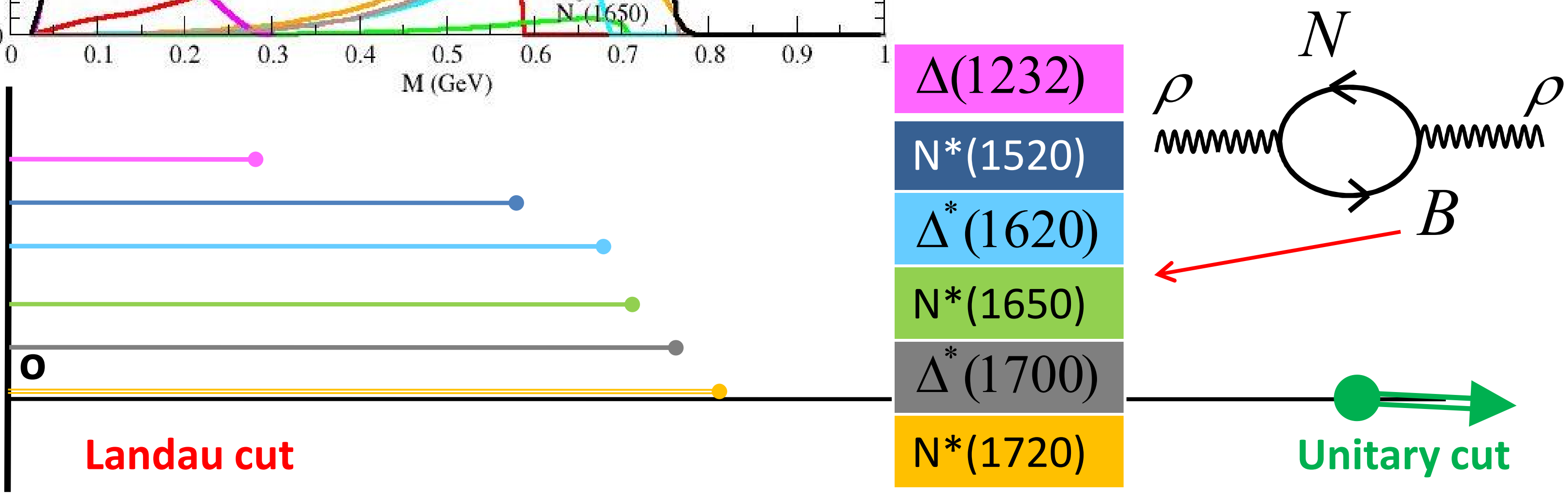
Unitary cut

$(m_B + m_N)$

=(0.94+0.94) GeV = 1.88 GeV [min]

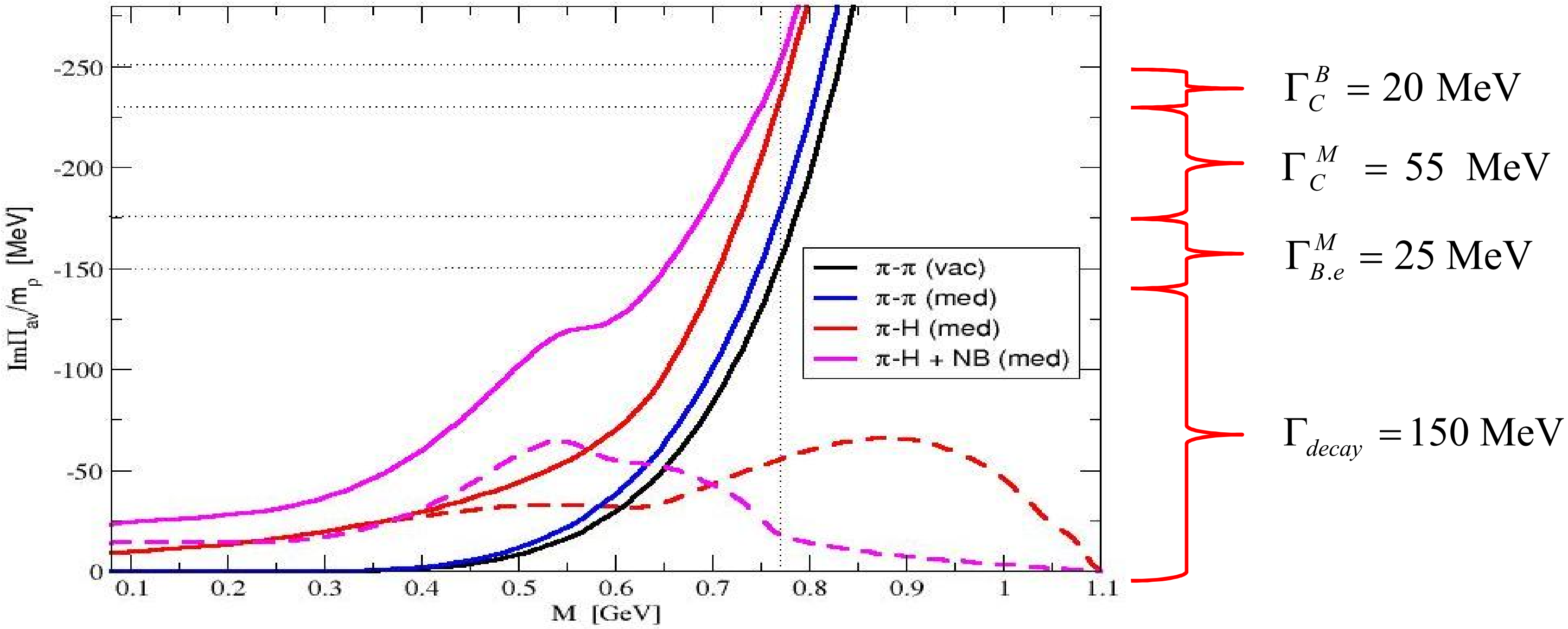
=(1.72+0.94) GeV = 2.66 GeV [max]





$M = \sqrt{q^2} \rightarrow$

$T = 150\,\text{MeV}\ \&\ \mu_B = 250\,\text{MeV}$



$\Gamma_{decay}^{\pi\pi} = 150\,\text{MeV}$ ---------------------------- vacuum part of $\pi\pi$ loop (unitary cut)	$\Gamma_{B.e.}^M = 25$ MeV ---------------------------- thermal part of $\pi\pi$ loops (unitary cut)	$\Gamma_C^M = 55$ MeV ---------------------------- $(\pi H = \pi\omega, \pi h_1, \pi a_1)$ loops (Landau cut)	$\Gamma_C^B = 20$ MeV ---------------------------- $(NB = NN, N\Delta, NN^*, N\Delta^*)$ loops (Landau cut)

Physical interpretation of imaginary part of in-medium self-energy :

$$\Gamma_{tot} = \Gamma_C^M + \Gamma_{B.e.}^M + \Gamma_C^B + \Gamma_{P.b.}^B = \frac{\mathrm{Im}\,\Pi(E_q,\vec{q})}{m_\rho}$$

rate at which ρ try to be thermalized with the thermal bath

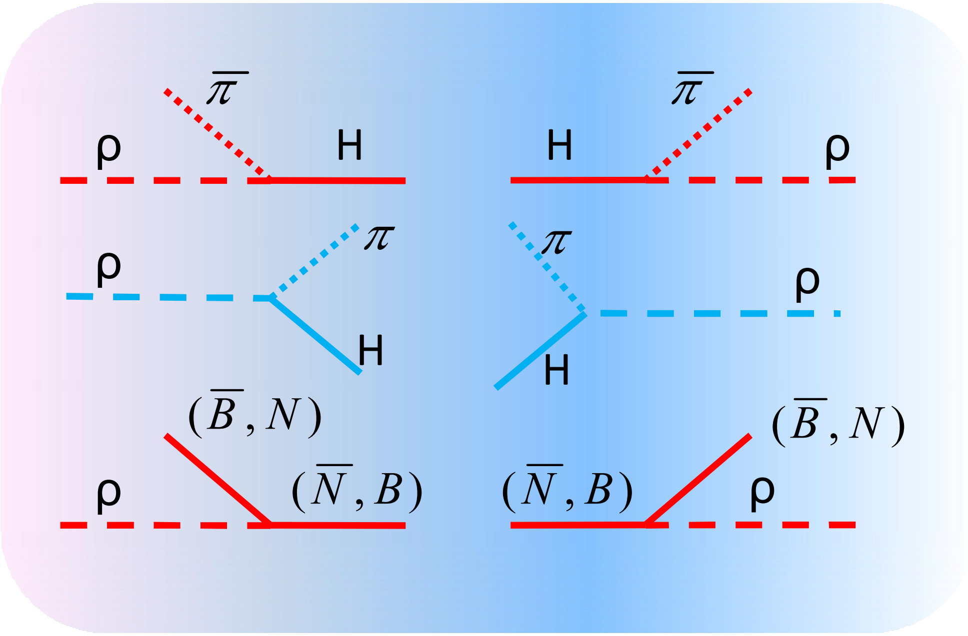
$$\frac{1}{\exp(E_q/T)-1} + c\exp(-\Gamma_{tot}t)$$

$$\frac{1}{\exp(E_q/T)-1}$$

ρ with $\vec{q}$

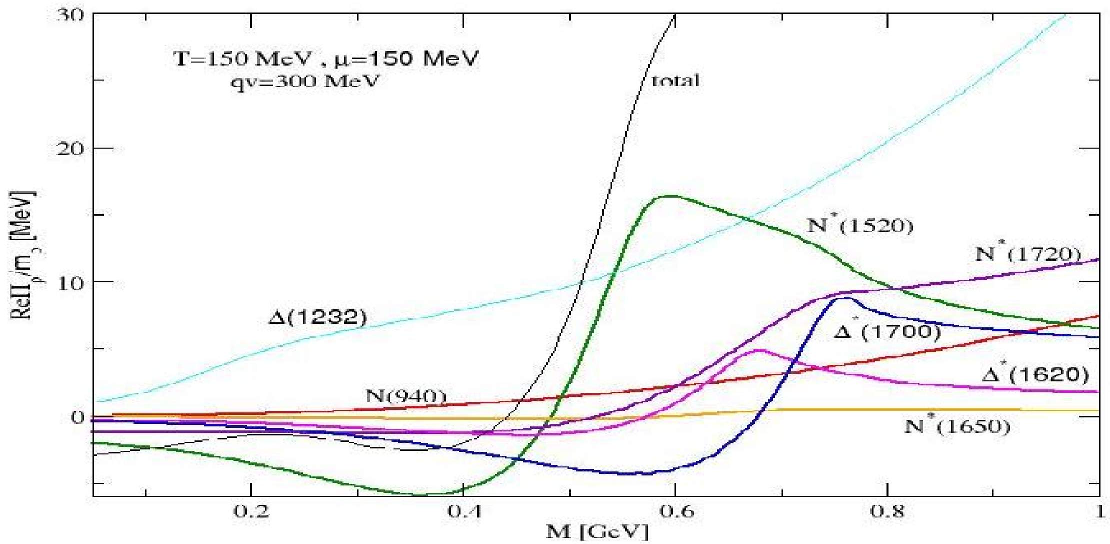
Thermalized Hadronic matter with mesons(H) and baryons(B)

mesons(H)	baryons(B)
$\pi(140)$	$\Delta(1232)$
$\omega(782)$	N*(1520)
$h_1(1170)$	$\Delta^*(1620)$
$a_1(1260)$	N*(1650)
	$\Delta^*(1700)$
	N*(1720)

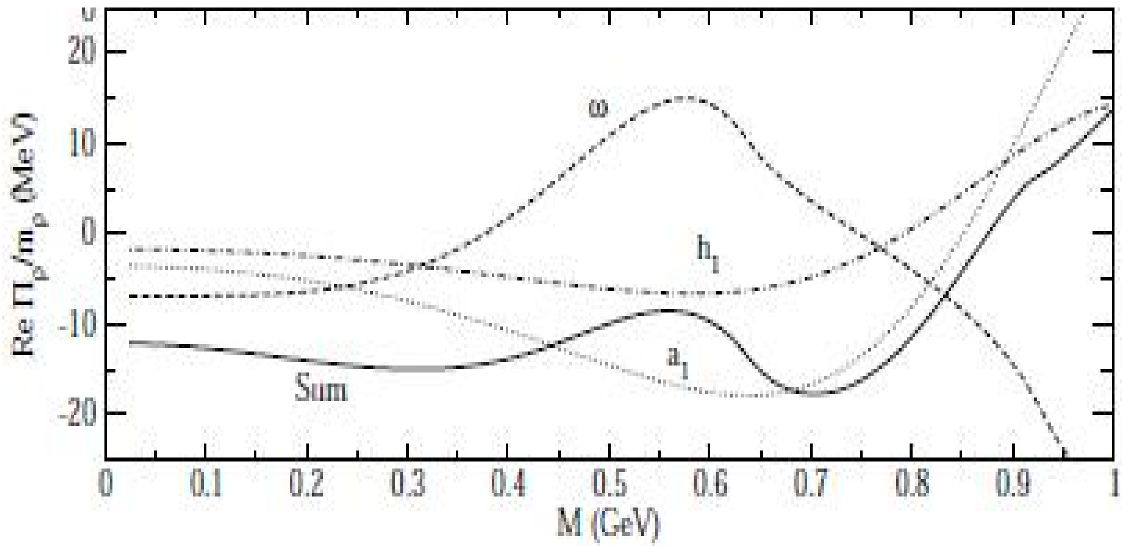


Real part of self-energy

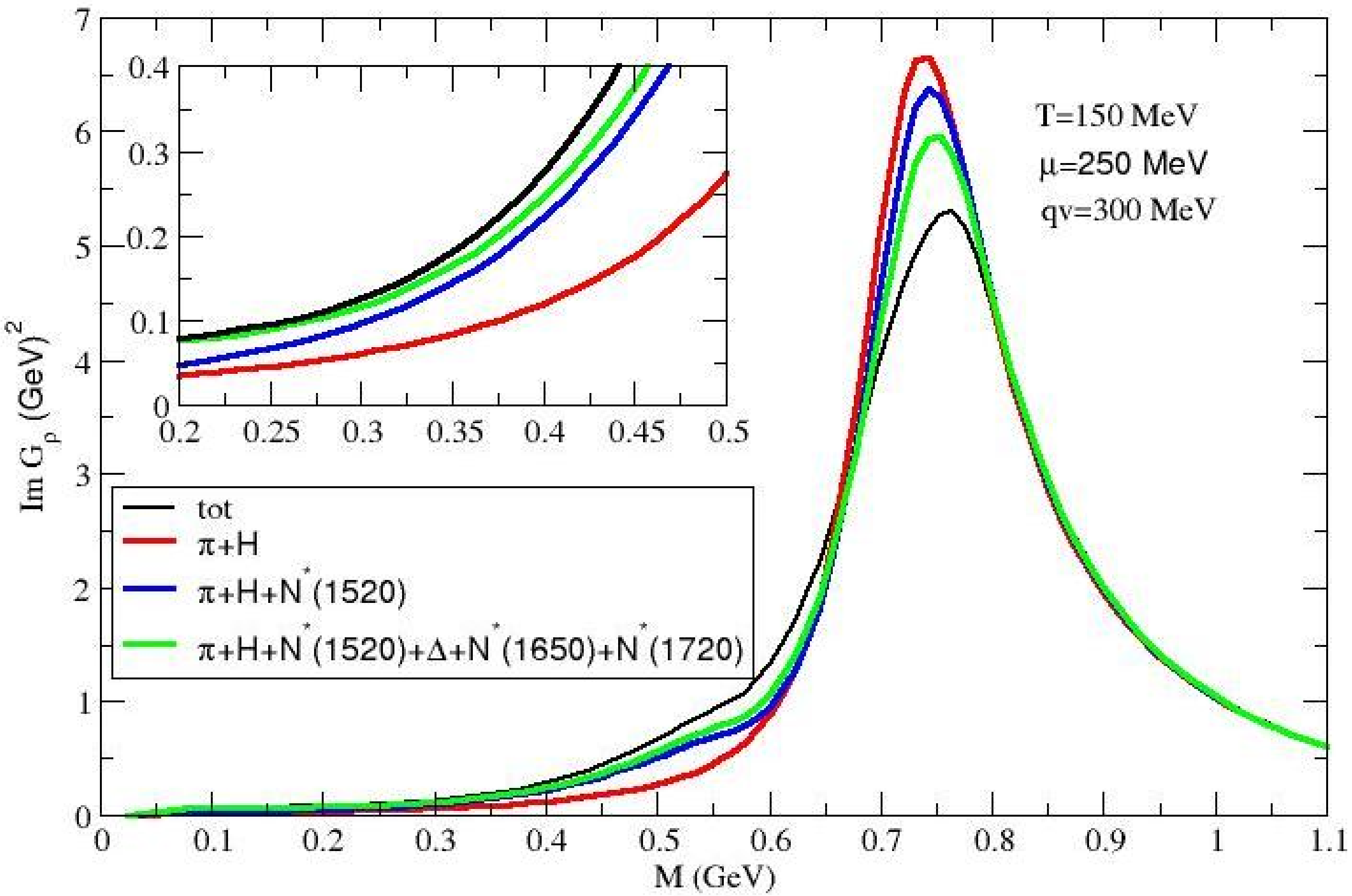
Baryonic loops



Mesonic loops

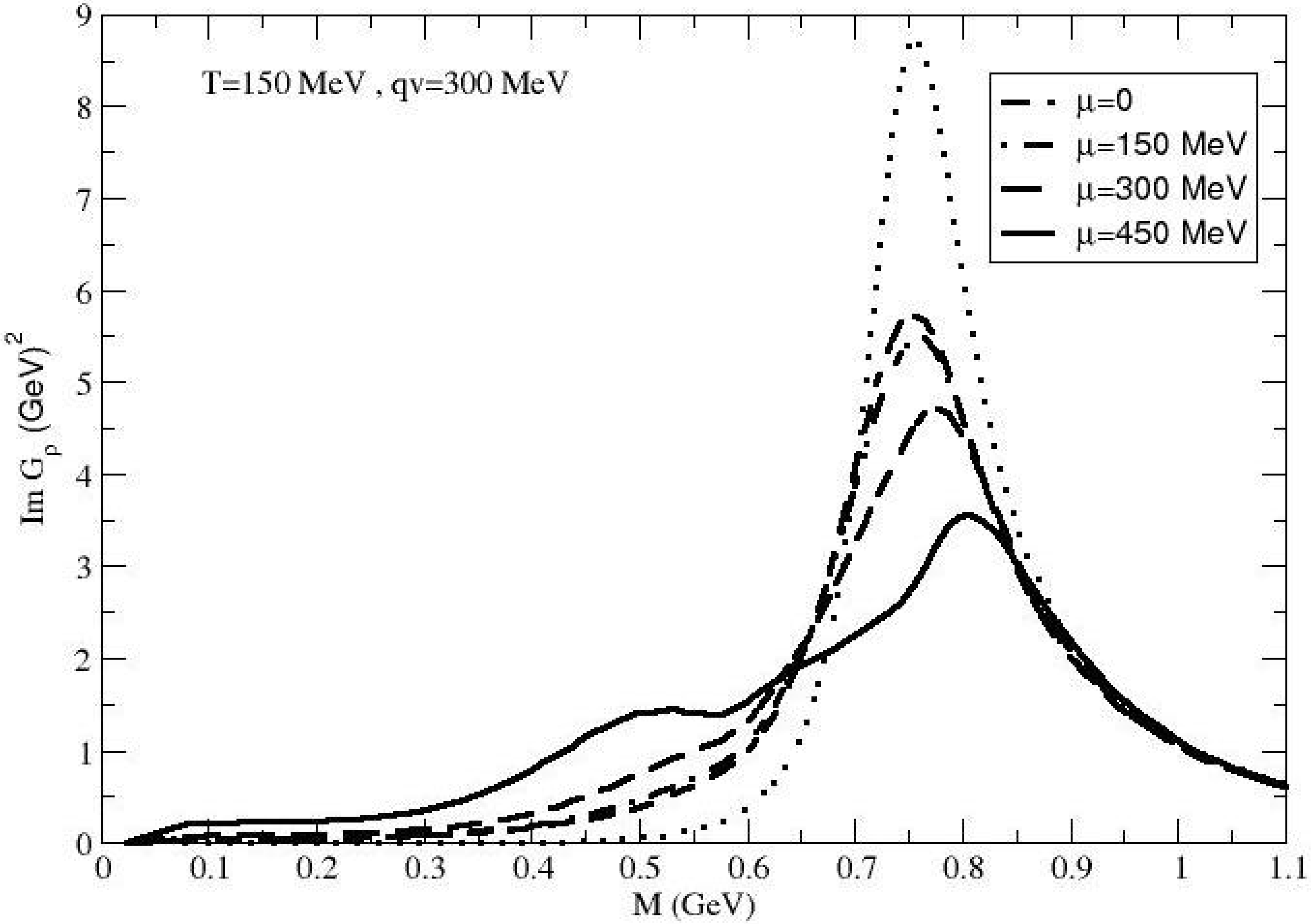


Effect of various loops on low mass invariant mass space in ρ spectral function :



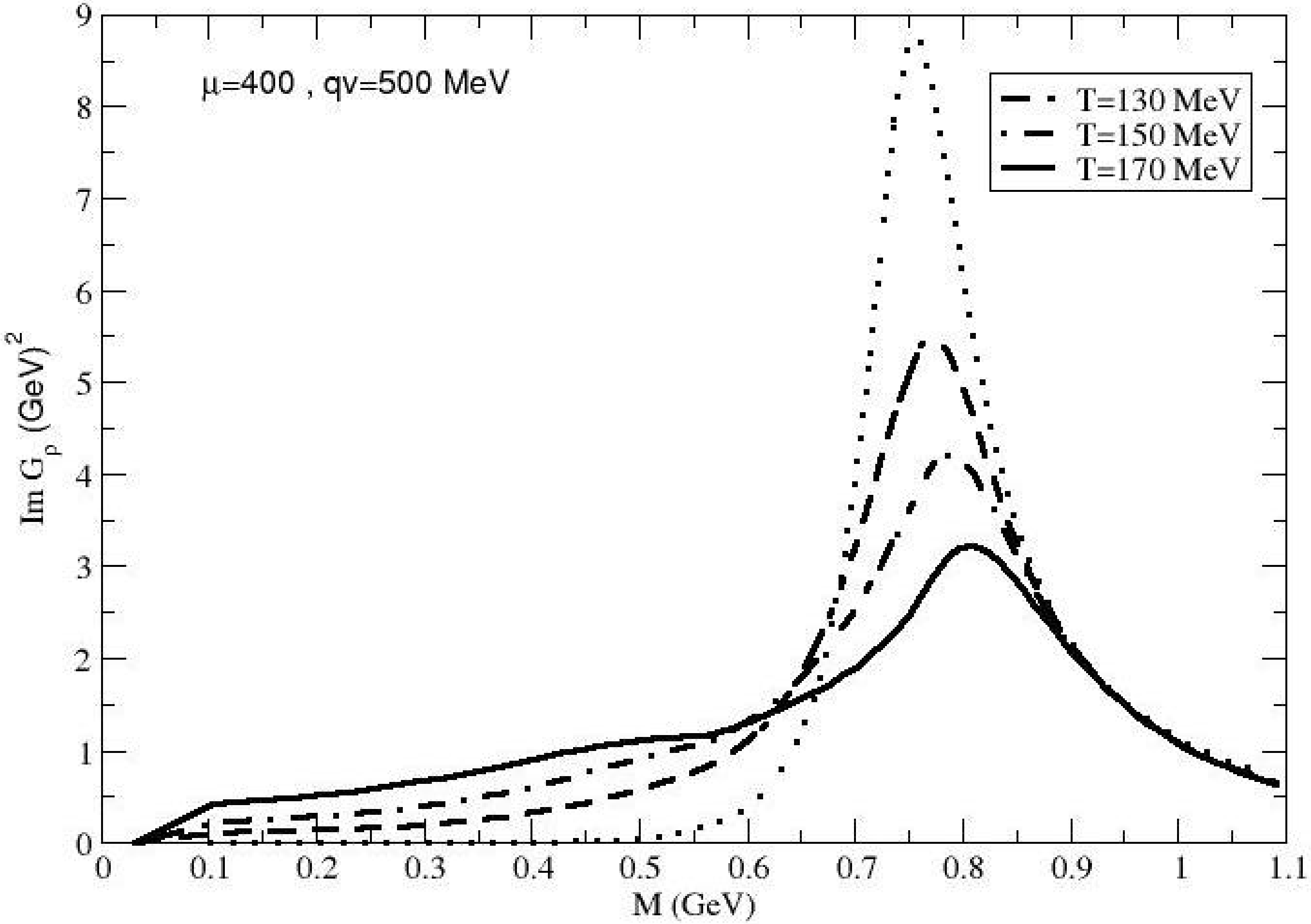
$\pi\omega$, NN*(1520) & NΔ(1232)

Effect of baryonic chemical potential on ρ spectral function in low mass region:



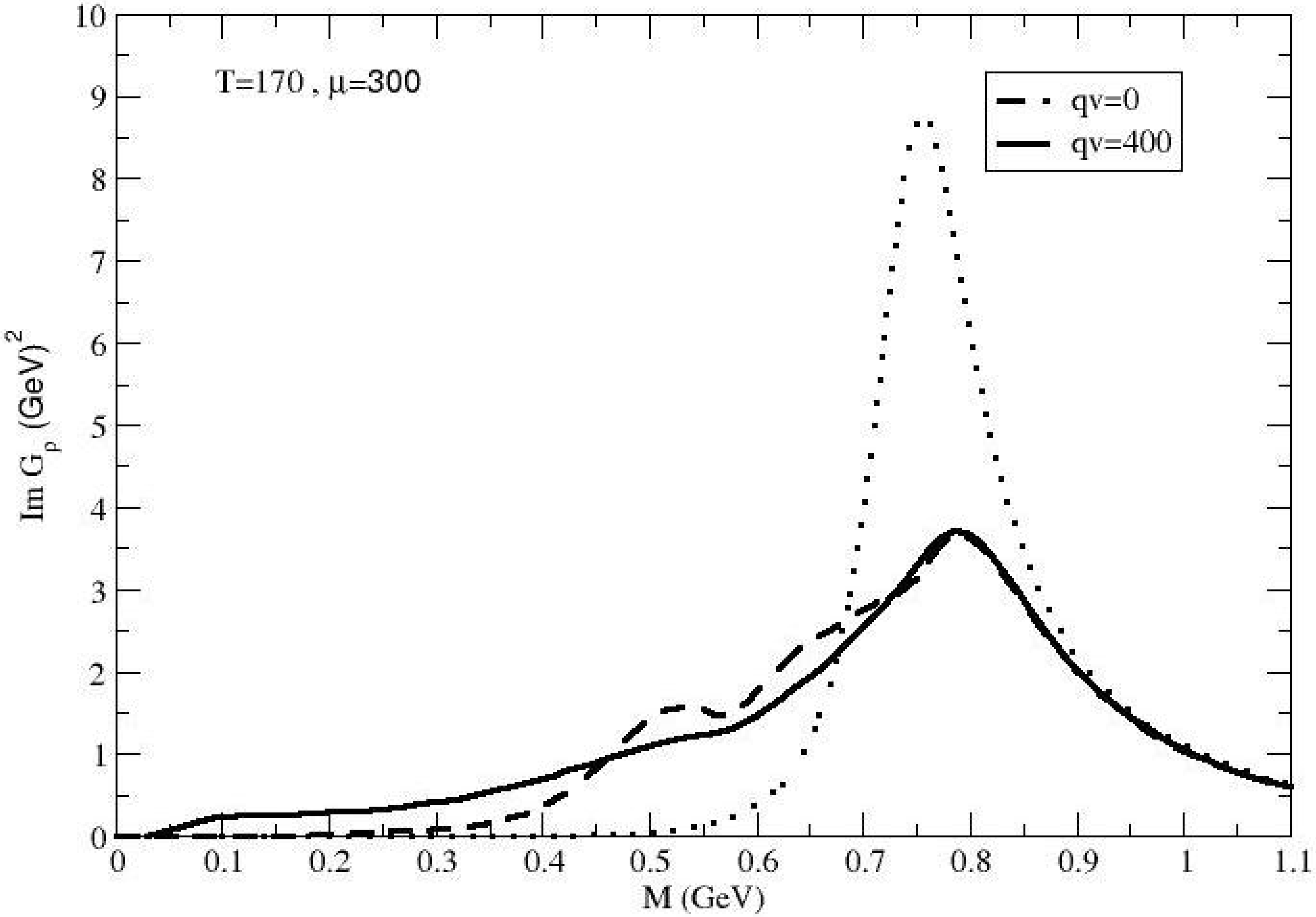
$$A_\rho(q^0, \vec{q}, T, \mu_B)$$

Effect of temperature on ρ spectral function in low mass region:



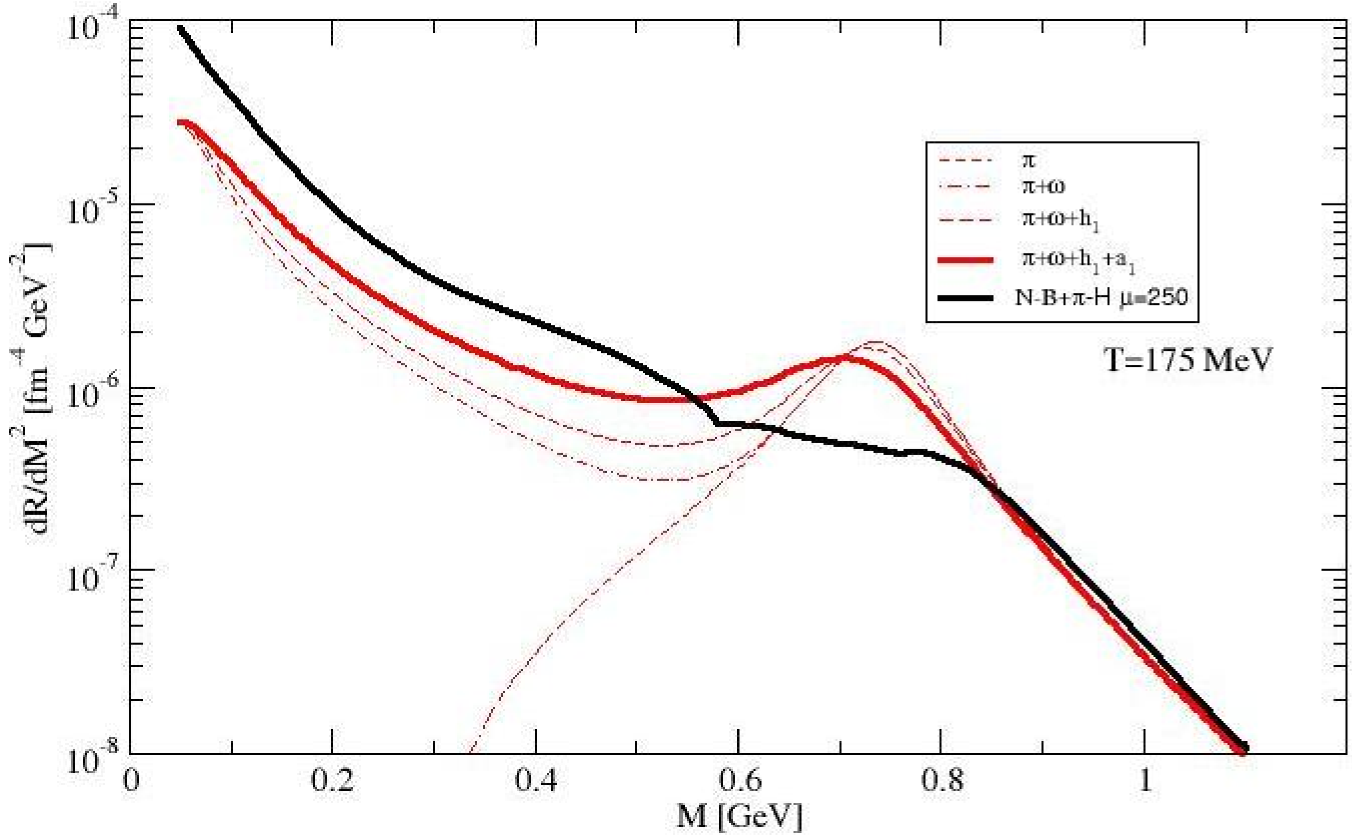
$$A_\rho(q^0, \vec{q}, T, \mu_B)$$

Effect of momentum of ρ in off mass shell on its spectral function in low mass region:



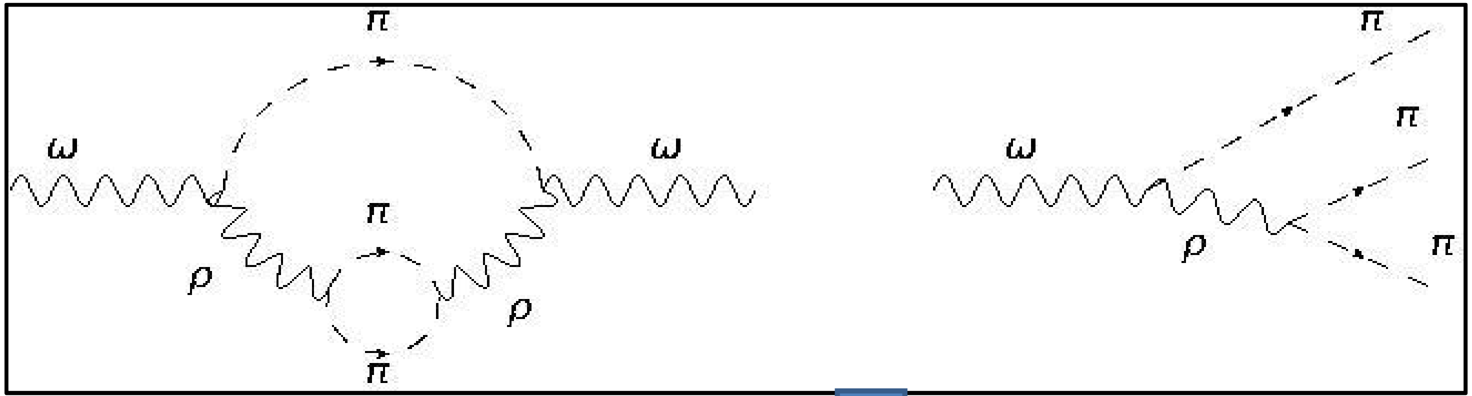
$$A_\rho(q^0, \vec{q}, T, \mu_B)$$

Effect of mesonic as well as baryonic medium modification of ρ on dilepton rate in low mass region :

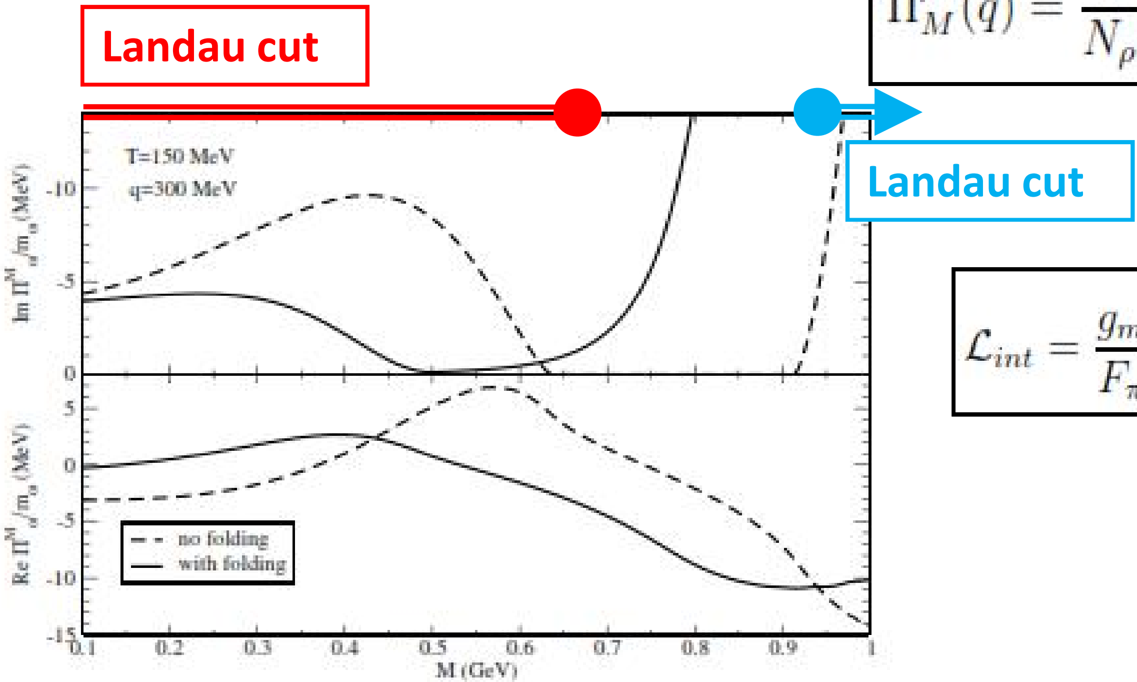


$$\frac{dR}{dM^2 d^2 q_T dy} = \frac{dR}{d^4 q} = (\ldots)\ L_{\mu\nu}\ \mathrm{A}_{\rho}^{\mu\nu}(q^0, \vec{q}, T, \mu_B)$$

Self Energy of ω for mesonic loops :

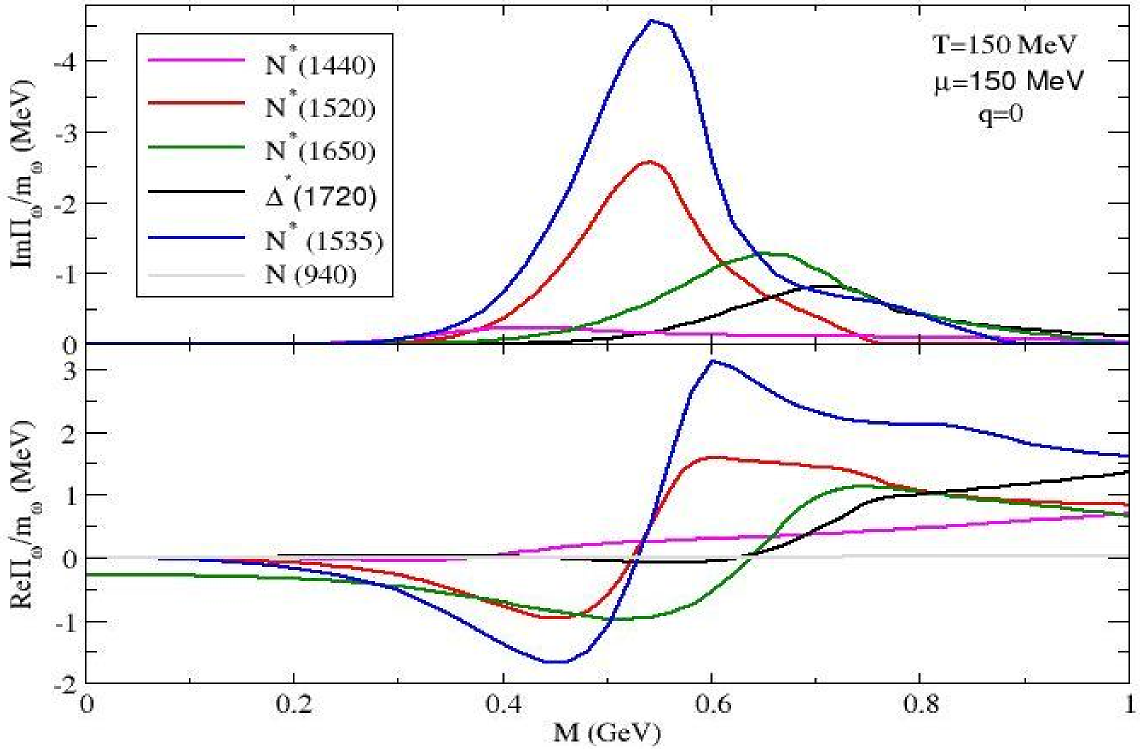


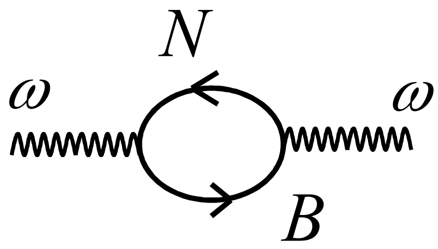
$$\overline{\Pi}^{\mu\nu}_M(q) = \frac{1}{N_\rho}\int_{4m_\pi^2}^{(q-m_\pi)^2} dM^2 [\overline{\Pi}^{\mu\nu}_{(\rho\pi)}(q,M)] A_\rho(M)$$



$$\mathcal{L}_{int} = \frac{g_m}{F_\pi}\epsilon_{\mu\nu\lambda\sigma}(\partial^\nu\omega^\mu\vec{\rho}^\lambda - \omega^\mu\partial^\nu\vec{\rho}^\lambda)\cdot\partial^\sigma\vec{\pi}$$

Self Energy of ω for baryonic loops :





S. Ghosh & S. Sarkar
Eur. Phys. J. A 49 (2013) 97

P. Muehlich, V. Shklyar, S. Leupold, U. Mosel, M. Post, Nucl. Phys. A **780**, 187 (2006).

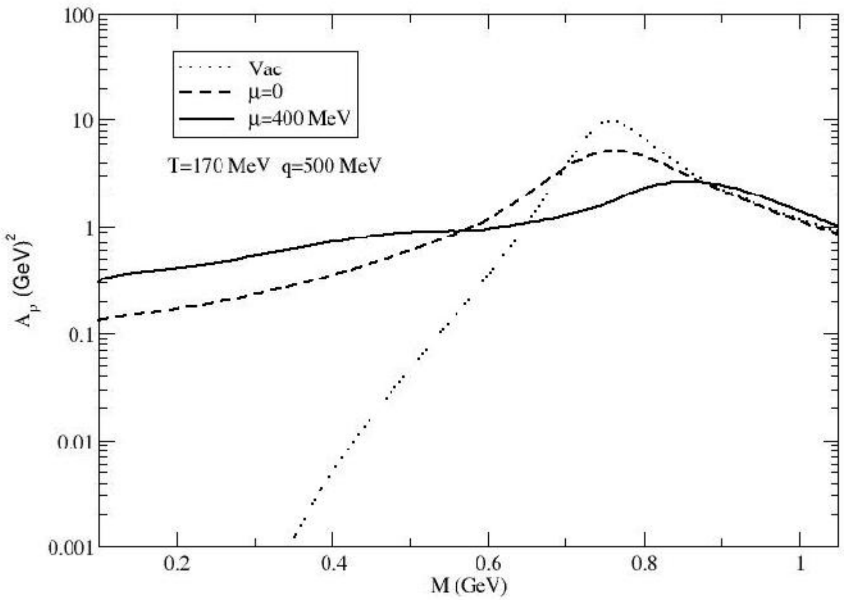
$$\mathcal{L} = -[\overline{\psi}_R(g_1\gamma_\mu - \frac{g_2}{2m_N}\sigma_{\mu\nu}\partial^\nu)\psi_N\omega^\mu + h.c.] \qquad J_R^P = \frac{1}{2}^+$$

$$\mathcal{L} = i[\overline{\psi}_R\gamma^5(g_1\gamma_\mu - \frac{g_2}{2m_N}\sigma_{\mu\nu}\partial^\nu)\psi_N\omega^\mu + h.c.] \qquad J_R^P = \frac{1}{2}^-$$

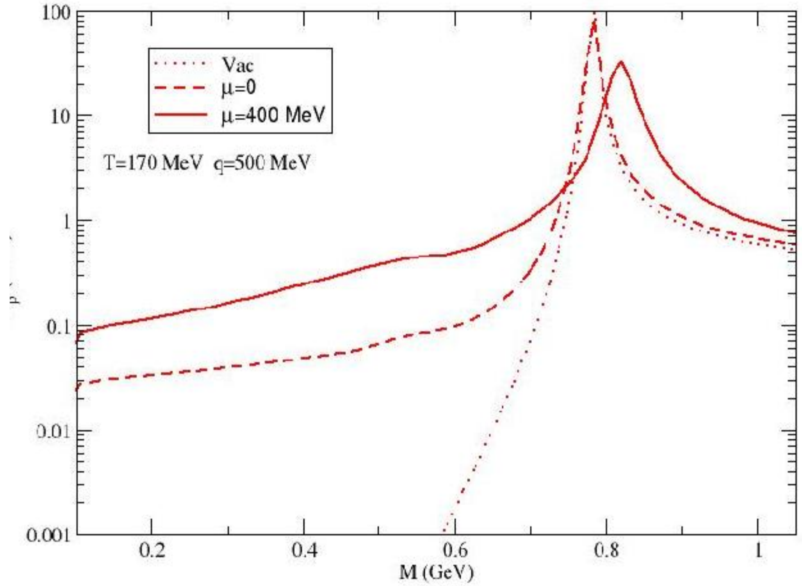
$$\mathcal{L} = -i[\overline{\psi}^\mu_R\gamma^5(\frac{g_1}{2m_N}\gamma^\alpha i\frac{g_2}{4m_N^2}\partial_N^\alpha + i\frac{g_3}{4m_N^2}\partial_\omega^\alpha)(\partial_\alpha^\omega\mathcal{O}_{\mu\nu} - \partial_\mu^\omega\mathcal{O}_{\alpha\nu})\psi_N\omega^\nu + h.c.] \qquad J_R^P = \frac{3}{2}^+$$

$$\mathcal{L} = -[\overline{\psi}^\mu_R(\frac{g_1}{2m_N}\gamma^\alpha i\frac{g_2}{4m_N^2}\partial_N^\alpha + i\frac{g_3}{4m_N^2}\partial_\omega^\alpha)(\partial_\alpha^\omega\mathcal{O}_{\mu\nu} - \partial_\mu^\omega\mathcal{O}_{\alpha\nu})\psi_N\omega^\nu + h.c.] \qquad J_R^P = \frac{3}{2}^-$$

ρ meson spectral function

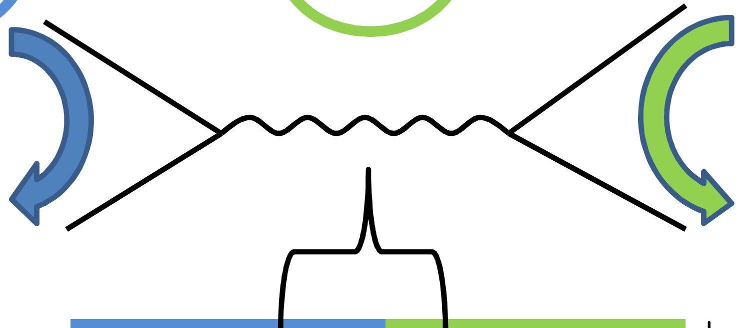


ω meson spectral function



Formalism of dilepton :

$$L_{\text{int}} = e\, J^h_\mu(x)\, A^\mu(x) + e\, J^l_\mu(x)\, A^\mu(x)$$



$$S_{\text{fi}} \sim \left\langle F \left| \int d^4x d^4y\, J^h_\mu(x) A^\mu(x)\, A^\nu(y) J^l_\nu(y) \right| I \right\rangle$$

$$\frac{dN}{d^4x d^4q} = (\ldots)\, L_{\mu\nu} W^{\mu\nu}$$

L. D. McLerran and T. Toimela, Phys. Rev. D **31**, 545 (1985).

H. A. Weldon, Phys. Rev. D **42**, 2384 (1990).

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$$W_{\mu\nu} = 2\epsilon(q_0)F_\rho^2 m_\rho^2 \mathrm{Im}\overline{D}^\rho_{\mu\nu} + 2\epsilon(q_0)F_\omega^2 m_\omega^2 \mathrm{Im}\overline{D}^\omega_{\mu\nu} + \cdots$$

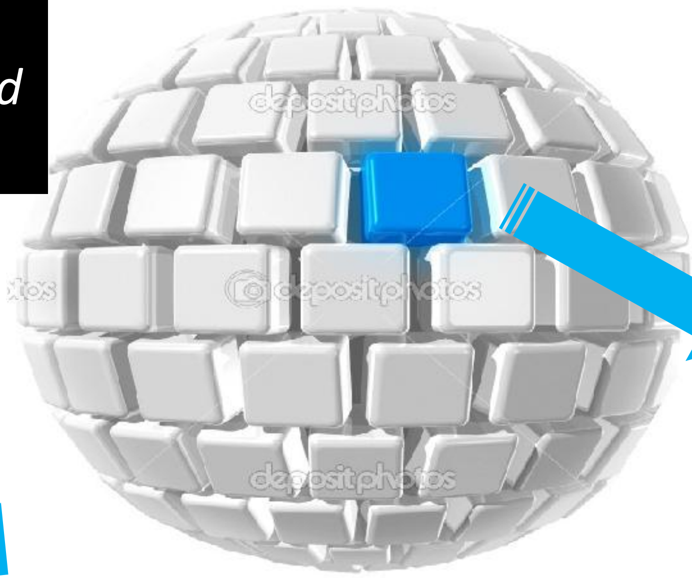
$$\overline{D}_{\mu\nu}(q) = -\frac{P_{\mu\nu}}{q^2 - m_\rho^2 - \overline{\Pi}_t(q)} - \frac{Q_{\mu\nu}/q^2}{q^2 - m_\rho^2 - q^2\overline{\Pi}_l(q)} - \frac{q_\mu q_\nu}{q^2 m_\rho^2}$$

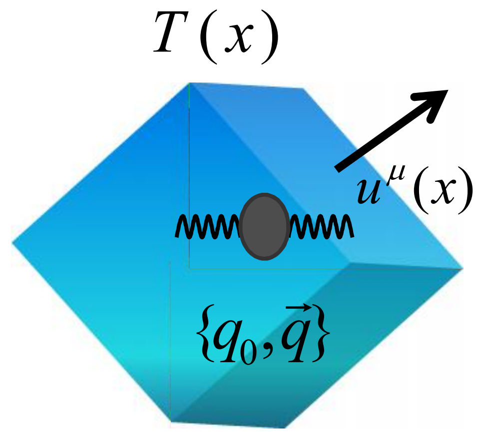
$$F_R^2 = \frac{3m_R \Gamma_{R\to e^+e^-}}{4\pi\alpha^2}$$ F_R=0.156 GeV, 0.046 GeV for ρ, ω

Contribution of ω is down by a factor ~ 10

Dilepton production in transverse momentum and invariant mass space :

$$\frac{dN\{q_0,\vec{q},T\}}{d^4x\,d^4q}$$





Fluid element

$$\frac{dN\{q_\mu u^\mu(x),\vec{q},T(x)\}}{d^4x\,d^4q}\otimes d^4x$$

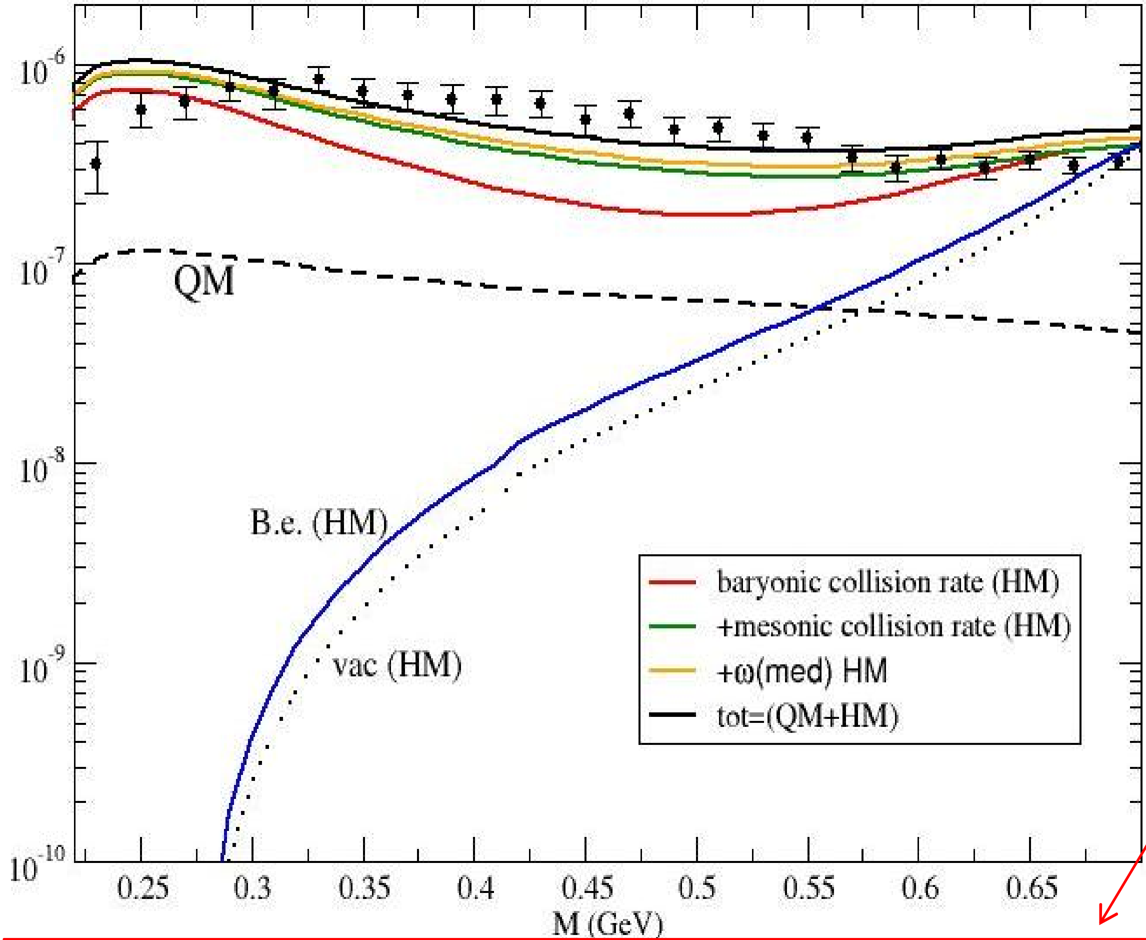
Invariant mass spectra

$$\frac{dN(M)}{dM}=\int d^4x(M\,dy\,d^2\vec{q}_T)\left[\frac{dN\{q_\mu u^\mu(x),\vec{q},T(x)\}}{d^4x\,d^4q}\right]$$

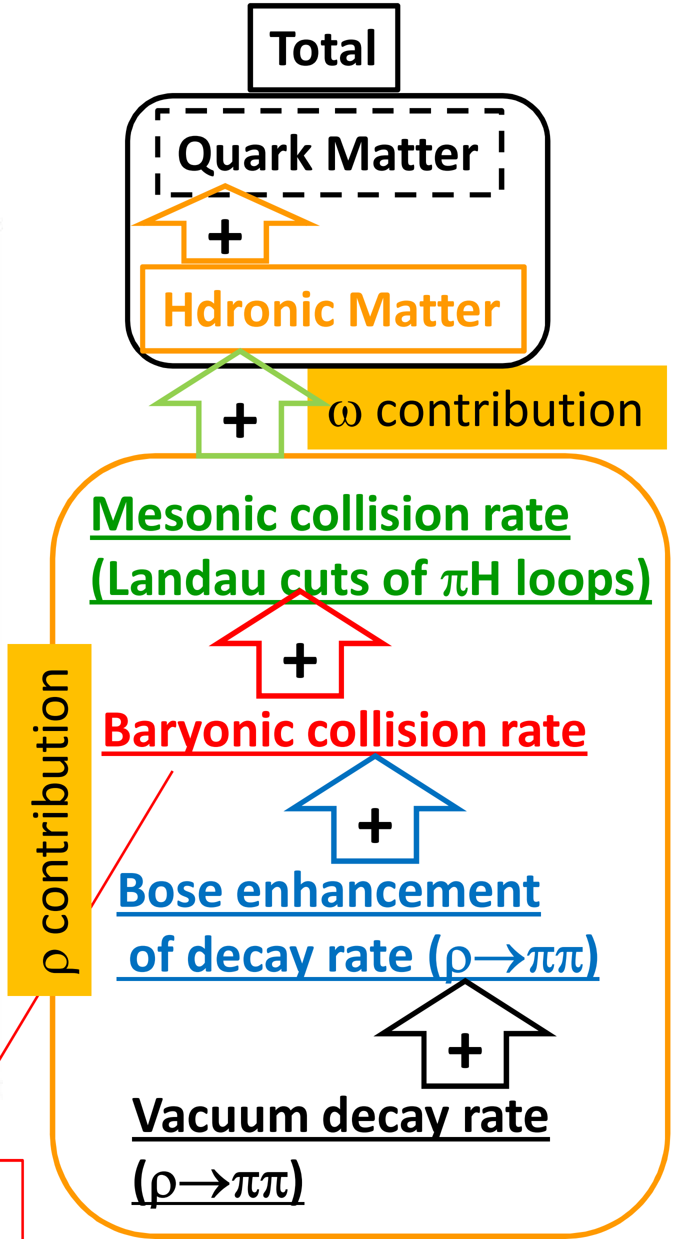
Transverse momentum spectra

$$\frac{dN(\vec{q}_T)}{\vec{q}_T\,d\vec{q}_T}=\int d^4x(2\pi M\,dM\,dy)\left[\frac{dN\{q_\mu u^\mu(x),\vec{q},T(x)\}}{d^4x\,d^4q}\right]$$

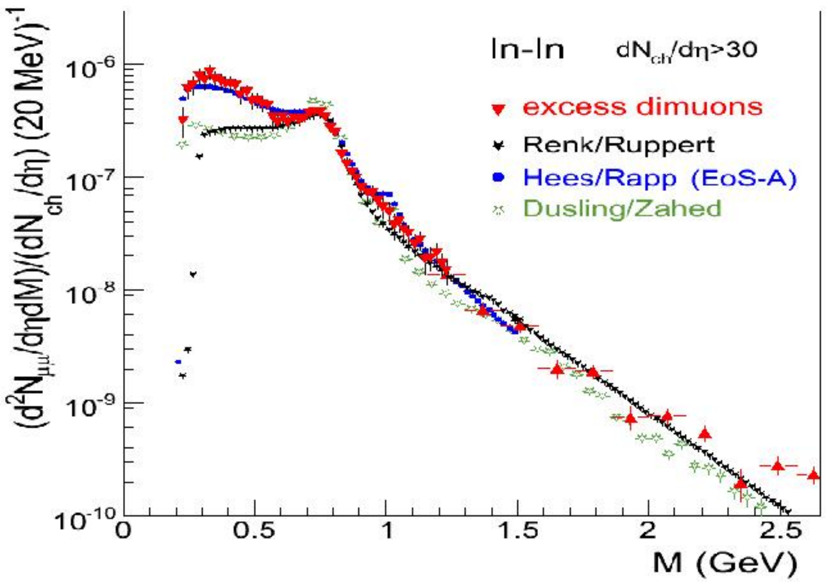
Understanding low mass enhancement in the language of Thermal Field Theory :

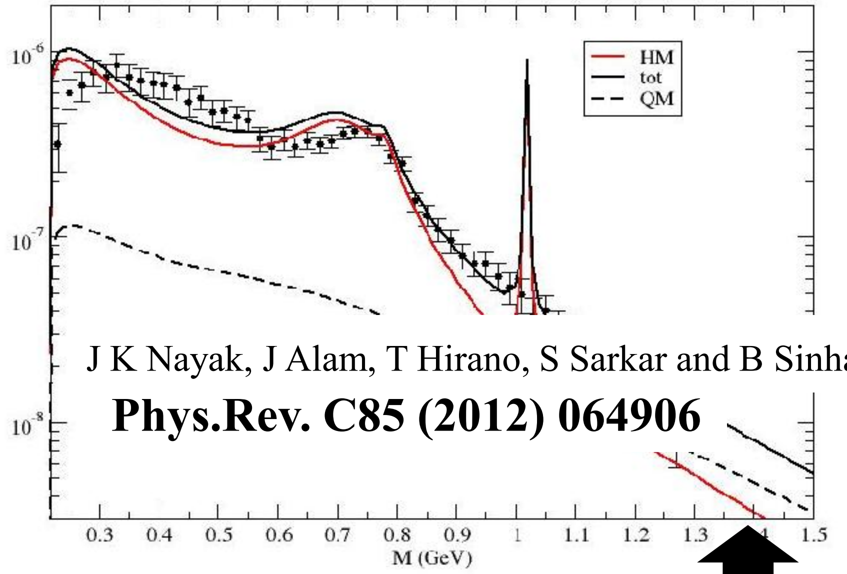


Baryon part from Eltesky et. al. [Phys. Rev. C 64, (2001) 035202]…..not our results of baryonic loops



Low mass enhancement at SPS :





J K Nayak, J Alam, T Hirano, S Sarkar and B Sinha

Phys.Rev. C85 (2012) 064906

Meson loop self-energies
(S.Ghosh, S.Mallik. S.Sarkar
Eur. Phys. C 70, (2010) 251)
+ Baryon part from Eltesky et. al.
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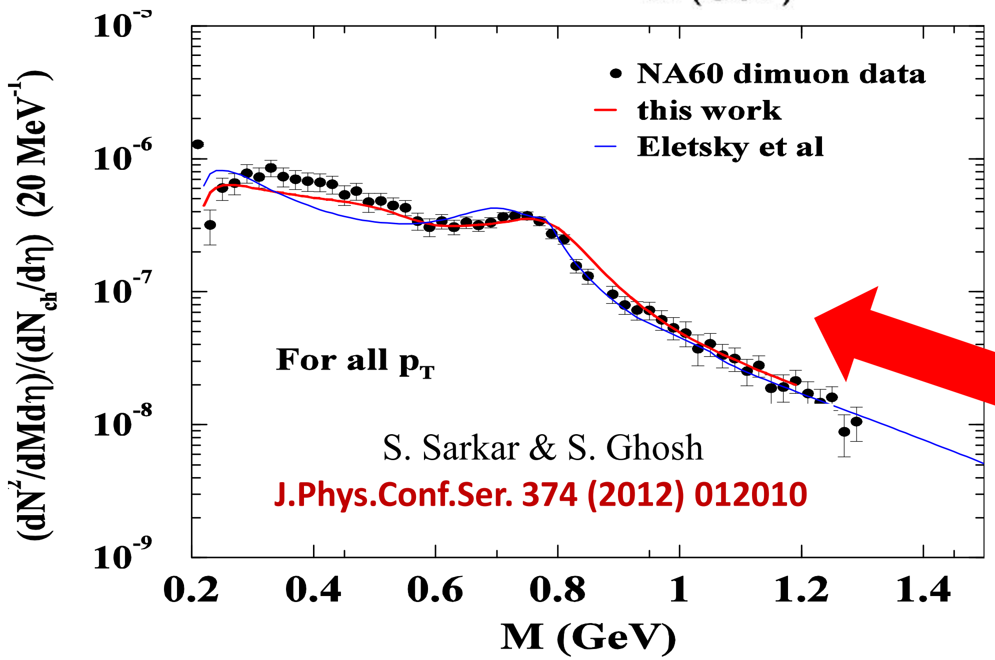

S. Sarkar & S. Ghosh

J.Phys.Conf.Ser. 374 (2012) 012010

Meson (S.Ghosh, S.Mallik. S.Sarkar
Eur. Phys. C 70, (2010) 251)
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Nucl. Phys. A 870, (2011) 94)
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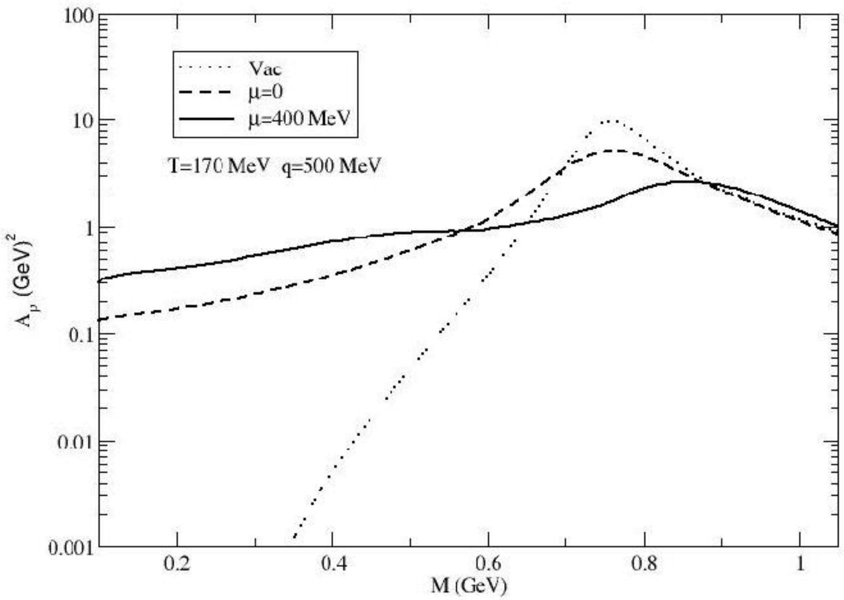

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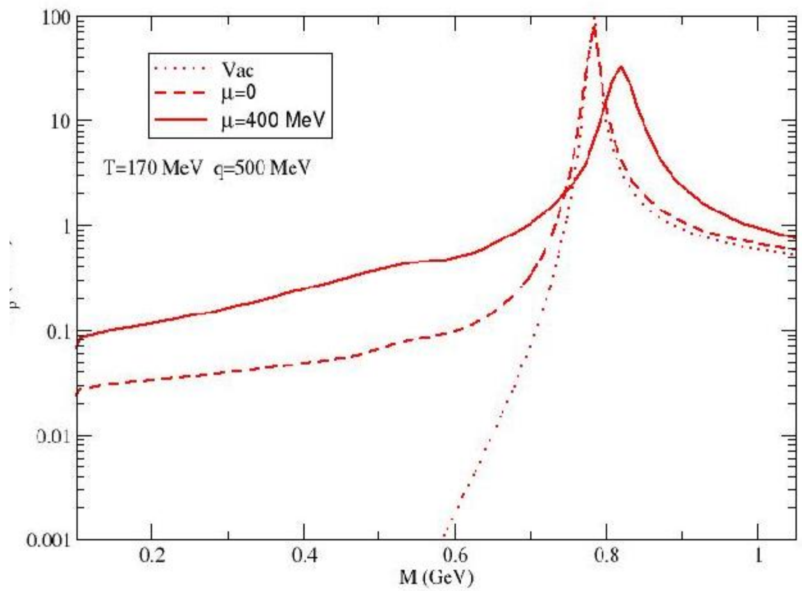

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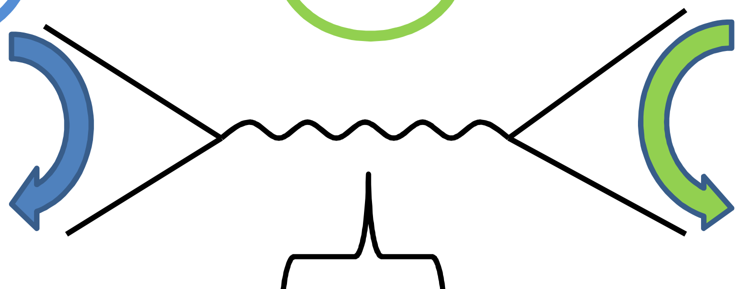

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