

# Microparasitic models II

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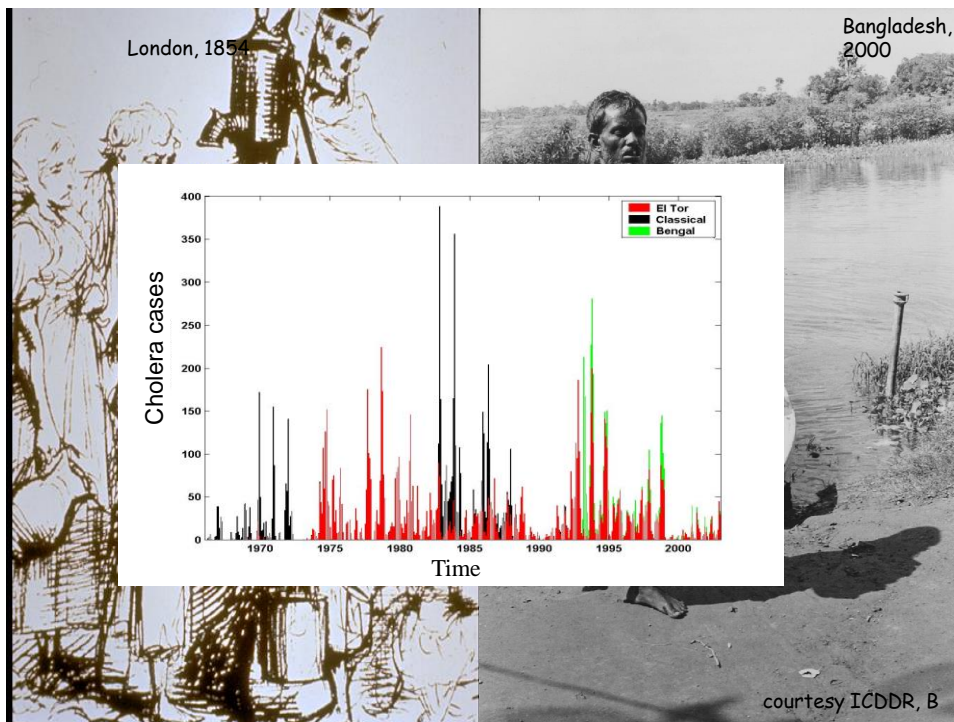
## questions:

- ❖ why do some diseases persist and become endemic?
- ❖ why do some diseases cycle?
- ❖ what leads to complex temporal patterns?

## Syphilis cycles in the US (NATURE highlights)



Repeated epidemics of syphilis across the United States during the past 50 years have followed a roughly ten-year cycle. These fluctuations have been attributed to changes in sexual behavior, but a new analysis, based on case report data collected from 68 cities since 1941, suggests that they are natural oscillations in disease incidence linked to host immunity. (Science Photo Library).



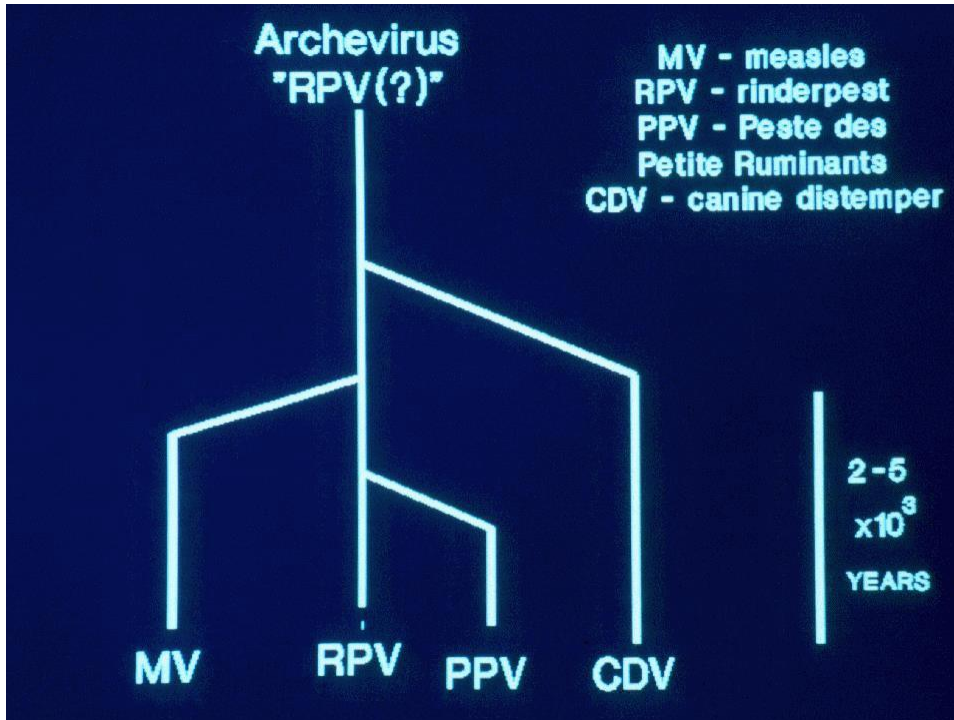


## Measles

- Airborne RNA virus
- Respiratory infection
- Mean latent period: 8 days
- Mean infectious period: 5 days
- Lifelong immunity after recovery
- Easy to diagnose

Still kills ~ 1 million people/year





## Harbour Seals & Phocine Distemper Virus (PDV)



1988: First Record- the best documented mass mortality event of sea mammals (18,000 killed)

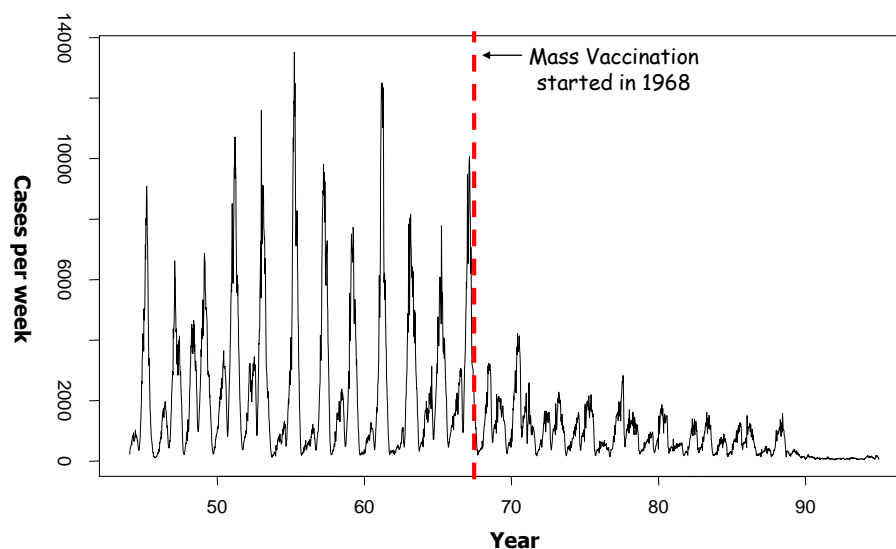
A morbillivirus, like measles  
Spreads through respiratory aerosol

Started in the Wadden sea  
Spread through Baltic & North sea

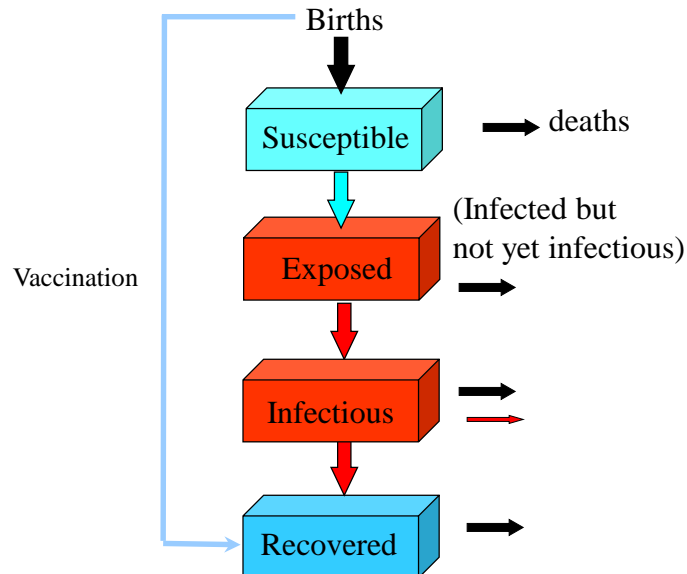
## Why has measles become the 'poster-child' for the population dynamics of infectious diseases?

- >50 years of weekly abundance data
- Data are spatially structured (>1000 spatial locations in England and Wales; e.g. Grenfell et al. 2001, *Nature* 414:716)
- Vaccination campaigns = major manipulative experiments

### Measles in England & Wales 1945-1995



## SEIR Model: Flow Chart



## SEIR Model: Equations

$$\frac{dS}{dt} = \vartheta(1-p) - \cancel{\beta IS} - \mu S$$

$\vartheta, \mu$  = birth, death rates  
 $p$  = proportion vaccinated  
 $\beta$  = mean transmission rate

$$\frac{dE}{dt} = +\cancel{\beta IS} - \cancel{\sigma E} - \mu E$$

$\sigma = 1/(\text{mean latent period})$

$$\frac{dI}{dt} = +\cancel{\sigma E} - \cancel{\gamma I} - \mu I$$

$\gamma = 1/(\text{mean infectious period})$

$$\frac{dR}{dt} = \vartheta p + \cancel{\gamma I} - \mu R$$

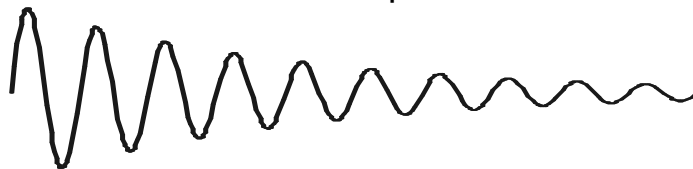
$\vartheta = \mu N, N=1$

## Assumptions:

- Background mortality balances births
- Transmission of infection follows a bilinear contact term  $\beta IS/N \rightarrow$  'mass-action transmission' in a homogeneously-mixed host population
- No disease-induced mortality, just morbidity
- An exponential distribution of incubation and infectious periods
- Recovered individuals are immune from infection for life

## SEIR Model: behavior

- *Endemic equilibrium* when  $R_0(1-p) > 1$
- *Explains persistence* of diseases (via births)
- Equilibrium approached by *damped oscillations*: recurrent epidemics



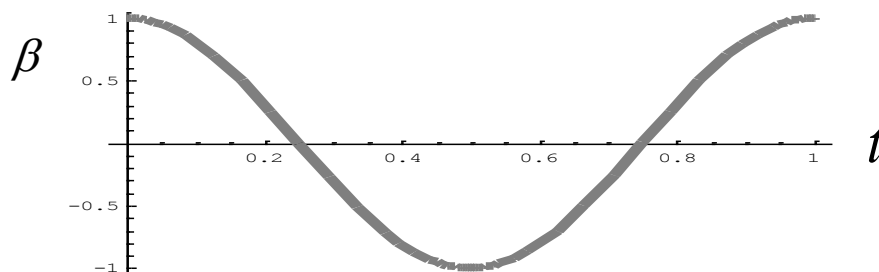
- Does not explain *persistent* oscillations



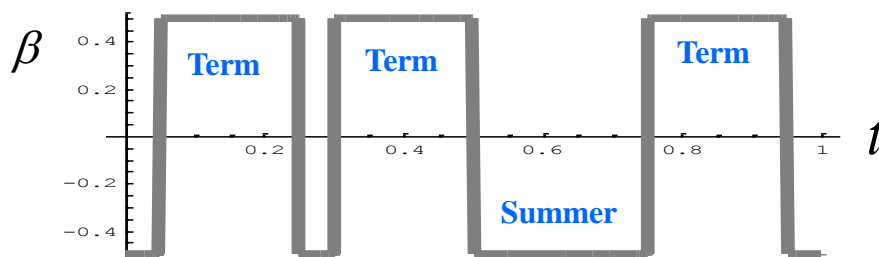
## We need to add seasonal contact rates

- We now have a *forced* system
- Forcing frequency can *resonate* with the natural timescales of the disease (e.g., damping period)
- Very rich dynamical system...  
(analogy: *forced pendulum*)

### Sinusoidal Forcing

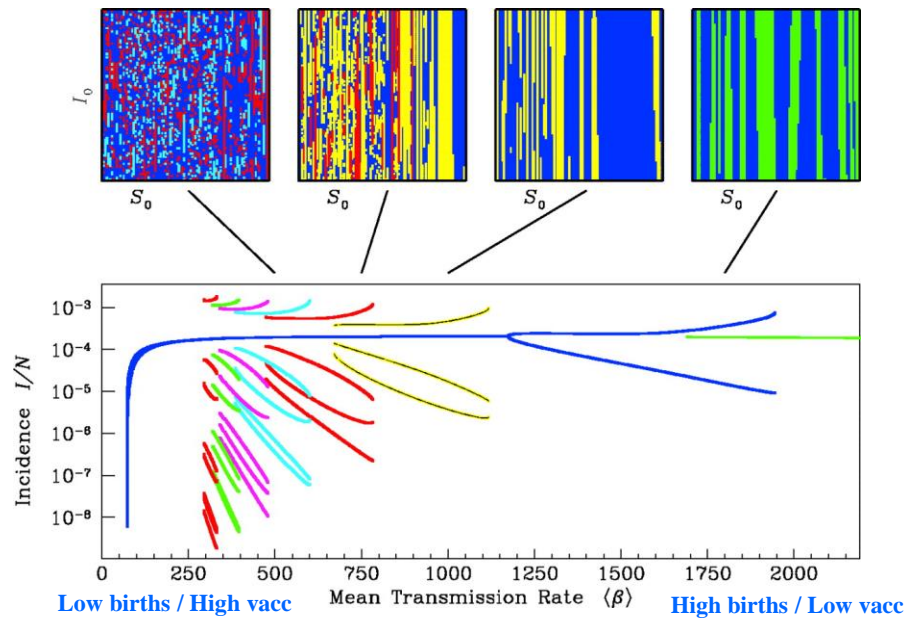


### Term-Time Forcing

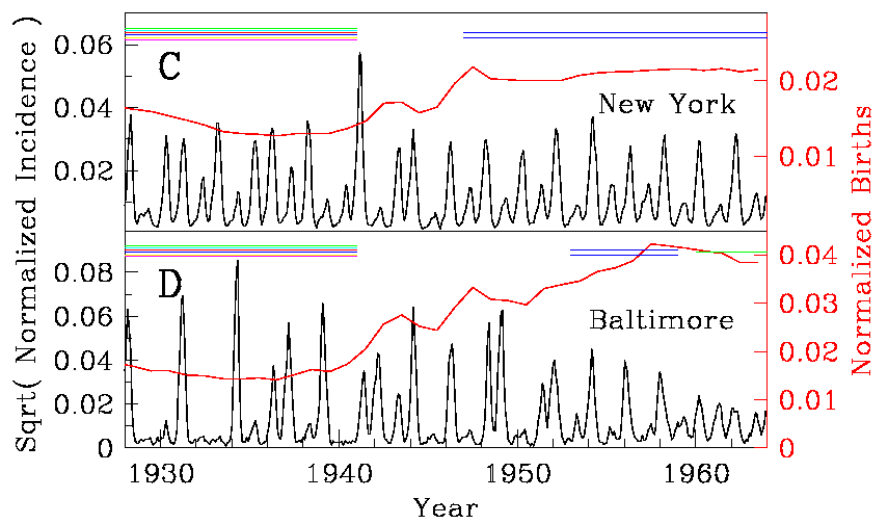


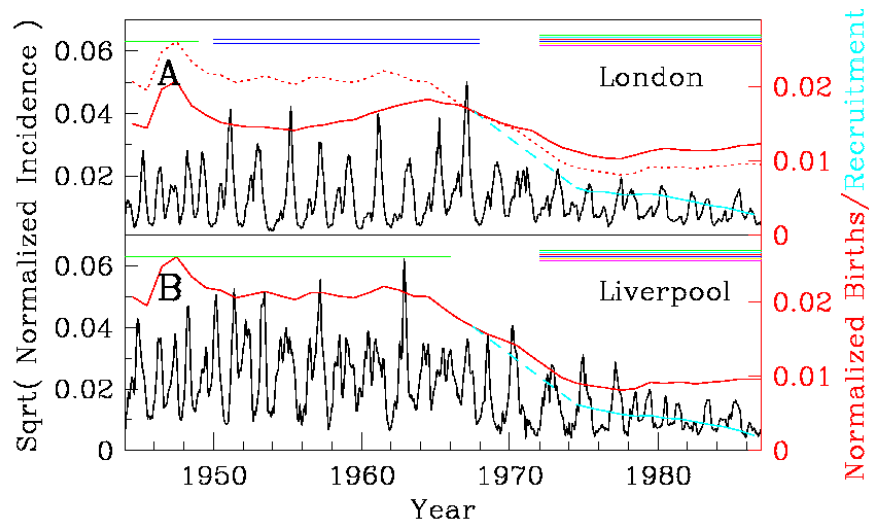


Earn, Rohani, Bolker, Grenfell, *Science* **287**, 667-670 (2000)



## Comparison of data and model predictions





# Measles The Movie

## articles

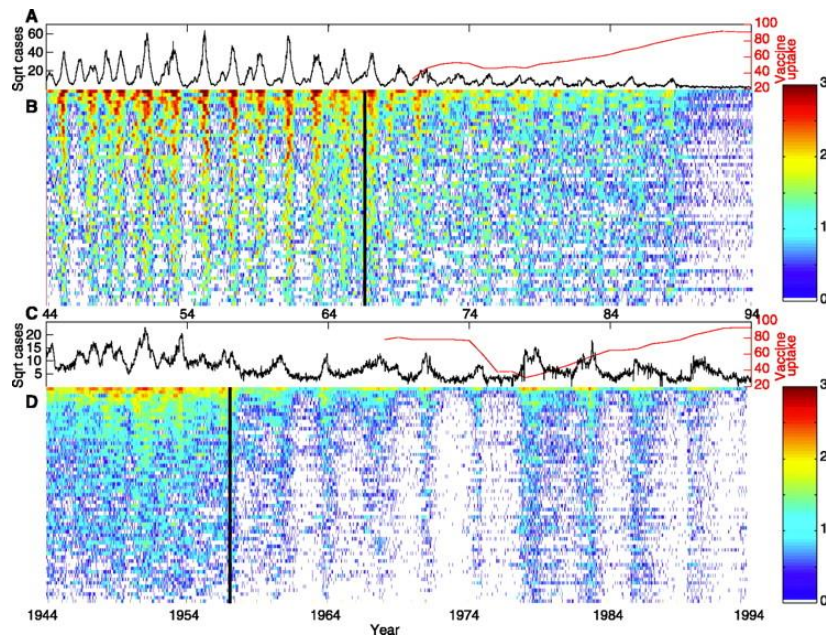
### Travelling waves and spatial hierarchies in measles epidemics

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Rohani, Earn, Grenfell, *Science* **286**, 968-971 (1999)



## References:

- 1) Earn, D.J.D., Rohani, P., Bolker, B.M., and Grenfell, B.T. 2000. A simple model of complex dynamical transitions in epidemics. *Science* 287: 667-670
- 2) Commentary by R. May in same issue