Toward a non-metric theory of gravity

playing around with the (affine) connection!

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Facts to consider:

- General Relativity (GR) has proven to be the most successful theory of gravity.
- It is not renormalizable, but there are problems with the choice of variables to be quantized and the choice of the Hilbert space to be used.
- Sum over all possible field configurations of the metric seems to be wrong (Euclidean + Minkowski).

How do people bypass (some) problems

Developing new alternatives theories:

LQG, Strings, Brans-Dicke, Teleparallel gravity, Modified Newtonian dynamics (MOND), Causal sets, Regge Calculus, Dynamical Triangulations, Penrose Twistor theory, Penrose spin networks, Connes Non–commutative geometry, Causal Fermion Systems…

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Defining our playground

Consider an affine connection $\hat{\Gamma}_{\rho}{}^{\mu}{}_{\sigma}$

- Curvature and Ricci tensor depend on $\hat{\Gamma}$.
- No $\mathcal{R}$ can be constructed (needs a metric).
- Parallel transport is well-possessed.
- Decomposition:

$$\hat{\Gamma}^{\mu}{}_{\rho\sigma} = \Gamma^{\mu}{}_{(\rho\sigma)} + T^{\mu}{}_{[\rho\sigma]} = \Gamma^{\mu}{}_{\rho\sigma} + \epsilon_{\rho\sigma\lambda\kappa} T^{\mu,\lambda\kappa} + A_{[\rho}\delta^{\mu}_{\nu]}$$

- We consider:
  - a power-counting renormalizable, and
  - diffeomorphism invariant model.

Three-dimensional model

$$S[\Gamma, T, A] = \int \mathrm{d}^3x \left(A_1 R_{\mu\nu}{}^{\mu}{}_{\rho} T^{\nu\rho} + A_2 \epsilon^{\mu\nu\rho} R_{\mu\nu}{}^{\sigma}{}_{\sigma} A_{\rho} \right.$$
$$+ A_3 \epsilon^{\mu\nu\rho} A_{\mu} \partial_{\nu} A_{\rho} + A_4 T^{\mu\nu} \nabla_{\mu} A_{\nu} + A_5 T^{\mu\nu} A_{\mu} A_{\nu}$$
$$+ A_6 \det(T^{\mu\nu}) + A_7 \epsilon^{\mu\nu\lambda} \Big(\Gamma^{\sigma}{}_{\mu\rho} \partial_{\nu} \Gamma^{\rho}{}_{\lambda\sigma}$$
$$\left. + \frac{2}{3} \Gamma^{\tau}{}_{\mu\rho} \Gamma^{\rho}{}_{\nu\sigma} \Gamma^{\sigma}{}_{\lambda\tau} \Big) + A_8 \epsilon^{\mu\nu\rho} \Gamma^{\sigma}{}_{\mu\sigma} \partial_{\nu} \Gamma^{\tau}{}_{\rho\tau} \right).$$

Eddington's trick

Identify

$$\frac{\delta}{\delta R_{(\mu\nu)}} S[\Gamma] \equiv \sqrt{g} g^{\mu\nu}$$

$$S[g, \Gamma, A] = \int \mathrm{d}^3x \left(\sqrt{g} \Big(A_1 R + A_4 \nabla^{\mu} A_{\mu} \right.$$
$$+ A_5 A_{\mu} A^{\mu} + A_6 \Big) + A_2 \epsilon^{\mu\nu\rho} R_{\mu\nu}{}^{\sigma}{}_{\sigma} A_{\rho}$$
$$\left. + A_3 \mathscr{L}_{\mathrm{CS}_v} + A_7 \mathscr{L}_{\mathrm{CS}_{na}} + A_8 \mathscr{L}_{\mathrm{CS}_a} \right)$$

And not! Something completely different.

$$
\begin{aligned}
S[\Gamma, T, A] = \int \mathrm{d}^4x \Big[& B_1 R_{\mu\nu}{}^{\mu}{}_{\rho} T^{\nu,\alpha\beta} T^{\rho,\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} + B_2 R_{\mu\nu}{}^{\sigma}{}_{\rho} T^{\beta,\mu\nu} T^{\rho,\gamma\delta} \epsilon_{\sigma\beta\gamma\delta} \\
& + B_3 R_{\mu\nu}{}^{\mu}{}_{\rho} T^{\nu,\rho\sigma} A_\sigma + B_4 R_{\mu\nu}{}^{\sigma}{}_{\rho} T^{\rho,\mu\nu} A_\sigma + B_5 R_{\mu\nu}{}^{\rho}{}_{\rho} T^{\sigma,\mu\nu} A_\sigma \\
& + C_1 R_{\mu\rho}{}^{\mu}{}_{\nu} \nabla_\sigma T^{\nu,\rho\sigma} + C_2 R_{\mu\nu}{}^{\rho}{}_{\rho} \nabla_\sigma T^{\sigma,\mu\nu} + D_1 T^{\alpha,\mu\nu} T^{\beta,\rho\sigma} \nabla_\gamma T^{(\lambda,\kappa)\gamma} \epsilon_{\beta\mu\nu\lambda} \epsilon_{\alpha\rho\sigma\kappa} \\
& + D_2 T^{\alpha,\mu\nu} T^{\lambda,\beta\gamma} \nabla_\lambda T^{\delta,\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\rho\sigma} + D_3 T^{\mu,\alpha\beta} T^{\lambda,\nu\gamma} \nabla_\lambda T^{\delta,\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\rho\sigma} \\
& + D_4 T^{\lambda,\mu\nu} T^{\kappa,\rho\sigma} \nabla_{(\lambda} A_{\kappa)} \epsilon_{\mu\nu\rho\sigma} + D_5 T^{\lambda,\mu\nu} \nabla_{[\lambda} T^{\kappa,\rho\sigma} A_{\kappa]} \epsilon_{\mu\nu\rho\sigma} + D_6 T^{\lambda,\mu\nu} A_\nu \nabla_{(\lambda} A_{\mu)} \\
& + D_7 T^{\lambda,\mu\nu} A_\lambda \nabla_{[\mu} A_{\nu]} + E_1 \nabla_{(\rho} T^{\rho,\mu\nu} \nabla_{\sigma)} T^{\sigma,\lambda\kappa} \epsilon_{\mu\nu\lambda\kappa} + E_2 \nabla_{(\lambda} T^{\lambda,\mu\nu} \nabla_{\mu)} A_\nu \\
& + T^{\alpha,\beta\gamma} T^{\delta,\eta\kappa} T^{\lambda,\mu\nu} T^{\rho,\sigma\tau} (F_1 \epsilon_{\beta\gamma\eta\kappa} \epsilon_{\alpha\rho\mu\nu} \epsilon_{\delta\lambda\sigma\tau} + F_2 \epsilon_{\beta\lambda\eta\kappa} \epsilon_{\gamma\rho\mu\nu} \epsilon_{\alpha\delta\sigma\tau}) \\
& + F_3 T^{\rho,\alpha\beta} T^{\gamma,\mu\nu} T^{\lambda,\sigma\tau} A_\tau \epsilon_{\alpha\beta\gamma\lambda} \epsilon_{\mu\nu\rho\sigma} + F_4 T^{\eta,\alpha\beta} T^{\kappa,\gamma\delta} A_\eta A_\kappa \epsilon_{\alpha\beta\gamma\delta} \Big]
\end{aligned}
$$

Up to a boundary term!

What does this action say?

- Scalar perturbations around a static, homogeneous and isotropic solution
  - Modifies the geodesic
  - Effective potential is Keplerian!!

  This is a general result!!!
- Contact with GR:
  - Take the limit ${T^\mu}_{\lambda\rho} \to 0$ (at EOM level).
  - The limit is a consistent truncation.
  - Only an EOM remains

$$C_1 \nabla_{[\sigma} {R_{\rho]\mu}}^{\mu}{}_{\nu} - C_2 \nabla_{\nu} {R_{\rho\sigma}}^{\mu}{}_{\mu} = 0$$

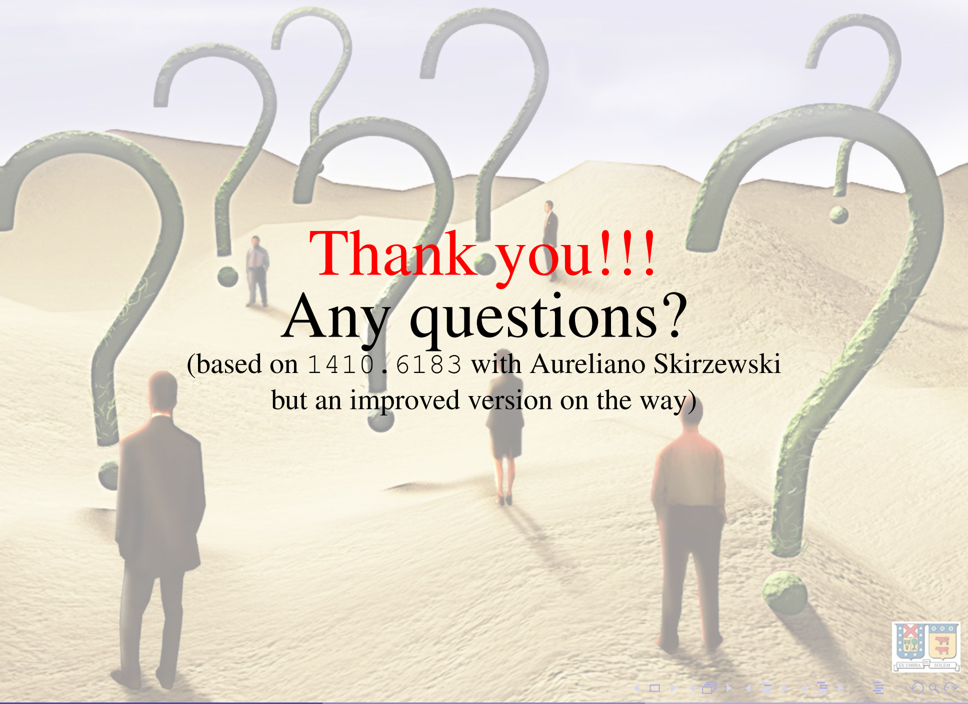
$$\nabla_{[\sigma} R_{\rho]\nu} = 0.$$

Only valid for vanishing torsion!!!

The vanishing torsion case

$$\nabla_{[\sigma}R_{\rho]\nu} = 0.$$

- It is a generalisation of Einstein's equation
  - $R_{\mu\nu} = 0$.
  - $R_{\mu\nu} \propto g_{\mu\nu}$, with or without Λ.
  - But $R_{\mu\nu} \neq R_{\mu\nu}(g)$.
  - We would like to study the physics in the new kind of solutions.
- Currently:
  - Analysing the "duality" provided by the Eddington trick.
  - Counting the number of degrees of freedom (new method `1406.1156`).
  - Coupling matter to gravity (EOM from `1411.2424`).



Thank you!!!

Any questions?

(based on `1410.6183` with Aureliano Skirzewski
but an improved version on the way)