Higher Spin Gravity and Entanglement Entropy

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Introduction and Motivation

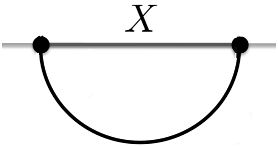
Higher spin theories are extensions of pure gravity coupled to an infinite tower of massless spin fields. They are formulated in an AdS background, and they have highly non linear equations of motion.

- Why Higher Spin Gravity?
  - AdS/CFT
- Why 2+1 dimensions?
  - No propagating degrees of freedom.
  - Chern Simons theories $SL(N,\mathbb{R}) \times SL(N,\mathbb{R})$

Introduction and Motivation

- Why Entanglement Entropy?

$$S_{EE} = \frac{L_X}{4\,G_3}$$



No geometry for higher spin theories → Ryu-Takayanagi proposal not valid

Could we find an emergent "geometry" for higher spin theories from entanglement entropy?

Gravity as Chern-Simons theory

$$S_{CS}[\mathcal{A}] = \frac{k}{4\pi} \int \mathrm{Tr} \left(\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \right)$$

- $G = SL(2, \mathbb{R}) \times SL(2, \mathbb{R})\,, \quad A, \bar{A} \in SL(2, \mathbb{R})$:

  Pure gravity is Chern-Simons

$$S_{EH} = S_{CS}[A] - S_{CS}[\bar{A}]$$

- $G = SL(N, \mathbb{R}) \times SL(N, \mathbb{R})\,, \quad A, \bar{A} \in SL(N, \mathbb{R})$:

  Chern-Simons is higher spin gravity, with a truncated tower of higher spins $(s = 2, 3, \ldots N)$

Holographic Entanglement entropy and Wilson lines

- Wilson line is entanglement entropy for pure gravity $(A, \bar{A} \in SL(2, \mathbb{R}))$ [Ammon-Castro-Iqbal '13]:

$$W_{\mathcal{R}}(f, i) = \mathrm{Tr}_{\mathcal{R}} \left(\mathcal{P} \exp \int_i^f (A + \bar{A}) \right) \quad \rightarrow \quad \text{lenght geodesic = EE}$$

- CONJECTURE: Wilson line is EE for higher spin $(A, \bar{A} \in SL(N, \mathbb{R}))$:

  CHECKS:

  - Wilson loop reproduces thermal entropy of higher spin black holes.
  - Results for specific higher spin background expected from CFT.

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Infinite-Dim Highest-Weight

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The problem of the two proposals

[deBoer-Jottar '13]	[Ammon-Castro-Iqbal '13]
$S_{EE} \sim k \log \left(\lim_{\rho_0 \to \infty} W_{\mathcal{R}}^{comp}(f,i) \right)$	$S_{EE} = -\log \left(W_{\mathcal{R}}(f,i) \right)$
$W_{\mathcal{R}}^{comp}(f,i) = \mathrm{Tr}_{\mathcal{R}} \left[\mathcal{P} \exp \left(\int_i^f \bar{A} \right) \mathcal{P} \exp \left(\int_f^i A \right) \right]$	$W_{\mathcal{R}}(f,i) = \mathrm{Tr}_{\mathcal{R}} \left(\mathcal{P} \exp \int_i^f (A + \bar{A}) \right)$

THE TWO PROPOSALS ARE THE SAME!!!

Castro-Llabres '14 [arXiv:1410.2870 [hep-th]]

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$2^{N(N-1)/2}$ dimensional

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Wilson line

Parallel transport operator:

$$\mathcal{P}\exp\left(\int_i^f dx^\mu \mathcal{A}_\mu\right)$$

Gauge transfomation:

$$\mathcal{P}\exp\left(\int_i^f dx^\mu \mathcal{A}_\mu\right) \xrightarrow[\mathcal{A}\to G^{-1}(\mathcal{A}+\mathrm{d})G]{} G^{-1}(f)\mathcal{P}\exp\left(\int_i^f dx^\mu \mathcal{A}_\mu\right) G(i)$$

Wilson line:

$$W_{\mathcal{R}}(C_{if}) = \mathrm{Tr}_{\mathcal{R}}\left[\mathcal{P}\exp\left(\int_i^f dx^\mu \mathcal{A}_\mu\right)\right]$$