

IV Southern-Summer School on Mathematical Biology



Eutrophication and parasitic infection of amphibians

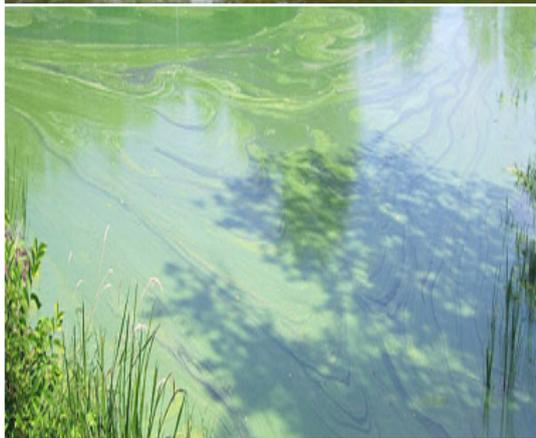
Group3

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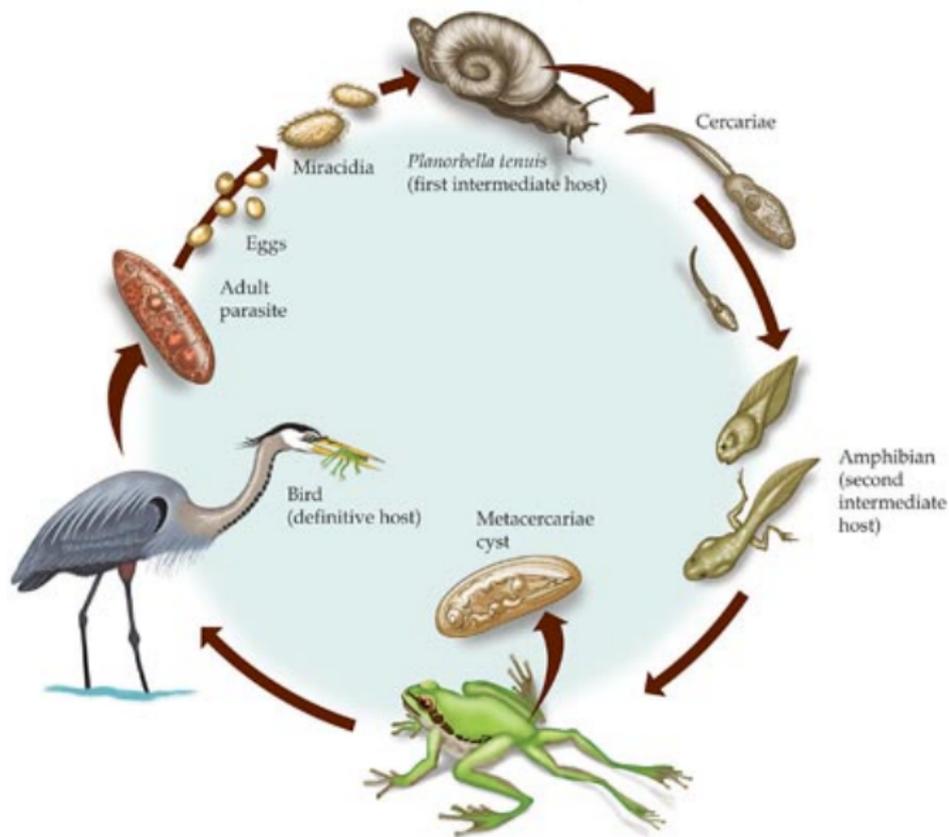
ICTP-SAIFR (SSMB)

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What is eutrophication?



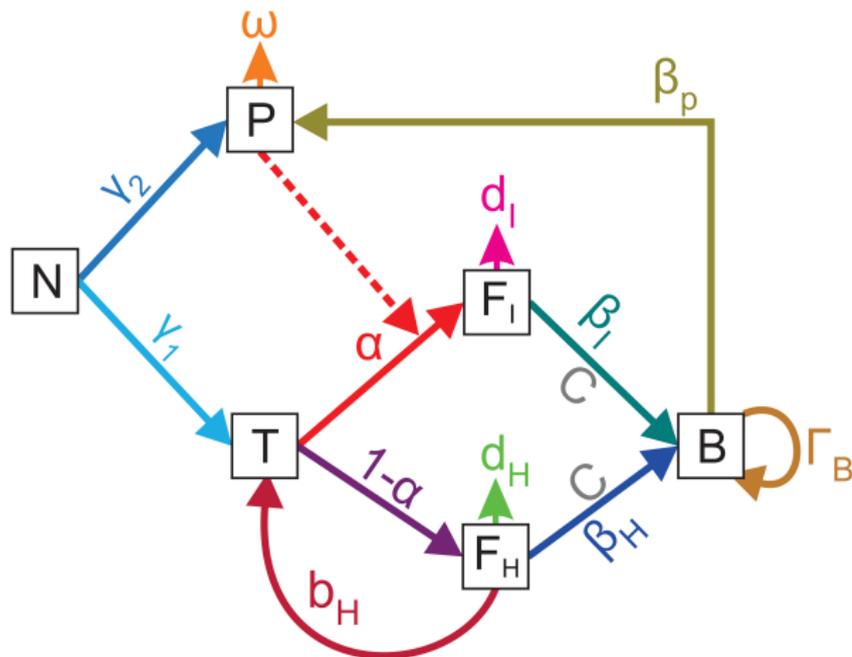
Infection cycle - *Ribeiroia ondatrae*



Our question

- Does eutrophication leads to frog extinction mediated by increasing prevalence of *R. ondatrae* infection?
- Objective: explore the dynamics of frog population in response to eutrophication.

Interaction diagram



Populations

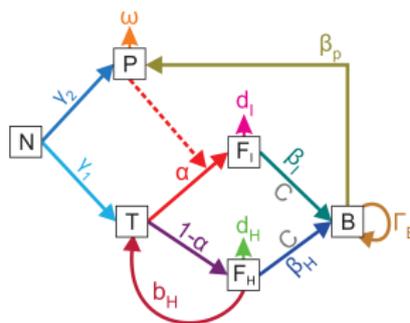
Population	Description
$B(t)$	Number of birds.
$P(t)$	Number of parasites.
$T(t)$	Number of tadpoles.
$F_H(t)$	Number of health frogs.
$F_I(t)$	Number of infected frogs.

Parameters

Summary of parameters that are used in the model.

Parameter	Description
d_H	Mortality rate in health frogs.
d_I	Mortality rate in infected frogs.
b_H	Birth rate of tadpoles produced by health frogs.
β_P	Rate of parasites shedded by birds.
β_I	Predation rate of infected frogs by birds.
β_H	Predation rate of health frogs by birds.
α	Infection rate.
γ_1	Rate of incorporation of N by tadpoles.
γ_2	Rate of incorporation of N by parasite.
ω	Mortality rate of parasite.
Γ_b	Survival rate of birds.
K	Carrying capacity for birds.
N	Amount of nutrients.
C	Conversion rate.

Equations



$$\frac{dB(t)}{dt} = \Gamma_B \left(1 - \frac{B}{K}\right) + C(\beta_H F_H + \beta_I F_I)B$$

$$\frac{dP(t)}{dt} = (\beta_p \gamma_2 N)B \frac{F_I}{F_I + F_H} - \left(\frac{\mu P}{1 + \mu P}\right)T - \omega P$$

$$\frac{dT(t)}{dt} = (b_H \gamma_1 N)F_H - T$$

$$\frac{dF_H(t)}{dt} = \left(\frac{1}{1 + \mu P}\right)T - (d_H + \beta_H B)F_H$$

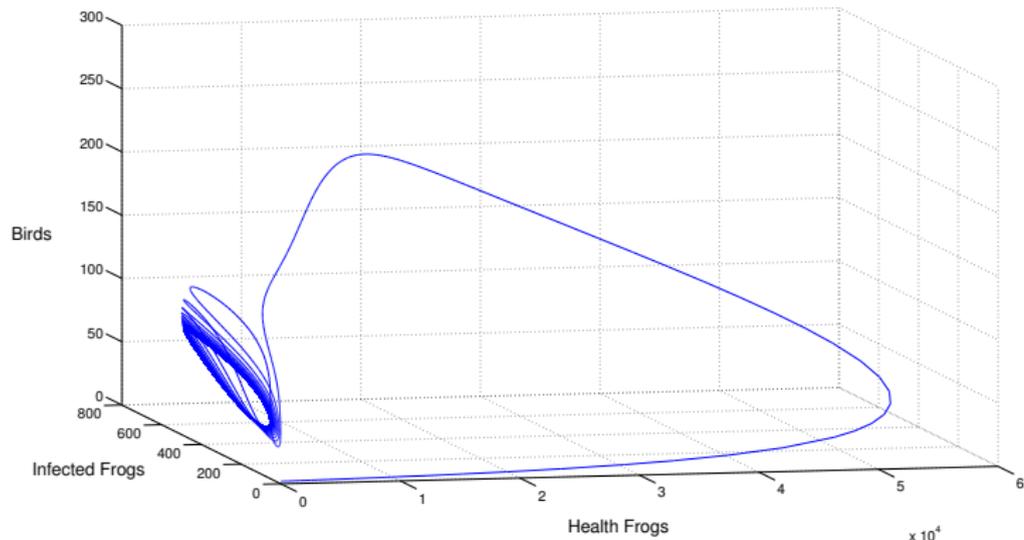
$$\frac{dF_I(t)}{dt} = \left(\frac{\mu P}{1 + \mu P}\right)T - (d_I + \beta_I B)F_I$$

Fixed points with B constant approximation

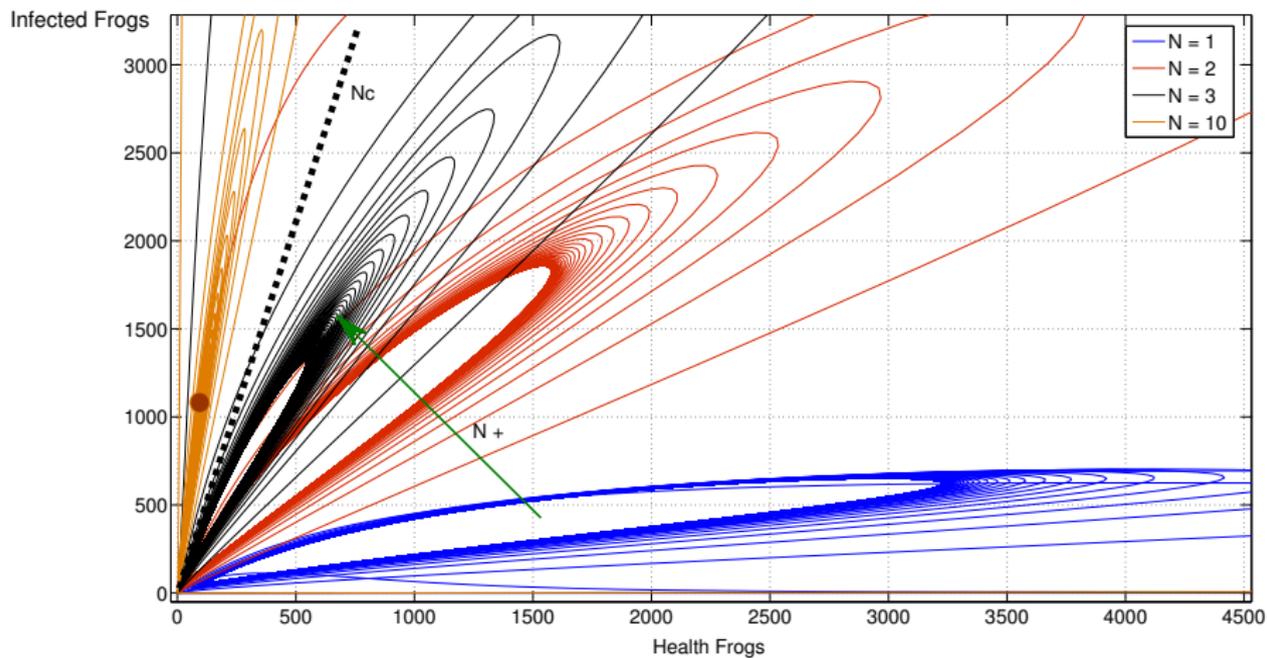
$$\begin{aligned}P^* &= \frac{1}{\mu} \left(\frac{b_H \gamma_1 N}{d_H + \beta_H B} - 1 \right) \\T^* &= b_H \gamma_1 N F_H^* \\F_I^* &= \frac{b_H \gamma_1 N \mu P^* F_H^*}{(d_I + \beta_I B)(1 + \mu P^*)} \\F_H^* &= \frac{\beta_p \gamma_2 N B \left(\frac{1+a}{a} \right) - \omega P^*}{\left(\frac{\mu P^*}{1 + \mu P^*} \right) b_H \gamma_1 N}\end{aligned}$$

Where $a = \frac{b_H \gamma_1 N \mu P^*}{(d_I + \beta_I B)(1 + \mu P^*)}$

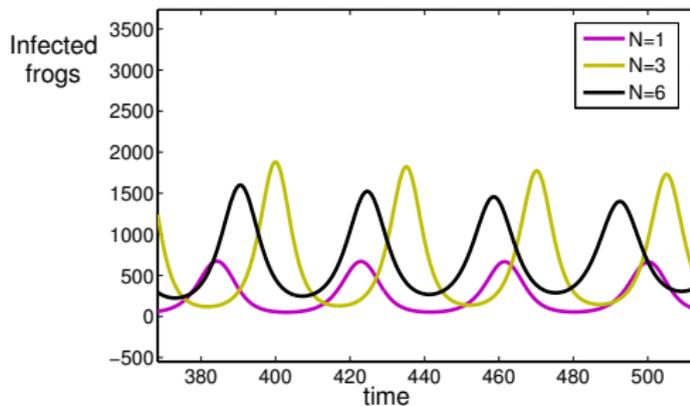
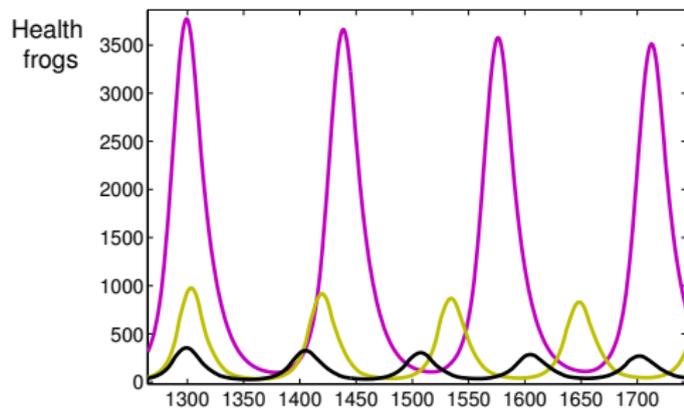
Results



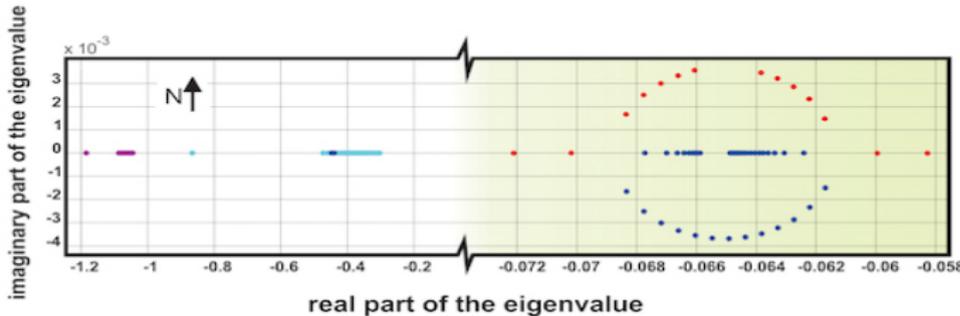
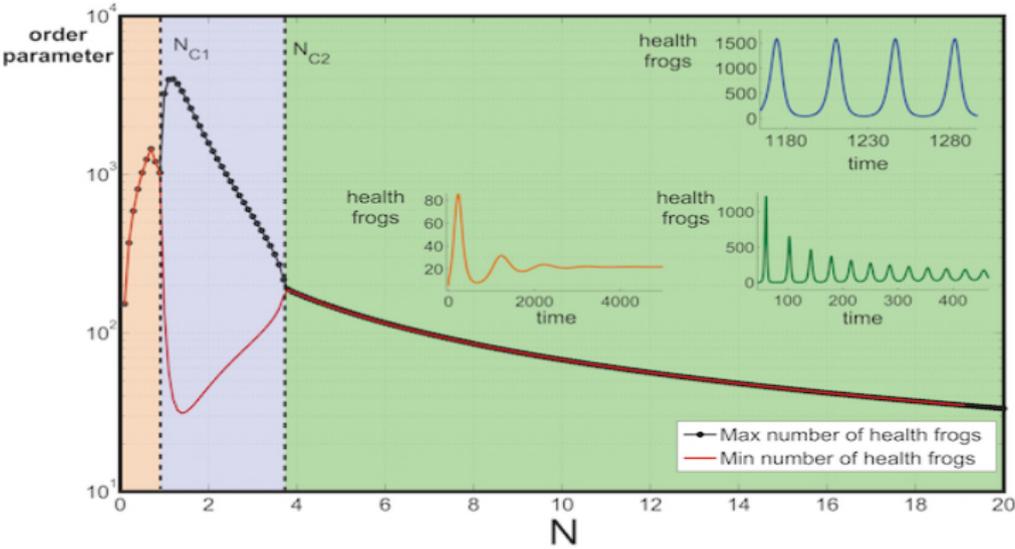
Results



Results



Results

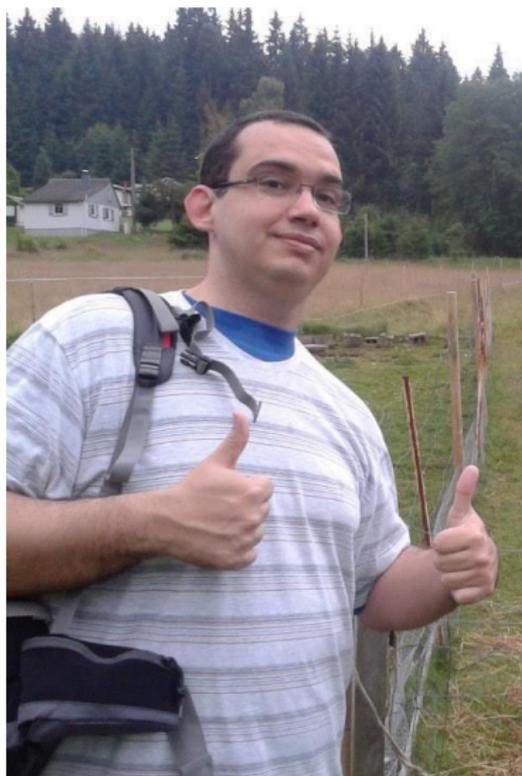


Conclusion

References



Johnson, P. T., Chase, J. M., Dosch, K. L., Hartson, R. B., Gross, J. A., Larson, D. J. *Aquatic eutrophication promotes pathogenic infection in amphibians*, Proceedings of the National Academy of Sciences, 104(40), 15781-15786, Carpenter, S. R. (2007).



Thanks for your attention!



Questions?



