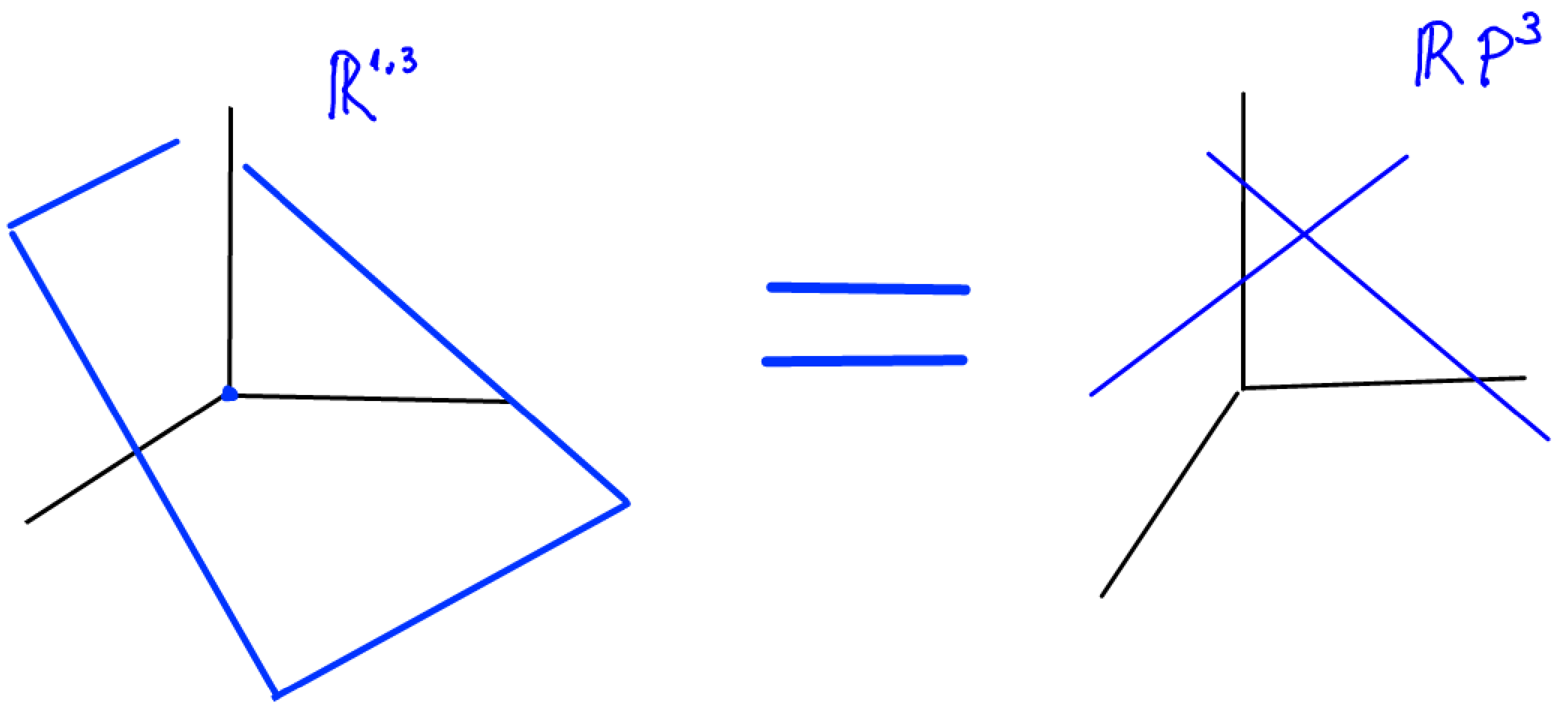
Quantum Mechanics of the Grassmannian Gr(2,4) & a theory of Distributions



based on

Spacetime mechanics:
- Quantum causal structure and expansive force arxiv:1501.03644

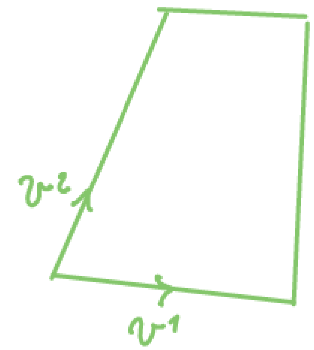
Spacetime mechanics:
- A theory of distributions. In progress.

Mauricio Valenzuela
Universidad Austral de Chile

ICTP-SAIFR 15.02.05

Plücker coordinates of Gr(2,4)

$$Z^{\mu\nu} = -Z^{\nu\mu} = (v^1)^\mu (v^2)^\nu - (v^2)^\mu (v^1)^\nu$$



Plücker Constraint $\boxed{\epsilon_{\mu\nu\lambda\rho} Z^{\mu\nu} Z^{\lambda\rho}}$

Quadric in $M_6 \ni \left\{ \begin{array}{l} (Z^{01}, Z^{02}, Z^{03}) = \vec{Y} \\ (Z^{12}, Z^{23}, Z^{31}) = \vec{Z} \end{array} \right\}$

We can find X^μ such that $\boxed{\epsilon_{\mu\nu\lambda\rho} X^\nu Z^{\lambda\rho} = 0}$

Quadric in $M_{10} \ni \{\underbrace{X^0, \vec{X}}_{X^\mu}, \vec{Y}, \vec{Z}\} = X^I$

in Subset of $\mathbb{R}^{10}$

P^9

Gr(2,4) is the space of light-rays

"Twistor Phase-space": parametrization

$$X^\mu = \frac{1}{4}\, \mathcal{L}^\alpha (\gamma^\mu)_\alpha{}^\beta \mathcal{L}_\beta$$

$$Z^{\mu\nu} = \frac{1}{2}\, \mathcal{L}^\alpha ([\gamma^\mu, \gamma^\nu])_\alpha{}^\beta \mathcal{L}_\beta$$

; $\mathcal{L}_\alpha$ = Real

$(\gamma^\mu)_\alpha{}^\beta$ = Real

$$\boxed{\mathcal{L}_\alpha = \begin{pmatrix} q_1 \\ q_2 \\ p_1 \\ p_2 \end{pmatrix}, \quad C_{\alpha\beta} = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ -\mathbb{1}_{2\times 2} & 0 \end{pmatrix}}$$

= Phase space

$$\vec X \perp \vec Y \perp \vec Z$$

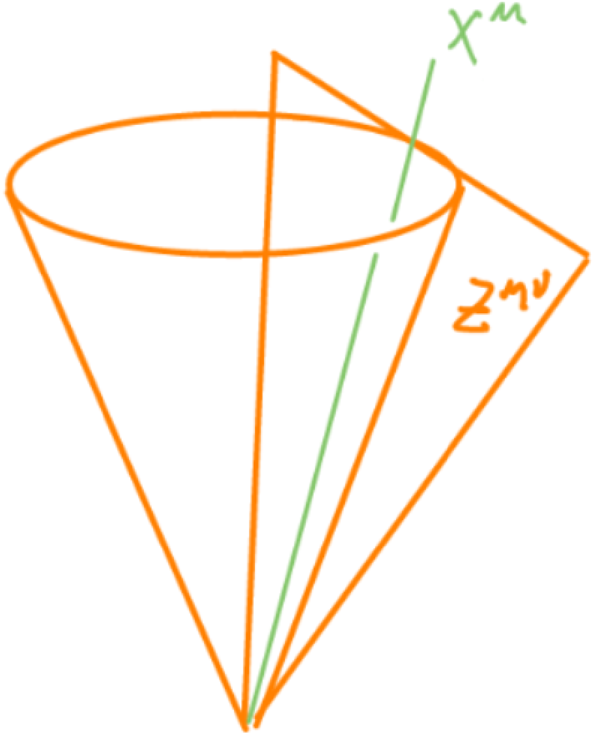
$$\vec X \times \vec Y = X^0 \vec Z, \qquad \vec X \cdot \vec X = (X^0)^2 ;$$

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[r] & 3 \arrow[ll, bend right] \end{tikzcd}$$

$$\to \left\{ \mathbb{1}, \frac{\vec X}{X^0}, \frac{\vec Y}{X^0}, \frac{\vec Z}{X^0} \right\} \in PM_4$$

Cf. Biot-Savart

$$\boxed{\vec B = \vec I \times \vec E \;;\quad \vec B \to \vec I \to \vec E}$$



Mechanical Observable $\longrightarrow$ Classical / Quantum

Quartic in PS

$\Rightarrow$ Dynamics

$\Rightarrow$ Statistical Mechanics

(Like in usual CM/QM)

$$\left(\mathcal{F}(\mathcal{L}_\alpha);\{\,,\}_{PB}\right) = \text{Poisson Manifold}$$

$$\|$$

$X^\mu, Z^{\mu\nu}, \ldots$

$\cup$

$$h_f g = \{f, g\}_{PB}$$

"Hamiltonians"

$$\left\{\begin{array}{l} X^\mu = \frac{1}{4}\mathcal{L}^\alpha(\gamma^\mu)_\alpha{}^\beta\mathcal{L}_\beta \\ Z^{\mu\nu} = \frac{1}{2}\mathcal{L}^\alpha([\gamma^\mu,\gamma^\nu])_\alpha{}^\beta\mathcal{L}_\beta \end{array}\right. ;\{\,,\}_{PB}\Bigg\}$$ is an "algebraic" Poisson Variety

"Classical Limit"

$\ell \to 0$

Deformation Quantization

$$\left(\mathcal{F}(\mathcal{L}_\alpha);\ \underbrace{\star_\ell}_{\text{Star product}}\right) \longrightarrow \left(\mathcal{F}(\mathcal{L}_\alpha);\ \underbrace{[\,,]_{\star_\ell}}_{\text{Moyal product}}\right)$$

$\{1, \mathcal{L}_\alpha, \ldots, \mathcal{L}_{\alpha}\cdots\mathcal{L}_{\alpha_m}, \ldots; \star\} =$ Weyl algebra

$\{W_n,\ n\in\mathbb{Z}^\ell; \star\} =$ Projector algebra

$\vdots$

$$\mathcal{L}_\alpha \star \mathcal{L}_\beta = \mathcal{L}_\alpha\mathcal{L}_\beta + i\ell C_{\alpha\beta}$$

Q-Mechanics Choose a "Hamiltonian", e.g. X^0,

$$\boxed{X^0 \star_\ell W = t\, W}$$

time-Energy duality $t \to \ell^2 E$, $E \to \ell^{-2} t$

t: $\star_\ell$- genvalue

W: Wigner function

$$\langle f(\mathcal{L})\rangle = \int d^4\mathcal{L}\,\big(f(\mathcal{L})\star W\big)$$

$$t = \ell\,(n_1+n_2+1) \quad ; \quad n_i = 0,1,2,\dots$$

Light-Cone

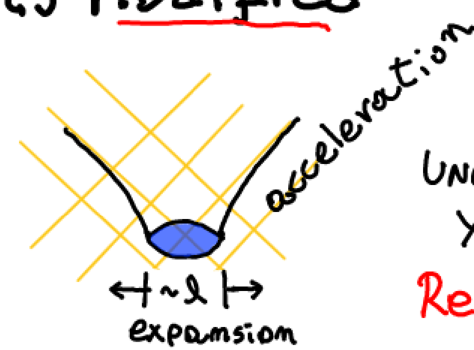
$$X^\mu X_\mu = 0$$

$\downarrow$ Deformed

$$X^\mu \star_\ell X_\mu = \ell^2$$

HyPerboloid

Space-time Causal structure is Modified



Uncertainty Yields Redshift

$$\epsilon_{\mu\nu\lambda\rho}\, X^\nu \star_\ell Z^{\lambda\rho} = 0$$

$$\epsilon_{\mu\nu\lambda\rho}\, Z^{\mu\nu} \star_\ell Z^{\lambda\rho} = 0$$

$\Downarrow$

Quantum Grassmanian

$$\{Gr(2,4), \star_\ell\}$$

Algebra

$$sp(2) = so(2,3) = \{ X^{\mu}, Z^{\mu\nu} ; [\ ,\ ]_{\star_\ell} \} \longrightarrow$$ "Singleton" irrep.

$-+++$

$$X^{\mu} \star_\ell X^{\nu} = X^{\mu} X^{\nu} + \frac{i\ell}{2} Z^{\mu\nu} + \underline{\frac{\ell^2}{8} \eta^{\mu\nu}}$$

$$X^{\mu} \star_\ell Z^{\nu\lambda} = X^{\mu} Z^{\nu\lambda} - \underline{\frac{i\ell}{2} (\eta^{\lambda\mu} X^{\nu} - \eta^{\nu\mu} X^{\lambda})}$$

$$Z^{\mu\nu} \star_\ell Z^{\nu\lambda} = Z^{\mu\nu} Z^{\nu\lambda} + \frac{i\ell}{2} (\eta^{\nu\lambda} Z^{\mu\rho} - \dots) + \underline{\frac{\ell^2}{8} (\eta^{\mu\rho} \eta^{\nu\lambda} - \eta^{\mu\lambda} \eta^{\nu\rho})}$$

$$\vdots$$

trace $\quad tr(f(x)) := f(0)$

$\star_\ell$-Matrix Model

$$\boxed{S = \mathrm{tr}\left([Q_\alpha, Y^I]_{\star_\ell} \star_\ell [Q_\beta, Y^J]_{\star_\ell} C^{\alpha\beta} g_{IJ} \right)}$$

$Q_\alpha = Q_\alpha(\mathcal{L})$, $Y^I = \{Y^\mu , Y^{\mu\nu}\}$ | 10 - Vector-Tensor Coordinates

$I = 0,1,2,3 ; 01, 02, 03, 12, 23, 34$

EoM

$$\boxed{\begin{aligned} Y^I \star_\ell [Y_I, Q_\alpha]_{\star_\ell} &= [Y_I, Q_\alpha]_{\star_\ell} \star_\ell Y^I \\ Q^\alpha \star_\ell [Y^I, Q_\alpha]_{\star_\ell} &= -[Y^I, Q_\alpha]_{\star_\ell} \star_\ell Q^\alpha \end{aligned}}$$

Some Solutions $\boxed{Q_\alpha = \mathcal{L}_\alpha \ , \ Y^I = \{X^\mu , Z^{\mu\nu}\}}$

4D-Grassmannian Subspace of 10D

Alternative to Compactification

Higuer Spin Symmetries $Q_\alpha, Y^I \rightarrow g^{-1}_{\star_\ell}(Q_\alpha, Y^I) \star_\ell g$, $g = \underbrace{\exp_{\star_\ell}(\epsilon(\mathcal{L}))}_{\text{Polynomial}}$

A theory of Distributions

$$\boxed{S = \int_{\substack{\text{Phase}\\\text{Space}}} [I, X]_{\star_\ell} \star_\ell \Omega \star_\ell [I, X]_{\star_\ell}}$$

Classification of Distributions in PS (Langlands)

(Cf. Wigner's Poincaré Irreps $\leftrightarrow$ Particles)

<u>Ex</u>

$\delta\Omega$: $[I, X]_{\star_\ell} = 0$

$X \star I = \underbrace{x}_{X\text{'s spectra}} I$

Groenewold-Moyal QM

The previous Model is obtained when

$\Omega = (\delta, g_{IJ}, C_{\alpha\beta}) =$ Measure

$I = Q_\alpha =$ Internal Vector

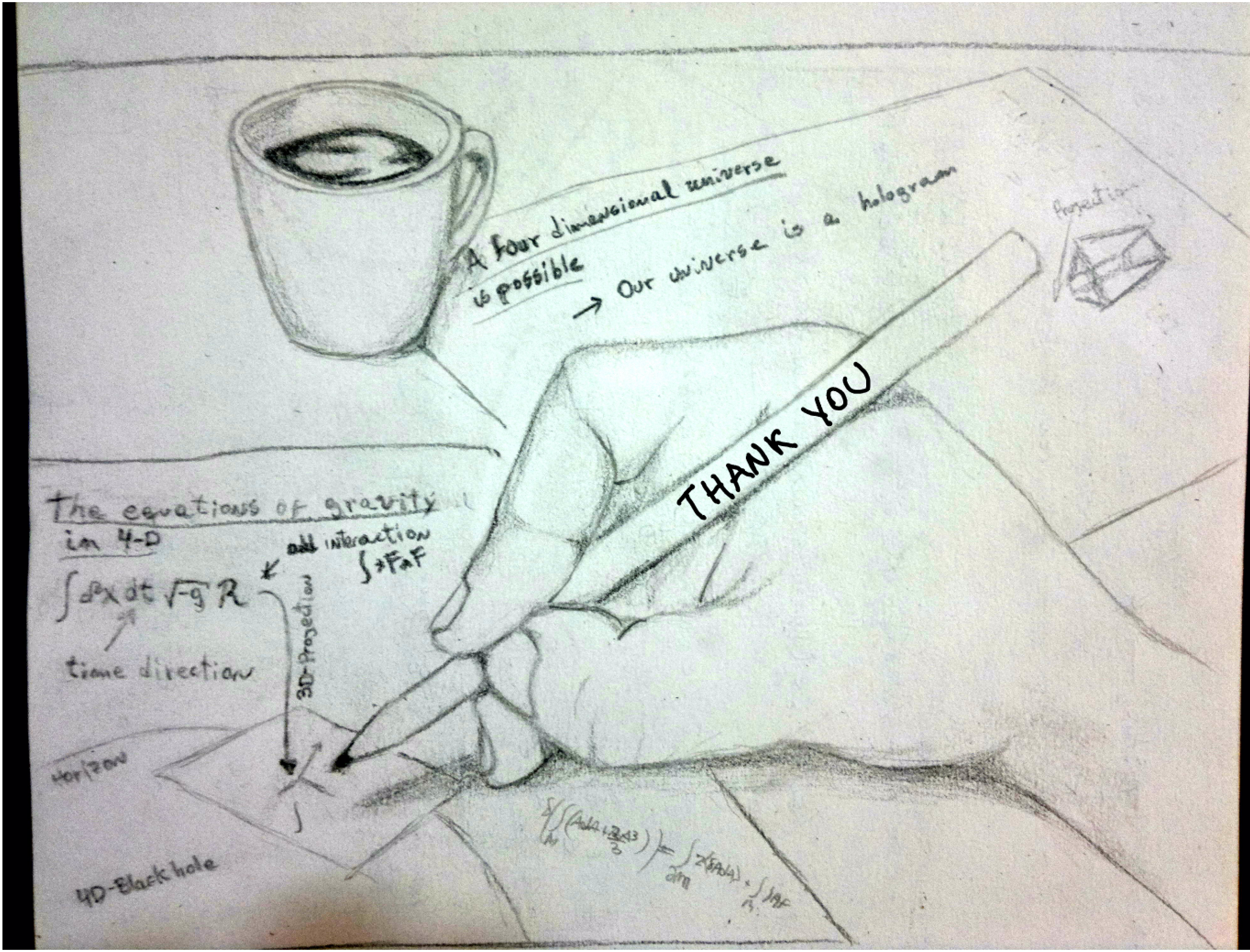
$X = y^I =$ Spacetime Vector

$\star_\ell =$ Interactions , $\ell =$ Quantum parameter

Note

$$\begin{tikzcd}
S \arrow[rr, "\text{Weyl Map}"] \arrow[dd] & & \text{Matrix Model} \arrow[dd, "\text{IKKT}"] \\
& \text{Classical limit } \ell \to 0 & \\
\text{Membrane theory} \arrow[rr] & & \text{String theory}
\end{tikzcd}$$

Further study $\longrightarrow$ Tools of Information theory


A four dimensional universe is possible
→ Our universe is a hologram
Projection
THANK YOU
The equations of gravity in 4-D
add interaction
∫ *F∧F
∫d³x dt √-g R
time direction
3D-Projection
Horizon
4D-Black hole